Beautiful and charming lattice QCD calculations

Oliver Witzel

Universität Siegen

Center for Particle Physics Siegen

KEK Flavor Factories workshop
February 10, 2023

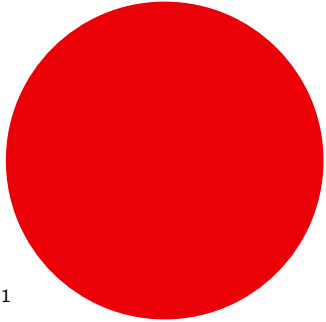
Heavy Flavors on the lattice

- Quark masses

up $\sim$ 0.002 GeV

charm $\sim$ 1.25 GeV

top $\sim$ 175 GeV



down $\sim$ 0.005 GeV

strange $\sim$ 0.095 GeV

bottom $\sim$ 4.2 GeV

- Lattice simulations have a cutoff a^{-1}
  - $\rightarrow$ Fully relativistic quarks require $am \ll 1$ i.e. $m \ll a^{-1}$
  - $\rightarrow$ Typically $a^{-1} \gtrsim 2$ GeV $\Rightarrow m_{\text{charm}} \lesssim a^{-1} \lesssim m_{\text{bottom}}$
  - $\rightarrow$ Charm but in particular bottom quarks require special considerations

Simulating heavy flavors

- Traditionally: simulate charm and bottom using effective actions
  - → Heavy quark effective Theory (HQET), Non-Relativistic QCD, Relativistic Heavy Quark (RHQ, Fermilab, Tsukuba)
  - → Allows to simulate charm and bottom quarks on coarser lattices
  - → Additional systematic uncertainties, partly perturbative renormalization, ...
  - → Few percent total errors

- State-of-the-art: fully relativistic simulations at $a^{-1} > 2$ GeV
  - → Heavy Highly Improved Staggered Quarks (HISQ), Heavy Domain-Wall Fermions (DWF), ...
  - → Same action for light (up/down/strange) as for heavy (charm/bottom) quarks
    - ⇝ Simulate heavier than charm and extrapolate
  - → Fully nonperturbative renormalization straight-forward, reduced systematic uncertainties
  - → Sub-percent precision feasible ⇝ QED effects become relevant

Cabbibo-Kobayashi-Maskawa (CKM) matrix

- Beautiful are of course calculations allowing us to extract CKM matrix elements
  $\rightarrow$ Combination of experimental measurement and theoretical prediction

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

Cabbibo-Kobayashi-Maskawa (CKM) matrix

- Beautiful are of course calculations allowing us to extract CKM matrix elements
  $\to$ Combination of experimental measurement and theoretical prediction
- Lattice QCD prefers processes with at most a single hadronic final state

$$\begin{bmatrix} \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & B_s \to K\ell\nu \\ D \to \ell\nu & D_s \to \ell\nu & B_{(s)} \to D_{(s)}\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B_{(s)} \to D^*_{(s)}\ell\nu \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{bmatrix}$$

Cabbibo-Kobayashi-Maskawa (CKM) matrix

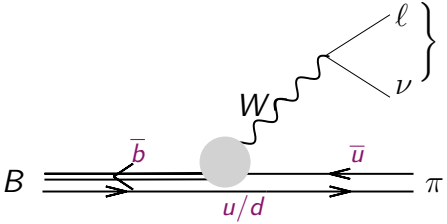
- Beautiful are of course calculations allowing us to extract CKM matrix elements
  → Combination of experimental measurement and theoretical prediction
- Lattice QCD prefers processes with at most a single hadronic final state

$$\begin{bmatrix} \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & B_s \to K\ell\nu \\ D \to \ell\nu & D_s \to \ell\nu & B_{(s)} \to D_{(s)}\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B_{(s)} \to D^*_{(s)}\ell\nu \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{bmatrix}$$

- A (biased) selection of highlights
  → V_{ub} from $B \to \pi\ell\nu$
  → V_{cb} form $B_s \to D_s^*\ell\nu$
  → V_{cd}, V_{cs} from leptonic $D_{(s)}$ decays
- Talk by Aida El-Khadra [Saturday]

