# Resistive wall impedance and incoherent tune shift for elliptical multilayer beam pipe

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Mini-workshop on impedance modeling and impedance effects at SuperKEKB and future colliders



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**OUTLINES**

1. **Impedance of the elliptical multilayer chamber**

2. **Incoherent tune shift due to quadrupolar impedance**

3. **Summary**

# 1. Impedance of the elliptical multilayer chamber

1. Traditional method to analytically calculate the impedance of an elliptical beam pipe

----the product of Yokoya form factor and the impedance of the circular chamber [1].

----IW2D (Form factor and impedance of the circular chamber)

2. Impedance of the elliptical chamber with thick wall is given by [2].

3. The chamber usually is treated with NEG coating

----to provide low dynamic outgassing with large pumping speed

----to mitigate electron cloud effects.

4. In Ref.[3], resistive wall impedance in elliptical multilayer vacuum chambers are derived, by means of the surface impedance.

[1] K. Yokya, Resistive wall impedance of beam pipes of general cross-section, Part. Accel. 41, 221(1993)
[2] A.Lutman, R.Vescovo and P.Craievich, Electromagnetic field and short-range wake function in a beam pipe of elliptical cross section, Phys. Rev. ST Accel. Beams, 074401 (2008)
[3] M. Migliorati, L. Palumbo, C. Zannini, N. Biancacci, V.G. Vaccaro, Resistive wall impedance in elliptical multilayer vacuum chambers, Phys. Rev. Accel. Beams, 22 (2019),121001

## Background

4. With the proposing of the high energy circular electron positron colliders, large impedance contribution from the resistive wall impedance of the NEG coated chambers has been reported [3,4].

----Considering the metric between the elliptical cylindrical coordinates and Cartesian coordinates, we develop the confocal elliptical beam pipe to investigate the non-uniform thickness beam pipe, by imposing precious field matching.

[3] Na Wang, Yuan Zhang, Yudong Liu, Saike Tian, Kazuhito Ohmi, and Chuntao Lin. Mitigation of coherent beam instabilities in CEPC. CERN Yellow Rep. Conf. Proc., 9:286, 2020.

[4] M. Migliorati, E. Belli, and M. Zobov. Impact of the resistive wall impedance on beam dynamics in the Future Circular e+e- Collider. Phys. Rev. Accel. Beams, 21(4):041001, 2018.

For a multilayer beam pipe, the interface between the vacuum region and metal wall, as well as those between adjacent metal layers, are given by $u = u_j$. Here, the outmost layer has infinite thickness.

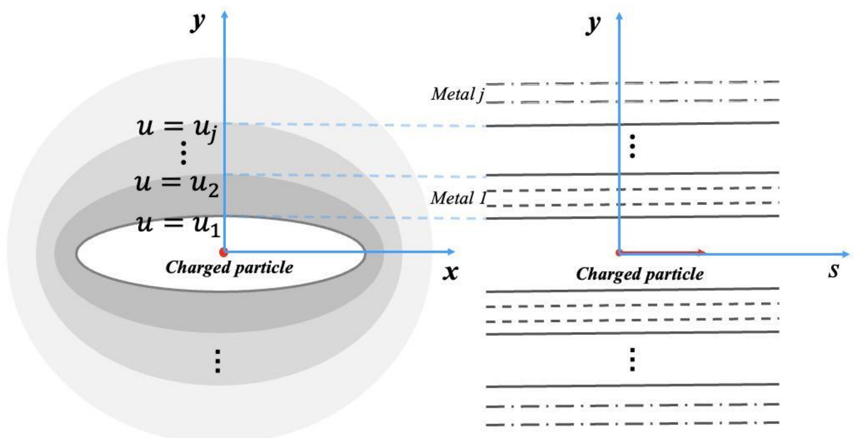



Fig.2 Cross section and side view of the elliptical beam pipe.

The relations between the elliptical and Cartesian coordinates in the transverse plane are given by

$$\begin{cases} x = F\cosh u \sin v, \\ y = F\sinh u \cos v. \end{cases}$$

Considering a point source particle at $(u_s\ , v_s)$, the charge density and the current density are

$$\rho = \frac{\delta(s-ct)\delta(u-u_s)\delta(v-v_s)}{h^2},\quad \boldsymbol{J} = qc\frac{\delta(s-ct)\delta(u-u_s)\delta(v-v_s)}{h^2}\hat{\boldsymbol{s}}.$$

where $h = F\sqrt{cosh^2 u - cos^2 v}$ is the metric between the elliptical cylindrical coordinates and Cartesian coordinates.

