Renormalon subtraction in $|V_{cb}|$ determination

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Collab. with Hayashi, Mishima, Takaura

☆Plan of Talk

1. Introduction

   OPE, Mass schemes, Renormalons

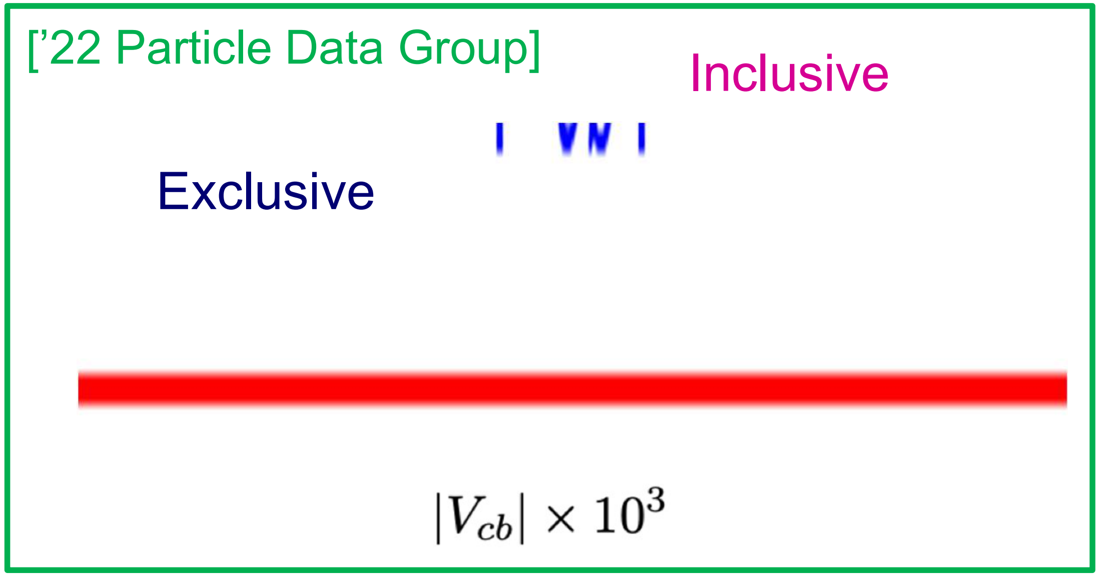
2. $|V_{cb}|$ determination in 1S mass scheme

3. $|V_{cb}|$ determination in $\overline{\mathrm{MS}}$ mass scheme

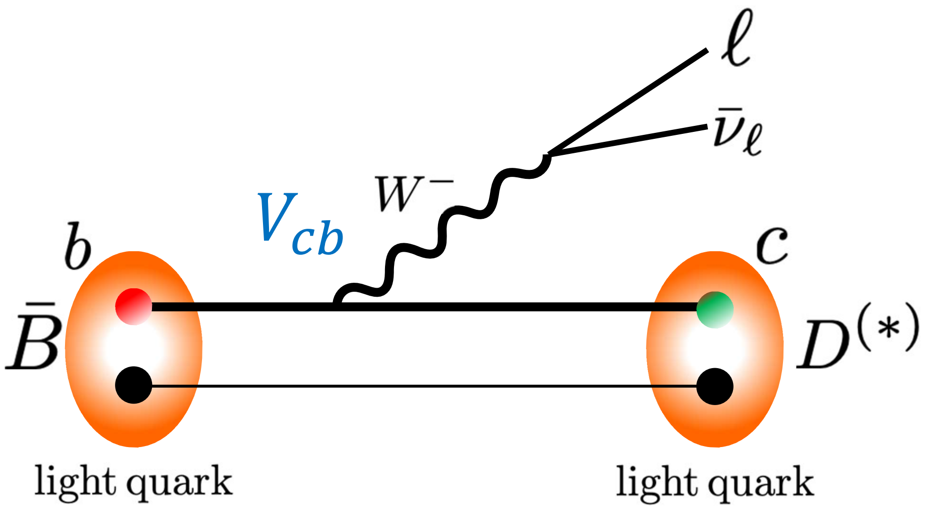
4. Summary & Conclusions

Current status of $|V_{cb}|$ determination and Our goal

- Inconsistency in $|V_{cb}|$ determinations



$\sim 2.4\sigma$ tension



Our present goal

Precise QCD calculation of
inclusive decay width
$\Gamma(B \to X_c \ell \bar{\nu}) \Rightarrow |V_{cb}|$

OPE of Inclusive Semileptonic B Decay Width

OPE in HQET: $1/m_b$-expansion

$$\Gamma_{\rm th} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{\rm short})^5 A_{EW} \left[C_{\bar{Q}Q} + C_{\rm kin} \frac{\mu_\pi^2}{(m_b^{\rm short})^2} + C_{\rm cm} \frac{\mu_G^2}{(m_b^{\rm short})^2} + \ldots \right]$$

$$C_i = c_{i,0} + c_{i,1}\, \alpha_s(m_b) + c_{i,2}\, \alpha_s(m_b)^2 + c_{i,3}\, \alpha_s(m_b)^3 + \cdots$$

$C_{\bar{Q}Q,3}$: Fael, Schoniwald, Steinhauser

$$\mu_\pi^2 = \frac{\langle B | \bar{b}_v D_\perp^2 b_v | B \rangle}{2m_B} \sim (\text{NR mom. of } b \text{ quark})^2 \sim \Lambda_{\rm QCD}^2$$

$$\mu_G^2 = \frac{\langle B | \bar{b}_v \frac{g}{2} \sigma^{\mu\nu} G_{\mu\nu} b_v | B \rangle}{2m_B} \sim (\text{spin-magnetic energy}) \sim \Lambda_{\rm QCD}^2$$

Double expansion in $\alpha_s(m_b)$ and $\Lambda_{\rm QCD}/m_b$

Mass schemes

$$\Gamma_{\rm th} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{\rm short})^5 A_{EW} \left[C_{\bar{Q}Q} + C_{\rm kin} \frac{\mu_\pi^2}{(m_b^{\rm short})^2} + C_{\rm cm} \frac{\mu_G^2}{(m_b^{\rm short})^2} + \ldots \right]$$

m_b : bottom on-shell (pole) mass   subject to IR instability

Well-defined mass [short-distance mass] should be used.

- $\overline{\rm MS}$ mass — Bardeen, Buras, Duke, Muta

Popular short-distance mass but it leads to slow perturbative convergence

- Kinetic mass — Bigi, Shifman, Uraltsev, Vainshtein

Can avoid renormalon problem by the introduction of factorization scale

- 1S mass — Hoang, Ligeti, Manohar

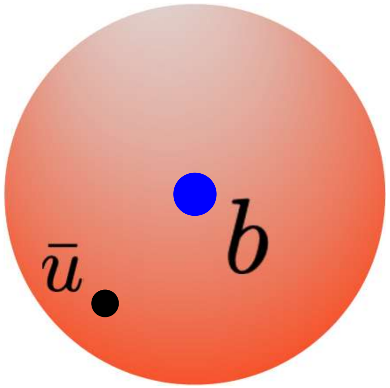
Physical mass defined as [Bottomonium(1S) mass]$_{\rm pert}$/2

⋮

The results of $|V_{cb}|$ determination should not depend on mass schemes at sufficiently high orders (if they converge).

