KEK theory meeting, 1st December 2022

The Branco Weiss Fellowship Society in Science

Self-Organised Localisation

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2105.08617 G. Giudice, M. McCullough, **TY**

Comments: 104 pages, 12 figures. v3: Measure problem discussion extended and references added. Version to be published in JHEP

Outline

- **Motivation**
  - EFT
  - Criticality
  - Quantum phase transitions (QPT)
- **Fokker-Planck Volume (FPV) equation**
  - FPV dynamics
- **FPV + QPT = SOL**
  - Discontinuity
  - Flux conservation
- **SOL solutions**
  - Metastability
  - Higgs mass
  - Cosmological constant
- **Conclusion**
  - Measure problem

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Effective Field Theory

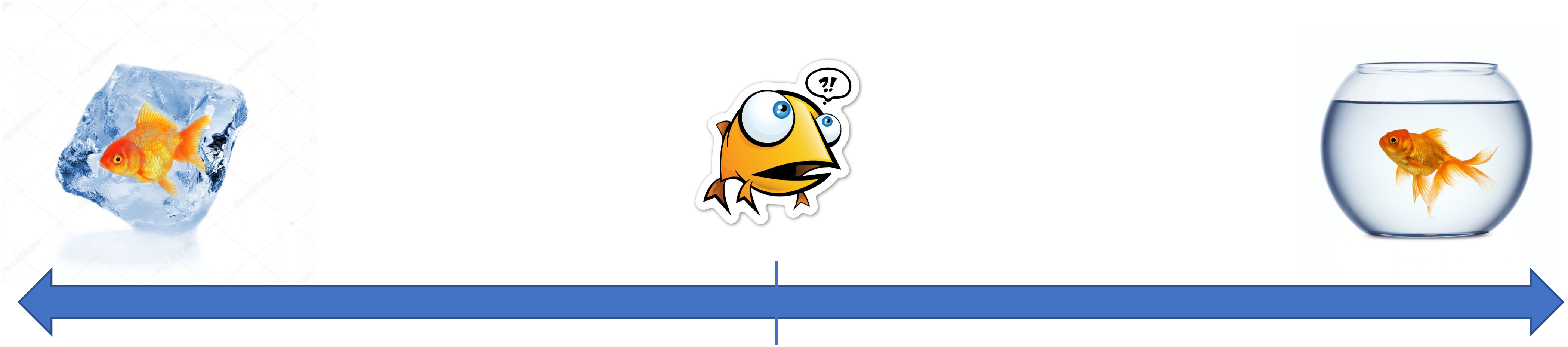
$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

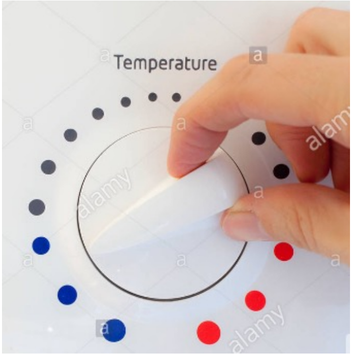
- Incredibly successful
- Explains many features of our theories
- **Natural expectations** for sizes of parameters
- *Sound reasoning*, vindicated many times in the past

- However: **hierarchy problem** and **cosmological constant**

Critical points

- To be at the **critical point** of a classical phase transition **requires tuning**



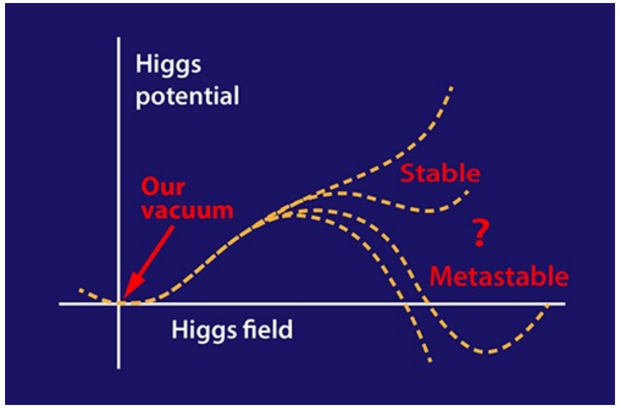


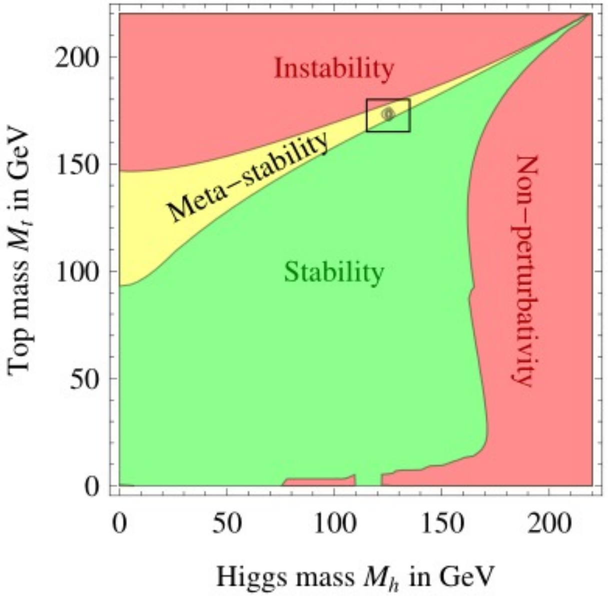
- Living **near criticality** is highly **non-generic**!

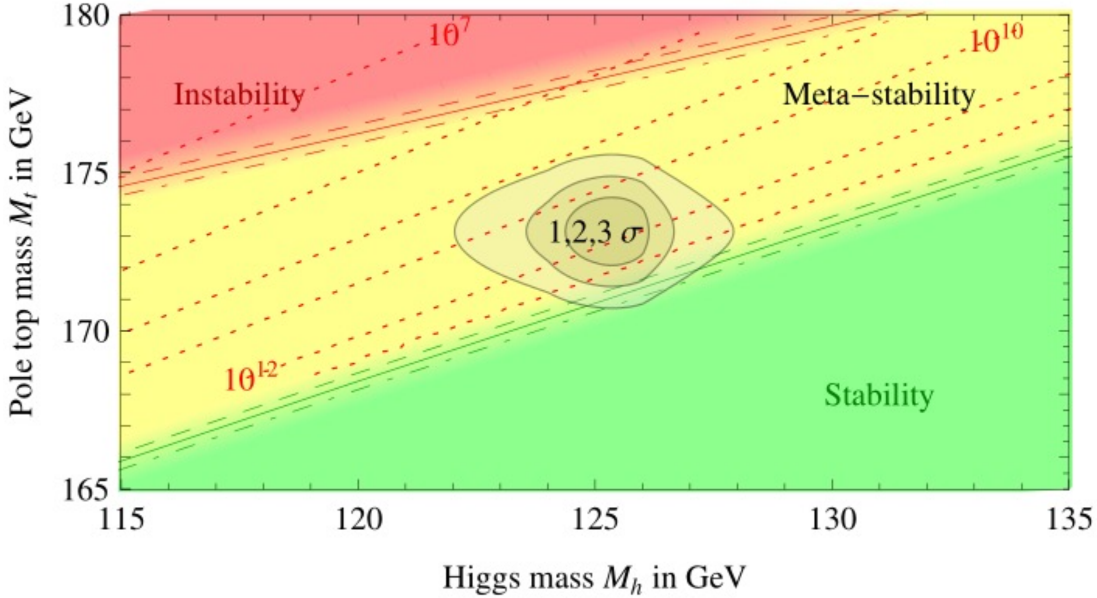
3 hints for **near-criticality** of our Universe

- 1) Higgs potential **metastability** in SM

1205.6497 Degrassi et al







- Living on critical boundary of **two phases coexisting**

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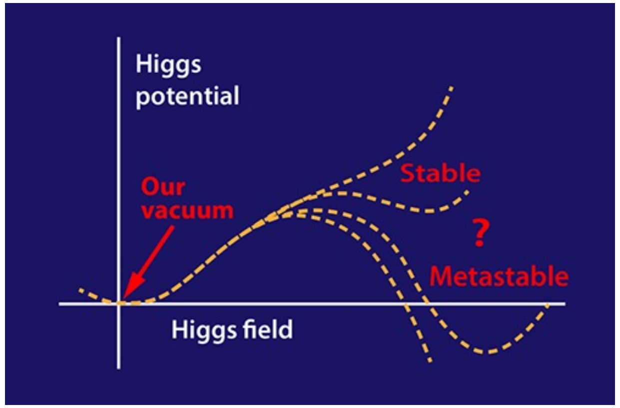

