Generalized Nyquist Criterion for Transverse Stability

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Introduction

- The dispersion relations for transverse instabilities with Landau damping are well known.

$$D_{q'm'}(I_s)$$

$$= -\,i\,q'\,\frac{\beta_x}{2T\nu_s E_{o/e}}\; i^{m'} F_{q'm'}(\nu) \sum_p \tilde{D}\,(p'')Z_T(p')J_{m'}(p''aI_s^{1/2})\; g_o(I_s)\;, \qquad (2.37)$$

where

$$F_{q'm'}(\nu) = -\,2\pi\nu_s \int_o^\infty \frac{\frac{df_o}{dI_x}\, I_x\, dI_x}{\nu - m'\nu_s - q'\nu_x(I_x)}\;. \qquad (2.38)$$

Substituting Eq. (2.36) into Eq. (2.37), we finally arrive at an equation for $D_{q'm'}$:

From Yong Ho Chin
CERN SPS/85-9

- The difficult part has been obtaining useful stability criteria from these equations.
- By using the beam response function a generalization of the Nyquist stability criteria leads to a simple technique, see M. Blaskiewicz, FR5RFP030 PAC09.

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Transverse dispersion relation

- Consider a symmetrically filled ring.
- The horizontal coherent force on a particle is

$$F_x(\theta,t) = i\frac{q^2\omega_0}{2\pi R}\sum_{k=-\infty}^{\infty}\left\{D_k Z_x(k) + d_k\right\}\exp\left[i(kM+s)(\theta-\omega_0 t) - i\Omega t\right]$$

- The external drive is d_k. The transverse impedance is evaluated at sidebands of Ω, the coherent betatron frequency.

$$Z_x(k) = Z_x\left[(kM+s)\omega_0 + \Omega\right]$$

- There are standard procedures to obtain a matrix equation for the dipole Fourier components, or D_ks.

Dispersion Relation without synchrotron frequency spread

- One gets a matrix $D_k - \sum_{m=-\infty}^{\infty} K_{k,m}(\Omega + i\varepsilon) D_m = drive$
- Where

$$K_{k,m}(\Omega + i\varepsilon) = \frac{\bar{I} q Z_x(k) \beta_x c}{4\pi R E_T} \exp\left(- \frac{\sigma^2 \left(w_m^2 + w_k^2 \right)}{2} \right) \sum_{n=-\infty}^{\infty} I_n(\sigma^2 w_k w_m) \tilde{I}(n, \Omega + i\varepsilon)$$

$$\tilde{I}(n, \Omega + i\varepsilon) = \iint \frac{\partial T_0(J_x, J_y)}{\partial J_x} \frac{J_x dJ_x dJ_y}{\Omega + i\varepsilon - \omega_{x0} + \alpha_x J_x + \alpha_y J_y + n\omega_s} \rightarrow \frac{1}{\Omega} \quad \text{as } \Omega \rightarrow \infty$$

$$w_k = kM + s + Q_x - \frac{\xi_x}{\eta}$$

One plots $Det\left[1 - K(\Omega)\right]$ parametrically with Ω on the complex plane. If it does not encircle 0, the system is stable

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Instability in the EIC Hadron Storage Ring (HSR) due to crabbing mode

- Voltage limited beta function at 275 GeV is 1300 meters. Injection β=130m.
- For the design IP
- There are 8, 197 MHz cavities with R/Q = 2400 Ω/m. There are 4, 394 MHz cavities with R/Q = 1270 Ω/m.
- Transverse impedance enters as $\beta_x Z_x$
- The crabbing voltage scales as $E\beta^{-1/2}$

$$qV_x = \frac{cE\tan(\theta/2)}{\omega_{crab}\sqrt{\beta_x^* \beta_x^{crab}}}$$

- 100 GeV is "easy"
- 41 GeV runs up against β=200m lower limit set by optics.

