On the connection between black hole ringdown and images

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- **CYC,** Hsu-Wen Chiang (LeCosPA, NTU), Jie-Shiun Tsao (NTNU), *Phys. Rev. D* 106 (2022) 4, 044068
- **CYC**, Yu-Jui Chen (NTU), Meng-Yuan Ho (NTU), Yung-Hsuan Tseng (NTHU), under preparation

- Geometric-optics approximations for black holes:
  - Eikonal correspondence
- Identifying the correspondence:
  - Deformed Schwarzschild spacetime
- Testing the correspondence:
  - A novel test using black hole quasinormal modes (QNMs) and images
- Conclusions

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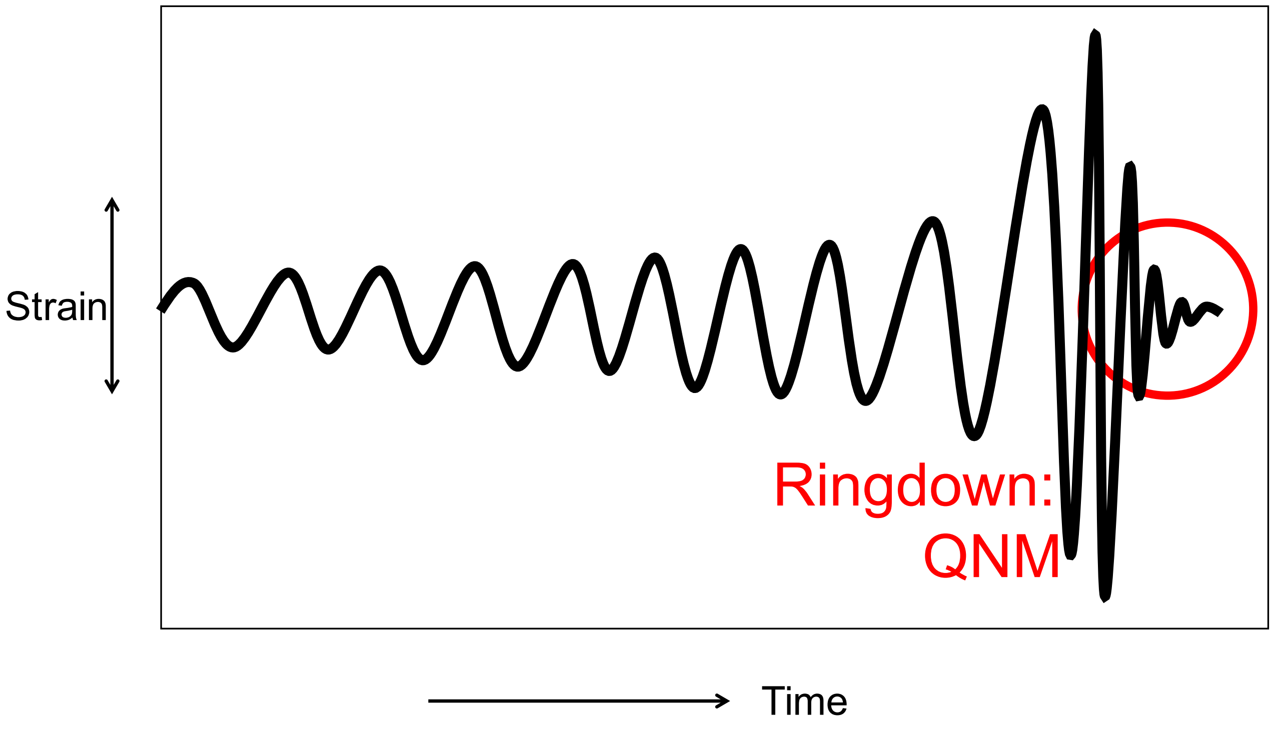
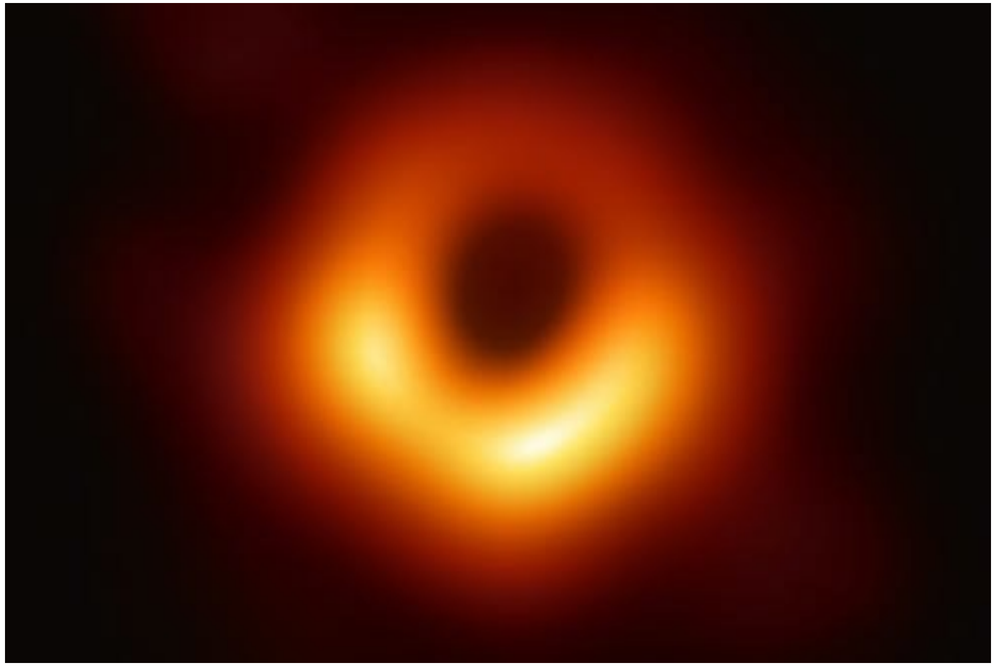
Geometric-Optics (Eikonal) Approximations





