QED corrections to $B \to \mu\bar{\nu}$

Claudia Cornella (JGU Mainz)

based on 2212.14430 and ongoing work with M. Neubert and M.König

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Motivation

Why $B \to \ell\nu$?

$$\mathcal{B}(B \to \ell\nu) = \tau_B G_F^2 f_{B_u}^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

- direct determination of $|V_{ub}|$,
- chirality suppressed → powerful probe of (pseudo)scalar NP.
- testing Lepton Flavor Universality in charged currents

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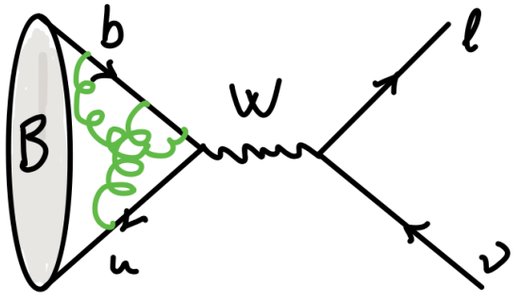
Why QED corrections?

Belle II will measure the τ, μ channels with $5 - 7\,\%$ uncertainty. [Belle II Physics Book]

Pure hadronic effects are simple: $\langle 0 | \bar{q}\gamma^\mu\gamma_5 b | B_q(p) | \rangle = i f_{B_q} p^\mu$

f_{B_u} is known with $\mathcal{O}(1\%)$ precision: $f_{B_u} = 189.4 \pm 1.4$ MeV

[FNAL/MILC 1712.09262]



With strong cuts on additional radiation, QED corrections are in the *same* ballpark

$\Rightarrow$ precise estimate needed!

QED corrections to exclusive B decays

QED effects are well under control for $\mu > m_b$ as well as for $\mu \ll \Lambda_{\rm QCD}$:

- above m_b QED can be included in the **weak effective Lagrangian** + renormalization group
- photons with energy much smaller than $\Lambda_{\rm QCD}$ cannot resolve the hadron structure and can be computed treating the B as **point-like**.

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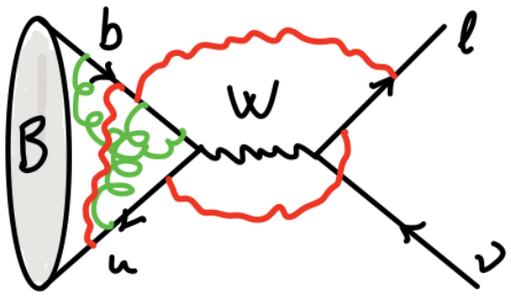
Things are more complicated for $\Lambda_{\rm QCD} < \mu < m_b$: very active research topic.
QED factorization theorems available only for a few processes:

- $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron, 1708.09152,1908.07011]
- $B \to \pi K, B \to D\pi$ [Beneke, Böer et al 2008.10615,2107.03819]
- $B_s \to \mu^+\mu^-\gamma$ [Beneke, Bobeth, Wang 2008.12494]

QED corrections to exclusive B decays

Main challenges in formulating a factorization theorem:

- unlike in QCD, **external states** can be **charged** in QED
  → "universal" hadronic quantities become process-dependent,
  e.g. decay constants are not constants anymore



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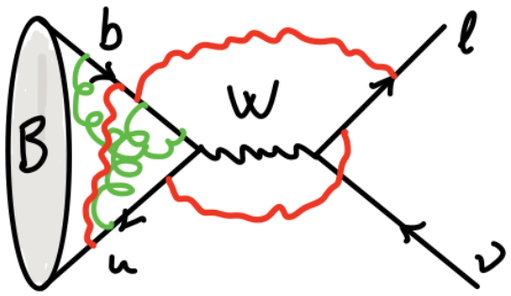
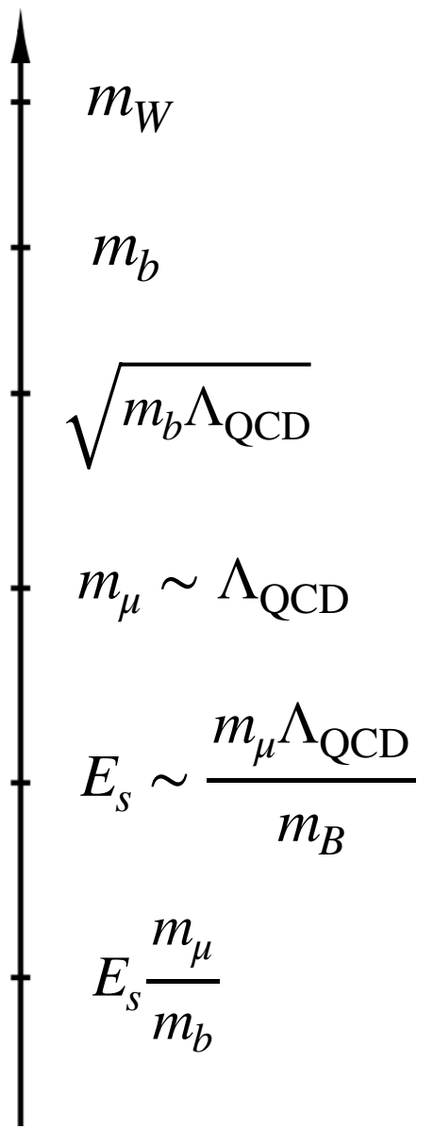


- Beyond leading power convolutions have **endpoint divergences**.
  These cannot be dealt with using standard renormalization techniques and require appropriate subtractions.