$B \to \pi \ell \nu$

Exclusive Semi-Leptonic Decays: Example $B \to \pi\ell\nu$



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parametrized placing the B meson at rest

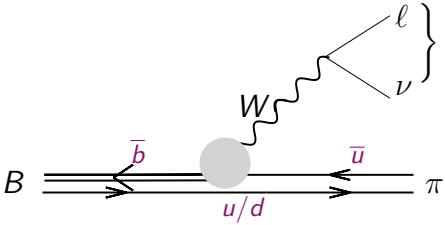
$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_B^2 (E_\pi^2 - M_\pi^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

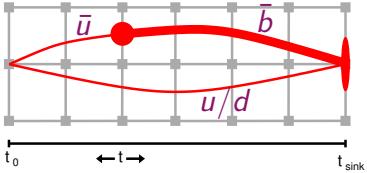
Exclusive Semi-Leptonic Decays: Example $B \to \pi\ell\nu$



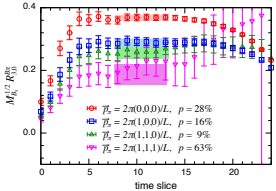
$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

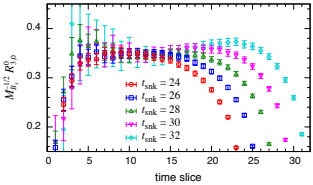
$$\langle\pi|V^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$



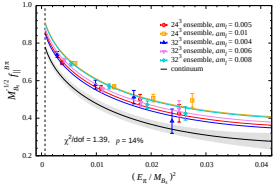
Typical steps of the calculation [Plots taken from Flynn et al. PRD 91 (2015) 074510]



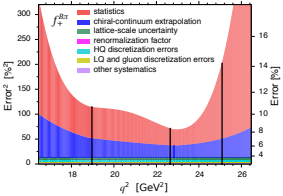
- Measure 2- and 3-pt for functions an ensemble set



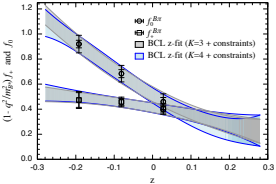
- Study systematic effects e.g. source-sink separation



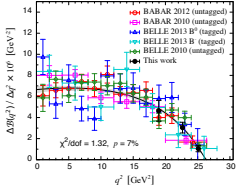
- Extrapolate to physical masses and the continuum



- Obtain full error budget

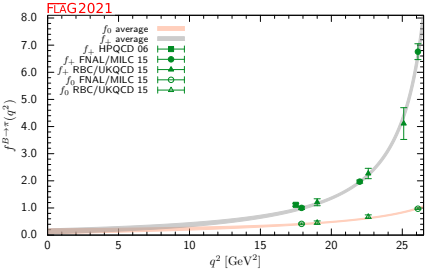


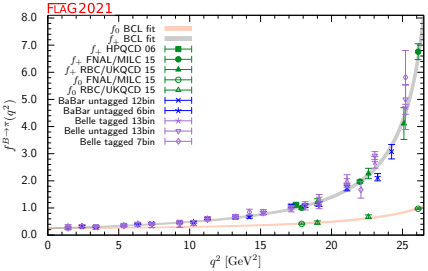
- Kinematical expansion to cover full q^2 range

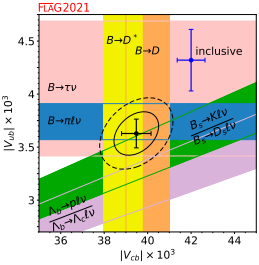


- Combine with experiment, compare to other determinations

FLAG average [FLAG 2021]