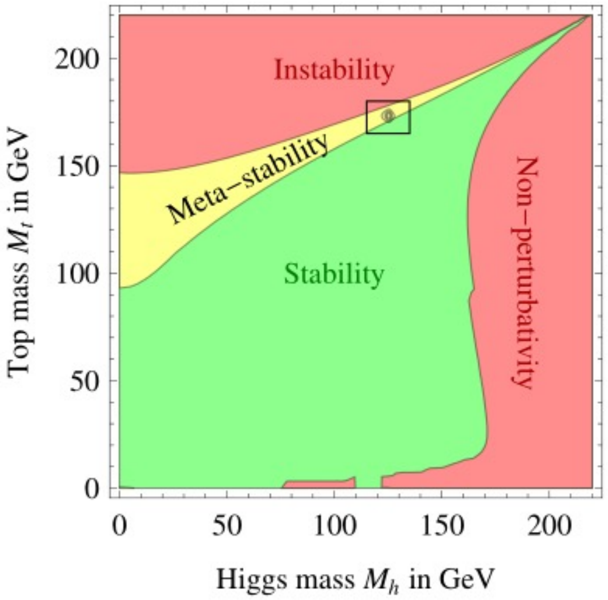

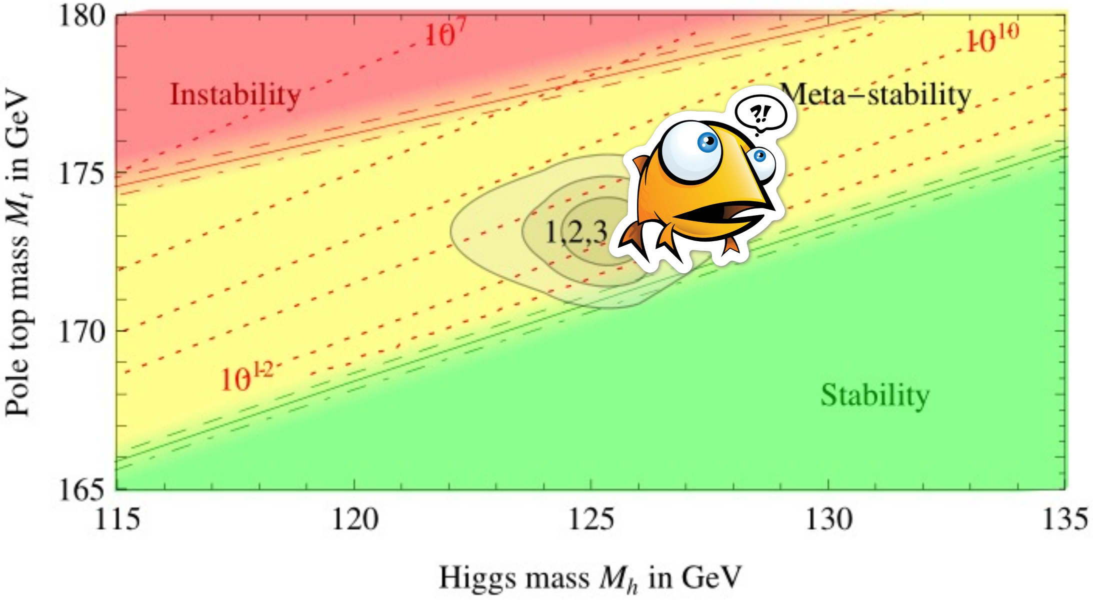

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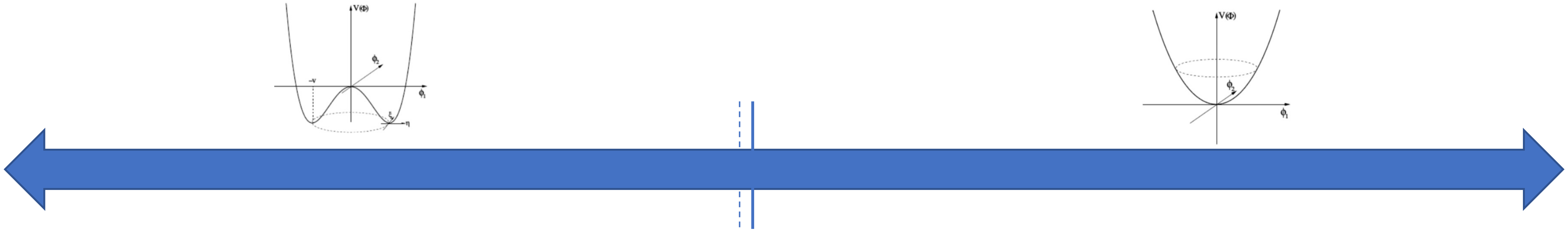
3 hints for **near-criticality** of our Universe

- 2) **Higgs mass**



$-M^2$ $+M^2$

- Tuned close to boundary between **ordered** and **disordered** phase

3 hints for **near-criticality** of our Universe

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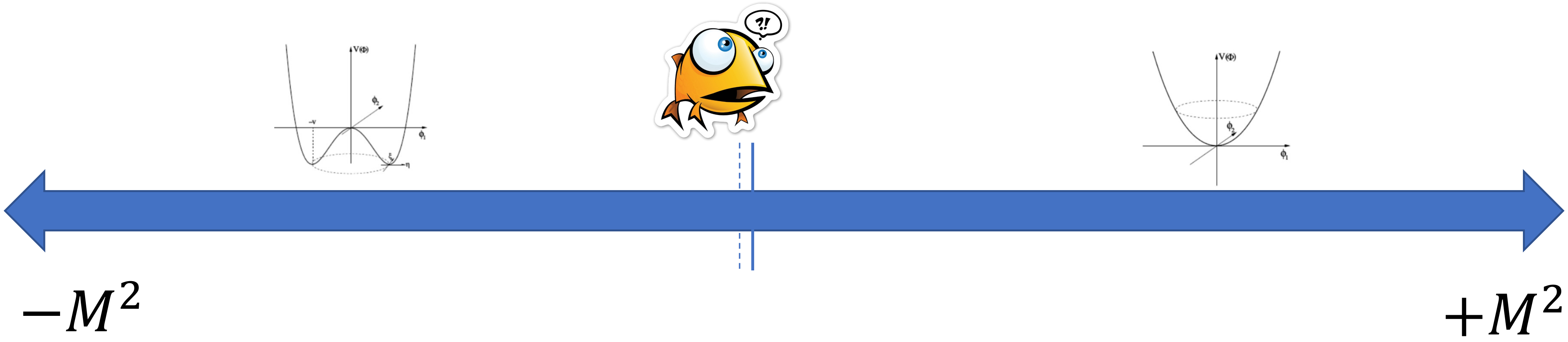

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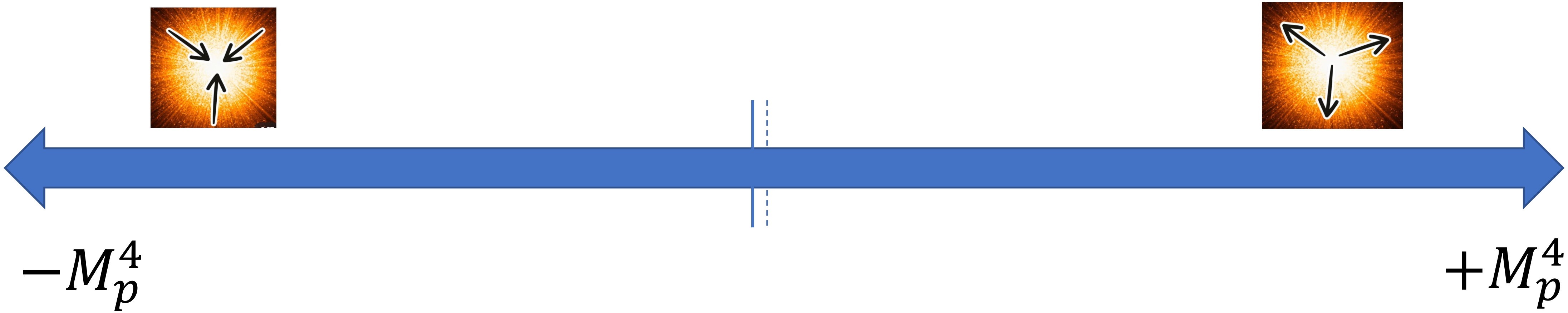
3 hints for **near-criticality** of our Universe

- 3) **Cosmological constant**



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3 hints for **near-criticality** of our Universe

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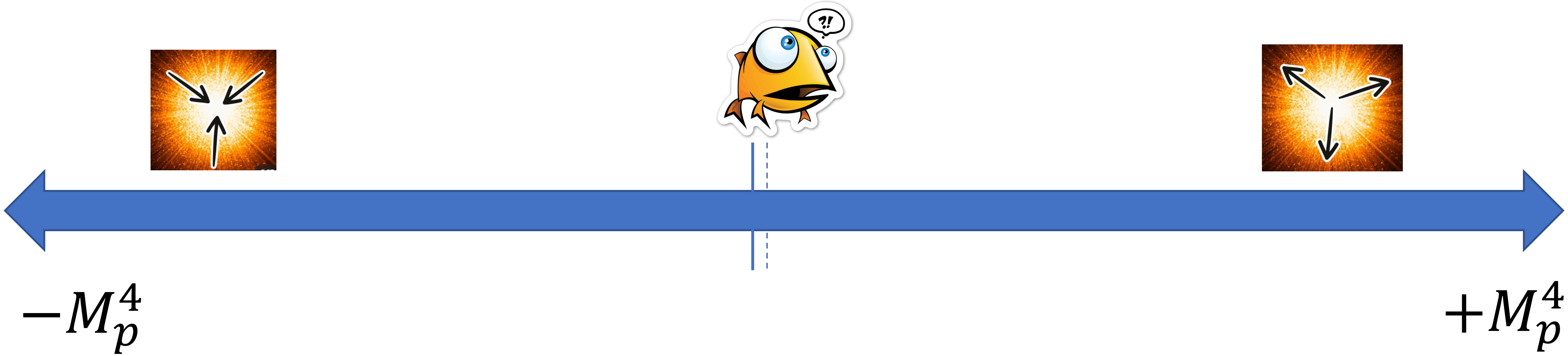


- Tuned close to boundary between **implosion** and **explosion**

3 hints for **near-criticality** of our Universe

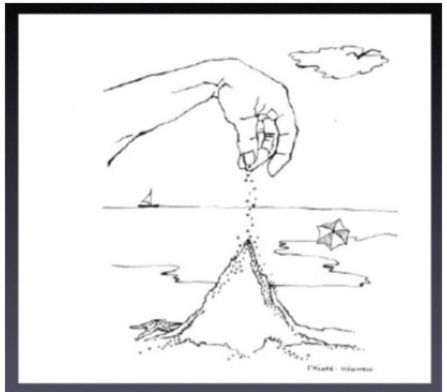
- Why do we appear to live at a **special point** close to criticality?

- Conventional explanations:
  - 1) **Metastability**: heavy new physics restores stability?
  - 2) **Higgs mass**: new symmetries?
  - 3) **Cosmological constant**: anthropics?

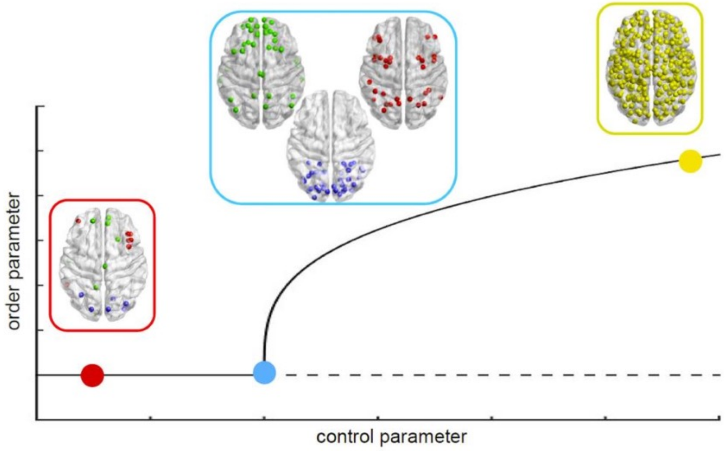
- Alternatively, hints for a **new principle** *beyond EFT expectations* at play?