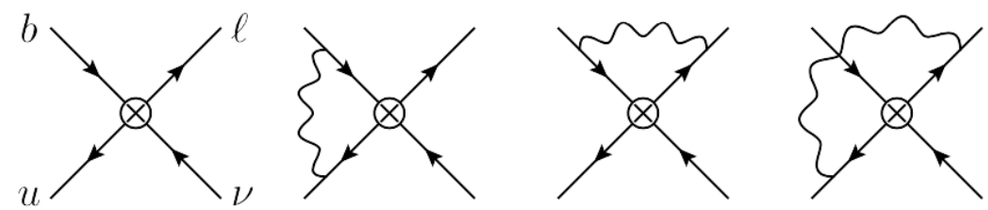
  Relevant for us: because of the chiral suppression, $B \to \mu\bar{\nu}$ is a genuine next-to-leading power process.

Scales

In the presence of QED corrections, $B \to \mu\bar{\nu}$ is sensitive to many scales:

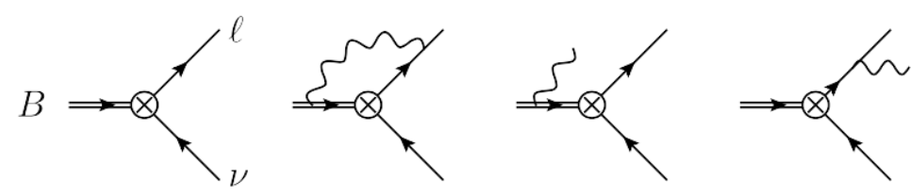


©Matthias König



description of meson by $|b\bar{q}\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

description of meson by point-like scalar



[Dai et al (*2022*), Phys.Rev.D 105 3]

Plan

We want to disentangle all these scales and the associated logarithms.
In practice, this means:

- Identify the **appropriate EFT** description for all relevant scales
- Derive a **factorization theorem** to break the multi scale process into a product of single-scale objects
- Use the **renormalization group** to evaluate each objects at its natural scale and evolve them to a common scale to resum logarithms

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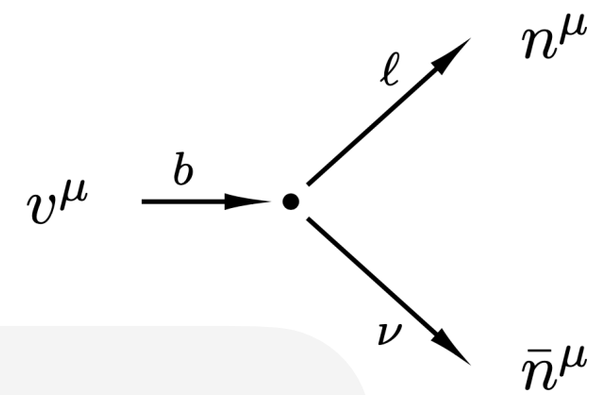
In this talk:

- general description of EFT construction
- focus on factorization of the amplitude above E_s (virtual corrections only)

Kinematics and power counting

To get the scaling of physical momenta,
analyze the kinematics of the non-radiative process.



$$p_B^\mu = m_B v^\mu\,, \qquad v^\mu = (1,0,0,0)$$

"collinear"

$$p_\mu^\mu = \frac{m_B}{2}\left(1+\lambda_\mu^2, 0, 0, +1-\lambda_\mu^2\right) \approx \frac{m_B}{2}(1,0,0,+1) = \frac{m_B}{2} n^\mu$$

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$$\lambda_\mu = \frac{m_\mu}{m_b} \ll 1$$

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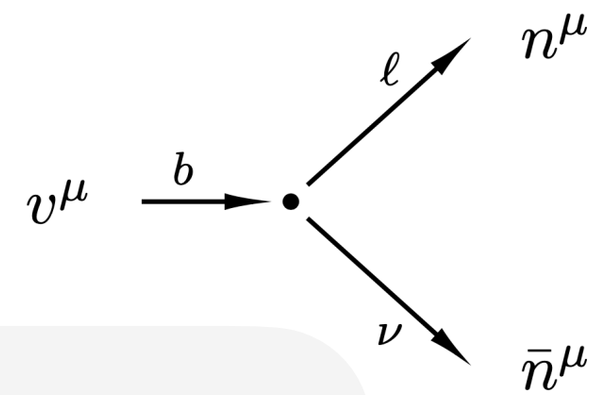

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"lightcone coordinates": $$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu \equiv \left((n\cdot p)\,,\,(\bar{n}\cdot p)\,,\,p_\perp\right)$$