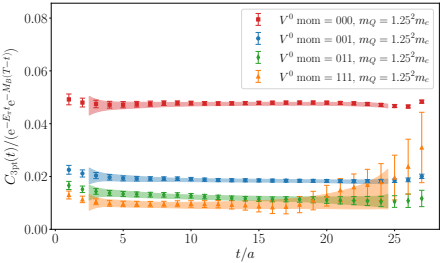


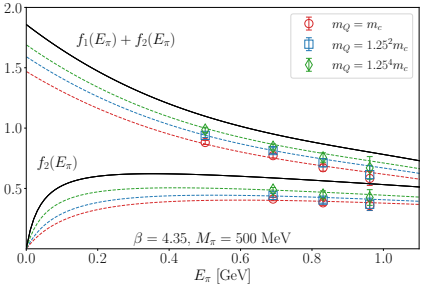
- FLAG average: Fermilab/MILC [Bailey et al. PRD92(2015)014024], RBC/UKQCD [Flynn et al. PRD 91 (2015) 074510]
  → Shown in addition HPQCD [Dalgic et al. PRD73(2006)074502][PRD75(2007)119906]
- Used effective actions only allowed determinations of form factors at large q^2
- Combined fit with experimental data gives $|V_{ub}^{\text{excl}}|$
  [BaBar PRD 83 (2011) 032007][PRD 86 (2012) 092004] [Belle PRD 83 (2011) 071101][PRD 88 (2013) 032005]
- Shape of lattice data consistent with experimental data

New in 2022: JLQCD [Colquhoun et al. PRD 106 (2022) 054502]

- Unitary setup
  - → MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
  - → Additional extrapolation in the heavy quark mass to reach m_b
  - → Fully nonperturbative renormalization

- $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- $M_\pi \gtrsim 230$ MeV



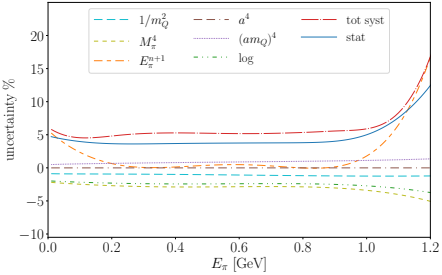


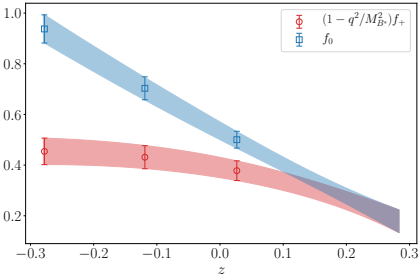
New in 2022: JLQCD [Colquhoun et al. PRD 106 (2022) 054502]

- Unitary setup
  - → MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
  - → Additional extrapolation in the heavy quark mass to reach m_b
  - → Fully nonperturbative renormalization

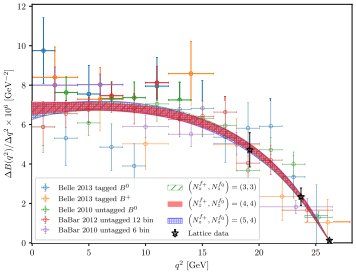
- $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- $M_\pi \gtrsim 230$ MeV

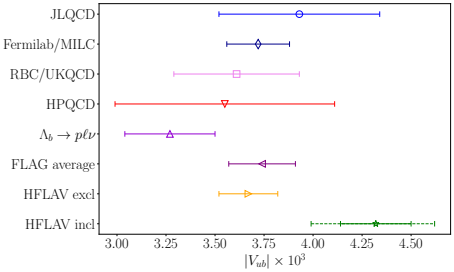
- Comparable stat. and sys. errors
  - → Total errors: $f_+ \sim 10\%$, $f_0 \sim 6\%$

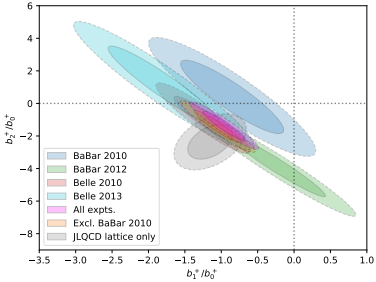




New in 2022: JLQCD [Colquhoun et al. PRD 106 (2022) 054502]







- Joint fit to determine $|V_{ub}|$
  $\Rightarrow |V_{ub}| = (3.93 \pm 0.41) \cdot 10^{-3}$
- Updates from other collaborations expected relatively soon
- Shape parameters of BCL z-fit
  $\rightarrow$ Tension with BaBar 2010
  $\rightarrow$ Looking forward to new data from Belle II