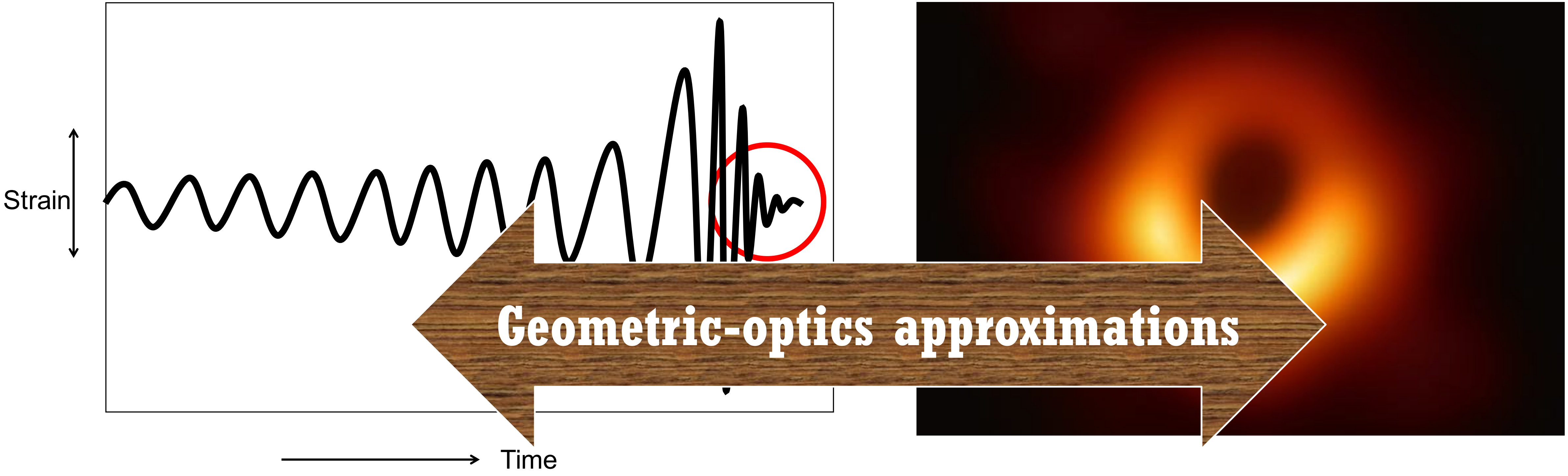
Field propagation in BH spacetimes

$$\nabla^\alpha \nabla_\alpha A = \cdots$$

Photon propagation in BH spacetimes

$$k^\alpha k_\alpha = 0$$

Geometric-Optics (Eikonal) Approximations



Field propagation in BH spacetimes

$$\nabla^{\alpha}\nabla_{\alpha}A = \boldsymbol{O(\lambda/L)} \sim \mathbf{0}$$

Photon propagation in BH spacetimes

$$k^{\alpha}k_{\alpha} = 0$$

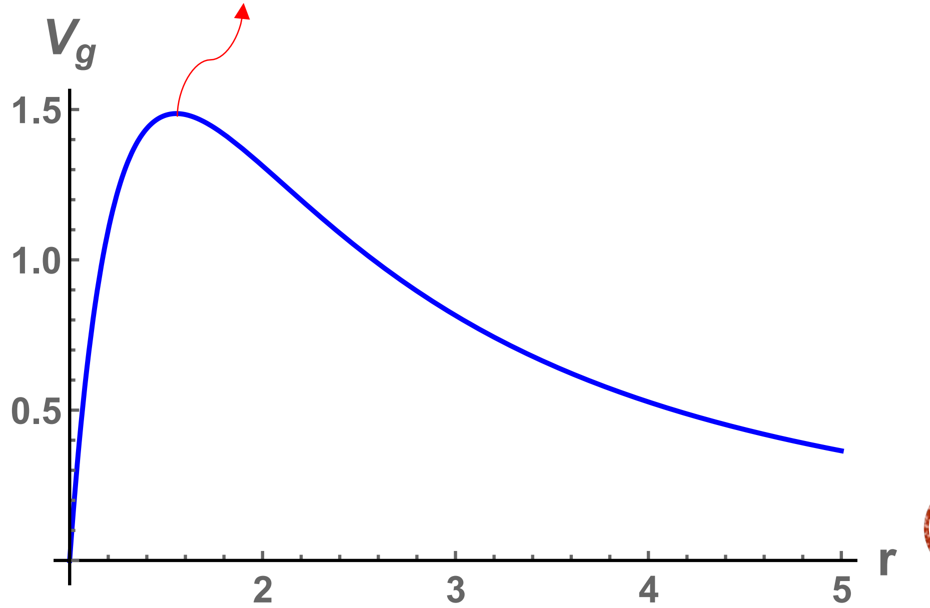
How does the correspondence manifest in BH spacetimes?

- Spacetime symmetry is crucial
- Non-rotating BH:

Static and spherically symmetric $ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)\Psi = V_g \Psi$$

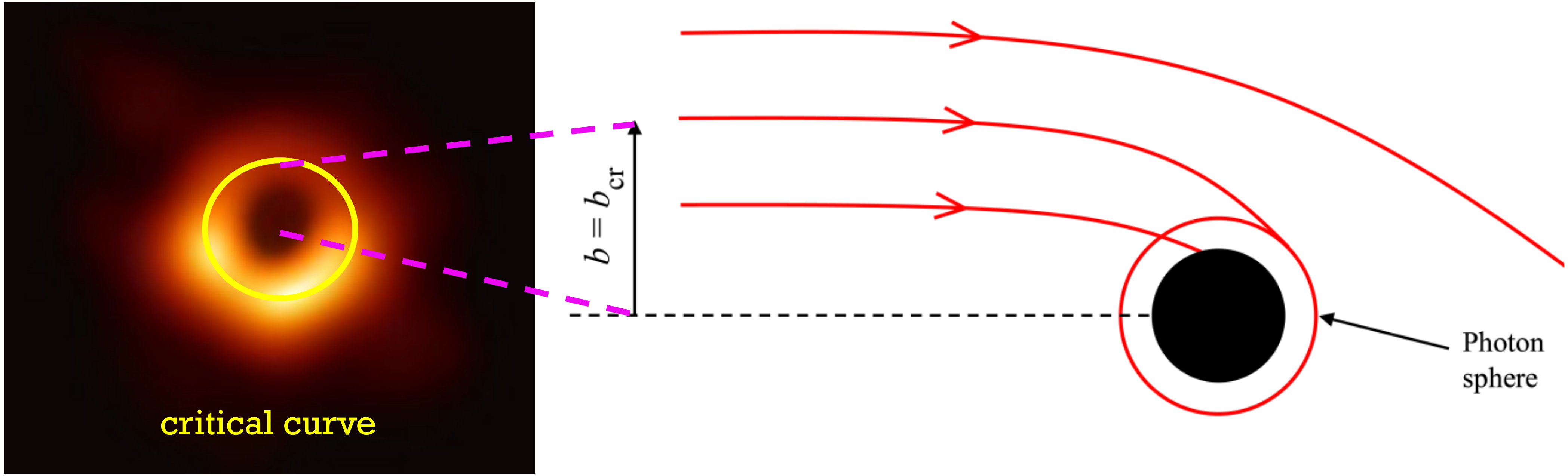
approach photon sphere when ($l \to \infty$)



- The potential for eikonal ($l \to \infty$) QNMs: $V \approx \frac{A(r)}{r^2} l^2$
- The peak of the potential coincides with the photon sphere
  - Photon sphere equation: $\partial_r[A(r)/r^2] = 0$

Eikonal QNMs Correspondence

- The eikonal QNMs ($l \to \infty$) and the photon sphere



$$\omega \approx \Omega_c l - i(n + 1/2)|\lambda_c|$$

- $Re(\omega) \to \Omega_c$ (orbital frequency of the photon sphere)
- $Im(\omega) \to \lambda_c$ (Lyapunov exponent)
- $\boxed{\gamma \equiv \lambda_c/\Omega_c \text{ (critical exponent)}}$

Cardoso, Miranda, Berti, Witek, Zanchin (2009)

Correspondence in Kerr Spacetime

- Separable geodesic equations (Carter constant), and separable wave equations

Wave Quantity	Ray Quantity	Interpretation
ω_R	$\mathscr{E}$	Wave frequency is same as energy of null ray (determined by spherical photon orbit).
m	L_z	Azimuthal quantum number corresponds to z angular momentum (quantized to get standing wave in ϕ direction).
A^R_{lm}	$\mathscr{Q}+L_z^2$	Real part of angular eigenvalue related to Carter constant (quantized to get standing wave in θ direction).
ω_I	$\gamma=-\mathscr{E}_I$	Wave decay rate is proportional to Lyapunov exponent of rays neighboring the light sphere.
A^I_{lm}	$\mathscr{Q}_I$	Nonzero because $\omega_I \neq 0$ (see Secs. II B 2 and III C 3 for further discussion).

Yang *et al*. (2012)

Recently extended to Kerr-Newman by Li *et al*. (2021)

Eikonal QNMs and BH Shadows

$\omega_R \quad \leftrightarrow$ Angular frequency on PS $\quad \leftrightarrow$ Size of shadow image

$\omega_I \quad \leftrightarrow$ Lyapunov exponent on PS $\quad \leftrightarrow$ Higher-order ring structures

Jusufi (2020), Cuadros-Melgar *et al.* (2020)

Jusufi (2020), Yang (2021)

- <u>What if the black hole spacetime has less symmetry?</u>
- <u>Can the eikonal correspondence be tested observationally?</u>

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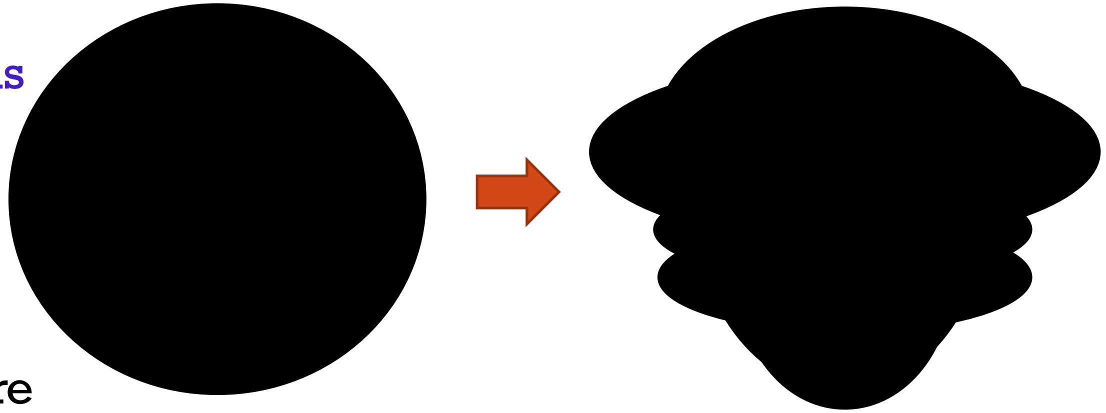
Deformed Schwarzschild Spacetime

- Consider small but general axisymmetric deformations of Schwarzschild BHs $\boxed{g_{\mu\nu} = g_{\mu\nu}^{Sch} + h_{\mu\nu}(r,\theta)}$



In the presence of deformations:

- Radial and latitudinal sectors of geodesic equations are NOT separable
- Generic photon orbits $r(\theta)$ do NOT have constant r
- Inseparable QNM equations if deformations are not small
  - Can be separable if deformations are small

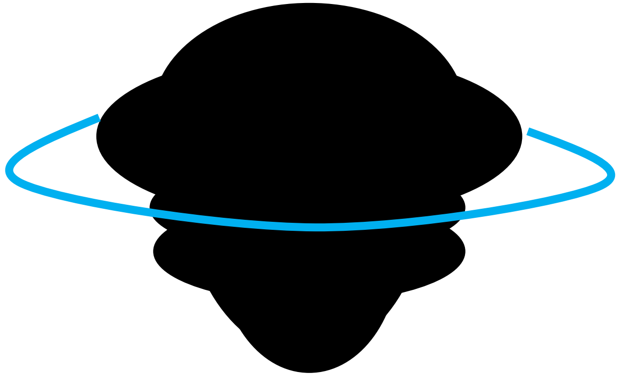
Cano, Fransen, Hertog (2020)