Self-Organised Criticality

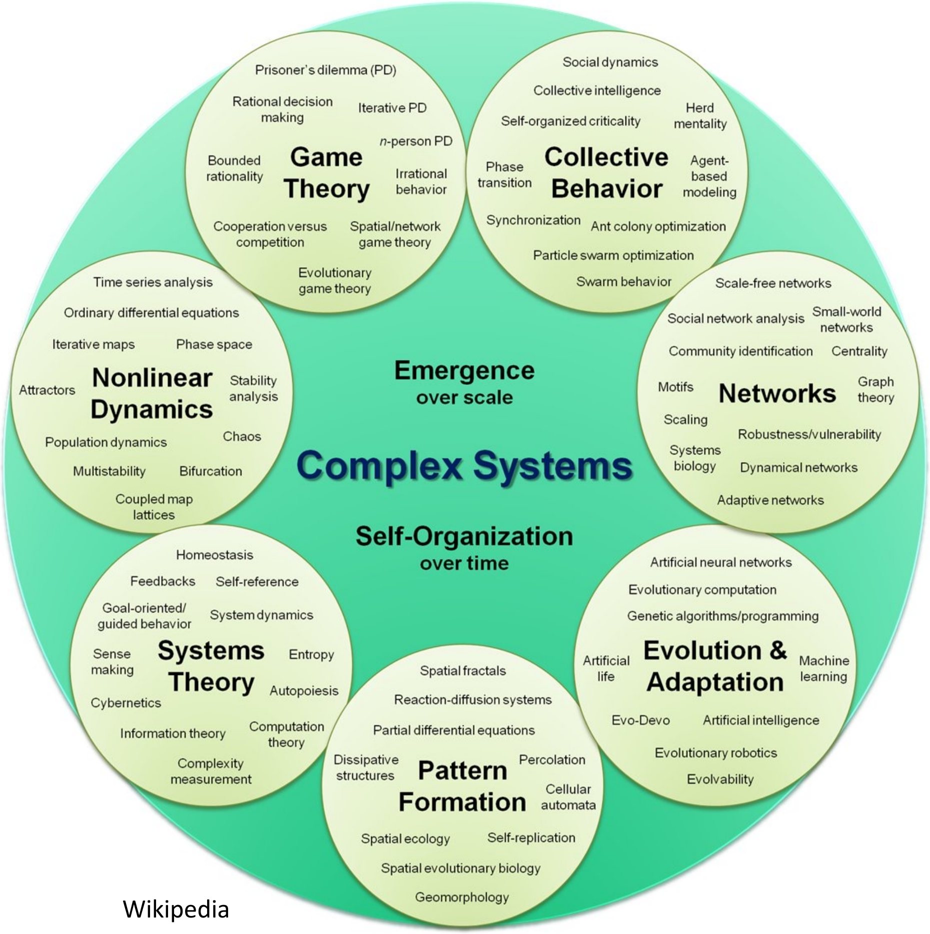
- Many systems in nature **self-tuned** to live near criticality



Bak, Tang and Weisenfeld (1989)



https://www.quantamagazine.org/toward-a-theory-of-self-organized-criticality-in-the-brain-20140403/



Wikipedia

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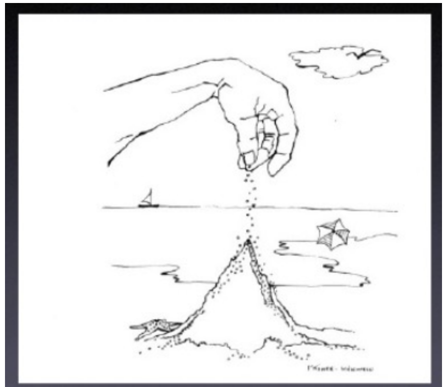

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“

How do we know that the creations of worlds are not determined by falling grains of sand?

”

Victor Hugo, *Les Misérables*

Self-Organised Criticality

- **Fundamental** self-organised criticality in our universe?

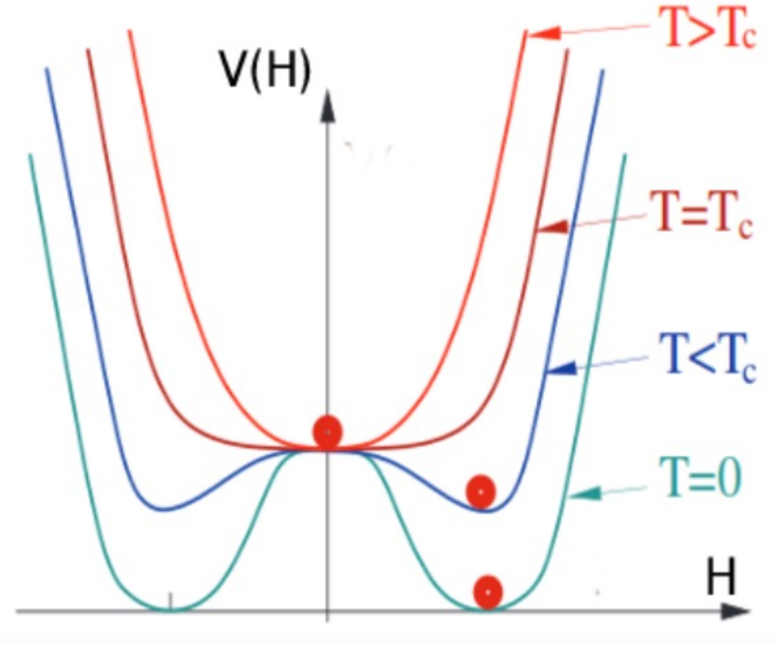
- Need a **mechanism** for self-organisation of **fundamental parameters**

  e.g. Self-Organized Criticality in eternal inflation landscape: J. Khoury et al 1907.07693, 1912.06706, 2003.12594

- **Self-Organised Localisation (SOL):**
  - cosmological **quantum phase transitions** localise fluctuating scalar fields during inflation at critical points

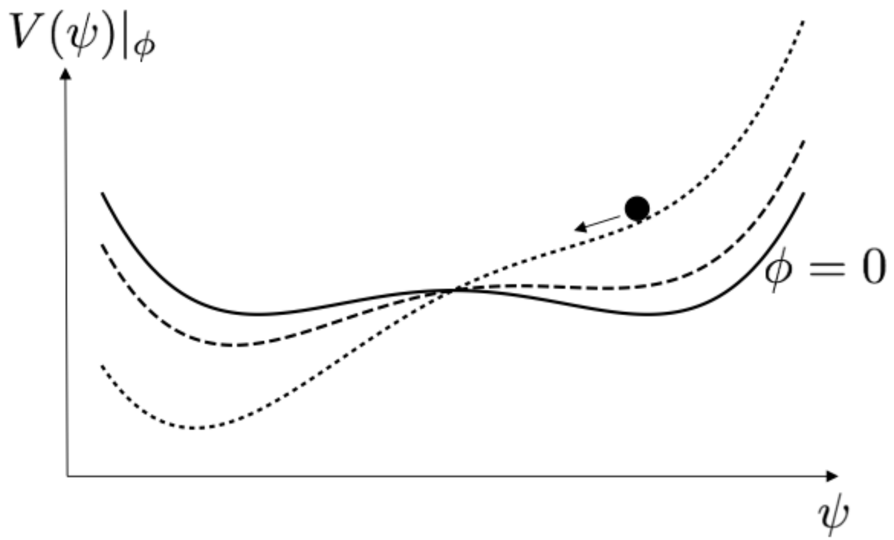
Phase Transitions (PT)

- **Classical** PT: varying background temperature



- **Quantum** PT: varying background field

$$V = \frac{\lambda}{4}\left(\psi^2 - \rho^2\right)^2 + \kappa\phi\psi$$



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Fokker-Planck Volume (**FPV**) equation

- **Langevin equation**: classical slow-roll + Hubble quantum fluctuations

$$\phi(t+\Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3P)}{\partial\phi} + \frac{V'P}{3H}\right] + 3HP = \frac{\partial P}{\partial t} \qquad H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

Quantum diffusion term

Classical drift term

Volume term

Fokker-Planck Volume (**FPV**) equation

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$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^{2+\xi}P)}{\partial\phi} + \frac{V'P}{3H^{2-\xi}}\right] + 3H^{\xi}P = H_0^{\xi-1}\frac{\partial P}{\partial t_\xi}$$

- **Ambiguity** in choosing time “gauge” $dt_\xi/dt = (H/H_0)^{1-\xi}$

FPV dynamics

- ϕ is *not* the inflaton: **apeiron** field scanning parameters
- Restrict to **EFT** field range f $\quad \varphi \equiv \frac{\phi}{f} \qquad V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi) \,, \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \varphi^n$
- Assume sub-dominant energy density
- Expand around constant inflationary background H_0 $\qquad H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6 M_p^2 H_0^2}\right)$

- FPV becomes

$$\frac{\alpha}{2} \frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \,, \qquad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \,, \qquad T \equiv \frac{t}{t_R} \,, \qquad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha \beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

Quantum diffusion

Volume

Classical drift

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- **Maximum** number of e-folds for **non-eternal** inflation: $N_{\text{e-folds}} < S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_\lambda e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda) p = 0$$

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- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

- Note: **boundary conditions** necessary input for solution

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Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

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e.g. D=0 at $\varphi = 1$ $\Rightarrow$ $\lambda_{\max} = \beta - \frac{\omega_1'^2}{2\alpha}$

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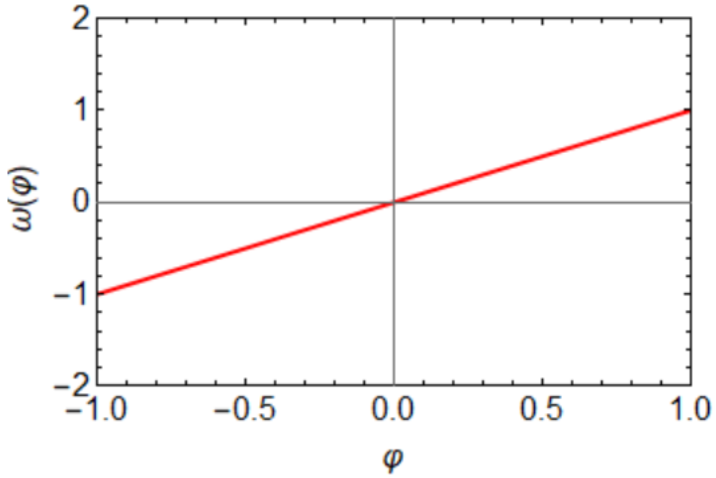

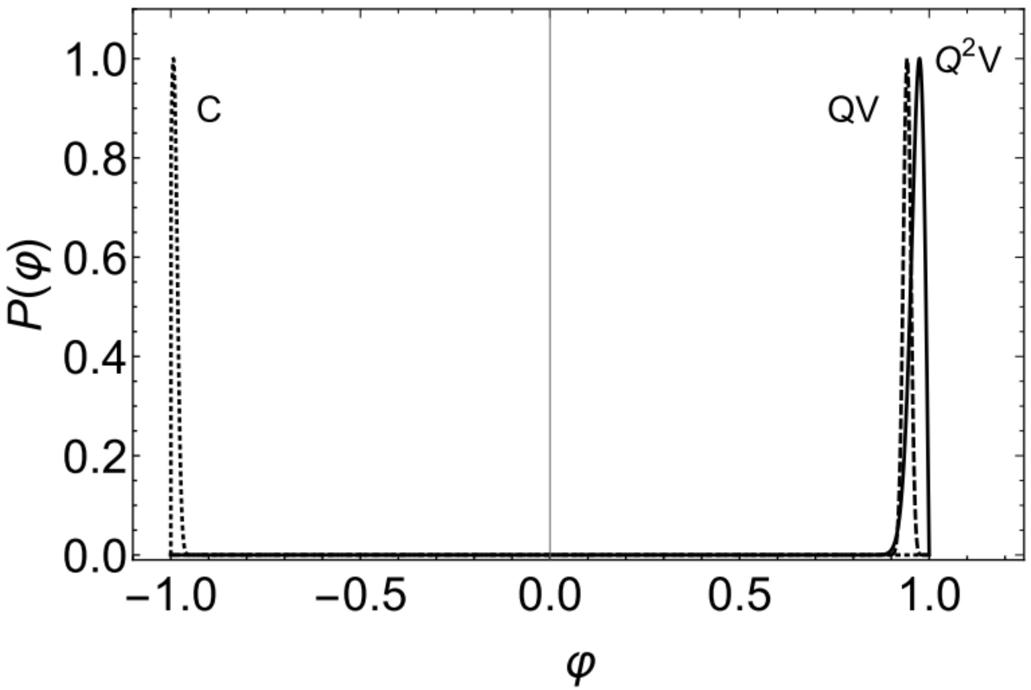

- C regime: $\alpha\beta \ll 1$. Peak is located as far down the potential as allowed by boundary condition.
- QV regime: $\alpha\beta \gg 1$, $\alpha^2\beta \ll 1$. Peak is a distance $1/(\alpha\beta)$ from the top with width $\sigma \simeq 1/\sqrt{\beta}$.
- Q^2V regime: $\alpha^2\beta \gg 1$. Peak as close to the top as possible, with a distance comparable to the width $\sigma \simeq (\alpha/\beta)^{1/3}$.