Expected properties of $\Gamma(B \to X_c \ell \bar{\nu})$

Mass scheme	Convergence	Fac. scale μ_{fac}	Renormalons	
			$O(\Lambda_{\mathrm{QCD}})$	$O(\Lambda_{\mathrm{QCD}}^2)$
Kinetic mass	fast	yes	no	no
1S mass	fast(?)	no	no	yes
$\overline{\mathrm{MS}}$ mass $\bar{m}_b \equiv m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$	slow	no	no	yes

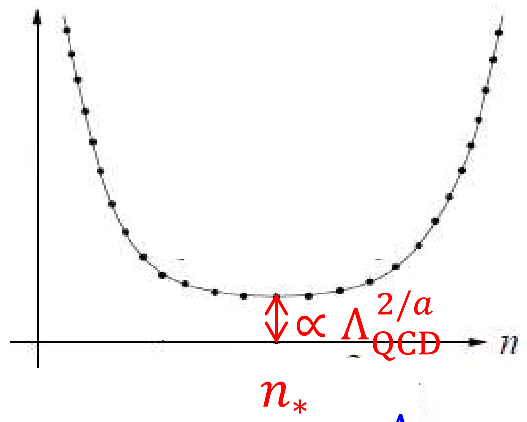


Renormalon uncertainty

't Hooft (See review by Beneke)

$$A = \sum_n c_n \alpha_s^n \quad ; \quad c_n \sim n!\, a^n$$

$c_n\, \alpha_s^n \sim n!\, a^n\, \alpha_s^n$



Asymptotic series
Limited accuracy

$\Lambda_{QCD} \sim 300$ MeV

$$\Gamma_{th} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{short})^5 A_{EW} \left[C_{\bar{Q}Q} + C_{kin} \frac{\mu_\pi^2}{(m_b^{short})^2} + C_{cm} \frac{\mu_G^2}{(m_b^{short})^2} + \ldots \right]$$

$$C_{\bar{Q}Q} = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \ldots + c_{n_*}\alpha_s^{n_*} + \ldots$$

$$c_{n_*}\alpha_s^{n_*} \sim \frac{\Lambda_{QCD}^2}{(m_b^{short})^2}$$

$$\frac{\mu_\pi^2}{(m_b^{short})^2} \sim \frac{\Lambda_{QCD}^2}{(m_b^{short})^2}$$

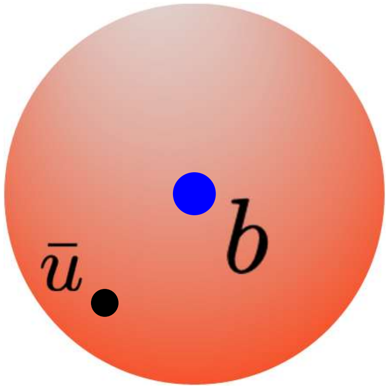
same order

High order effect in UV computation and IR nonperturbative effect cannot be distinguished. (Renormalon problem)

The results of $|V_{cb}|$ determination should not depend on mass schemes at sufficiently high orders (if they converge).

Expected properties of $\Gamma(B \to X_c \ell \bar{\nu})$

Mass scheme	Convergence	Fac. scale $\mu_{\rm fac}$	Renormalons $O(\Lambda_{\rm QCD})$	Renormalons $O(\Lambda_{\rm QCD}^2)$	
Kinetic mass	fast	yes	no	no	Bordone, et al. Bernlochner, et al.
1S mass	fast(?)	no	no	yes	Hayashi, et al.
$\overline{\rm MS}$ mass	slow $\Rightarrow$ improve?	no	no	~~yes~~ subtract	Hayashi, et al. (in preparation)



2. $|V_{cb}|$ determination in 1S mass scheme

1S mass scheme: Importance of nonperturbative effects

$$\Gamma_{\rm th} = \frac{G_F^2|V_{cb}|^2}{192\pi^3}(m_b^{\rm short})^5 A_{EW}\left[C_{\bar{Q}Q} + C_{\rm cm}\frac{\mu_G^2}{(m_b^{\rm short})^2} + (C_{\rm kin}\frac{\mu_\pi^2}{(m_b^{\rm short})^2})\right]$$

PT: LO NLO NNLO NNNLO

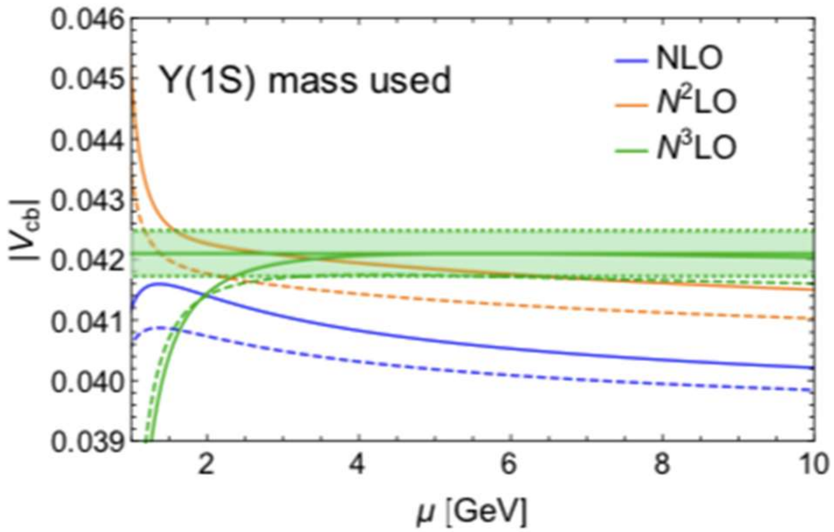
$$= \frac{G_F^2|V_{cb}|^2}{192\pi^3}(m_b^{\rm short})^5 A_{EW}\Big[0.5903 - 0.0836 - 0.0281 - 0.007$$

NP: μ_G^2 μ_π^2

$$-0.0151 + (0.005)\Big]$$

Nonperturbative effects are of the same order of magnitude as the current highest order PT contribution.