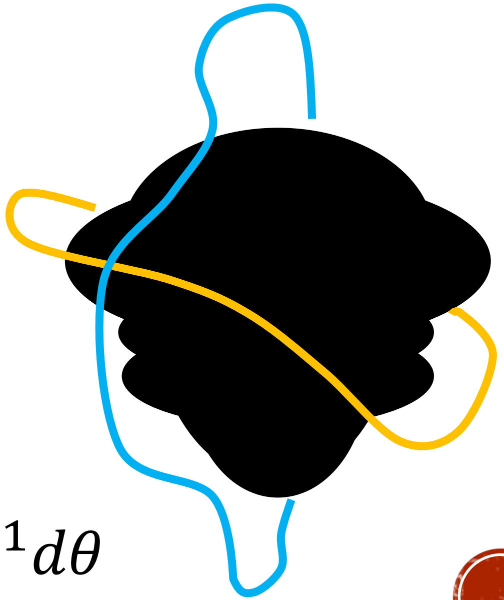
$$\boxed{(\partial_{r_*}^2 + \omega^2)\Psi_{lm} = V_{\mathrm{eff}}(r,l,m)\Psi_{lm}} \qquad \boxed{V_{\mathrm{eff}}(r,l,m) = V_{Sch} + \delta V}$$

Two Kinds of Photon Orbits

- Planar circular photon orbits with a constant radius:
  - The peak of $V_{\text{eff}}(r)$ is precisely on these orbits

$(|m| = l \gg 1)$





- Generic photon orbits do not have constant r
- These photon orbits should
  - be periodic
  - form a class of limit cycles
- We can integrate the orbits along full periods $\oint d\lambda = \oint \dot{\theta}^{-1} d\theta$

Generic Orbits

$$\left\langle \frac{d}{d\lambda}(g_{rr}\dot{r}) \right\rangle = \langle \partial_r F(r^*, \theta)(r - r^*) \rangle$$

definition of limit cycle

$$= 0$$

- The peak of $V_{\text{eff}}(r)$ coincides with the root of this integrated equation

($|m| < l$ and $l \gg 1$)

Chen, Chiang, Tsao (2022)

Generic Orbits

$$\left\langle \frac{d}{d\lambda}(g_{rr}\dot{r}) \right\rangle = \langle \partial_r F(r^*, \theta)(r - r^*) \rangle$$

definition of limit cycle

$$= \partial_r F_0(r_P) \langle r - r^* \rangle + o(\epsilon)$$

Lyapunov exponent is $O(1)$

averaged radius along one period

$$= 0$$

- The peak of $V_{\text{eff}}(r)$ coincides with the averaged radius of these orbits along one period

Chen, Chiang, Tsao (2022)

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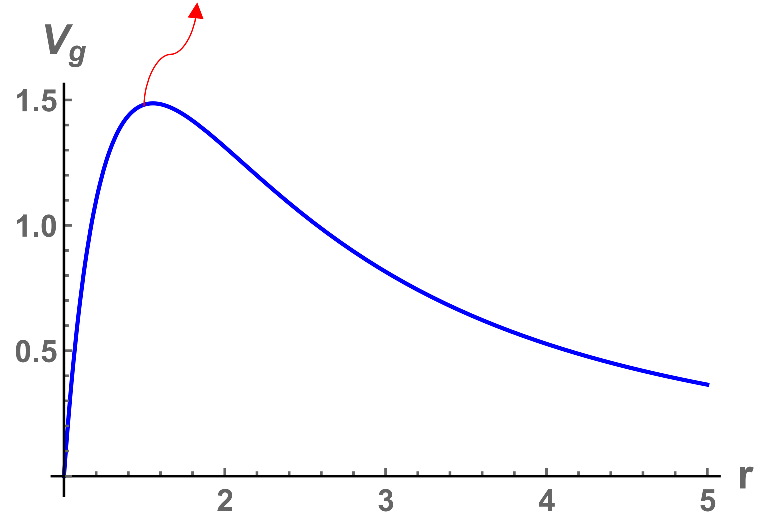
Eikonal Correspondence Violation

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

approach photon sphere when ($l \to \infty$)



- In GR, the potential for eikonal ($l \to \infty$) QNMs: $V \approx \frac{A(r)}{r^2} l^2$
- The peak of the potential coincides with the photon sphere
  - Photon sphere equation: $\partial_r [A(r)/r^2] = 0$

- This may not be true for modified gravity:

$$V \approx \alpha(r) \left(\frac{A(r)}{r^2} l^2 \right)$$

- The peak of the potential may differ from the photon sphere of BHs
  - Non-minimal coupling between matter and curvature — Chen, Bouhmadi-López, Chen (2019) (2021) Chen, Chen (2020)
  - String-inspired models — Cardoso, Gualtieri (2010) Konoplya, Stuchlik (2017) Moura, Rodrigues (2021)

- <u>A novel test of eikonal correspondence based on ringdown and image observations of a single black hole</u>

QNM Observables

$$\gamma_l^{QNM} \equiv 2l \frac{|\omega_I|}{\omega_R}$$

<u>If the eikonal correspondence is satisfied:</u>

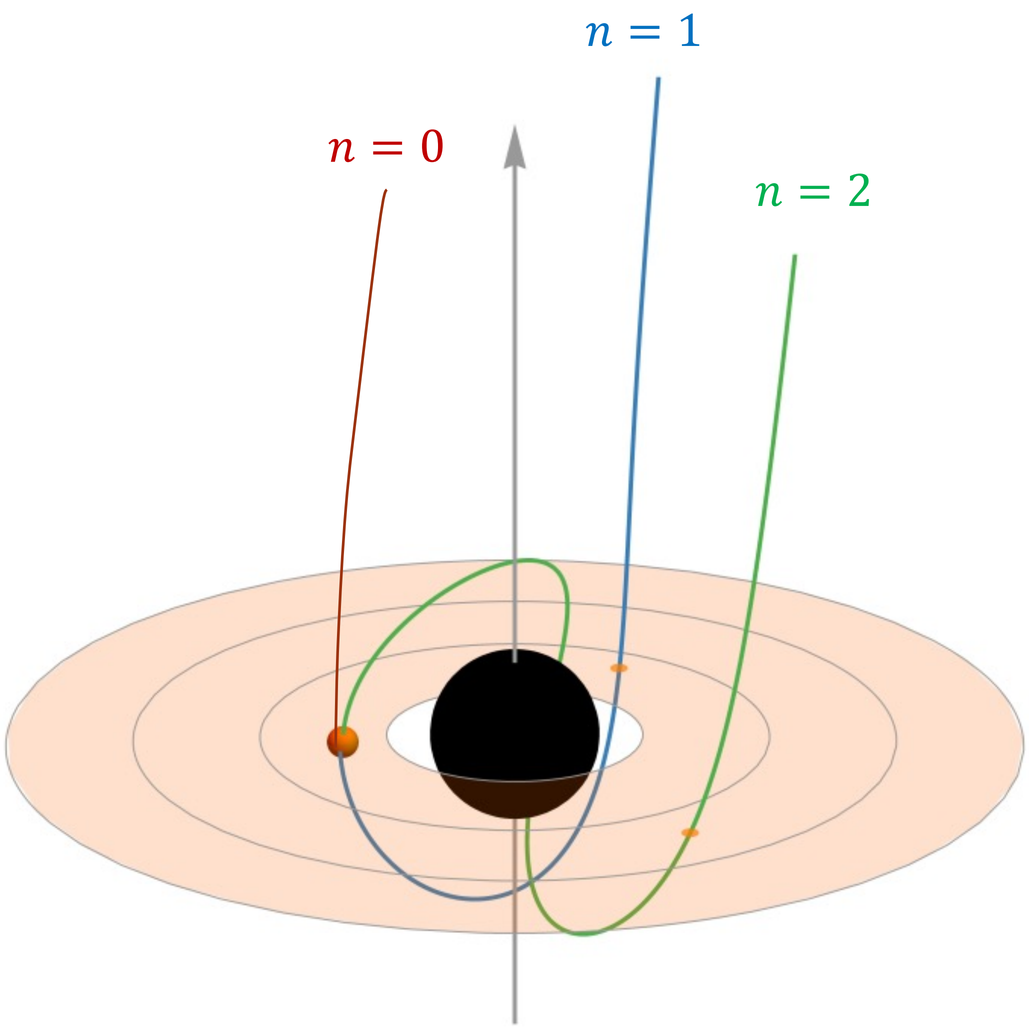
$$\gamma_l^{QNM} = \left(1 - \frac{1}{2l}\right)\gamma + O(l^{-2})$$