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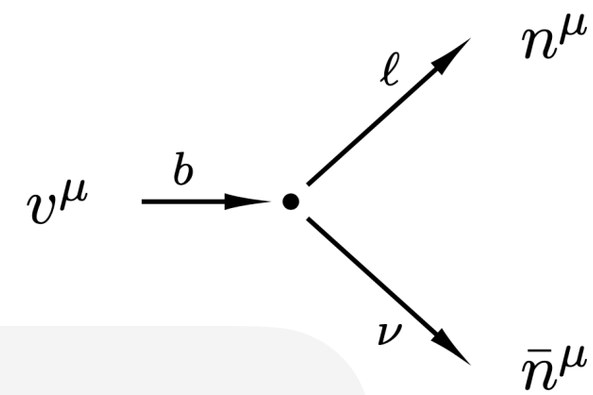

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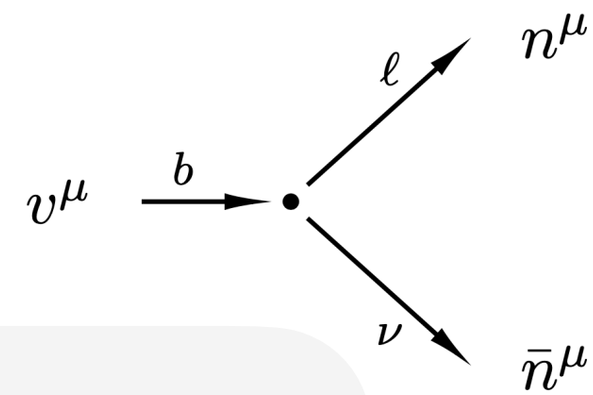

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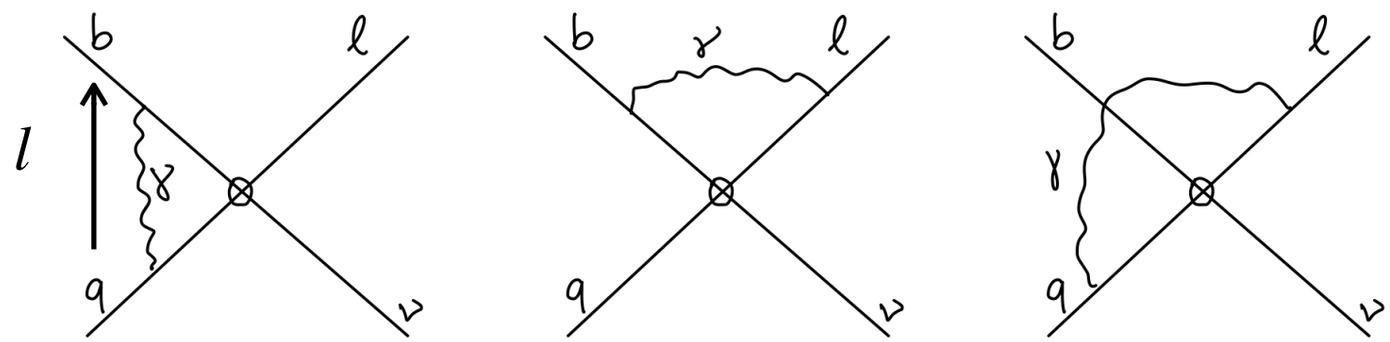
Initial state quarks are bound in the meson, with residual momenta of $\mathcal{O}(\Lambda_{\rm QCD})$:

$$p_b = m_b v + k_b\,, \quad p_q = k_q \qquad\qquad k_b, k_q \sim m_b(\lambda,\lambda,\lambda) \qquad\qquad \lambda = \frac{\Lambda_{\rm QCD}}{m_b}$$

Region analysis of virtual corrections

Compute one loop graphs in the Fermi theory,



$$\mathscr{L}_{\text{fermi}} = \frac{4G_F}{\sqrt{2}}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu)$$

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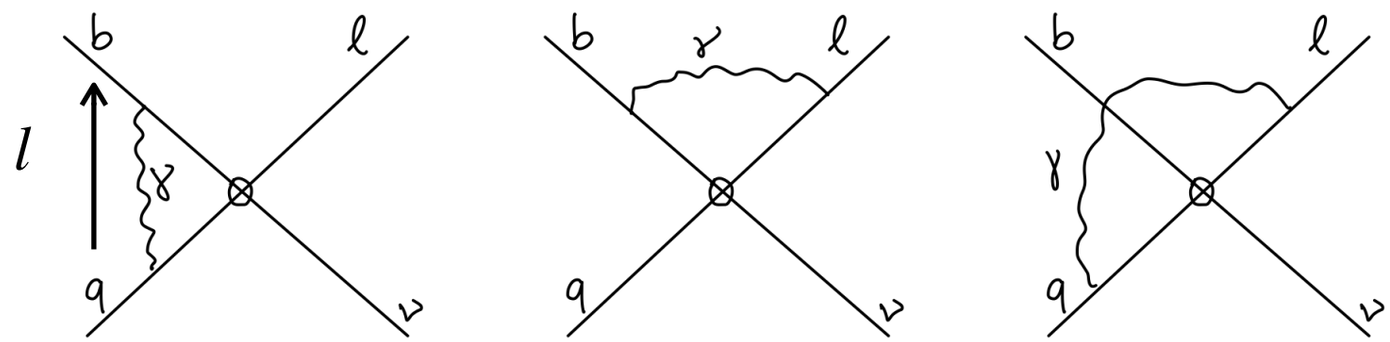

$$\mathscr{L}_{\text{fermi}} = \frac{4G_F}{\sqrt{2}}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu)$$