Determination of $|V_{cb}|$ using 1S mass

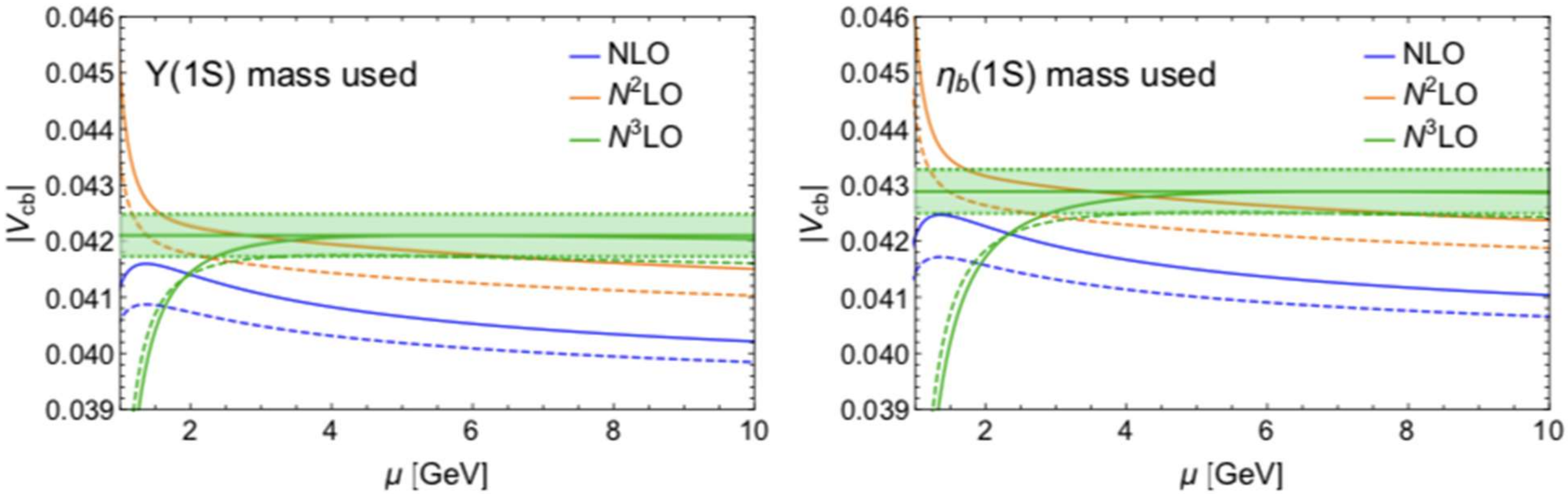


Result using Υ(1S) mass

$$|V_{cb}| = 42.1(4)_{\rm pert}(1)_{\alpha_s}(4)_{\bar{m}_c}(0)_{\mu_G^2}(0)_{m_{\Upsilon(1S)}}(3)_{\rm Br}(1)_{\tau_B}(1)_{\rm h.o.C_{cm}}(2)_{\mu_\pi^2} \times 10^{-3}$$

$C_{\bar{Q}Q,2}^{(1S)}$, $C_{\bar{Q}Q,3}^{(1S)}$: dep. on bottomonium 1S spin

Determination of $|V_{cb}|$ using 1S mass



Result using $\Upsilon(1S)$ mass

$$|V_{cb}| = 42.1(4)_{\mathrm{pert}}(1)_{\alpha_s}(4)_{\bar{m}_c}(0)_{\mu_G^2}(0)_{m_{\Upsilon(1S)}}(3)_{\mathrm{Br}}(1)_{\tau_B}(1)_{\mathrm{h.o.C_{cm}}}(2)_{\mu_\pi^2} \times 10^{-3}$$

Result using $\eta_b(1S)$ mass

$$|V_{cb}| = 42.9(4)_{\mathrm{pert}}(1)_{\alpha_s}(4)_{\bar{m}_c}(0)_{\mu_G^2}(0)_{m_{\Upsilon(1S)}}(3)_{\mathrm{Br}}(1)_{\tau_B}(1)_{\mathrm{h.o.C_{cm}}}(2)_{\mu_\pi^2} \times 10^{-3}$$

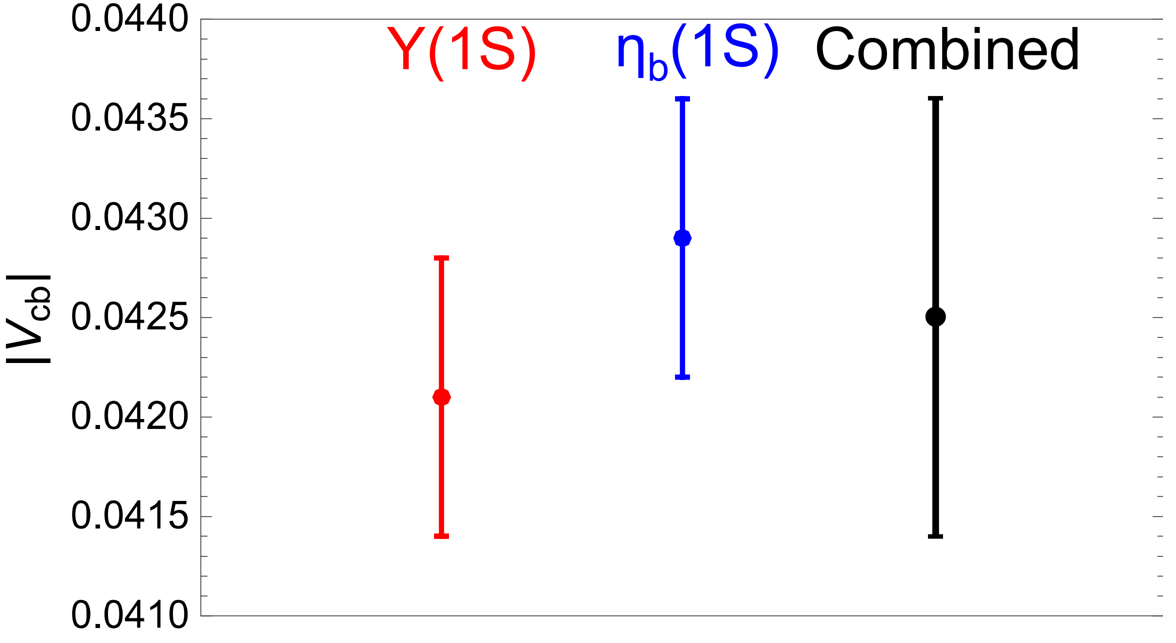
Combined result

$\Upsilon(1S)$ mass: $|V_{cb}| = (42.1 \pm 0.7) \times 10^{-3}$

$\eta_b(1S)$ mass: $|V_{cb}| = (42.9 \pm 0.7) \times 10^{-3}$

Combined: $|V_{cb}| = (42.5 \pm 0.7 \pm \overset{\nearrow 0.4_{\text{pert}}}{0.8}_{\text{spin dependence}}) \times 10^{-3}$

$$= \boxed{(42.5 \pm 1.1) \times 10^{-3}}$$



3. $|V_{cb}|$ determination in $\overline{\mathrm{MS}}$ mass scheme

$\overline{\mathrm{MS}}$ mass scheme + $O(\Lambda_{\mathrm{QCD}}^2)$ renormalon subtraction

Our method for renormalon subtraction “Dual space approach”:

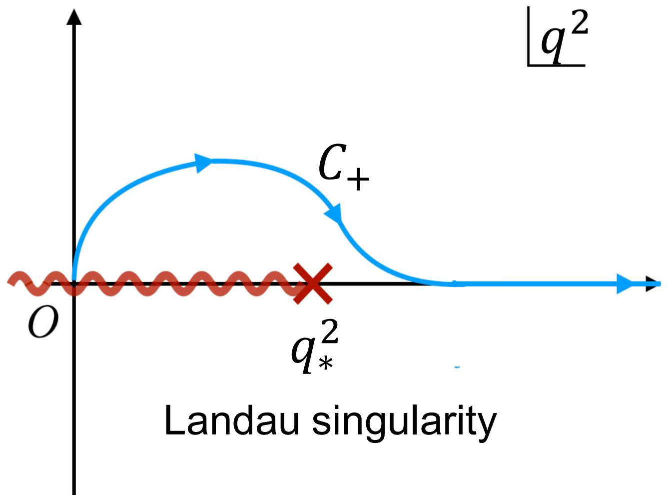
$$C_{\bar{Q}Q} \sim \mathrm{Re}\, \frac{1}{\bar{m}_b^2} \int_{C_+} dq^2 \; e^{-q^2/\bar{m}_b^2} \sum_n \tilde{c}_n\, \alpha_s(q^2)^n$$

renormalons suppressed

- Improved version of FTRS method.

  c.f. Hayashi, YS, Takaura

- Equivalent to the conventional renormalon subtraction scheme: “PV scheme.” (⇐ Borel resummation)



2D Non-linear σ model

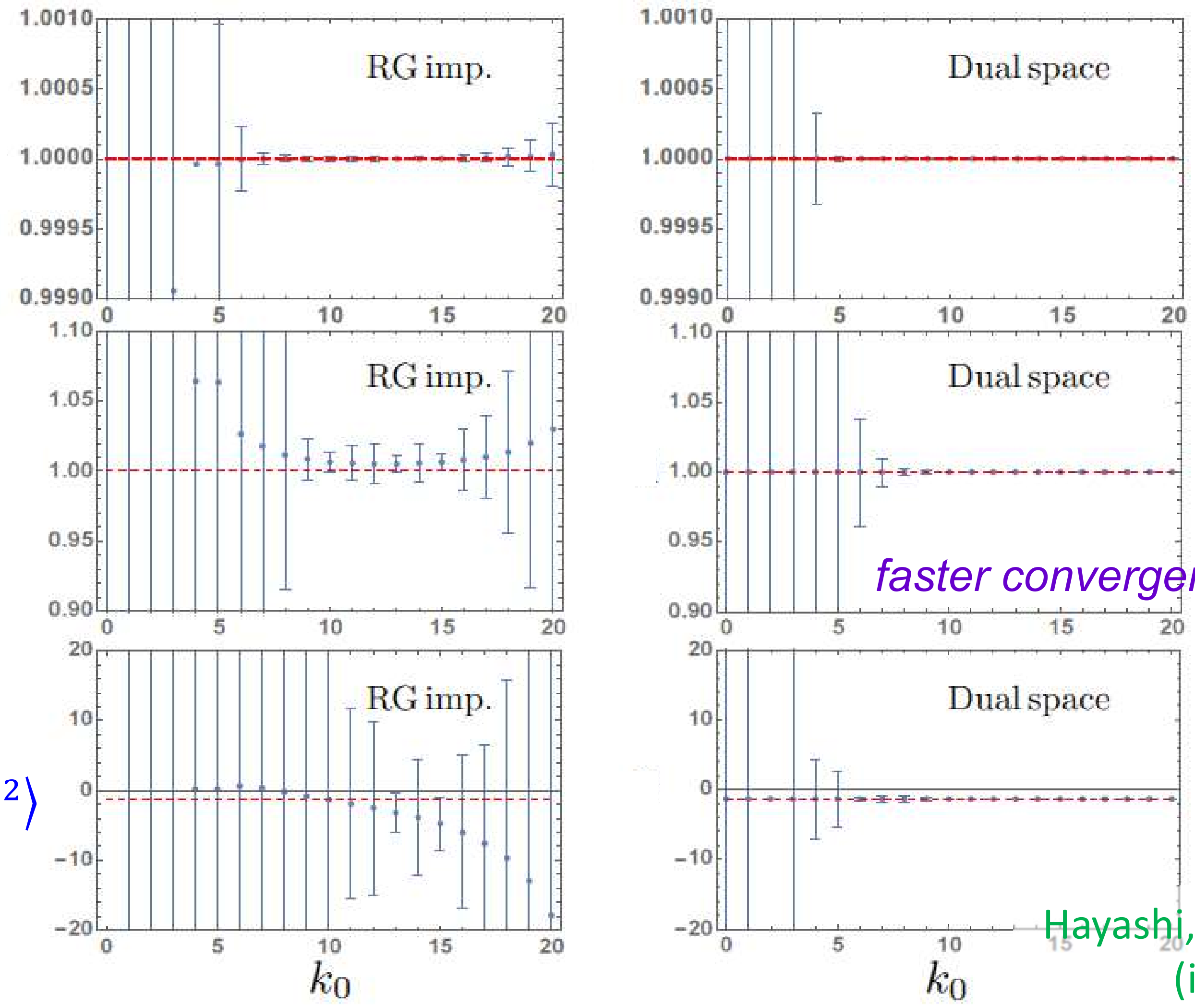
Simulation: Extracting non-pert. parameters by a fit to exact $A(Q)$ (experimental data)

$$A(Q)_{\mathrm{OPE}} = C_{\mathbf{1}}(Q)\,\langle \mathbf{1} \rangle + C_{\alpha}(Q)\,\frac{\langle \alpha \rangle}{Q^2} + C_{\alpha^2}(Q)\,\frac{\langle \alpha \rangle^2}{Q^4} + C_{\delta\alpha^2}(Q)\,\frac{\langle (\delta\alpha)^2 \rangle}{Q^4} + \cdots$$