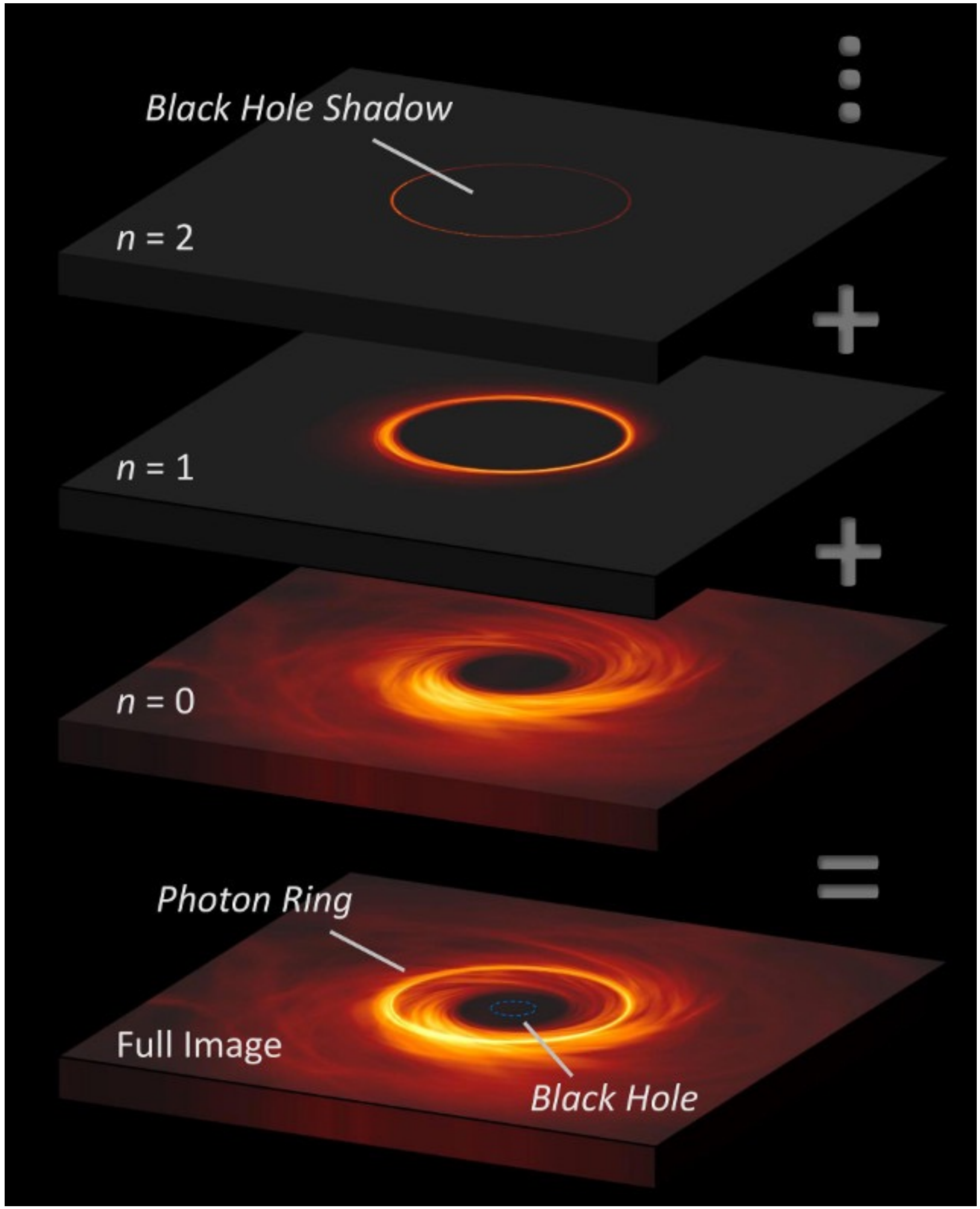
$\boxed{\gamma \equiv \lambda_c/\Omega_c \text{ (critical exponent)}}$

γ_l^{QNM} converges to γ <u>from below</u> when $l \to \infty$

Chen *et al*. under preparation

Photon Ring Observables





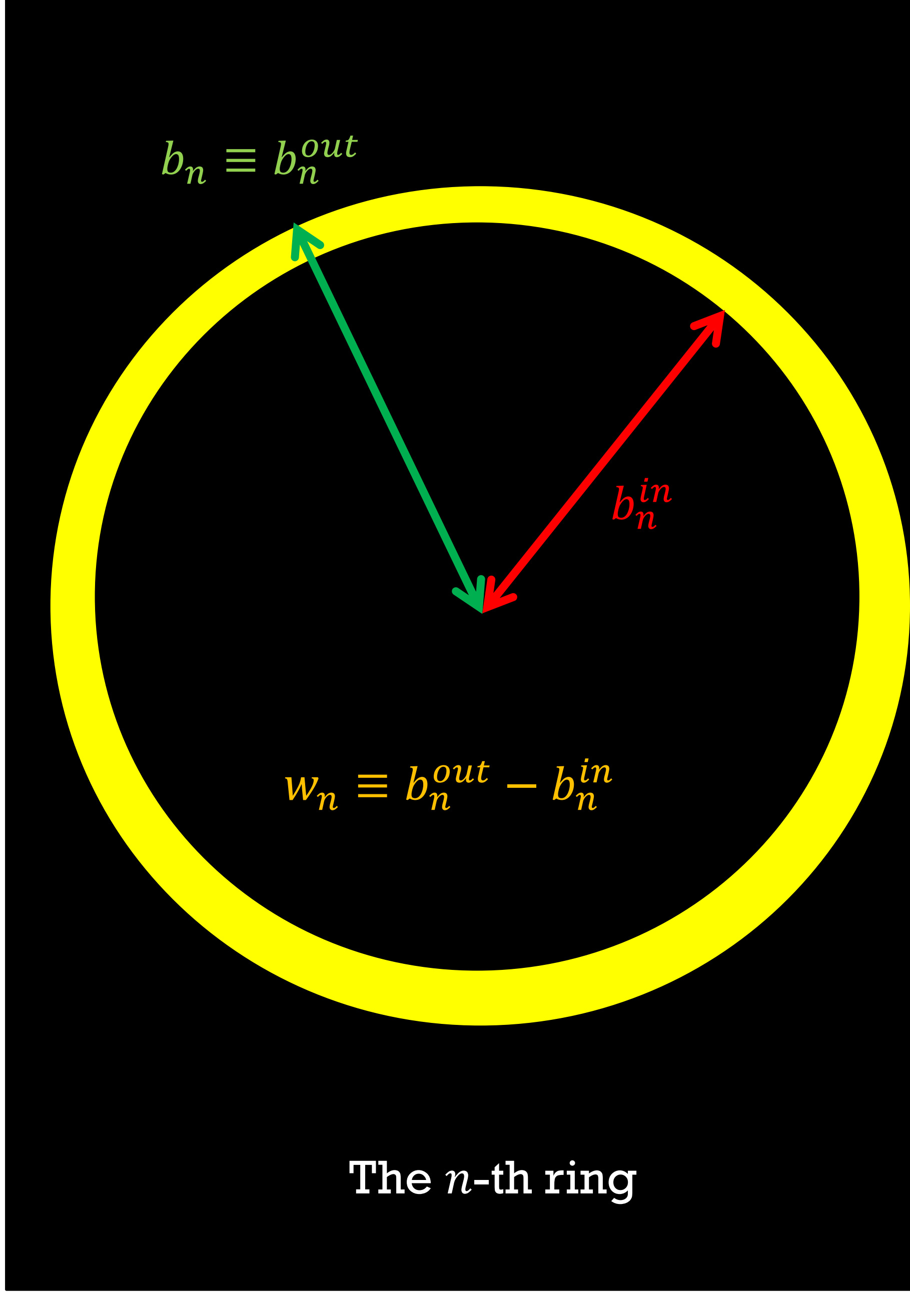
Gralla, et al. (2020)