Find momentum **regions** responsible for singularities:

hard	$l_h^\mu \sim m_B(1,1,1)$	m_B
hard-collinear	$l_{hc}^\mu \sim m_B(\lambda,1,\sqrt{\lambda})$	$\sqrt{m_B\Lambda_{\text{QCD}}} \sim \sqrt{m_B m_\mu}$
collinear	$l_c^\mu \sim m_B(\lambda^2,1,\lambda)$	$m_\mu \sim \Lambda_{\text{QCD}}$
soft	$l_s^\mu \sim m_B(\lambda,\lambda,\lambda)$	
soft-collinear	$l_{sc}^\mu \sim m_B(\lambda^3,\lambda,\lambda^2)$	$\frac{m_\mu \Lambda_{\text{QCD}}}{m_B}$

$\mu \sim m_b$: from the Fermi Theory to HQET $\otimes$ SCET 1

Below the hard scale, radiation is too soft to affect the b momentum.
The appropriate description is **HQET**:

$$b(x) \to e^{-im_b v\cdot x} h_v(x)$$

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Relevant momentum modes as in the last slide: hard-collinear, collinear, soft

The right EFT is **Soft Collinear Effective Theory** (of type 1): "**SCET 1**"

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The right EFT is **Soft Collinear Effective Theory** (of type 1): "**SCET 1**"

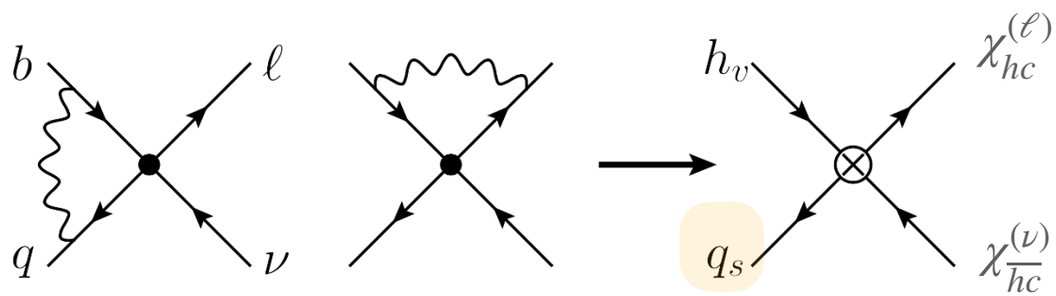
In practice: split each field in modes, integrate out the hard ones:

$$\ell = \cancel{\ell_h} + \ell_{hc} + \ell_c + \ell_s$$

Different modes are different fields, with distinct momentum scaling!

$\mu \sim m_b$: from Fermi to HQET $\otimes$ SCET 1

Loops with virtuality $p_h^2 \sim \mathcal{O}(m_b^2)$ match onto four-fermion operators in SCET 1 involving a soft spectator **("A-type"** or direct contributions)



Only one contributes at tree level:

$$O_A^{(5)} = m_\ell \frac{n^\mu}{2} \left(\bar{q}_s \gamma_\mu P_L h_v \right) \left(\bar{\chi}_c^{(\ell)} \frac{1}{in \cdot \overleftarrow{\partial}} P_L \chi_c^{(\ell)} \right)$$

$$\chi_c \sim \mathcal{O}(\lambda)$$
$$q_s, h_v \sim \mathcal{O}(\lambda^{3/2})$$

The hard matching generates more operators, e.g. $\left(\bar{q}_s \gamma_\perp^\mu P_L h_v \right) \left(\bar{\chi}_c^{(\ell)} \gamma_\perp^\mu P_L \chi_{\bar{c}}^{(\nu)} \right)$, but these have **no projection** on the B.

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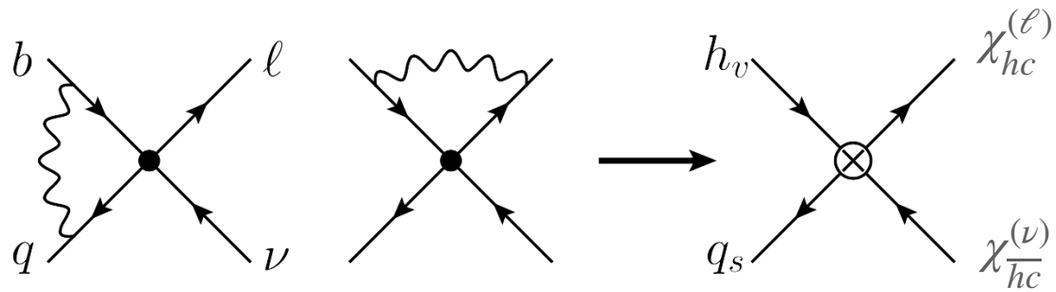