Scale unit Λ

$\langle \alpha \rangle$

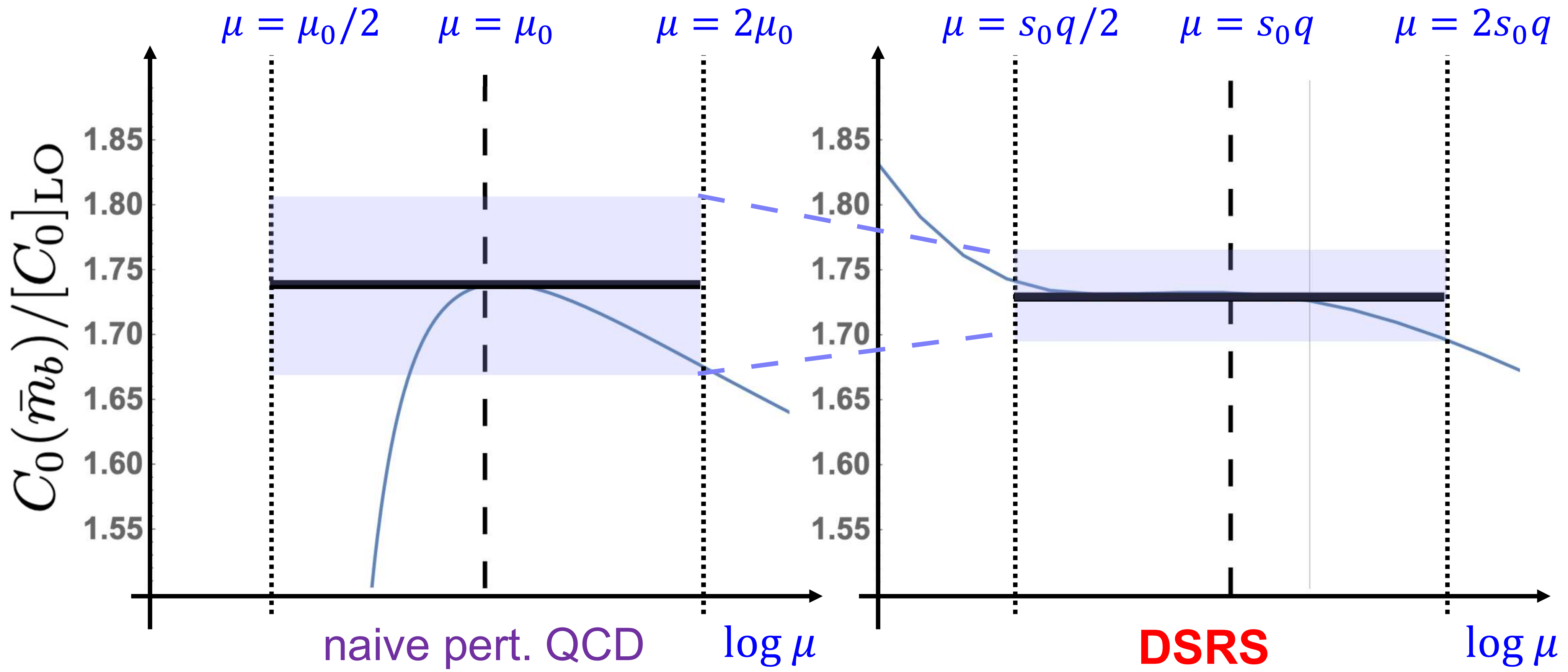
$\langle (\delta\alpha)^2 \rangle$



faster convergence

Hayashi, Mishima, YS, Takaura (in preparation)

Comparison with naive pert. QCD
(without $O(\Lambda_{\rm QCD}^2)$ renormalon subt.)



$[C_0(\bar{m}_b)]_{\rm PT}/[C_0]_{\rm LO} = 1.739\,(70)$ $\quad$ ${\rm Re}\,[C_0(\bar{m}_b)]/[C_0]_{\rm LO} = 1.733\,(35)$

pert. uncertainty reduced

$|V_{cb}|$ determination using $\overline{\mathrm{MS}}$ mass

$$|V_{cb}| = \sqrt{\frac{\Gamma_{\rm exp}}{\Gamma_0 m_b^5 \left[\mathrm{Re}\,[C_0]\left(1 - \frac{\mathrm{Re}\,\mu_\pi^2}{2m_b^2}\right) + \frac{\mu_G^2}{m_b^2}\right]}}$$

$$\Rightarrow \boxed{|V_{cb}| = 0.04147(43)_{\rm PT}(^{+87}_{-98})_{\rm sys}}$$

NNNLO

Preliminary

- Perturbative uncertainty for DSRS ~1% < naive pQCD ~2%

  $\ni$ Large off-shellness, ~~**Renormalons**~~,… $\vee$

  non-pert contr. μ_π^2, μ_G^2 ~ 0.2-0.5%

Large systematic uncertainty

Preliminary

$$|V_{cb}| = 0.04147\,(43)_{\rm PT}\,\left({}^{+87}_{-98}\right)_{\rm sys}$$

from $\bar{m}_b,\ \bar{m}_c,\ \alpha_s,\ \Gamma_{\rm exp},\ {\rm Re}\,\mu_\pi^2,\ \mu_G^2,\cdots$

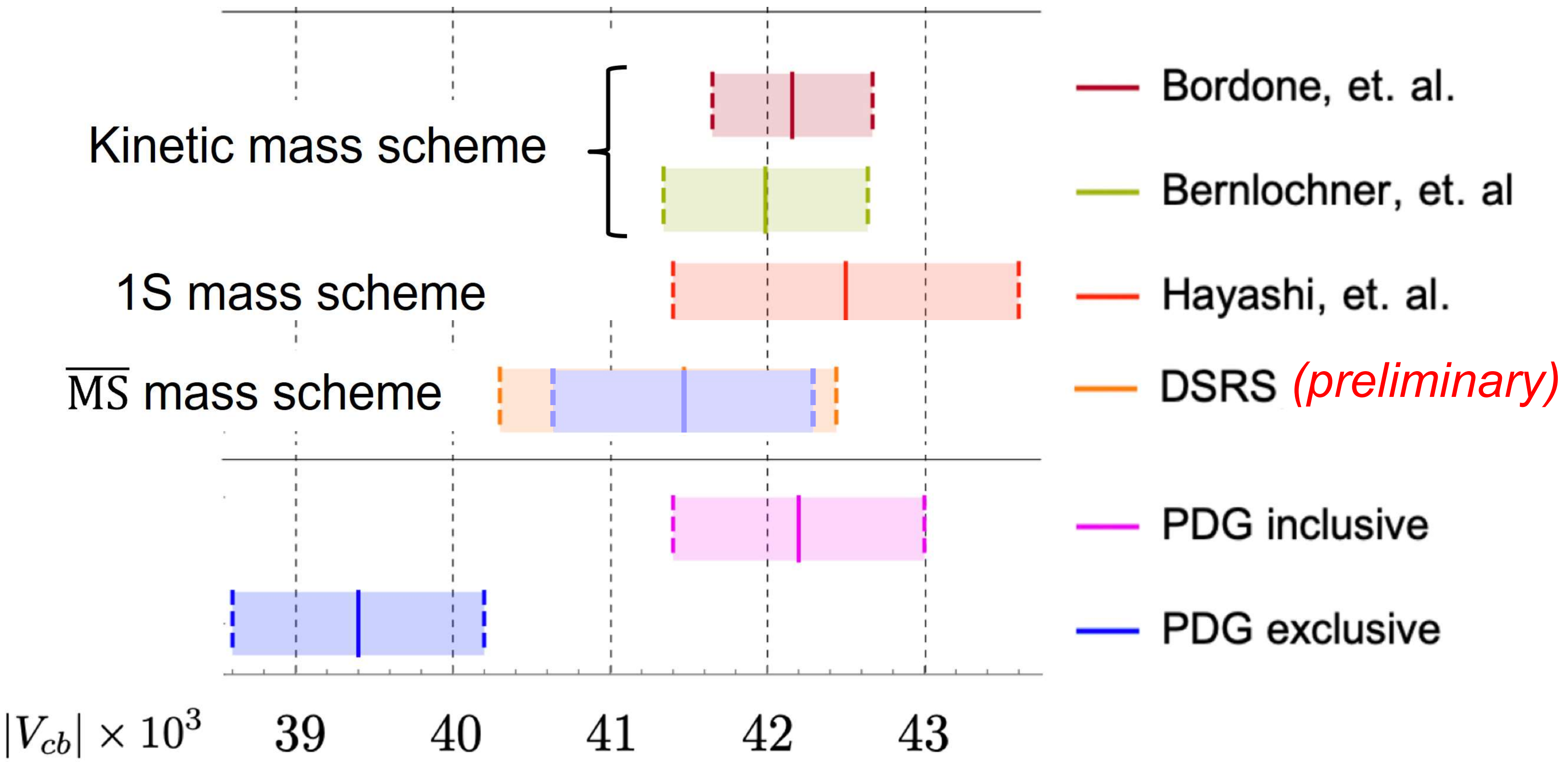
$$|V_{cb}| \propto \bar{m}_b^{-5/2}$$

$$\bar{m}_b = 4.18^{+0.03}_{-0.02}\ {\rm GeV} \to |V_{cb}| = 0.04147\,\left({}^{+61}_{-89}\right)_{\bar{m}_b}$$

['22 Particle Data Group]

More precise value of $\bar{m}_b$ desired.