Photon Ring Observables

$$\gamma_n^w \equiv \frac{1}{\pi} \ln \frac{w_n}{w_{n+1}}$$

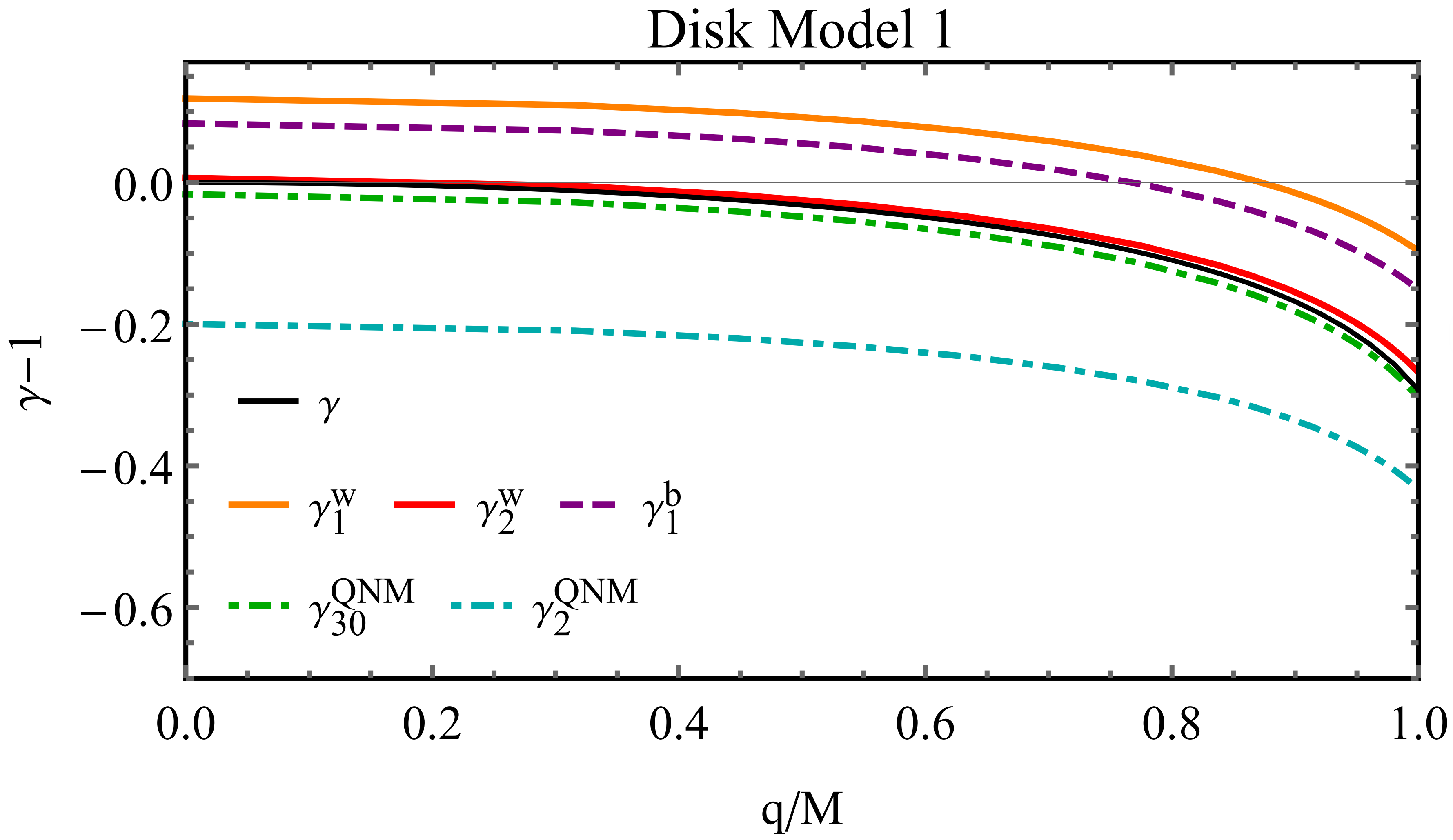
$$\gamma_n^b \equiv \frac{1}{\pi} \ln \frac{b_n - b_{n+1}}{b_{n+1} - b_{n+2}}$$

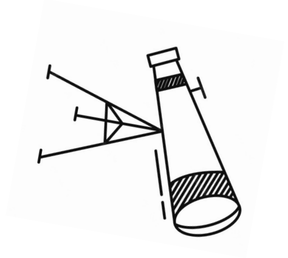
- Two ring observables converge to γ <u>from above</u> when $n \to \infty$

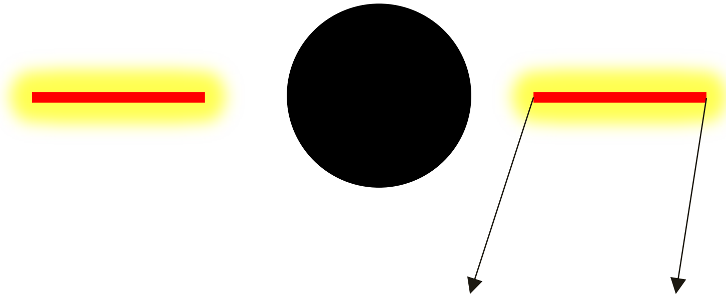


Chen *et al.* under preparation

Example: Reissner-Nordström Black Holes







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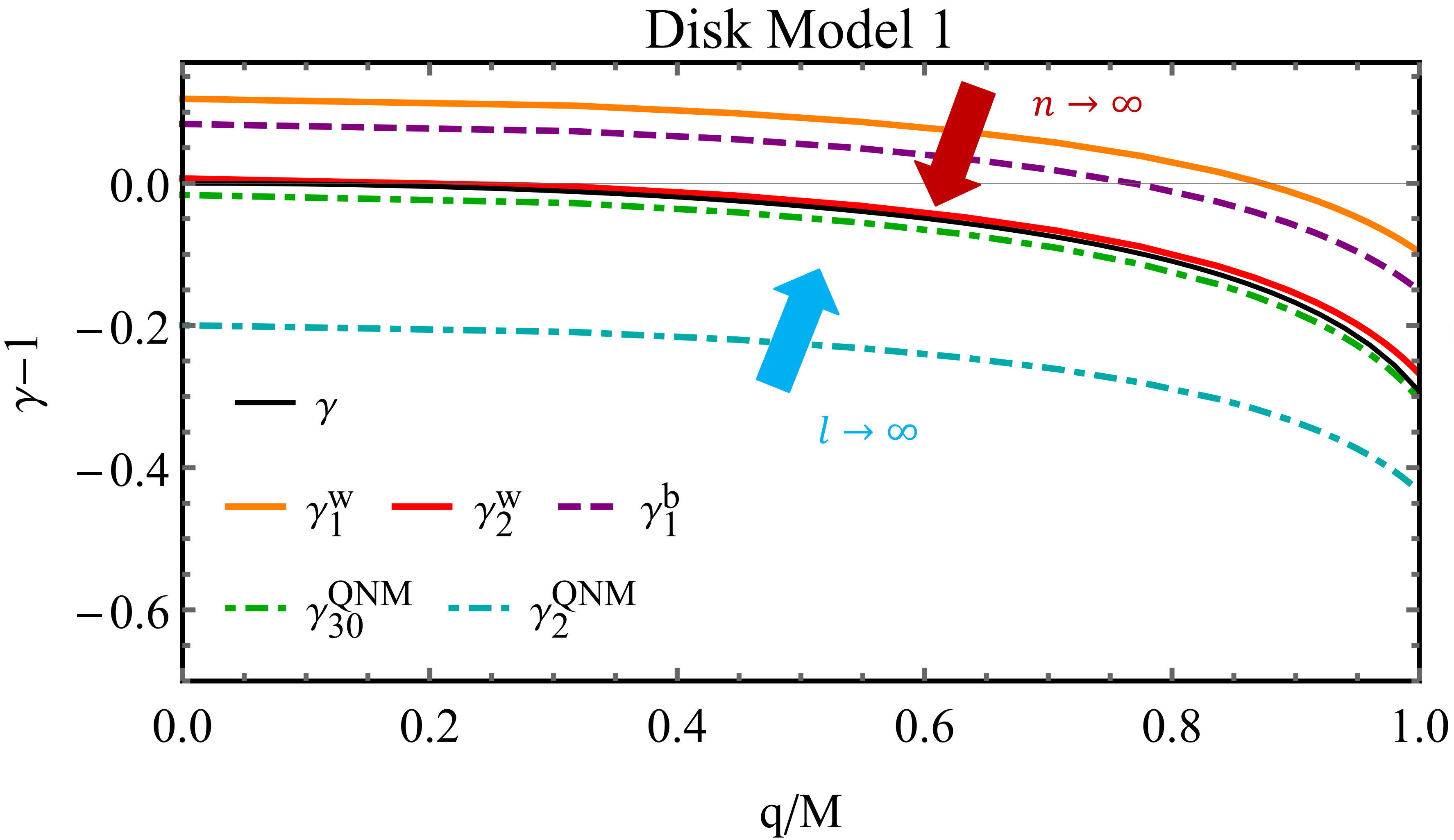