# Solution of Maxwell equations

For a multilayer elliptical beam pipe, the general expressions of the electromagnetic fields in the vacuum region are the same as that derived by A. Lutman in the case of thick wall[1]. The longitudinal electric and magnetic fields are

$$\tilde{E}_z = A_0 + \sum_{n=1}^{\infty} A_n \cosh nu \cos nv + \sum_{n=1}^{\infty} B_n \sinh nu \sin nv\,,$$

$$c\tilde{B}_z = B_0 + \sum_{n=1}^{\infty} B_n\, cosh\, nu \cos nv - \sum_{n=1}^{\infty} A_n \sinh nu \sin nv.$$

where $A_n$ and $B_n$ are the unknown coefficients to be determined by imposing field matching.

[1] A.Lutman, R.Vescovo and P.Craievich, Electromagnetic field and short-range wake function in a beam pipe of elliptical cross section, Phys. Rev. ST Accel. Beams, 074401 (2008)

The Maxwell's equations in the *j*-th layer of the resistive wall can be written as

$$\frac{\partial^2 \tilde{E}_z^j}{\partial u^2} + \frac{\partial^2 \tilde{E}_z^j}{\partial v^2} + h^2\lambda_j^2 \tilde{E}_z^j = 0,$$

$$\frac{\partial^2 c\tilde{B}_z^j}{\partial u^2} + \frac{\partial^2 c\tilde{B}_z^j}{\partial v^2} + h^2\lambda_j^2 c\tilde{B}_z^j = 0,$$

$$h\tilde{E}_u^j = \frac{ik}{\lambda_j^2}\frac{\partial \tilde{E}_z^j}{\partial u} + \frac{ik}{\lambda_j^2}\frac{\partial c\tilde{B}_z^j}{\partial v},$$

$$h\tilde{E}_v^j = \frac{ik}{\lambda_j^2}\frac{\partial \tilde{E}_z^j}{\partial v} - \frac{ik}{\lambda_j^2}\frac{\partial c\tilde{B}_z^j}{\partial u}$$

$$hc\tilde{B}_u^j = -\left(\frac{ik}{\lambda_j^2} + \frac{i}{k}\right)\frac{\partial \tilde{E}_z^j}{\partial v} + \frac{ik}{\lambda_j^2}\frac{\partial c\tilde{B}_z^j}{\partial u},$$

$$hc\tilde{B}_v^j = \left(\frac{ik}{\lambda_j^2} + \frac{i}{k}\right)\frac{\partial \tilde{E}_z^j}{\partial u} + \frac{ik}{\lambda_j^2}\frac{\partial c\tilde{B}_z^j}{\partial v}.$$

where $\lambda_j^2 = ikZ_0\sigma_j$, $Z_0$ is the impedance of free space.

The above equations can be solved by separating the variables u and v, and the solutions can be written as the product of functions *U(u)* and *V(v),* which yields the standard Mathieu angular and radial equations

$$\frac{d^2}{du^2}U(u) - (a - 2Q\cosh 2u)U(u) = 0, \qquad \frac{d^2}{dv^2}V(v) + (a + 2Q\cos 2v)V(v) = 0.$$

where $Q = F^2\lambda^2/4$ is the parameter of Mathieu funciton.

Since the electromagnetic fields must be 2π periodic in v, the periodic solutions of Mathieu angular function are expected.
As to the solutions of the Mathieu radial equations, the electromagnetic fields excited by the driving beam contain both the outward and inward propagating waves. For the outmost layer, only the outward propagating wave is expected.