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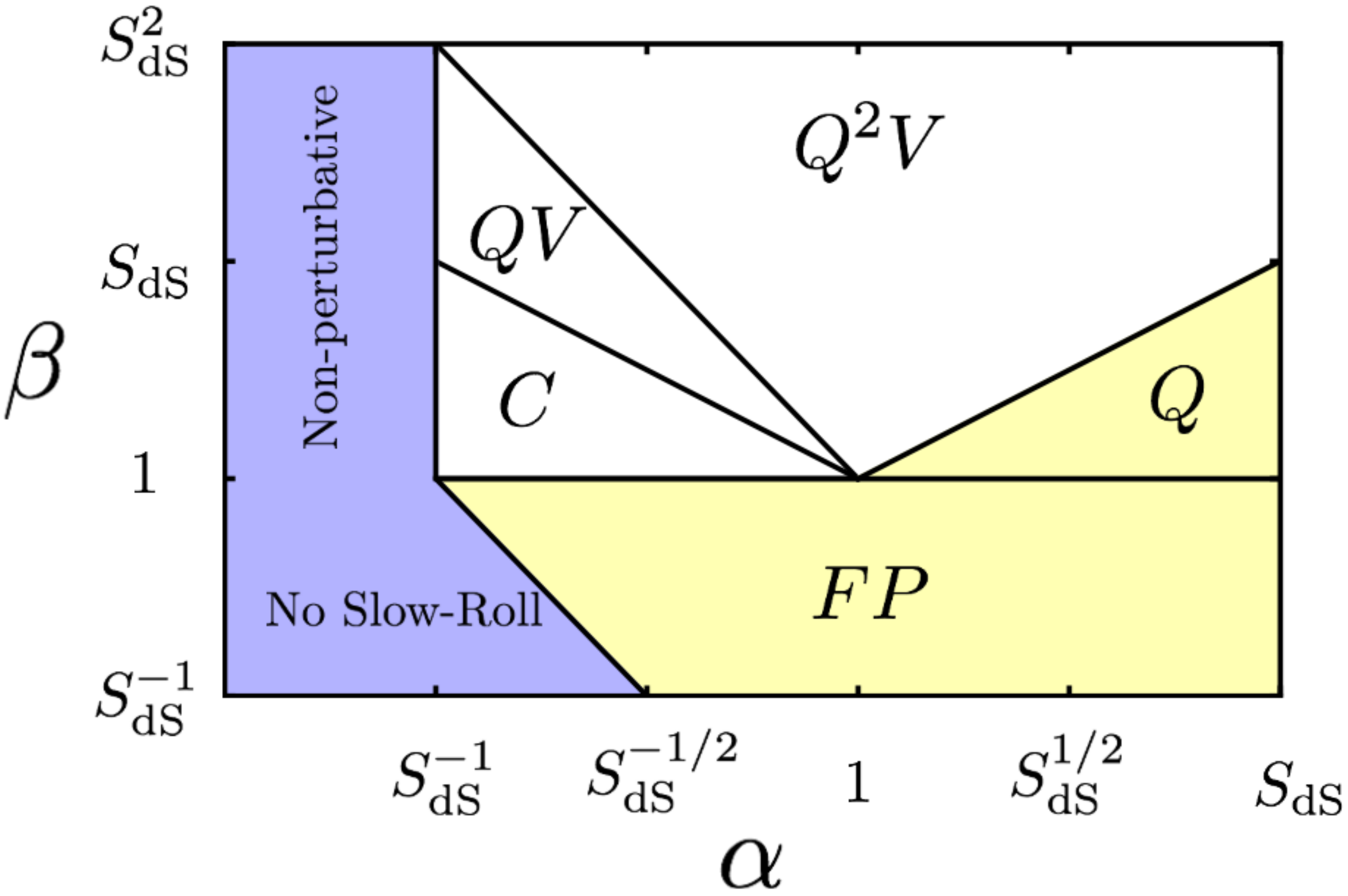

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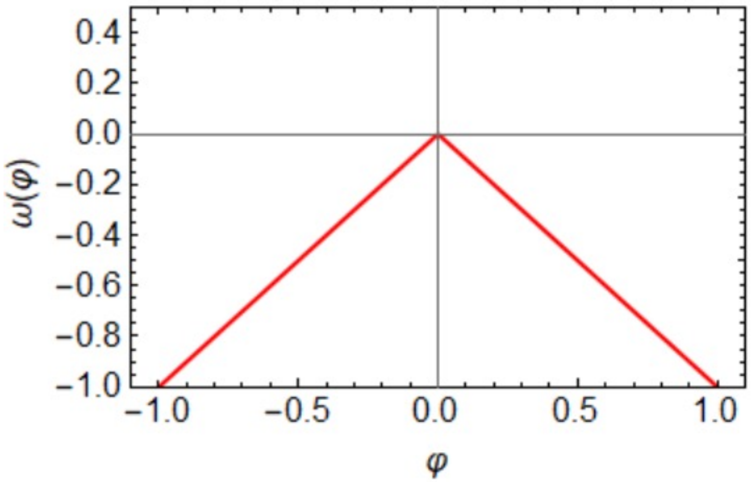
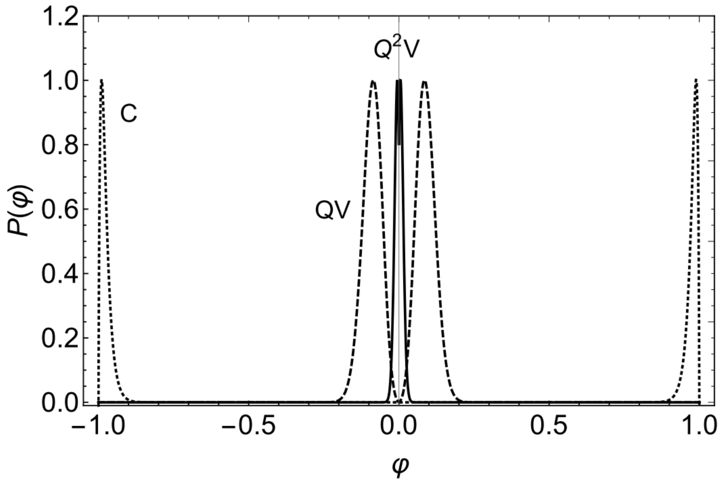
Junction conditions at phase transitions





- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon\to 0}\int_{\phi_c-\epsilon}^{\phi_c+\epsilon} d\phi \frac{\partial}{\partial\phi}\left[\frac{V'P}{3H}+\frac{\hbar}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^3P\right)\right]=0 \quad \Rightarrow \quad \frac{\Delta P'}{P(\varphi_c)}=-\frac{2\Delta\omega'}{\alpha}$$

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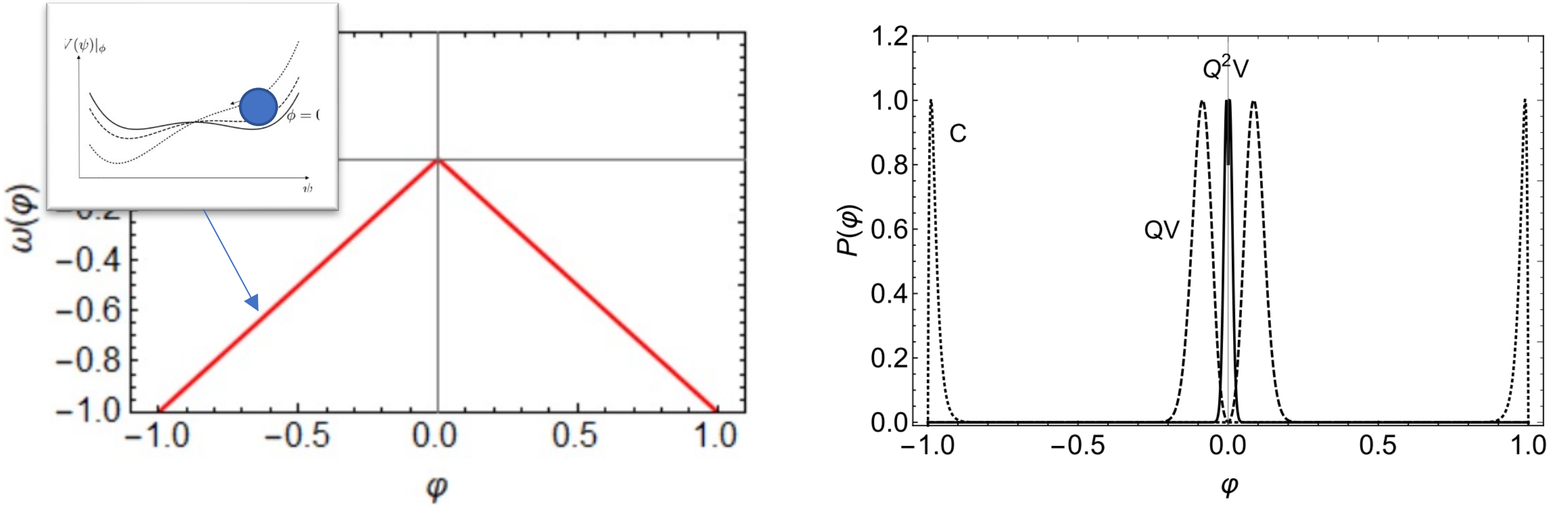