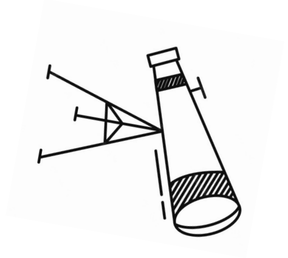

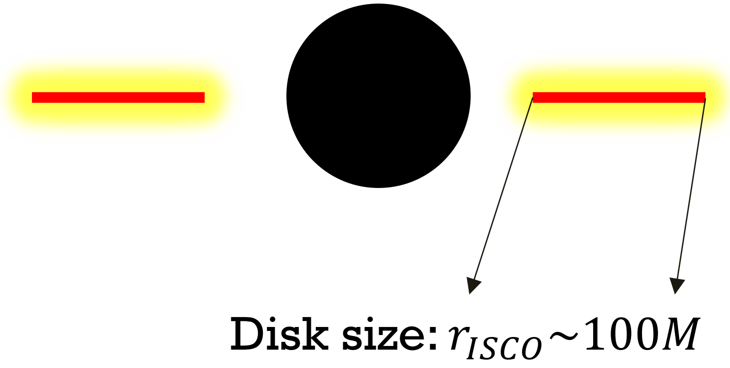

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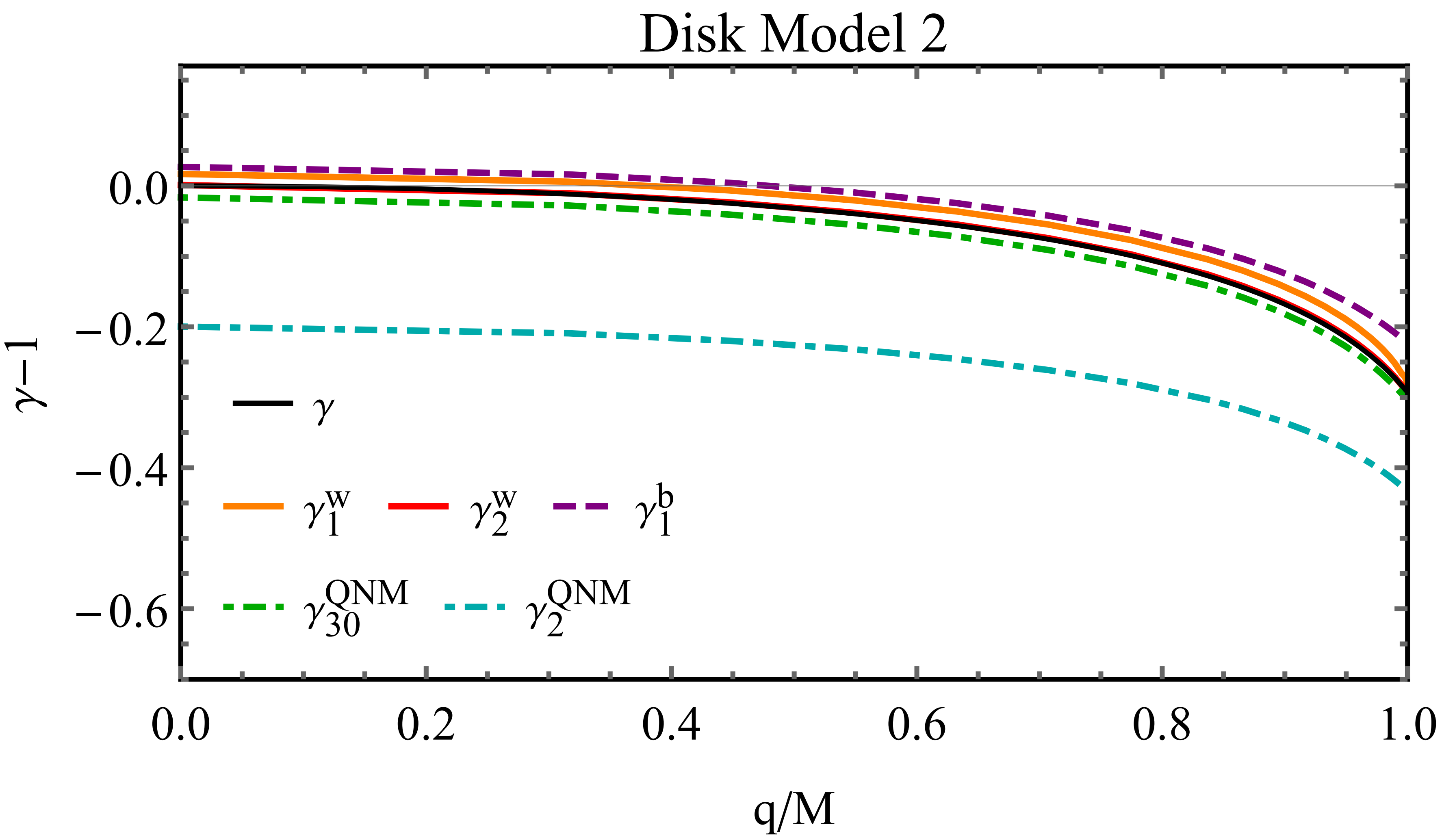

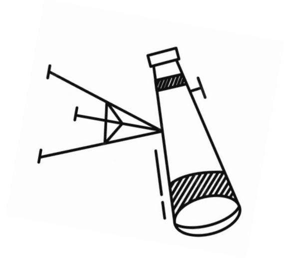

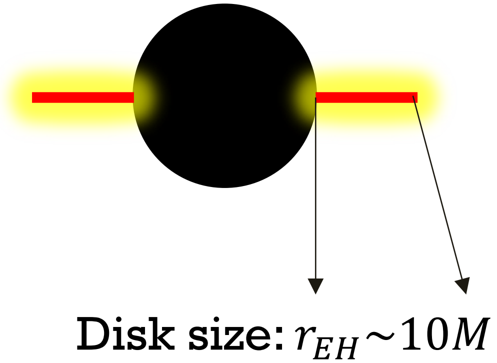

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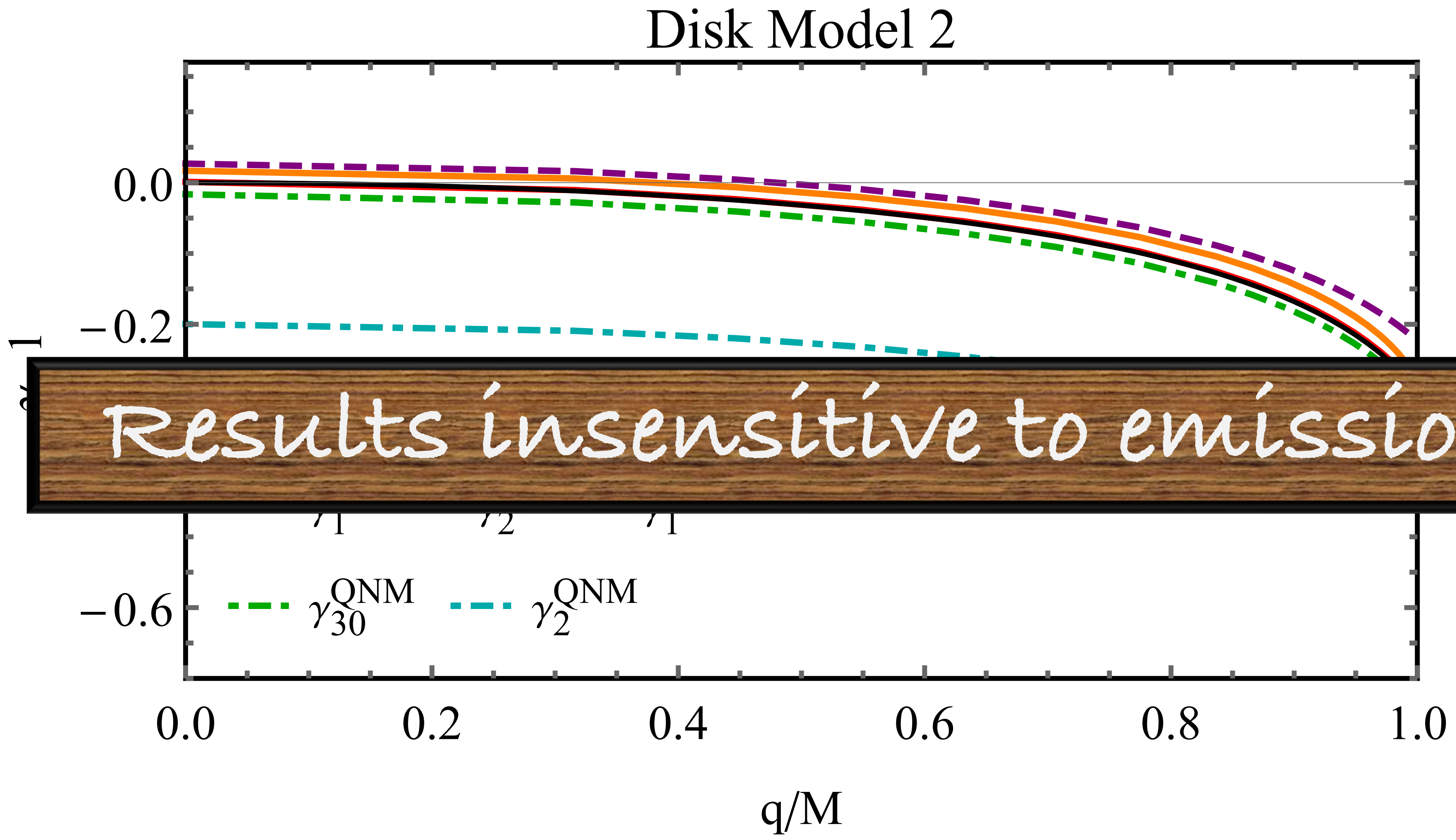
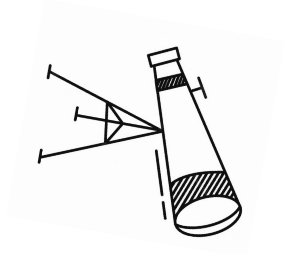
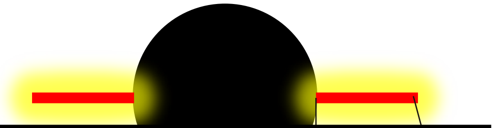


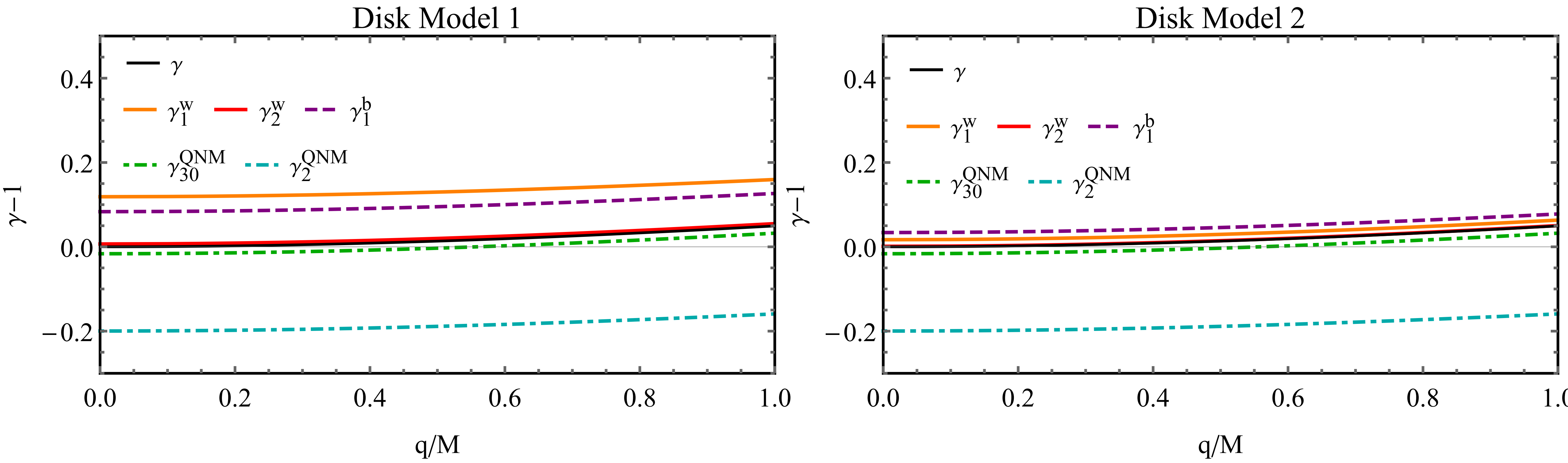




Results insensitive to emission models!