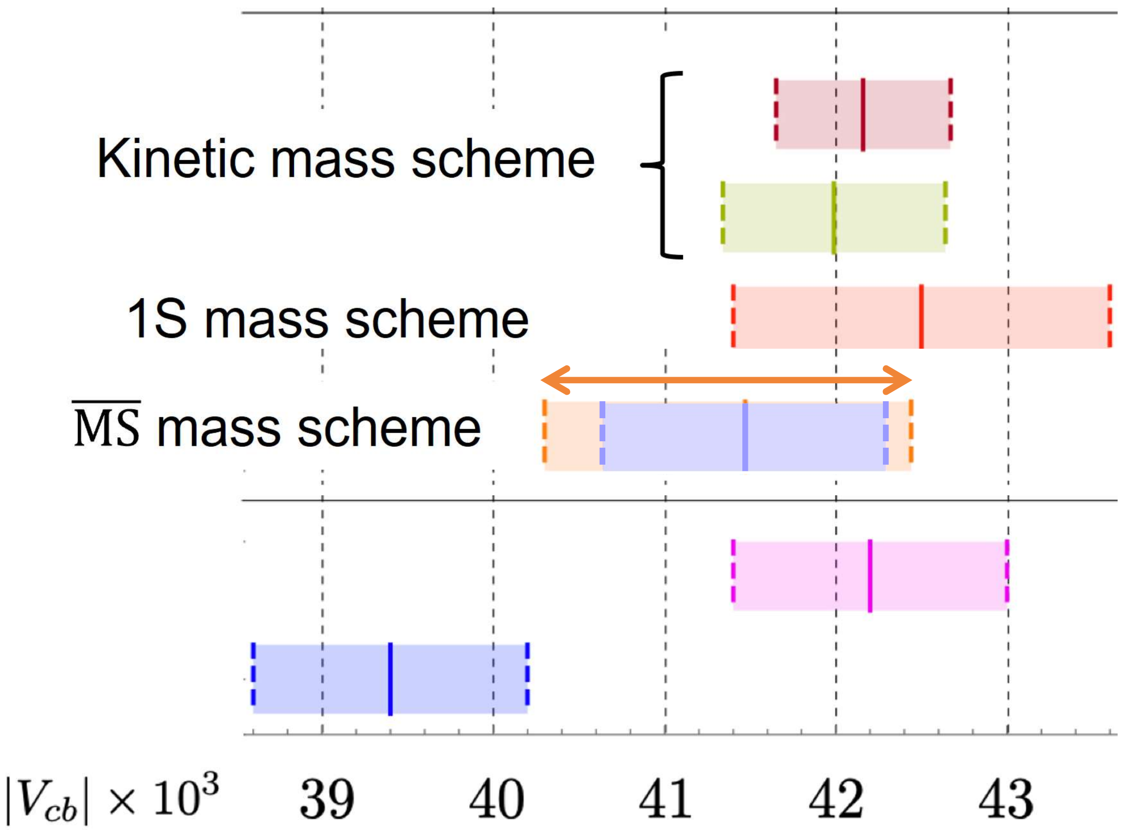
Comparison of $|V_{cb}|$ determinations



Agree with previous inclusive determinations ※ tension with exclusive

Precise determination of $\overline{m}_b$ is important

Input quark mass dependence



Their input uncertainty

$$\delta \bar{m}_b = 12\ \mathrm{MeV}$$

['19 FLAG]

Our input uncertainty

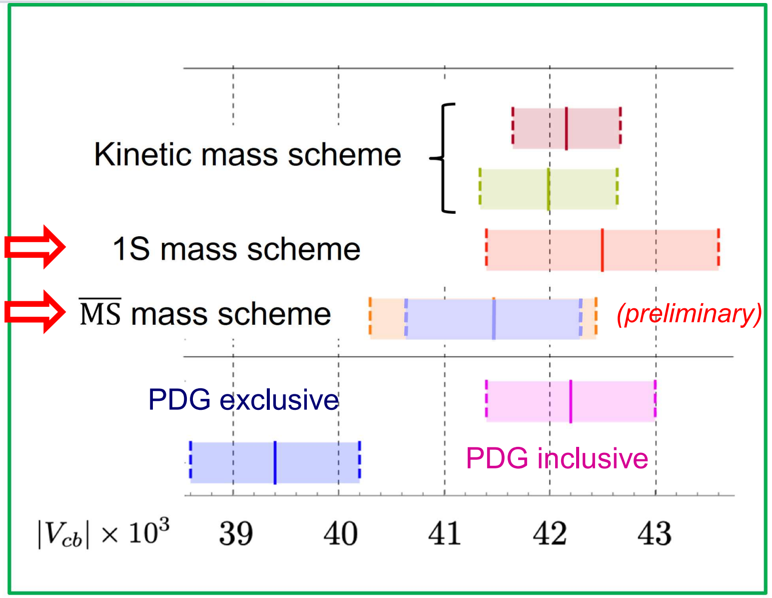
$$\delta \bar{m}_b = 30\ \mathrm{MeV}$$

(Conservative) ['22 PDG]

Agree with previous inclusive determinations ※ tension with exclusive

Precise determination of $\bar{m}_b$ is important

Summary & Conclusion



We determined $|V_{cb}|$ using $\Gamma(B \to X_c \ell \bar{\nu})$

- **1S mass scheme** ($O(\Lambda^2_{\rm QCD})$ renormalon)

$$|V_{cb}| = (42.5 \pm 1.1) \times 10^{-3}$$

Nonperturbative contr. μ_π^2, μ_G^2 ~ pert. error.

Large difference between $|V_{cb}|$ from $\Upsilon(1S)$ and $\eta_b(1S)$. (pert. or nonpert.?)

- **$\overline{\text{MS}}$ mass scheme** ($O(\Lambda^2_{\rm QCD})$ renormalon subtracted)

$$|V_{cb}| = (41.5{\pm}^{1.0}_{1.2}) \times 10^{-3}$$ *(preliminary)*

Accuracy of pert. QCD prediction improves by renormalon subtraction.

Large error from $\delta\overline{m}_b$. Precise $\overline{m}_b$ is important.

$|V_{cb}| \propto m_b^{-5/2}$

We confirm theoretical calculation of inclusive $|V_{cb}|$.