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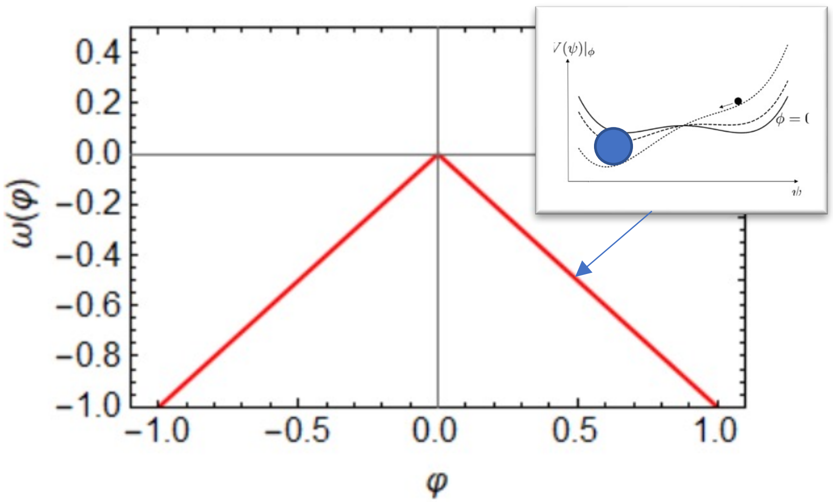

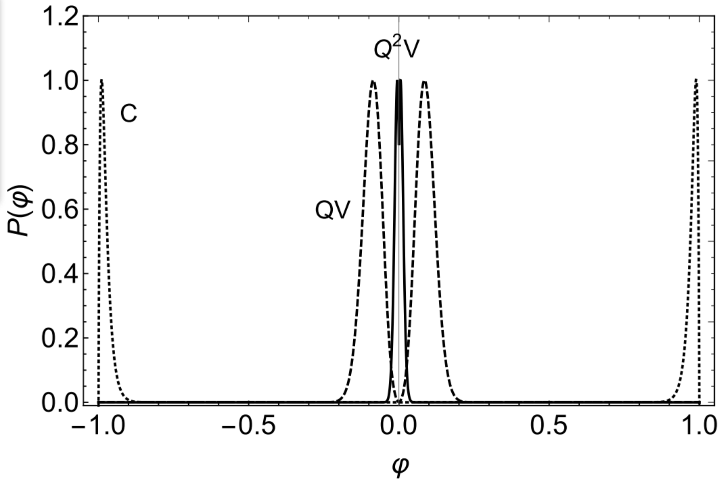

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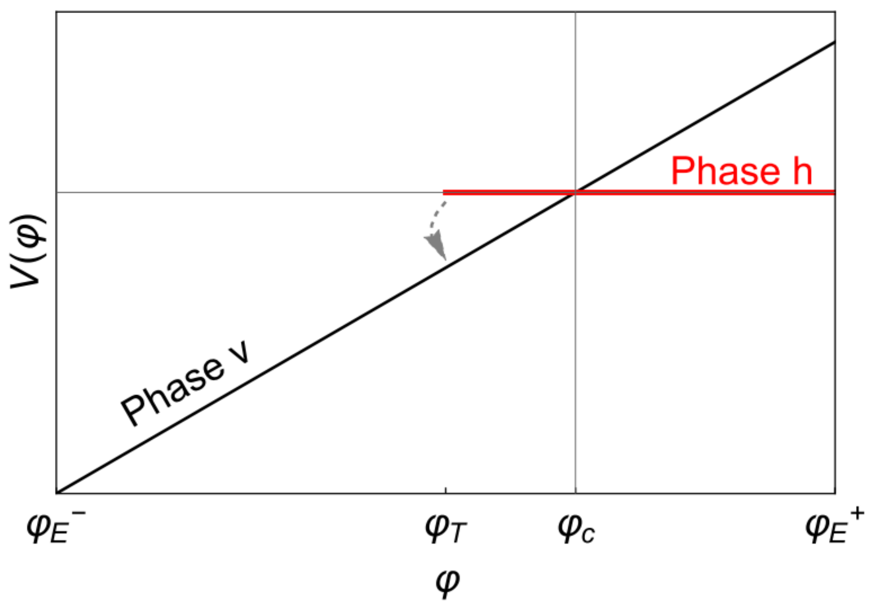

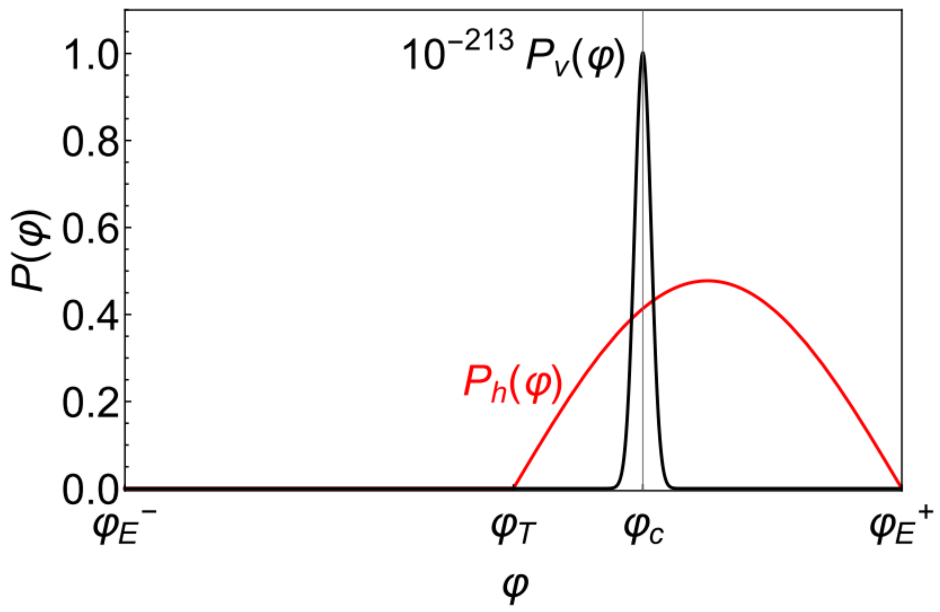

- **Coexistence** of branches of different phases, require continuity of P_V and $P_V + P_h$ in FPV at ϕ_T: **flux conservation** junction conditions

$$P_h(\phi_T) = 0 \qquad \Delta P_v' = -P_h'(\phi_T) \qquad \Delta P_v = 0$$

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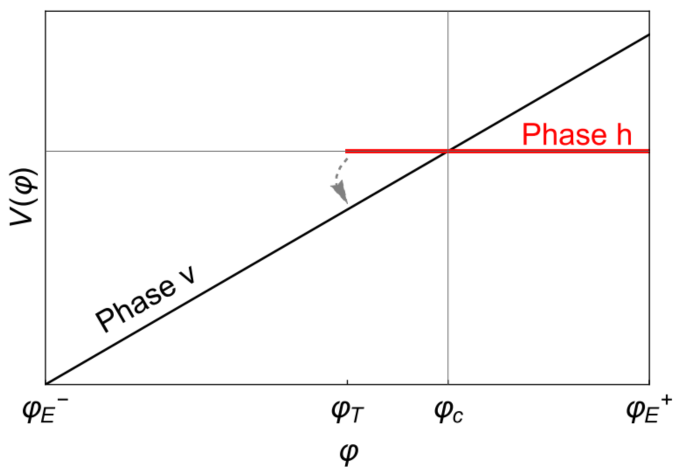
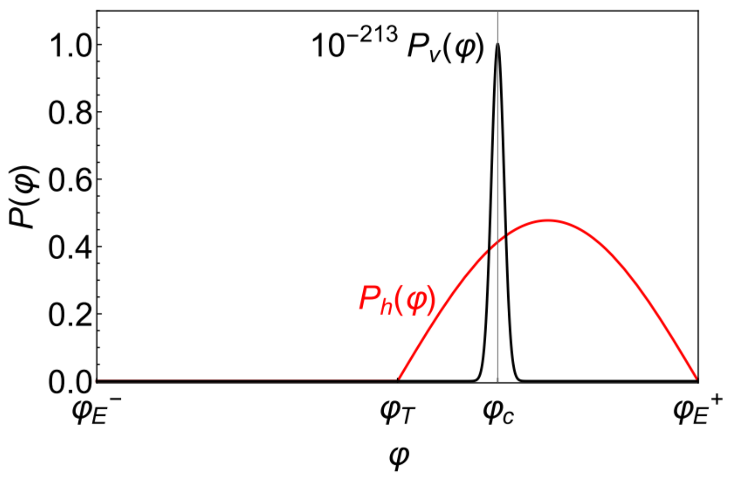




- **Phase v** *must be in C regime*
- **Boundary conditions** pick out diffusionless solution over Gibbs solution
- Require **flux at boundary**

Solve FPV: $\frac{\alpha}{2}p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)p = 0\,, \qquad \omega_h(\varphi) = 0 \quad , \quad \omega_v(\varphi) = \varphi\,.$

Phase h:

$$p_h(\varphi_E^+) = 0 \quad , \quad p_h(\varphi_T) = 0\,. \qquad p_h'(\varphi_T) = \kappa_h\,.$$

$$p_h(\varphi) = \frac{(\varphi_E^+ - \varphi_T)}{\pi}\kappa_h \sin\left(\frac{\pi(\varphi - \varphi_T)}{\varphi_E^+ - \varphi_T}\right) \quad , \quad (\varphi > \varphi_T)\,.$$

$$\lambda = -\frac{\alpha}{2}\frac{\pi^2}{(\varphi_E^+ - \varphi_T)^2}\,.$$

Phase v:

$$1)\ P_v^-(-1) = 0 \quad , \quad 2)\ P_v^{+\prime}(1) = -k_v\,,$$

$$3)\ P_v^+(\varphi_T) = P_v^-(\varphi_T) \quad , \quad 4)\ P_v^{+\prime}(\varphi_T) = P_v^{-\prime}(\varphi_T) - k_h\,,$$