Reducing apparent impedance with RF feedback (transverse, high Q)

$$D_T(t) = D_B(t) + x_0 I_{LL}(t) - x_0 Y_{op} V_x(t - T_{delay})$$

$$F(t) = \hat{F}\exp(-i\omega t), \qquad \hat{V}_x = \tilde{W}_x(\omega)\hat{D}_T$$

$$\left(\omega_r^2 - \omega^2 - i\frac{\omega\omega_r}{Q}\right)\hat{V}_x = \frac{R_x}{Q}\omega_r^2\left(\hat{D}_B + x_0\hat{I}_{LL} - x_0\tilde{Y}_{op}(\omega)\hat{V}_x\exp(i\omega T_{delay})\right)$$

$$\left(\frac{-i}{R_x}\frac{\omega}{\omega_r} + \frac{Q}{R_x}\left(1 - \frac{\omega^2}{\omega_r^2}\right) + x_0\tilde{Y}_{op}(\omega)\exp(i\omega T_{delay})\right)\hat{V}_x = x_0\hat{I}_{LL} + \hat{D}_B$$

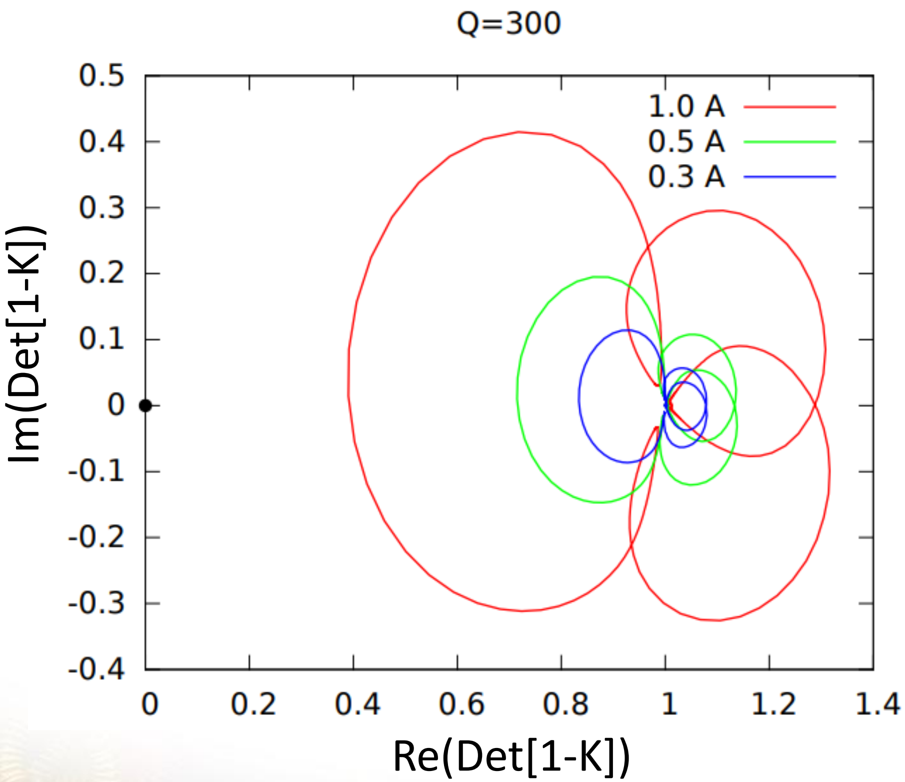
x_0 is related to the loaded quality factor, Q.

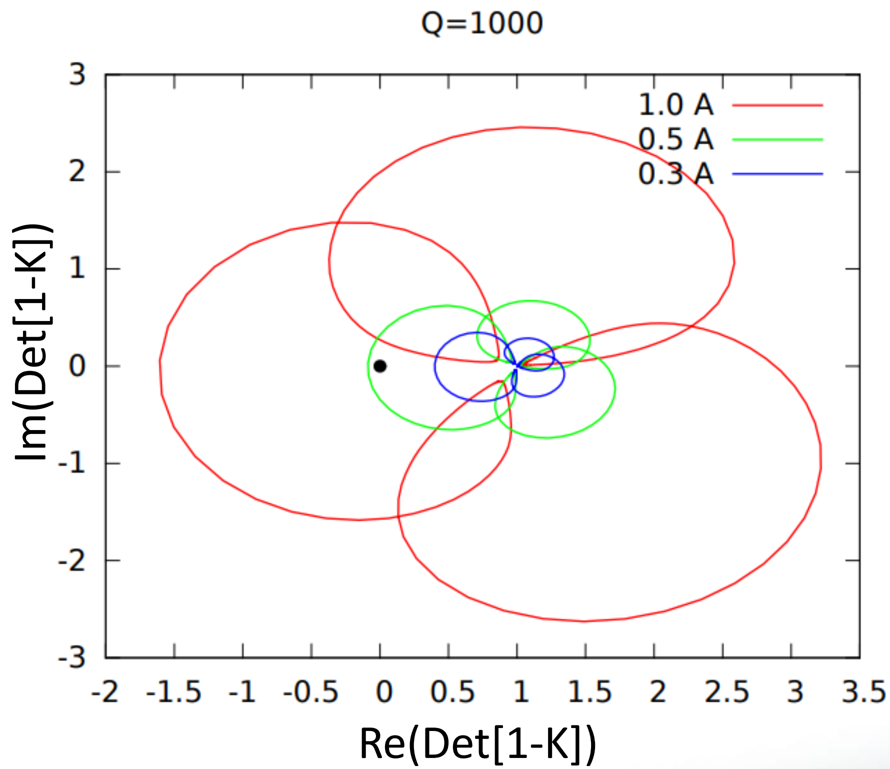
Take $x_0 Y_{op} = -i\omega/(\omega_r R_{eff})$, which is direct coupling to V_z.