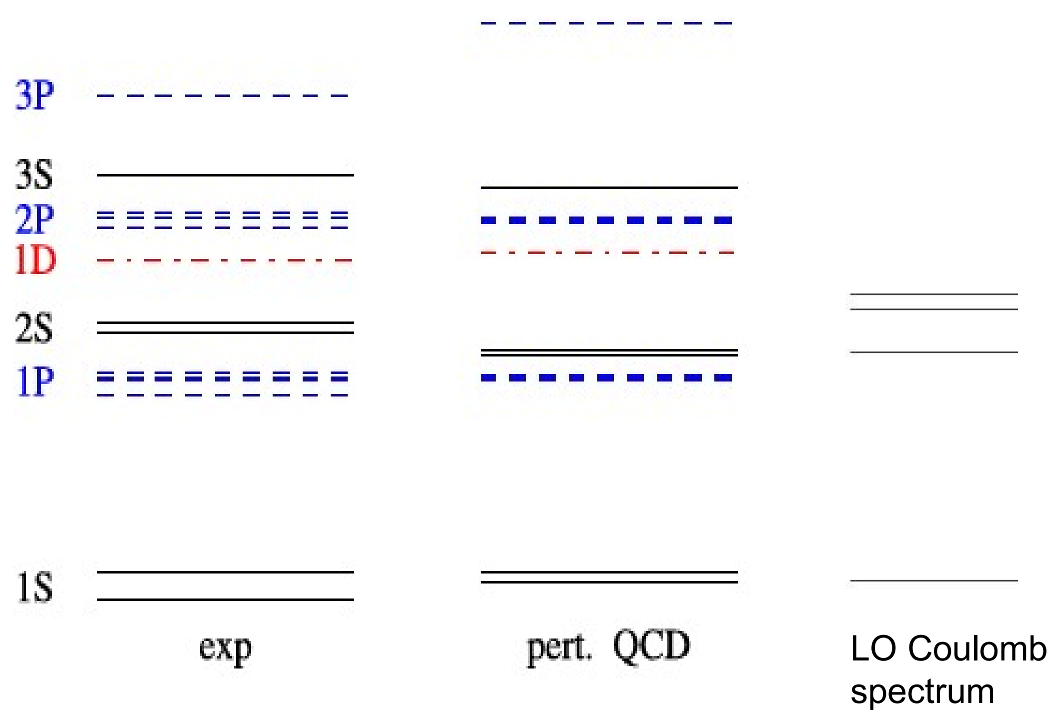
$\overline{\mathrm{MS}}$ mass scheme ($O(\Lambda^2_{\mathrm{QCD}})$ renormalon subtracted)

$$|V_{cb}| = 0.04147\,(43)_{\mathrm{PT}}\,({}^{+61}_{-89})_{\overline{m}_b}\,(43)_{\overline{m}_c}\,(23)_{\alpha_s}\,(38)_{\mathcal{B}}\,(5)_{\tau_B}\,(10)_{\mu^2_\pi}\,(5)_{\mu^2_G}\,(1)_{1/m_b^3}\,(1)_{\mathrm{sub}\,u=1}$$

$$= 0.04147\,({}^{+98}_{-117}) \quad (\text{from PDG inputs}), \qquad (4.123)$$

Bottomonium spectrum by fixed order calc.

Reproduces global structure
but not fine structures



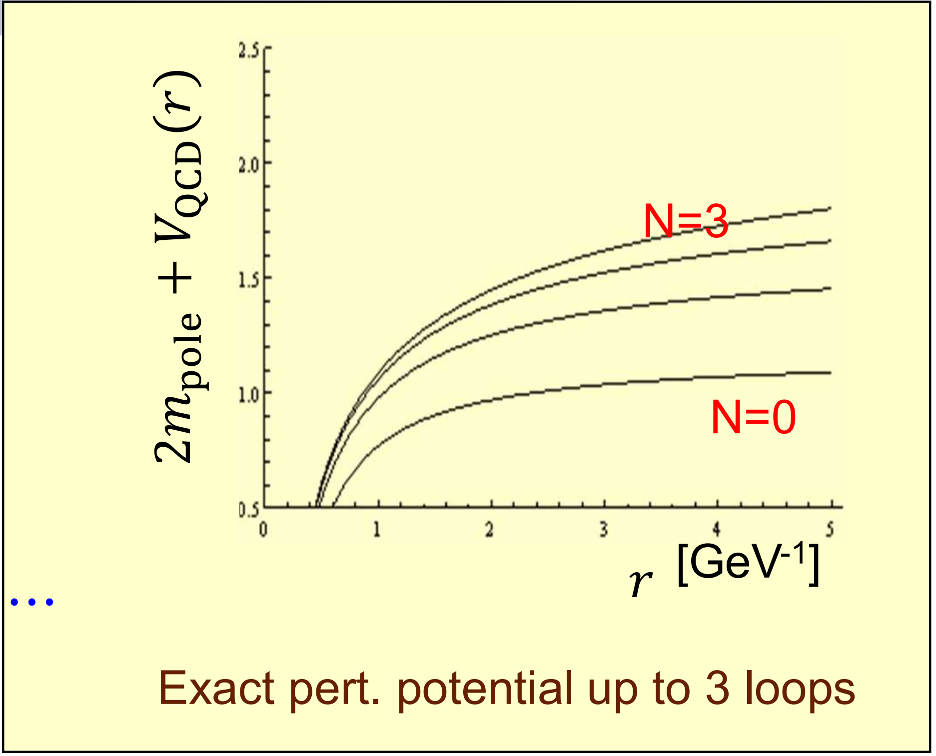
@N^3LO

Kiyo, YS

LO

$$\Delta M_{1S}^{\text{spin}} \approx \frac{4\pi C_F \alpha_s(\mu)}{3m_b^2}\,\vec{S}^2\,|\psi_{1S}(\mathbf{0})|^2 + \cdots$$