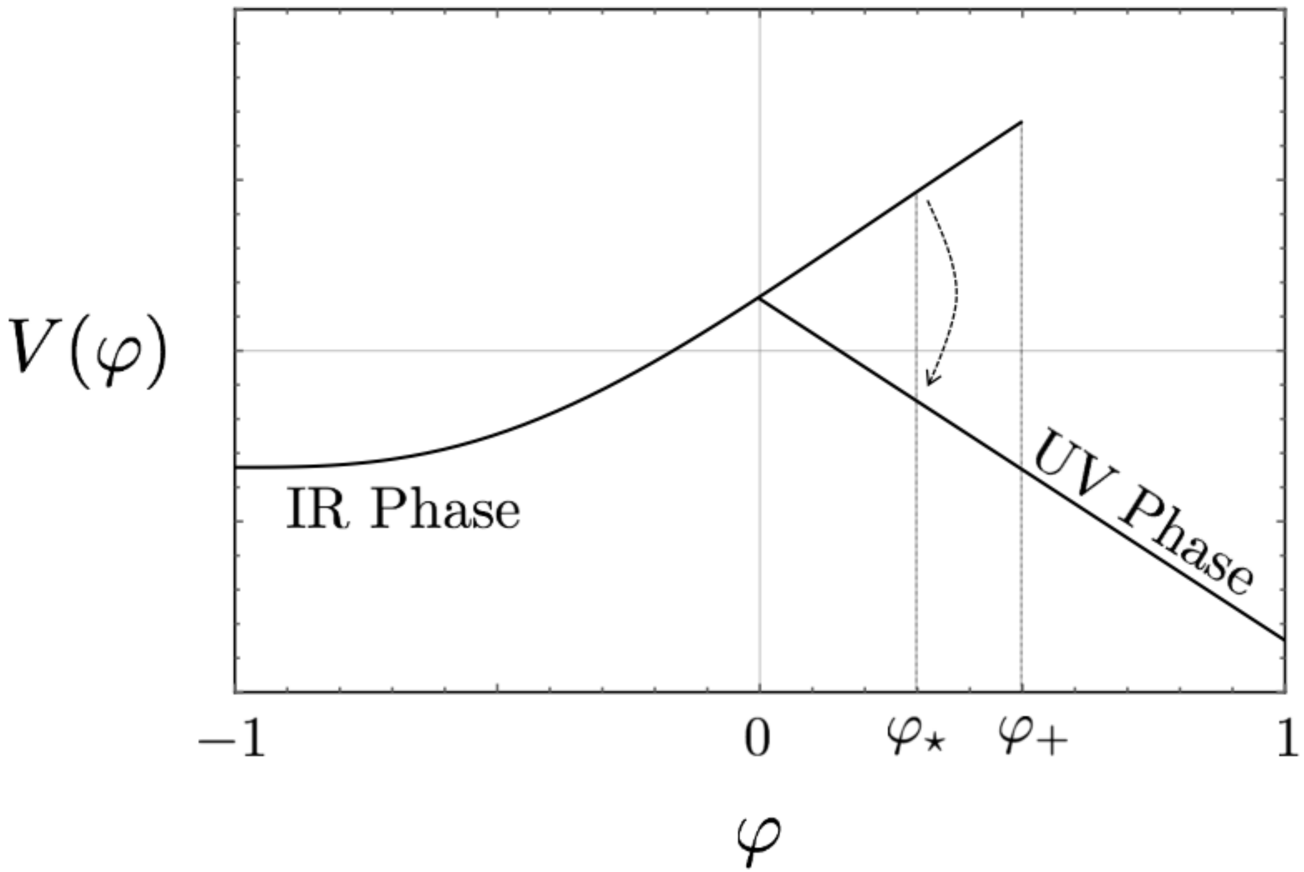
$$P_v^\pm(\varphi, \lambda) = e^{-\frac{\varphi}{\alpha}}\left[g_a^\pm(\lambda)Ai(x) + g_b^\pm(\lambda)Bi(x)\right]\,,$$
$$x = \frac{1 + 2\alpha\lambda - 2\alpha\beta\varphi}{(2\alpha^2\beta)^{2/3}}\,.$$

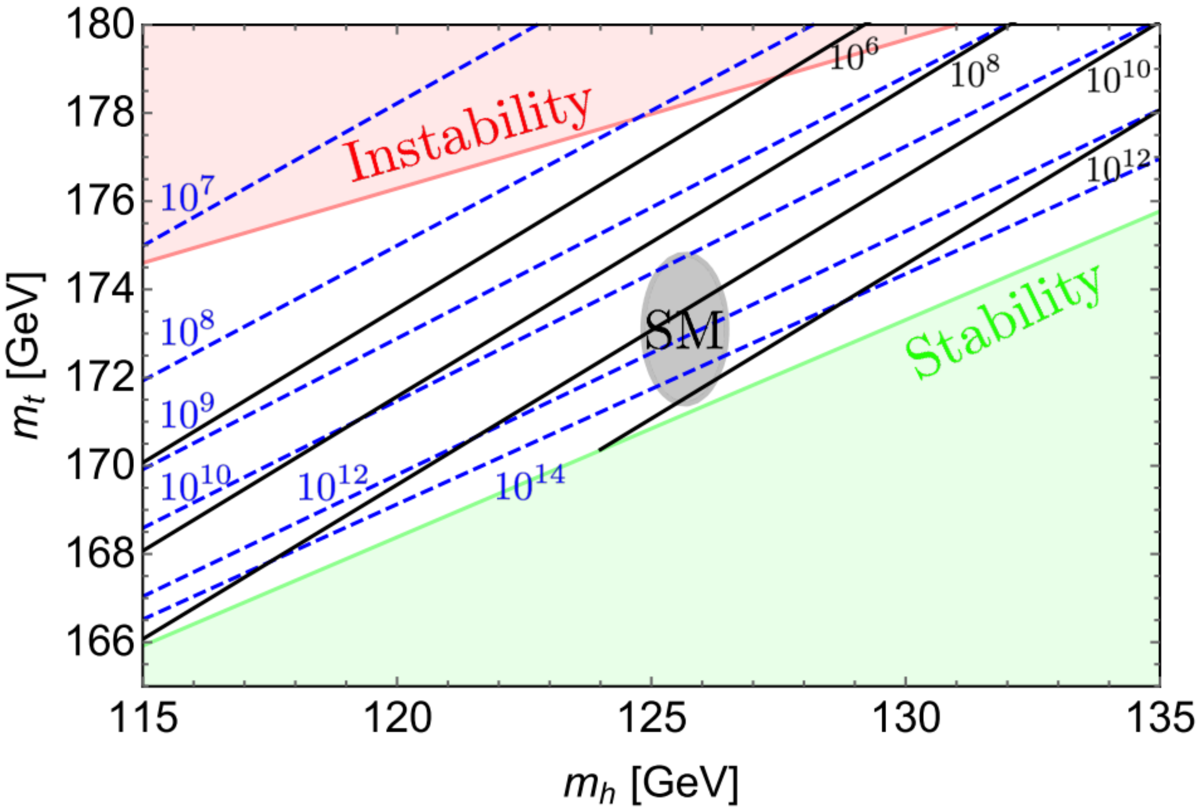
Outline

- Motivation
  - EFT
  - Criticality
  - Quantum phase transitions (QPT)
- Fokker-Planck Volume (FPV) equation
  - FPV dynamics
- FPV + QPT = SOL
  - Discontinuity
  - Flux conservation
- **SOL solutions**
  - Metastability
  - Higgs mass
  - Cosmological constant
- Conclusion
  - Measure problem

Higgs metastability

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2 \qquad \lambda(\varphi, M/g_*) = -g_*^2\,\varphi$$





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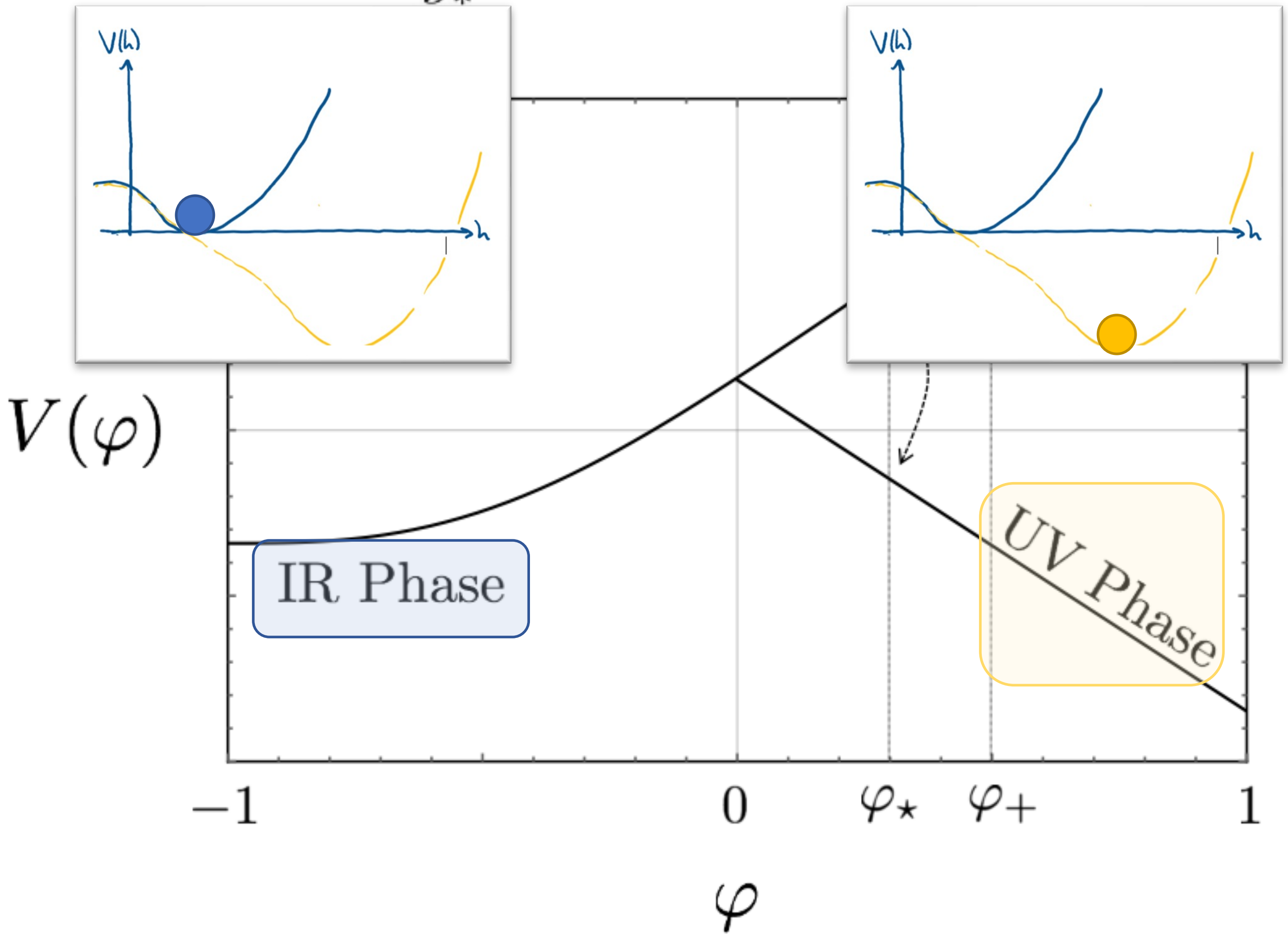