Disk size: $r_{EH} \sim 10M$

Example: Kazakov-Solodukhin Black Holes



- Robust qualitative results in different metrics

Example: Dynamical Chern-Simons Gravity

$$S = \int d^4x\sqrt{-g}\left(\kappa R + \frac{\alpha}{4}\vartheta R R^*\right) - \frac{\beta}{2}\int d^4x\sqrt{-g}(\partial\vartheta)^2$$

CS correction dynamical scalar field

- Parity-violating term from the CS correction

Jackiw, Pi (2003) Alexander, Yunes (2009)

- Motivated from string theory

Campbell, Kaloper, Madden, Olive (1993) Moura, Schiappa (2006)

- Schwarzschild metric still a solution
- Schwarzschild perturbations: Axial mode coupled to scalar modes

Cardoso, Gualtieri (2010) Molina, Pani, Cardoso, Gualtieri (2010) Motohashi, Suyama (2011)(2012) Kimura (2018)

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CS correction dynamical scalar field

coupled QNM equation: $$\left(\frac{d^2}{dr_*^2} + \omega^2\right)\begin{pmatrix}\Psi \\ \Theta\end{pmatrix} = \begin{pmatrix}V_{11} & V_{12} \\ V_{21} & V_{22}\end{pmatrix}\begin{pmatrix}\Psi \\ \Theta\end{pmatrix}$$

Ψ : axial mode

Θ : scalar mode

$$V_{11} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right), \quad V_{22} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2}\left(1 + \frac{36M^2}{\kappa\beta r^6}\right) + \frac{2M}{r^3}\right)$$

$$V_{12} = V_{21} = \left(1 - \frac{2M}{r}\right)\sqrt{\frac{(l+1)!}{\beta\kappa(l-1)!}}\frac{6M}{r^5}$$

- Schwarzschild perturbations: Axial mode coupled to scalar modes

Cardoso, Gualtieri (2010) Molina, Pani, Cardoso, Gualtieri (2010) Motohashi, Suyama (2011)(2012) Kimura (2018)

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break eikonal correspondence

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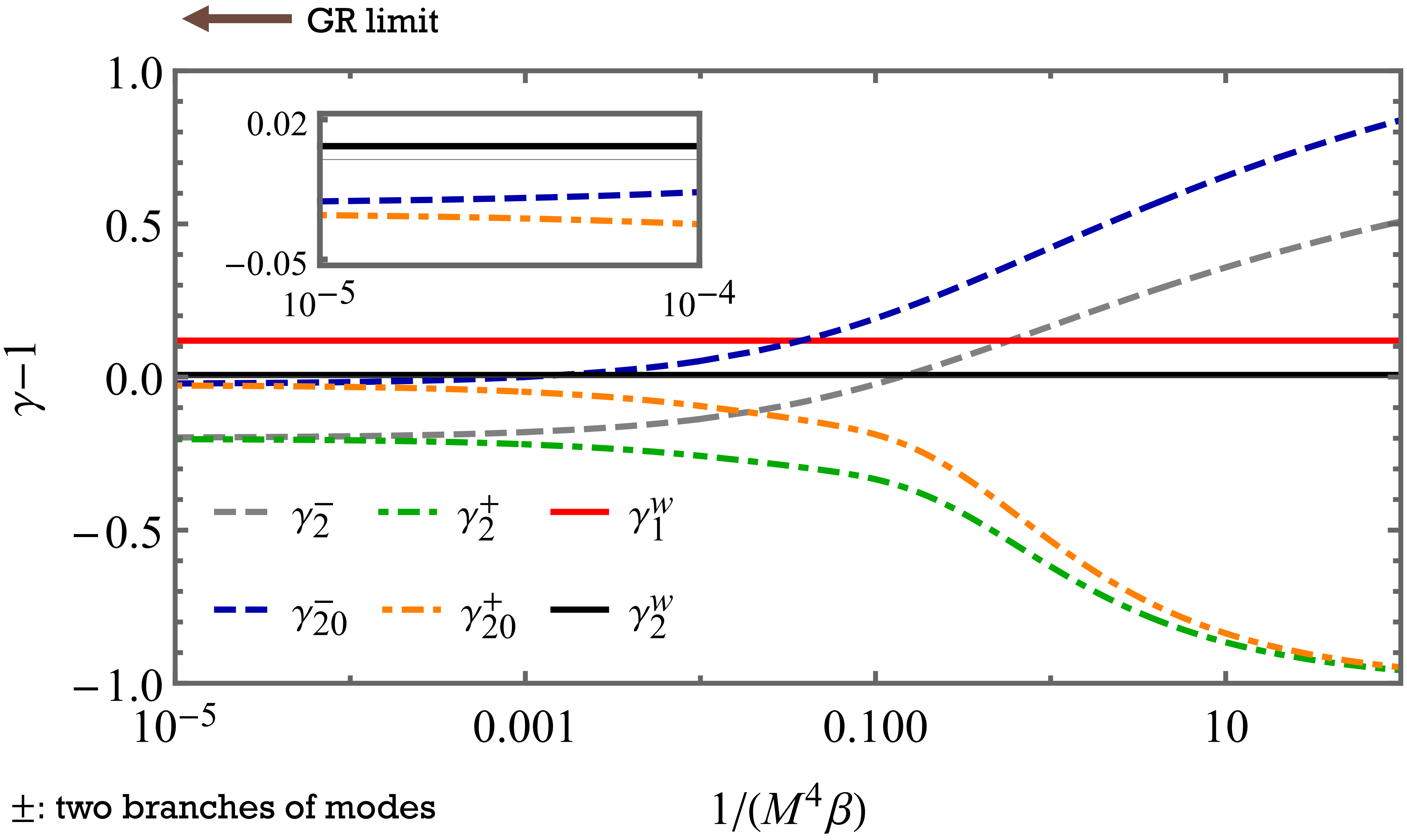

$\pm$: two branches of modes

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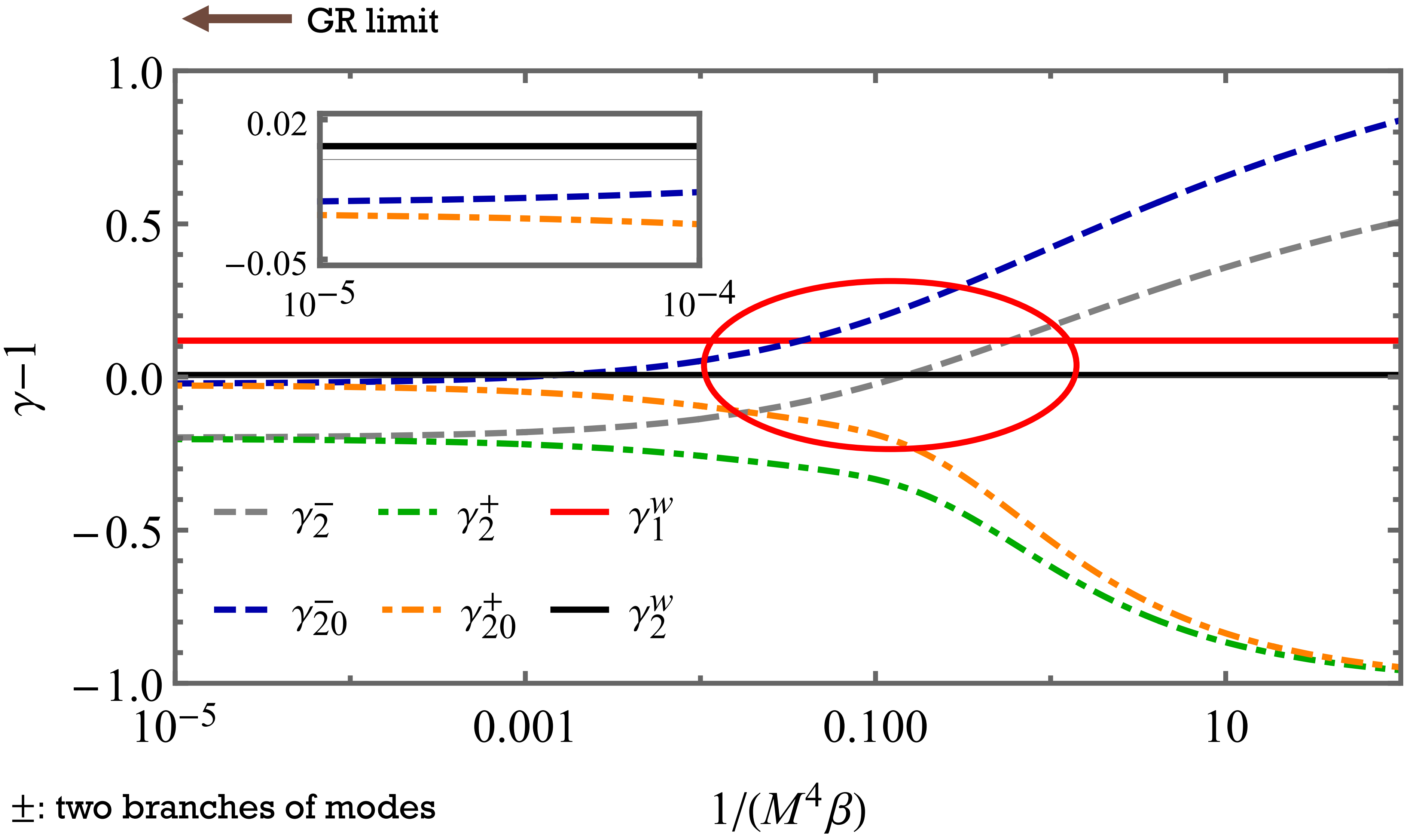