NNLO



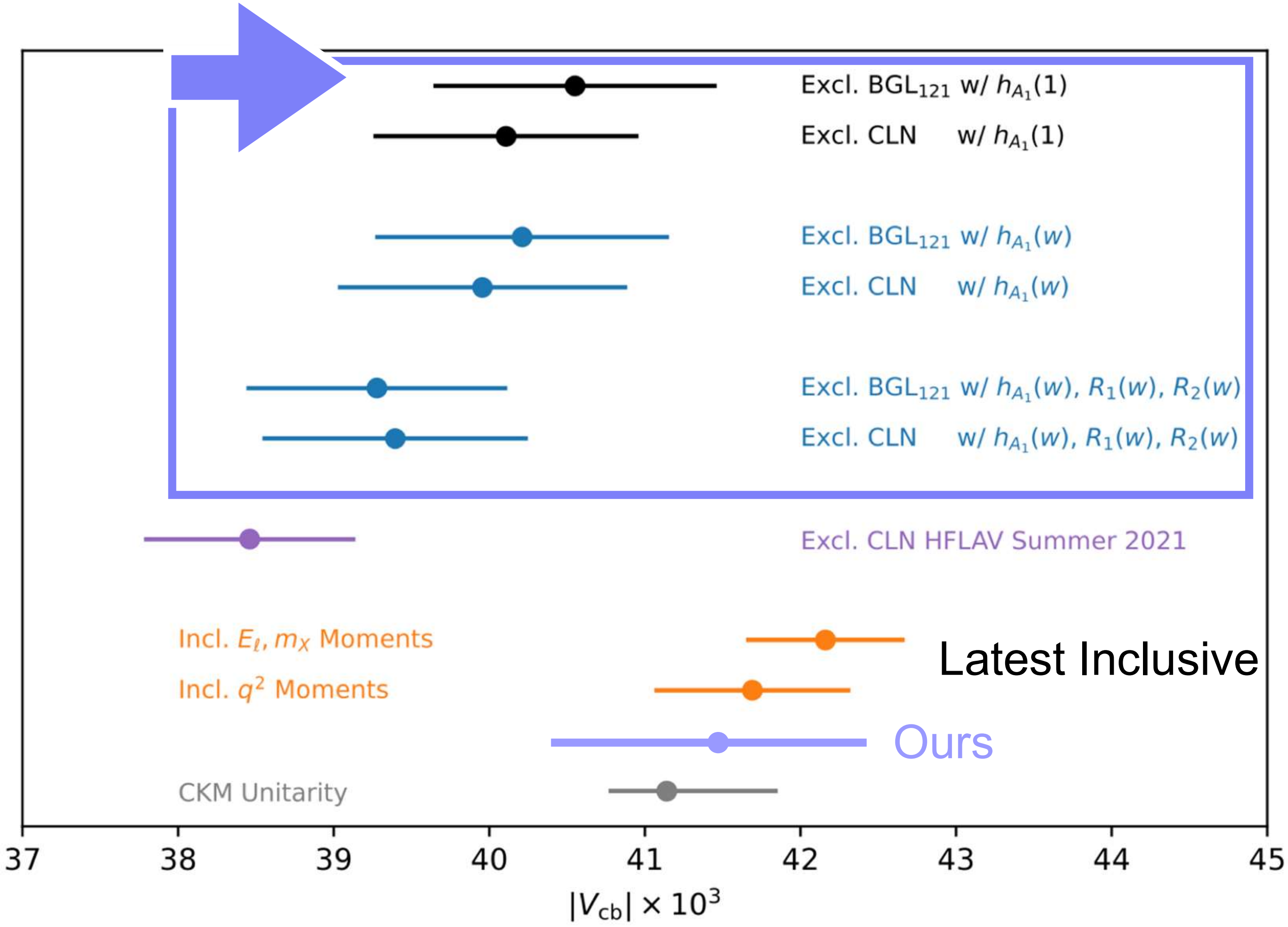
- Higher order corr. expected to enhance $\Delta M_{1S}^{\text{spin}}$.
- This effect is further enhanced by $|V_{cb}| \propto m_b^{-5/2}$.

Exclusive $|V_{cb}|$ update

[Submitted on 18 Jan 2023] arXiv:2301.07529

Measurement of Differential Distributions of $B \to D^* \ell \bar{\nu}_\ell$ and Implications on $|V_{cb}|$

Belle Collaboration, M. T. Prim, F. Bernlochner, F. Metzner, K. Lieret, T. Kuhr, I. Adachi, H.

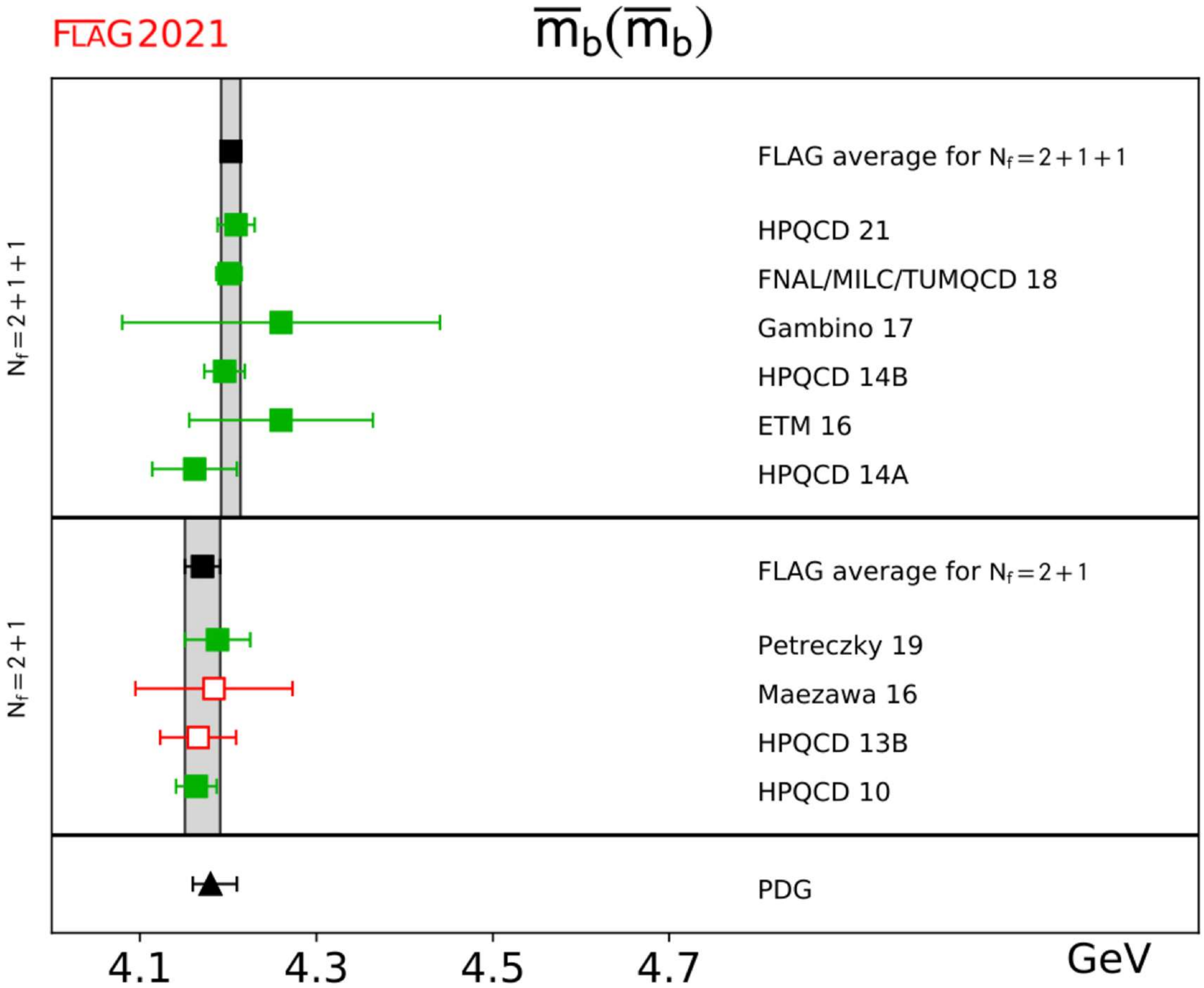


New Exclusive

(various lattice inputs)

Previous Exclusive

FLAG report of $\bar{m}_b$



FLAG report of $\bar{m}_c$