$B_s \to D_s^* \ell \nu$

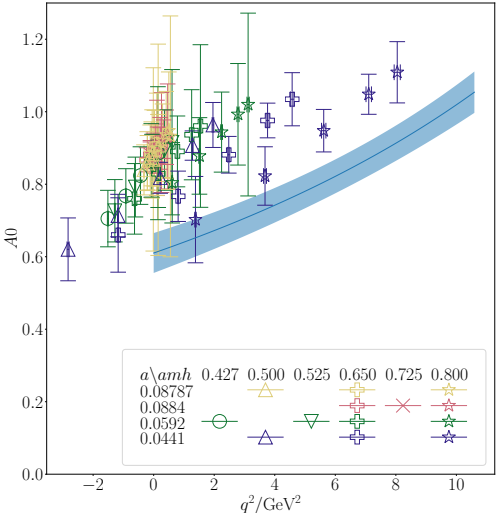
Exclusive Semi-Leptonic Decays: Example $B_s \to D_s^* \ell \nu$

$$\langle D_s^*(k,\varepsilon_\nu)|\mathcal{V}^\mu|B_s(p)\rangle = V(q^2)\frac{2i\varepsilon^{\mu\nu\rho\sigma}\varepsilon_\nu^* k_\rho p_\sigma}{M_{B_s}+M_{D_s^*}}$$

$$\begin{aligned}\langle D_s^*(k,\varepsilon_\nu)|\mathcal{A}^\mu|B_s(p)\rangle = & A_0(q^2)\frac{2M_{D_s^*}\varepsilon^*\cdot q}{q^2}q^\mu \\ & + A_1(q^2)(M_{B_s}+M_{D_s^*})\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ & - A_2(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_{D_s^*}}\left[k^\mu+p^\mu-\frac{M_{B_s}^2-M_{D_s^*}^2}{q^2}q^\mu\right]\end{aligned}$$

- Determine the four form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$ or in HQE convention $h_V(w)$, $h_{A_0}(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$
- Narrow-width approximation i.e. D_s^* is treated as a QCD-stable particle
- Proxy for $B \to D^* \ell \nu$ (only spectator quark differs)

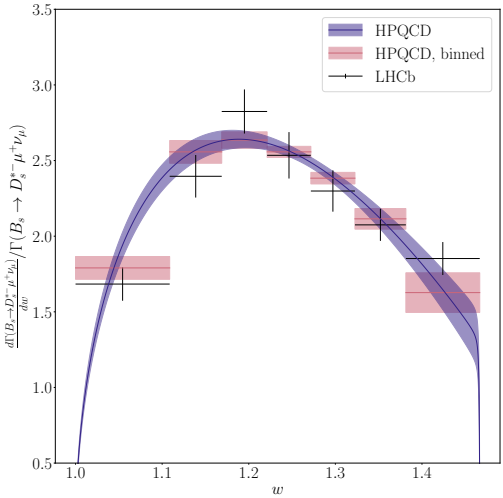
HPQCD $B_s \to D_s^* \ell \nu$ [Judd, Davies PRD105(2022).094506]



- All-HISQ setup
  - → Fully non-perturbative renormalization
  - → Simulate heavier-than-charm → close-to-bottom
  - → Directly cover most of the allowed q^2 range at the finest lattice spacing
  - ⇝ Parametrize pole mass for different charm masses

- $B \to D^* \ell \nu$
  - → Fermilab/MILC [Bazavov et al. EPJC 82(2022)1141]
  - → JLQCD [Kaneko et al. PoS Lattice2021 (2022) 561]
  - → LANL/SWME [Jang et al. PoS Lattice2019 (2020) 056]
  - → HPQCD [Harrison Talk Barolo 2022]

HPQCD $B_s \to D_s^* \ell \nu$ [Judd, Davies PRD105(2022).094506]