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$\mu \sim m_b$: from Fermi to HQET $\otimes$ SCET 1

To reproduce the hard-collinear region of the partonic one-loop graphs we need operators with a hard-collinear spectator (**indirect** or "**B type**")

$$\mathcal{O}_{B,1}^{(7/2)} = \left(\boxed{\bar{\chi}_{hc}^{(q)}} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} i\overleftarrow{\not{D}}_{\perp s} \frac{\not{\bar{n}}}{2}\gamma_\perp^\mu P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\gamma_{\perp\mu}P_L\chi_{\bar{C}}^{(\nu)}\right),$$

$$\mathcal{O}_{B,2}^{(7/2)} = \left(\bar{\chi}_{hc}^{(q)}\frac{\not{\bar{n}}}{2}P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} i\overleftarrow{\not{D}}_{\perp s}P_L\chi_{\overline{hc}}^{(\nu)}\right),$$

$$\mathcal{O}_{B}^{(4)} = m_\ell\left(\bar{\chi}_{hc}^{(q)}\frac{\not{\bar{n}}}{2}P_L h_v\right)\left(\bar{\chi}_{hc}^{(\ell)}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}P_L\chi_{\overline{hc}}^{\nu}\right).$$

These operators are power-enhanced with respect to the A ones, but the matrix elements need one insertion of the power-suppressed soft-collinear interactions → they contribute at the same order.

$\mu \sim \sqrt{m_b \Lambda}$: from SCET 1 to SCET 2

Below $\mu \sim \sqrt{m_b \Lambda}$ we integrate out hard-collinear modes and lower the virtuality.
Now collinear and soft modes live at the same scale:

$$p_c \sim (1, \lambda^2, \lambda)\,, \qquad p_s \sim (\lambda, \lambda, \lambda)\,, \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}\left(\lambda^2\right)$$

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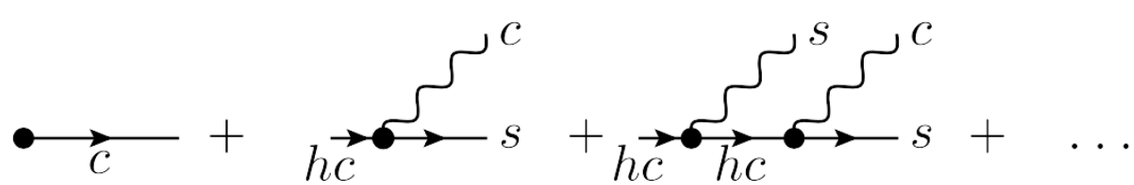
When integrating out hard-collinear modes, the intermediate propagators introduce **non-local** operators:

$$\psi_{hc} \quad \rightarrow \quad \psi_c \quad + \quad \psi_c \cdot \psi_s \quad + \quad \psi_c \cdot \psi_s^2 + \ldots$$



$$\frac{1}{n \cdot \partial} q_s, \quad \left(\frac{1}{n \cdot \partial} \mathcal{A}^{\mu}_{\perp s}\right)\left(\frac{1}{n \cdot \partial} q_s\right), \quad \ldots$$

$\Rightarrow$ contain more fields, but are of the same order!

(Not) overcoming the chiral suppression

Inverse derivative operators can **probe the meson structure**:

$$\left\langle 0 \left| \left(\frac{1}{n \cdot \partial} q_s \right) \ldots h_v \right| B \right\rangle \sim \frac{1}{\lambda_B} \sim \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^{-1} \right)$$

and possibly overcome the chiral suppression!
Happens for $B_s \to \mu\mu$, but not for $B \to \mu\bar{\nu}$. Why?

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For left-handed currents, these contributions come with **evanescent** Dirac structures:

$$\left(\bar{v} \frac{\not{n}}{2} \gamma_\perp^\mu \gamma_\perp^\nu P_L u \right)_h \left(\bar{u} \gamma_\mu^\perp \gamma_\nu^\perp \left[\frac{v - a\gamma_5}{2} \right] v \right)_\ell = 2(v-a) \left(\bar{v} \frac{\not{n}}{2} P_L u \right)_h \left(\bar{u} P_R v \right)_\ell + \mathcal{O}(\epsilon)$$

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$\Rightarrow$ structure-dependent contributions to $B \to \mu\bar{\nu}$ carry the **same suppression** as the tree level result!