±: two branches of modes

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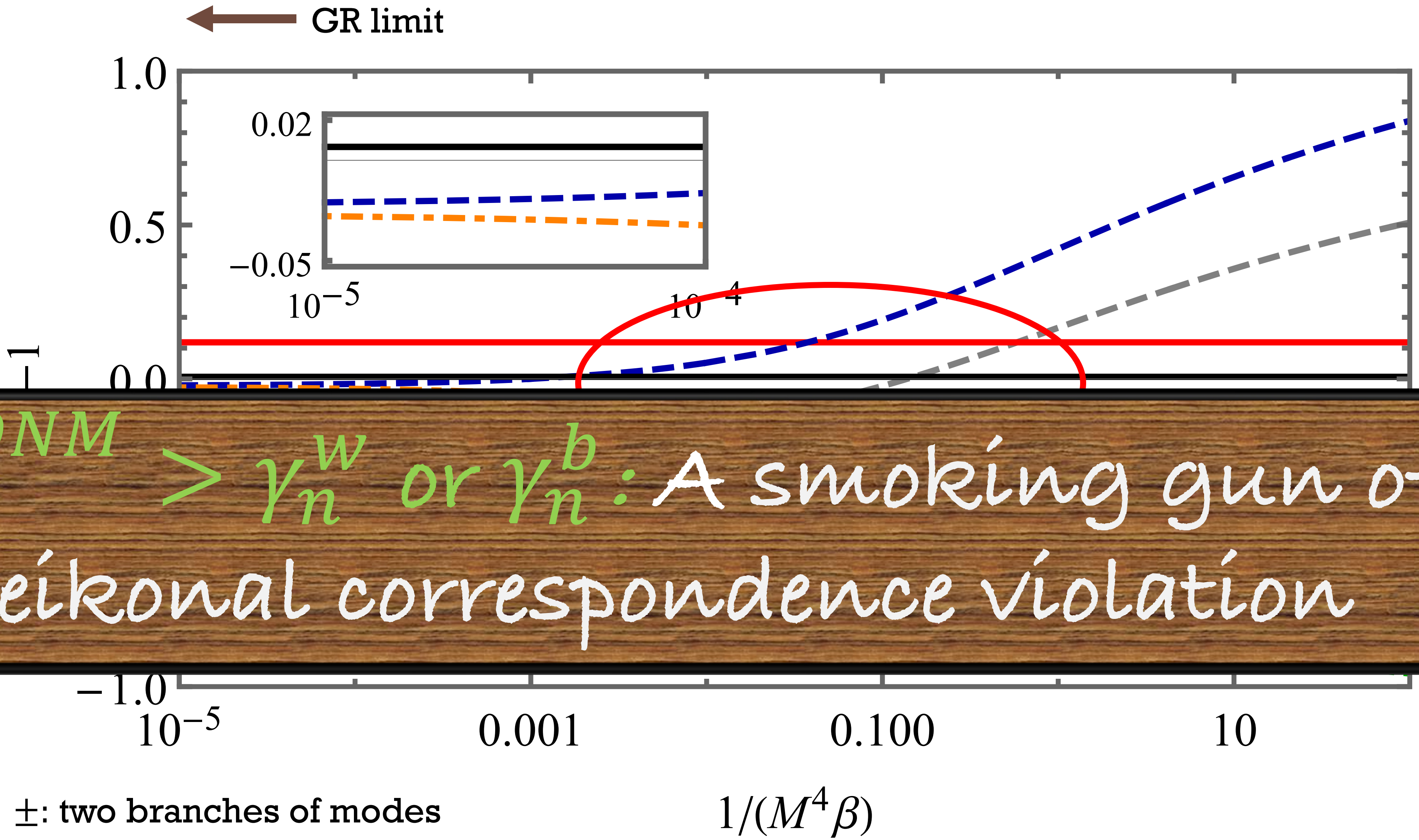

$\gamma_l^{QNM} > \gamma_n^w$ or γ_n^b: A smoking gun of eikonal correspondence violation

±: two branches of modes

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Conclusions I

- Geometric-optics approximation adopted in BH spacetime
  - Correspondence between eikonal QNMs and BH images
- Identifying the correspondence
  - Non-rotating BHs
  - Kerr BHs
  - Deformed BHs
    - Eikonal correspondence through the definition of averaged radius along full closed photon orbits
- Future:
  - Non-axisymmetric deformations
  - Deformed Kerr

Conclusions II

- Geometric-optics approximation adopted in BH spacetime
  - Correspondence between eikonal QNMs and BH images
- Testing the correspondence
  - QNM observables and photon ring observables
  - They converge to critical exponent γ from opposite directions
  - Smoking gun of eikonal correspondence violation, place constraints… etc
- Future:
  - Rotating cases
  - General inclinations and emission models

Conclusions III

Thank you for your attention!

Deformed Schwarzschild Spacetime

$$g_{tt} = -\left(1 - \frac{2M}{r}\right)\left(1 + \epsilon A_j(r)\cos^j\theta\right)\ ,$$

$$g_{rr} = \left(1 - \frac{2M}{r}\right)^{-1}\left(1 + \epsilon B_j(r)\cos^j\theta\right)\ ,$$

$$g_{\theta\theta} = r^2\left(1 + \epsilon C_j(r)\cos^j\theta\right)\ ,$$

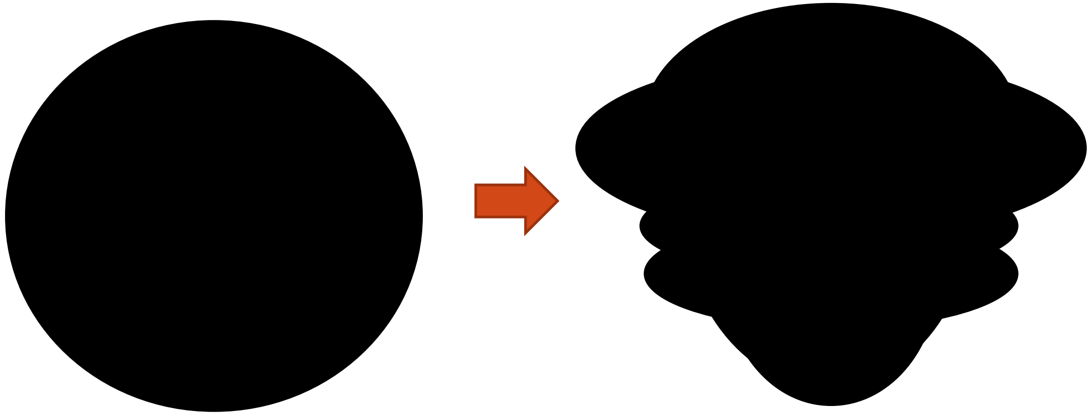
$$g_{\varphi\varphi} = r^2\sin^2\theta\left(1 + \epsilon D_j(r)\cos^j\theta\right)\ ,$$

$$g_{tr} = \epsilon a_j(r)\cos^j\theta\,, \qquad g_{t\theta} = \epsilon b_j(r)\cos^j\theta\,,$$

$$g_{r\theta} = \epsilon c_j(r)\cos^j\theta \qquad g_{r\varphi} = \epsilon d_j(r)\cos^j\theta\,,$$

$$g_{\theta\varphi} = \epsilon e_j(r)\cos^j\theta\,.$$

- A general axisymmetric deformation which excludes frame-dragging effects



- Small deformation: $|\epsilon| \ll 1$

Chen, Chiang, Tsao (2022)