$R_{eff} = Q_{eff}(R/Q)$, $Q_{eff} = 4 f_{res} T_{delay}$ for reasonable phase margin (Boussard & Lambert 1983).

Stability of crabbing mode

The data for the plots below were made in under a minute of CPU time. Including synchrotron frequency spread will slow things down a bit.





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Other options

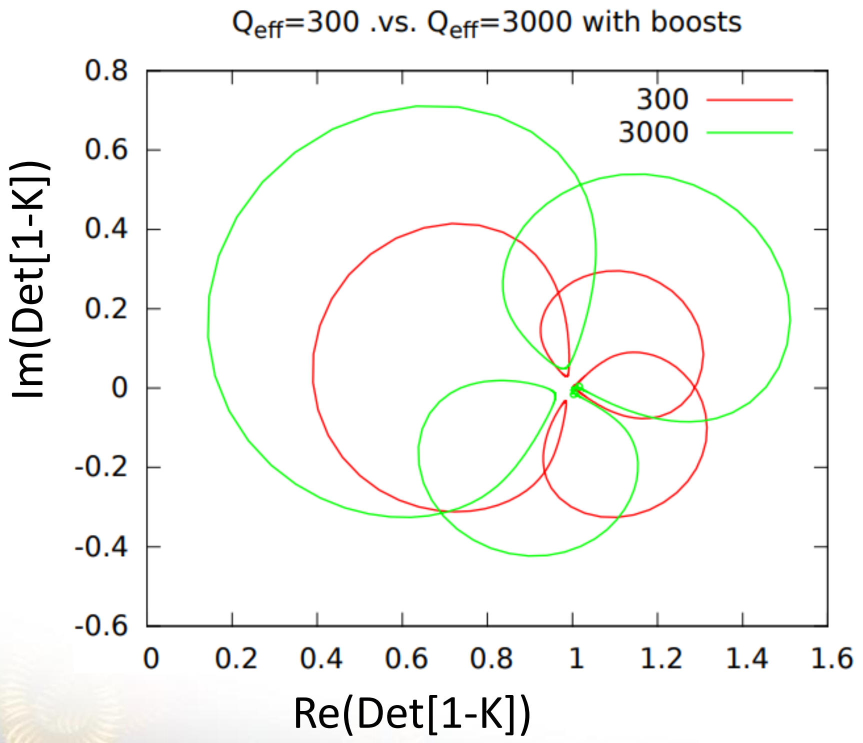
- The power of this technique is the speed.
- Consider a lower feedback gain, but include boosts in the feedback path