Factorization formula for virtual corrections

convolution

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j H_j\, S_j\, K_j + \sum_i H_i \otimes J_j \otimes S_i \otimes K_i\,,$$

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- hard function: matching corrections at $\mu \sim m_b$

Factorization formula for virtual corrections

convolution

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j H_j\, S_j\, K_j + \sum_i H_i \otimes J_j \otimes S_i \otimes K_i\,,$$

- hard function: matching corrections at $\mu \sim m_b$
- hard-collinear function: matching corrections at $\mu \sim (m_b \Lambda_{\text{QCD}})^{1/2}$

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SCET I operators with soft spectator (A-type)

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Factorization formula for virtual corrections

convolution

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Factorization formula for virtual corrections

Neglecting $\mathcal{O}(\alpha\alpha_s)$ corrections, two main contributions:

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$
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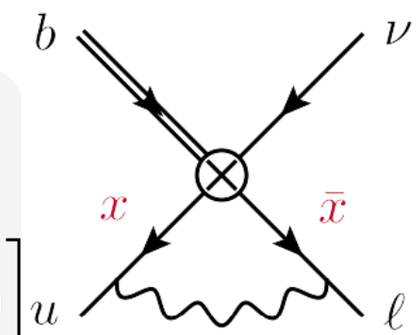

with $\quad O_A = \bar{n}_\mu\, \bar{u}_s \gamma^\mu P_L h_v\, S^\dagger_{\nu_\ell}$

$$O_B(\omega) = \bar{n}_\mu \int \frac{dt}{2\pi} e^{i\omega t} \bar{u}_s(tn)[tn,0]\gamma^\mu P_L h_v(0)\, S^\dagger_{\nu_\ell}(0) \qquad \omega = n \cdot p_u$$

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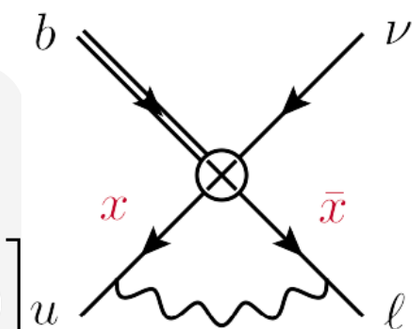

(collinear) momentum fraction carried by the virtual hc spectator

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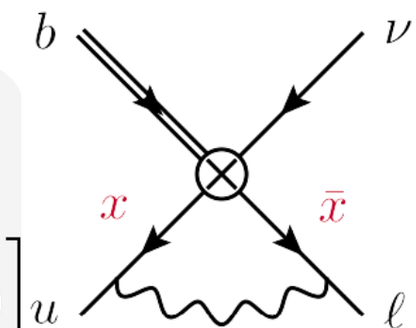

$\sim x^{-\epsilon}$ $\sim x^{-1-\epsilon}$

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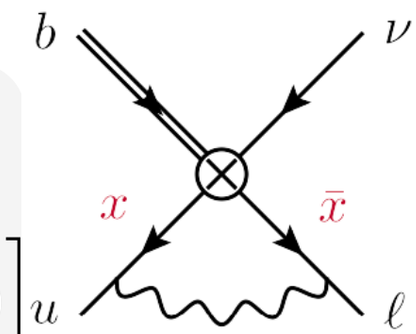

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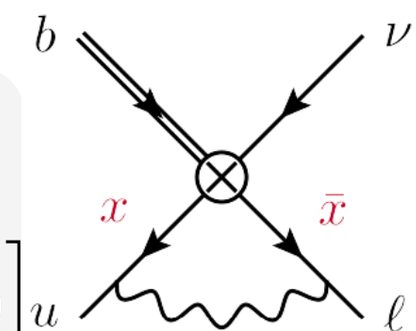

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- endpoint divergences → poles that <u>cannot</u> be removed with standard RG techniques.
- standard problem of factorization beyond leading power, systematically treatable with refactorization-based subtraction (RBS) scheme [Liu, Neubert 1912.08818, Liu, Mecaj, Neubert, Wang 2009.04456]

Factorization formula for virtual corrections

In practice:

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$$\int_0^1 dx\, \left[H_B(m_b, x) J_B(m_b\omega, x) - \theta(\lambda - x)[[H_B(m_b, x)]][[J_B(m_b\omega, x)]] \right]$$

$0 < \lambda < 1$

$[[f]]$ = singular part of f for $x \to 0$

- remove the divergence from $H_B \otimes J_B$ with a plus subtraction

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- remove the divergence from $H_B \otimes J_B$ with a plus subtraction
- add it back, combining it with the other term in the factorization formula
- define renormalized **decay constant**:

$$S_A^{(\lambda)} = \langle 0 | O_A^{(\lambda)} | B^-(v) \rangle = -\frac{i\sqrt{m_B}}{2} F(\mu, \lambda m_b, v \cdot v_\ell)$$

Factorization formula for virtual corrections

$\mathcal{O}(\alpha)$ result (for $\lambda = 1$):