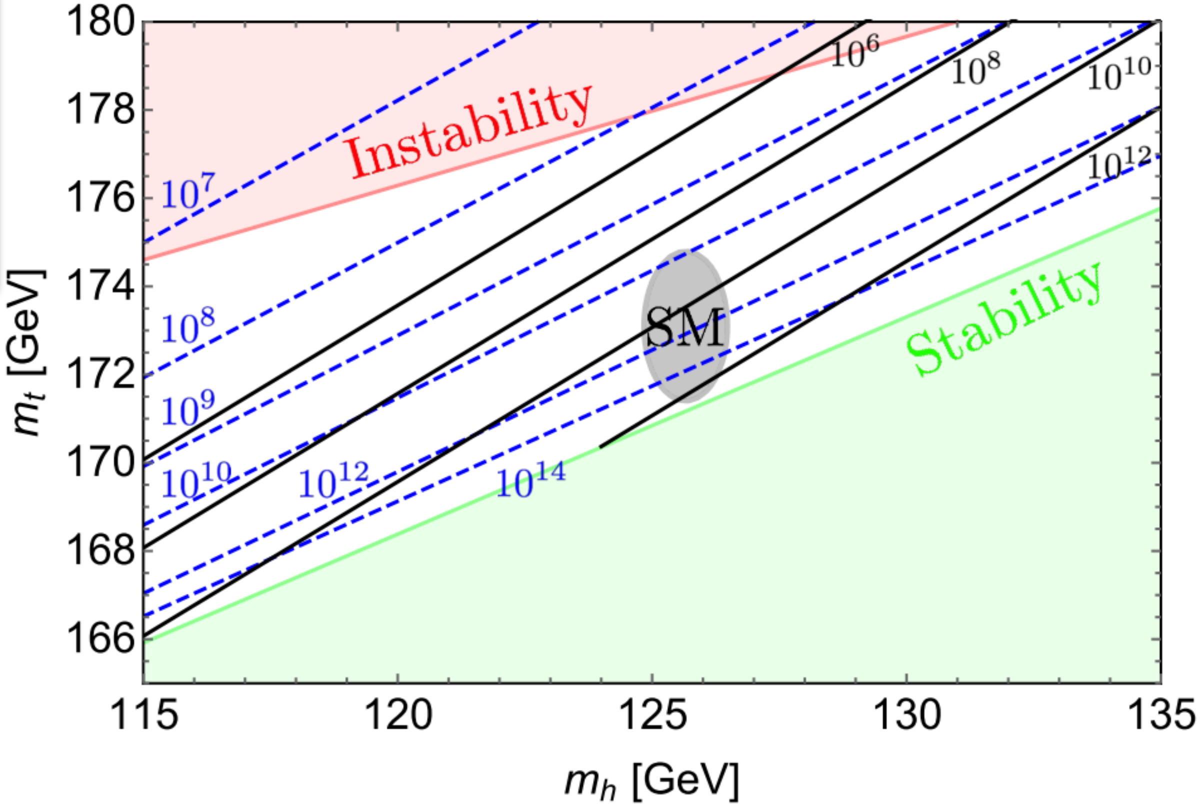

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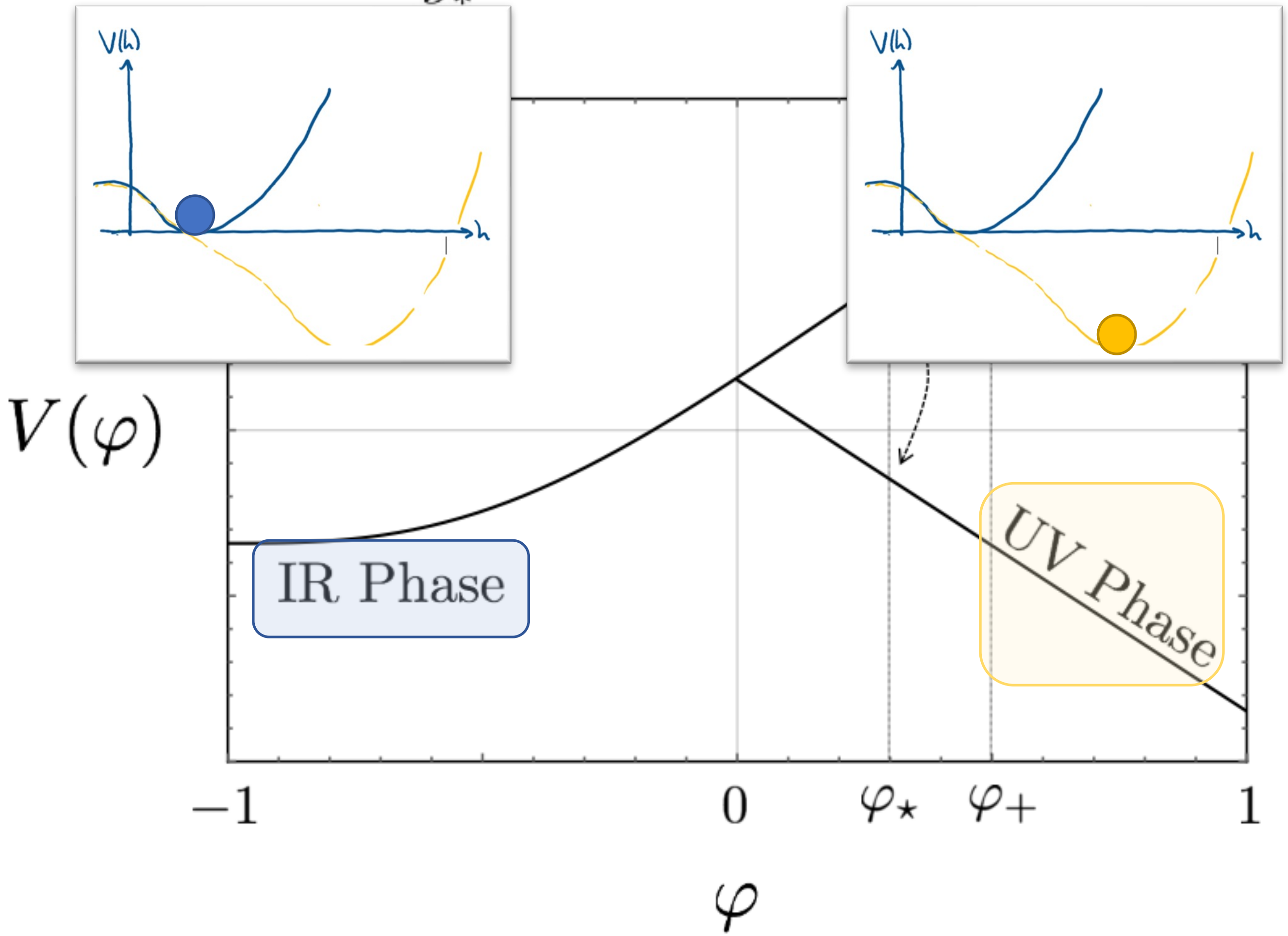
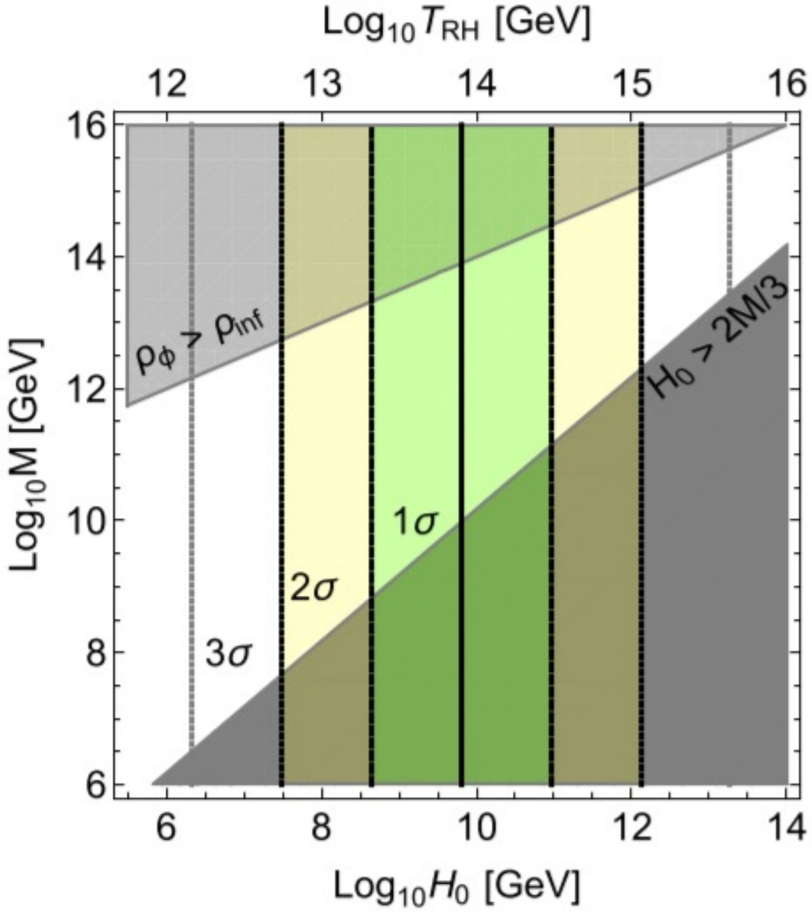