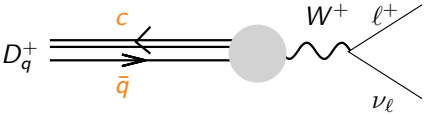


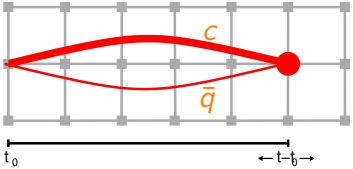
- All-HISQ setup
  - → Fully non-perturbative renormalization
  - → Simulate heavier-than-charm → close-to-bottom
  - → Directly cover most of the allowed q^2 range at the finest lattice spacing
  - ⇝ Parametrize pole mass for different charm masses
- Combination with LHCb [LHCb JHEP12(2020)144]
  - → $|V_{cb}|^{\text{excl}}_{\text{BGL}} = 42.7(1.5)_{\text{lat}}(1.7)_{\text{exp}}(0.4)_{\text{em}} \times 10^{-3}$
  - → $|V_{cb}|^{\text{excl}}_{\text{CLN}} = 41.6(1.5)_{\text{lat}}(1.6)_{\text{exp}}(0.4)_{\text{em}} \times 10^{-3}$
- PDG 2022 $|V_{cb}|^{\text{excl}} = 39.4(0.8) \times 10^{-3}$
  $|V_{cb}|^{\text{incl}} = 42.2(0.8) \times 10^{-3}$
  [PDG, Workman et al. PTEP (2022) 083C01]

f_D and f_{D_s}

Extracting $|V_{cd}$ and $|V_{cs}|$ from leptonic decays

- Branching fractions of leptonic decay modes well measured
  - → D^+: CLEO-c, BESIII
  - → D_s^+: CLEO-c, BaBar, Belle, BESIII
- Allows to infer $|V_{cq}| \cdot f_{D_q^+}$ for $q = d, s$
- Calculating $f_{D_q^+}$ on the lattice gets us the CKM matrix elements
  - → Only simple 2-pt functions needed





Lattice calculations of f_D and f_{D_s}

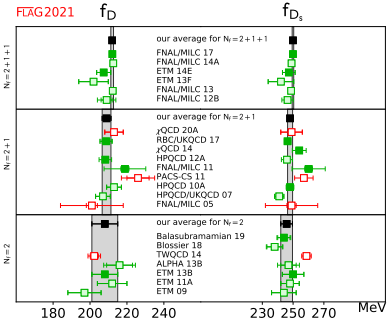
- Consistent determinations by many groups [FLAG 2021]

  2+1+1 flavor: $f_D = 212.0(0.7)$ MeV

  $f_D = 249.0(0.5)$ MeV

  [Fermilab/MILC PRD 98 (2018) 074512] [ETMC PRD 91 (2015) 054507]

  $\Rightarrow |V_{cd}| = 0.2179(7)(57)$ and $|V_{cs}| = 0.983(2)(18)$

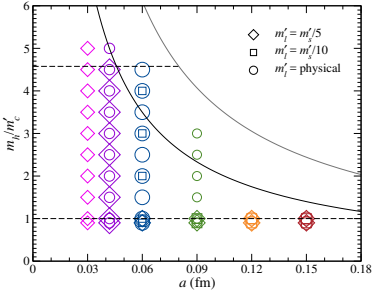


- Impressive sup-percent precision by Fermilab/MILC

  [Bazavov et al. PRD 98 (2018) 074512]

- Exploiting full power of 2+1+1 flavor HISQ ensembles

  $\to$ 6 different lattice spacings with physical quark masses



Summary

- $B \to \pi \ell \nu$
  - → Consistent determinations of $|V_{ub}|^{\text{excl}}$ from different lattice collaborations
  - → New calculation by JLQCD with larger errors
  - → Puzzling tension between $|V_{ub}|^{\text{excl}}$ and $|V_{ub}|^{\text{incl}}$
  - → Desirable: calculation with physical light quarks to largely eliminate chiral extrapolation
  - ⇝ New development: $|V_{ub}|^{\text{incl}}$ using lattice QCD see e.g. [Kellermann et al. PoS Lattice2022 (2023) 414]
- $B_{(s)} \to D_{(s)}^* \ell \nu$
  - → Finally lattice calculations over the full q^2 range for decays with vector final state
  - → More calculations underway
- f_D and f_{D_s}
  - → Sub-percent level precision ⇝ desirable to see more results at that level of precision
  - → Precise experimental measurements of $B_{(c)}^+$ leptonic branching fraction highly desirable

Charming views