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = i\sqrt{2}G_F K_{\text{EW}}(\mu)V_{ub}\,\frac{m_\ell}{m_b}\sqrt{m_B}\,F(\mu, m_b, v\cdot v_\ell)\cdot\bar{u}(p_\ell)P_L v(p_\nu)\sum_j \mathcal{M}_j(\mu,\lambda,v\cdot v_\ell)$$

$$\begin{aligned}\mathcal{M}_{2-\text{part.}}(\mu) = 1 + \frac{\alpha}{4\pi}\Bigg\{ & Q_b^2\left[-\frac{3}{2}\ln\frac{\mu^2}{m_b^2} - 2\right] + Q_\ell(Q_\ell - Q_b)\int_0^\infty d\omega\,\phi_-(\omega)\left[2\ln\frac{\mu^2}{m_b\omega} + \frac{\pi^2}{3} + 3\right] \\ & + Q_b Q_\ell\left[-\frac{1}{2}\ln^2\frac{\mu^2}{m_b^2} + 2\ln\frac{\mu^2}{m_b^2} - 3\ln\frac{\mu^2}{m_\ell^2} - \frac{5\pi^2}{12} - 1\right] + Q_\ell^2\,K_\epsilon(\mu, m_\ell, v\cdot v_\ell)\Bigg\} \\ & + \frac{C_F\alpha_s(\mu)}{4\pi}\left[-\frac{3}{2}\ln\frac{\mu^2}{m_b^2} - 2\right],\end{aligned}$$

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→ **QED corrections suffer from *relevant* hadronic uncertainties**! (*to be quantified)

$\mu \sim \Lambda_{\rm QCD} \sim m_\mu$: quarks become hadrons, the muon gets heavy

Below $\mu \sim \Lambda_{\rm QCD}$ quarks hadronize can move to an effective description with a **Yukawa theory**, with the meson playing the role of a heavy scalar.

$$\mathcal{L}_{\rm y} = y\, e^{-i m_B (v\cdot x)} \varphi_B \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) + {\rm h.c.}$$

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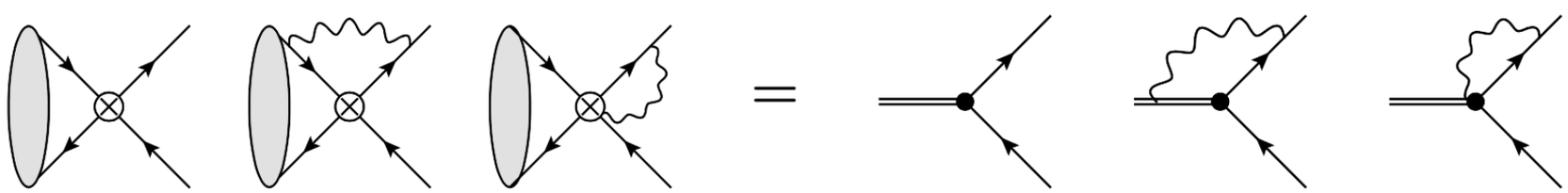
The Yukawa coupling is fixed by matching hadronic matrix elements between this and the previous description:

$$\langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II \otimes HQET} \,|\, B \rangle \quad = \quad \langle \ell\, \nu \,|\, \mathcal{L}_{\rm SCET\,II \otimes HSET} \,|\, B \rangle$$



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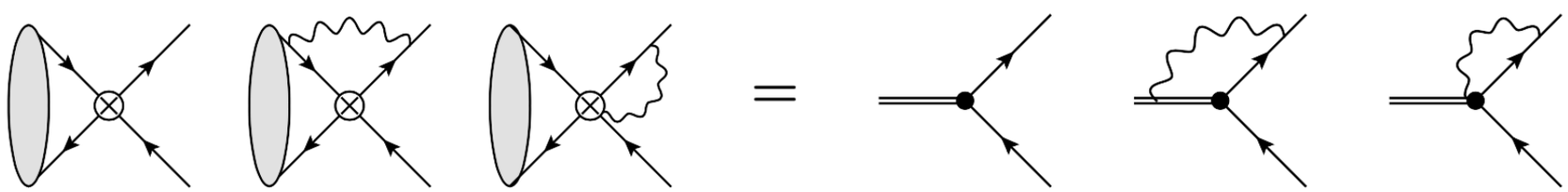

Since $\Lambda_{\rm QCD} \sim m_\mu$, we can integrate out the **muon** in the same matching step.
Below m_μ, the muon looks infinitely heavy to the remaining radiation, and we can describe it with a HQET-like field.

Region analysis of the real corrections

To understand the d.o.f. of the low-E theory, do a region analysis of the real corrections.

Which photons can survive the cut?

- need at least one $\sim E_s$ component to probe the cut
- either all components are of this order, or other components are smaller
- the largest component (if existing) should point in the collinear direction

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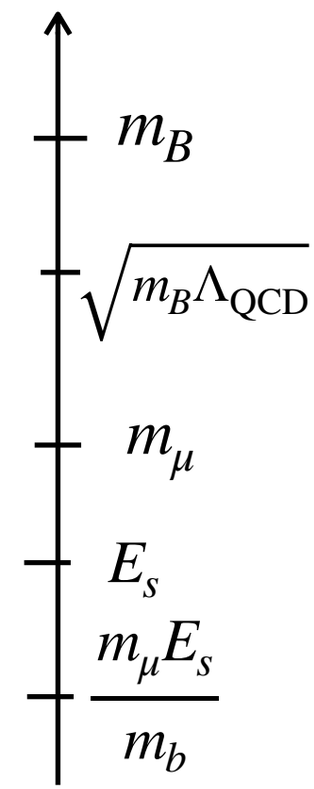