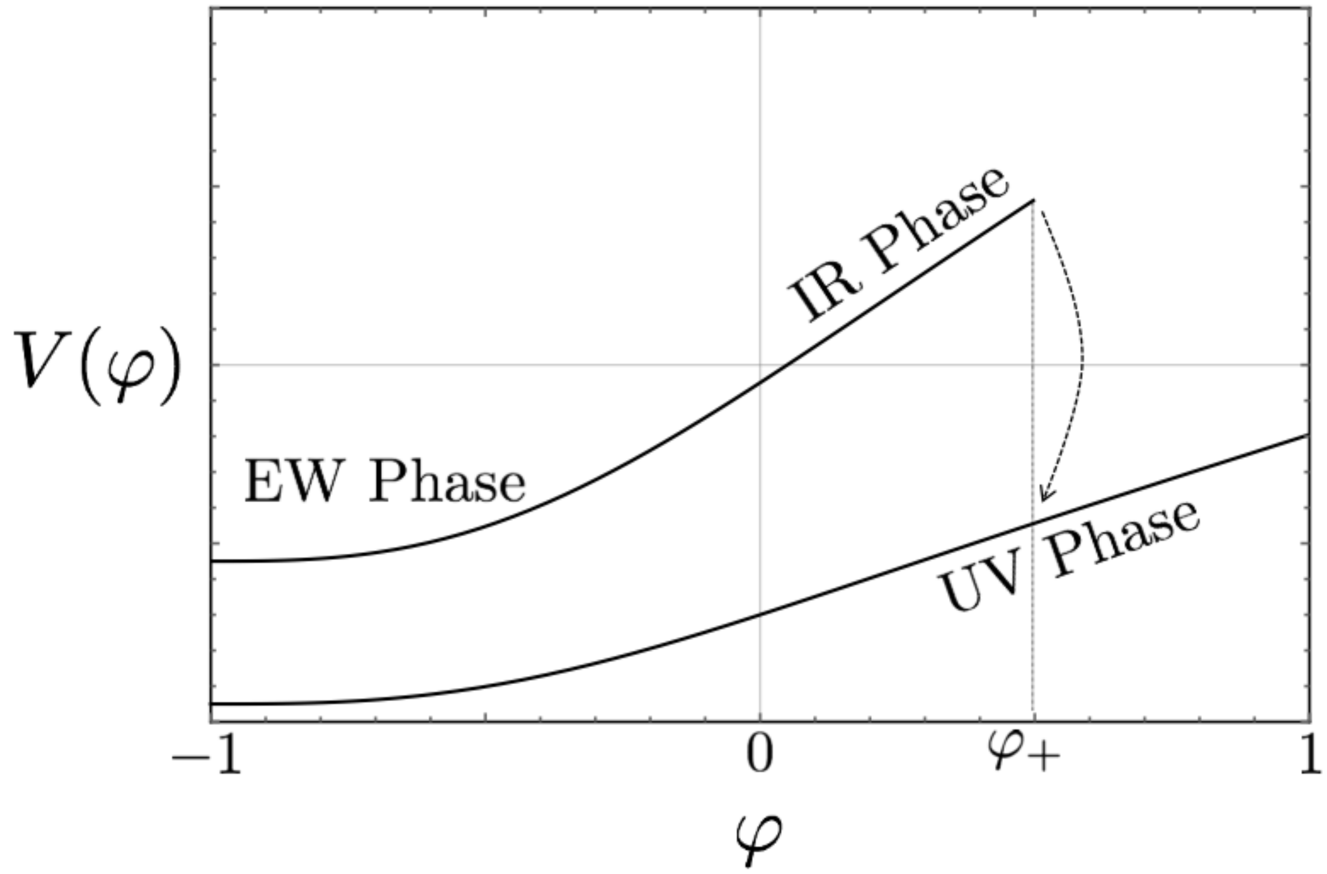


Higgs mass naturalness

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

$$\frac{V(\varphi,\langle h\rangle)}{M^4} = \begin{cases} \kappa_{\rm EW}\varphi + \kappa_2\varphi^2 + \dots & \text{for } \varphi<0 \quad (\text{unbroken EW: } \langle h\rangle = 0) \\ \kappa_{\rm EW}\varphi + \kappa_{\rm IR}\varphi^2 + \dots & \text{for } 0<\varphi<\varphi_+ \quad (\text{IR phase: } \langle h\rangle = v) \\ -\kappa_0 + \kappa_{\rm UV}\varphi + \kappa_2\varphi^2 + \dots & \text{for any } \varphi \quad (\text{UV phase: } \langle h\rangle = c_{\rm UV}M) \end{cases}$$

$$\kappa_{\rm EW} = \frac{\omega'(0)}{g_*^2}\,, \quad \kappa_2 = \frac{\omega''(0)}{2g_*^2}\,, \quad \kappa_{\rm IR} = \kappa_2 - \Delta\kappa\,, \quad \kappa_0 = \frac{-\lambda_{\rm UV}c_{\rm UV}^4}{4}\,, \quad \kappa_{\rm UV} = \kappa_{\rm EW} - \frac{c_{\rm UV}^2}{2}$$



- Unbroken to broken transition **not sufficient**

- Use **broken IR to broken UV** phase transition

$$\varphi_+ = \frac{-\beta_I\, e^{-\frac{3}{2}}\Lambda_I^2}{M^2} \quad \Rightarrow \quad v = e^{-\frac{3}{4}}\,\Lambda_I$$

- Need **lower instability scale $\boldsymbol{\Lambda_I}$**: ~TeV through VL fermions

- (Naturalness motivation: scalars and vectors heavy, **only VL fermions at TeV scale**)

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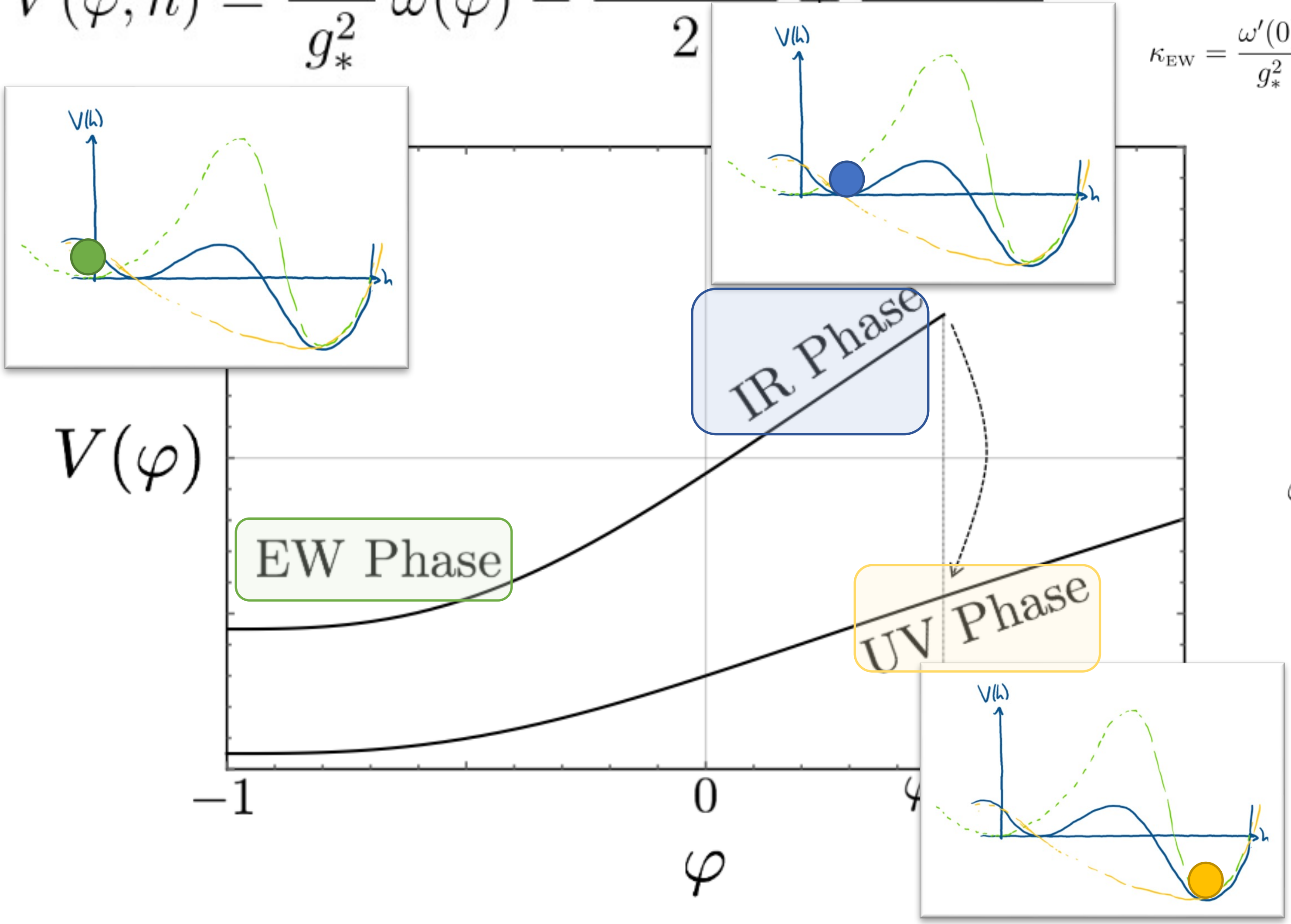

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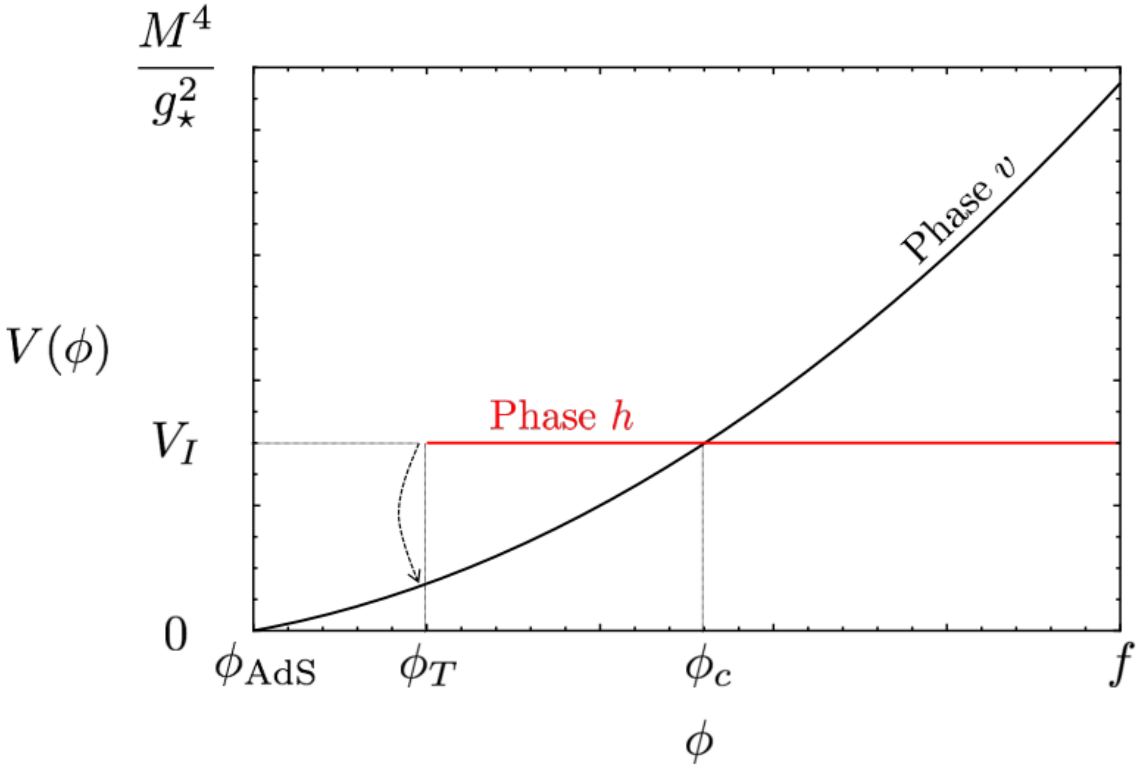
Cosmological constant

- **Hidden phase**: vanishing cosmological constant by R-symmetry
- **Visible phase**: SOL localises at vacuum degeneracy point



$$p_h(\phi) = \sin\left[\sqrt{\frac{6(1-\lambda_H)}{\hbar}}\,\frac{2\pi(\phi-\phi_T)}{H_I}\right]$$

$$\lambda_H = 1 - \frac{\hbar\, H_I^2}{24(f-\phi_T)^2}$$

$$V_v(\bar\phi) = V_I\,\lambda_H^{2/\xi}\,, \qquad \sigma = \sqrt{\frac{2}{3\xi}}\,M_P$$

$$\Rightarrow \quad V_v(\bar\phi) = V_I\left(1 - \frac{\hbar\, H_I^2}{12\xi f^2}\right)$$

- Solution must be in **C regime** with appropriate **boundary conditions**

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Take-home message

- Scalar fields undergoing quantum fluctuations during inflation can be **localised at the critical points** of quantum phase transitions: **SOL**

- SOL suggests **our Universe lives at the critical boundary** of coexistence of phases

- **Measure problem**: ambiguous choice of time parametrisation (recall $\beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$)
- Related to regularisation of **infinite reheating surface**
- We have **not specified** the inflaton sector: decoupled from our scalar
- SOL prediction is quantitative but dependent on chosen solution of measure problem: **exponential localisation can remain a feature**