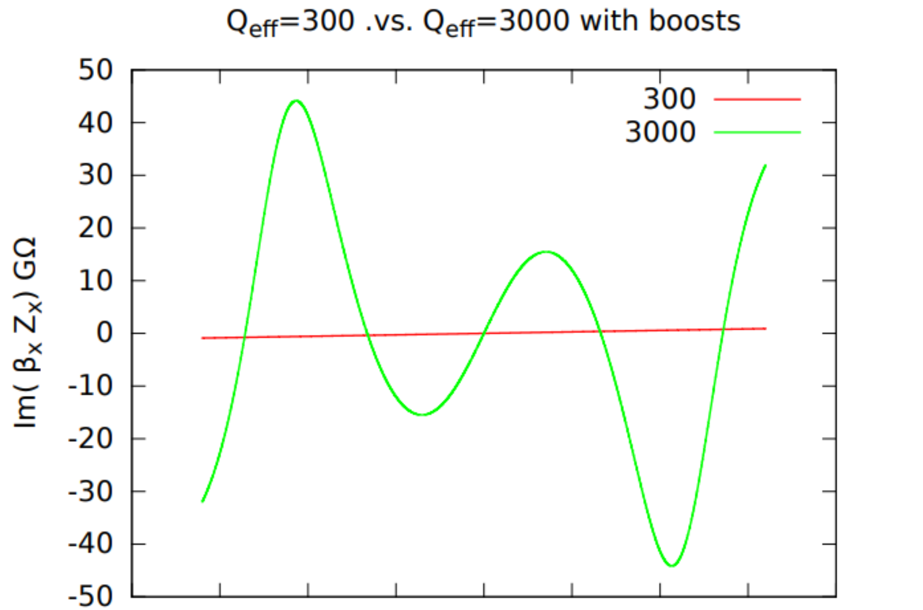
$$e^{i\omega T_{delay}} \to e^{i\omega T_{delay}} \left[1 - b\frac{id\omega}{(\omega_r - \omega_-)^2 - \omega^2 - id\omega}\right]\left[1 - b\frac{id\omega}{(\omega_r + \omega_-)^2 - \omega^2 - id\omega}\right]$$

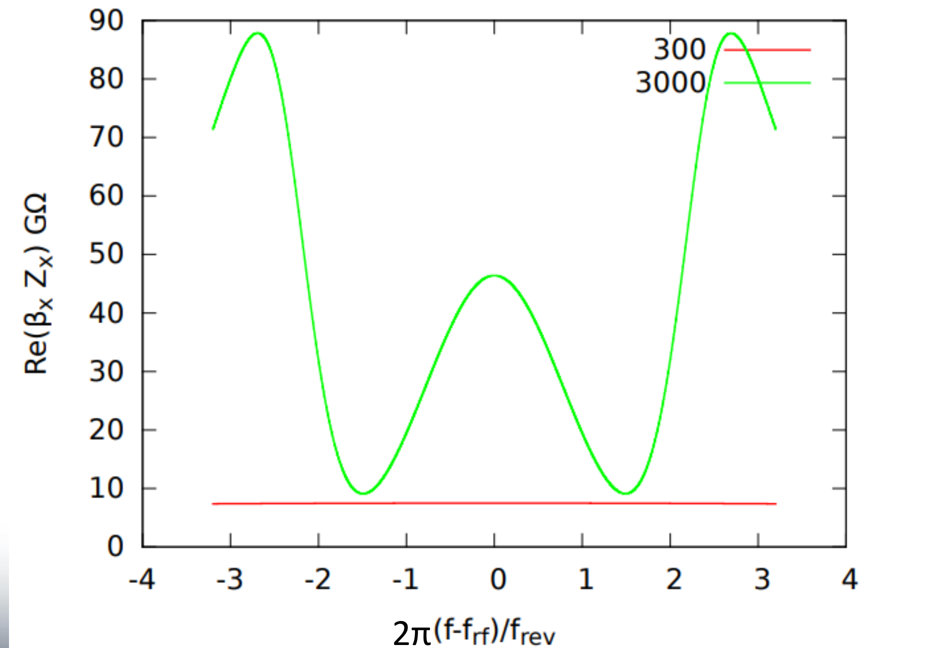
- Where ω_- is the frequency of the lowest betatron sideband, b is the boost of the gain and d is the width of the boost.
- One can easily modify the code
- Just for interest, note that a positive imaginary part for ω is always allowed. This is equivalent to driving the system with an expontially growing signal.

Results with boosts

Used b=6, d=0.05ω_{rev}.







Conclusions

- A generalized Nyquist criteria has been used to test the stability of transverse modes
- It is straightforward to include tune spread from all 3 degrees of freedom.
- Including localized impedance looks tractable too.

Eq (37) F. Ruggerio Particle Accelerators 1986

$$\det\{\delta_{nm} - KC_n(\nu)M_{nm}(\nu)\} = 0.$$

- Convergence needs to be studied, but convergence for appropriate basis expansions under very stressful conditions has been demonstrated.

V. Balbekov, PRAB 20, 034401 (2017)