Two options:

$p_{us} \sim m_B(\lambda^2, \lambda^2, \lambda^2)$ $\quad p_{us}^2 \sim \lambda^4$ ultrasoft

$p_{sc} \sim m_B(\lambda^4, \lambda^2, \lambda^3)$ $\quad p_{sc}^2 \sim \lambda^6$ (ultra) soft-collinear

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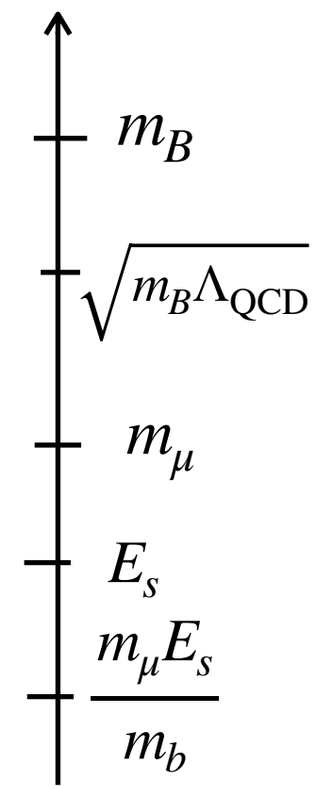

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$p_{us} \sim m_B(\lambda^2, \lambda^2, \lambda^2)$ $\quad p_{us}^2 \sim \lambda^4$ ultrasoft

$p_{sc} \sim m_B(\lambda^4, \lambda^2, \lambda^3)$ $\quad p_{sc}^2 \sim \lambda^6$ (ultra) soft-collinear

What is this soft-collinear mode?

In lepton rest frame: $p'_{sc} \sim m_\ell(\lambda^2, \lambda^2, \lambda^2)$

⇢ It is just an ultrasoft photon to the muon, just as the other mode is to the B!

⇢ muon needs to be described by a "**boosted HQET**" construction, "bHLET"

Understanding the low-energy theory

→ our low energy theory is a

heavy scalar effective theory $\otimes$ boosted heavy lepton effective theory

$$\Phi_B(x) = e^{-im_B\,(v\cdot x)}\varphi_B \qquad\qquad \ell(x) = e^{-im_\ell\,(v_\ell\cdot x)}\chi_{v_\ell}(x)$$

Interactions of the B and muon with **ultrasoft and soft-collinear photons** can be moved into Wilson lines

$$Y_v^{(\mathrm{s})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v\cdot A_s(x+sv)\right\}$$
$$Y_v^{(\mathrm{sc})}(x) = \mathcal{P}\exp\left\{ie\int_{-\infty}^{0} ds\, v\cdot A_{\mathrm{sc}}(x+sv)\right\}$$

and **decoupled** from them via field redefinitions.

Understanding the low-energy theory

Real corrections are matrix elements of these Wilson lines:

$$W_s(\omega_s,\mu) = \left[\sum_{n_s=0}^{\infty}\prod_{i=1}^{n_s}\int d\Pi_i(q_i)\right]\left|\langle n_s\gamma_s(q_i)|\, Y_v^{(\mathrm{s})}Y_n^{(\mathrm{s})\dagger}\,|0\rangle\right|^2\delta\left(\omega_s - q_0^{(\mathrm{s})}\right)\,,$$

$$W_{sc}(\omega_{sc},\mu) = \left[\sum_{n_{sc}=0}^{\infty}\prod_{j=1}^{n_s}\int d\Pi_j(q_j)\right]\left|\langle n_{sc}\gamma_{sc}(q_j)|\, Y_{\bar{n}}^{(\mathrm{sc})\dagger}Y_{v_l}^{(\mathrm{sc})}\,|0\rangle\right|^2\delta\left(\omega_{sc} - q_0^{(\mathrm{sc})}\right)$$

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The convolution with a measurement function containing the experimental cut yields the radiative function of the process:

$$S(E_s,\mu) = \int_0^\infty d\omega_s\int_0^\infty d\omega_{sc}\,\theta\left(\frac{E_s}{2}-\omega_s-\omega_{sc}\right)W_s(\omega_s,\mu)W_{sc}(\omega_{sc},\mu)$$

Integration and renormalisation of the bare functions can be carried out in Laplace space and allow to resum soft and soft-collinear logarithms.

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Full factorization formula:

$$\Gamma = \underbrace{|\mathcal{A}^{\mathrm{virtual}}|^2}_{\text{non-radiative}}\otimes\underbrace{W_{us}(\mu)\otimes W_{usc}(\mu)}_{\text{radiative}}$$

Conclusions

- We derived a factorization formula for QED corrections to $B \to \mu\nu$, which **separates all scales** in the process and allows for the resummation of the associated logarithms.

  We used a multi-step matching procedure involving a combination of **HQET**, **SCET 1 & 2,** and **boosted HLET**. The derivation can be systematically extended to higher orders in the coupling constants and power counting.

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- Channels with **other lepton flavors** cannot be obtained simply by replacing the muon mass: they have different scale hierarchies, matching threshold, EFT constructions….