$$V_{a_{2n}}(v) = \sum_{m=0}^{\infty} A_{2m}^{a_{2n}} \cos 2mv\,,$$

$$U_{a_{2n}}^{(1),(2)} = \frac{p^{2n}}{[A_0^{(2n)}]^2} \sum_{k=0}^{\infty} (-1)^k A_{2k}^{(2n)}(Q)\; J_k(v_1) H_m^{(1),(2)}(v_2),$$

$$V_{a_{2n+1}}(v) = \sum_{m=0}^{\infty} A_{2m+1}^{a_{2n+1}} \cos(2m+1)v\,,$$

$$U_{a_{2n+1}}^{(1),(2)} \frac{s^{2n+1}}{\sqrt{q}\left[B_1^{(2n+1)}\right]^2} \sum_{k=0}^{\infty} (-1)^k B_{2k+1}^{(2n+1)}(Q)\,[J_k(v_1) H_m^{(1),(2)}(v_2) - J_{k+1}(v_1) H_m^{(1),(2)}(v_2)]],$$

$$V_{b_{2n}}(v) = \sum_{m=0}^{\infty} B_{2m}^{b_{2n}} \sin 2mv\,,$$

$$U_{b_{2n}}^{(1),(2)} = -\frac{s^{2n+2}}{q\left[B_2^{(2n+2)}\right]^2} \sum_{k=0}^{\infty} (-1)^k B_{2k+2}^{(2n+2)}(Q)\,[J_k(v_1) H_m^{(1),(2)}(v_2) - J_{k+2}(v_1) H_m^{(1),(2)}(v_2)],$$

$$V_{b_{2n+1}}(v) = \sum_{m=0}^{\infty} B_{2m+1}^{b_{2n+1}} \sin(2m+1)v\,.$$

$$U_{b_{2n+1}}^{(1),(2)} = -\frac{p^{2n}}{\sqrt{q}\left[A_1^{(2n+1)}\right]^2} \sum_{k=0}^{\infty} (-1)^k A_{2k+1}^{(2n+1)}(Q) \left[ J_k(v_1) H_m^{(1),(2)}(v_2) + J_{k+1}(v_1) H_m^{(1),(2)}(v_2) \right].$$

where $v_1 = \sqrt{Q}e^{-u}, \;\; v_2 = \sqrt{Q}e^{u}$

## Field matching

For a metal conductor, considering $|Q| = ikZ_0\sigma F^2/4 \gg 1$, the asymptotic functions of Mathieu functions are introduced

$$\boxed{\begin{aligned} U^{(1)}(\sigma,u) &\cong e^{i2\sqrt{Q}\,\sinh u}, \\ U^{(2)}(\sigma,u) &\cong e^{i2\sqrt{Q}\,\cosh u}. \end{aligned}}$$

In order to determine the field inside the vacuum region, the continuity of the tangential fields on different interfaces of $u=u_j$ are imposed

$$\begin{aligned}
\widetilde{E}_z^j\left(u=u_j\right) &= \widetilde{E}_z^{j-1}\left(u=u_j\right), \\
c\widetilde{B}_z^j\left(u=u_j\right) &= c\widetilde{B}_z^{j-1}\left(u=u_j\right), \\
h\widetilde{E}_v^j\left(u=u_j\right) &= h\widetilde{E}_v^{j-1}\left(u=u_j\right), \\
hc\widetilde{B}_v^j\left(u=u_j\right) &= hc\widetilde{B}_v^{j-1}\left(u=u_j\right).
\end{aligned}$$

With both source and witness particles locating at $(0, \frac{\pi}{2})$, the longitudinal impedance per unit length can be expressed as

$$Z_{\parallel} = \frac{1}{\mathrm{qc}} \tilde{E}_z|_{(u_s,v_s)=(0,\pi/2)}.$$

The transverse impedance is defined by

$$Z_x = -\frac{1}{\omega P_x}\frac{\partial \tilde{E}_z}{\partial x}, \qquad Z_y = -\frac{1}{\omega P_y}\frac{\partial \tilde{E}_z}{\partial y}.$$

With the source particles having offset of $\Delta x_s = F\sin\Delta v$ , $\Delta y_s = F\sinh\Delta u$ , and the witness particle locating at $(u_w,\ v_w) = (0, \frac{\pi}{2})$, the transverse dipole impedance per unit length are

$$Z_x^D = \frac{1}{qF\omega\Delta x_s}\frac{\partial \tilde{E}_z}{\partial v}|_{(u_s,v_s)=(0,\frac{\pi}{2}-\Delta v)},$$

$$Z_y^D = \frac{1}{qF\omega\Delta y_s}\frac{\partial c\tilde{B}_z}{\partial v}|_{(u_s,v_s)=(\Delta u,\pi/2)}.$$

With the source particles having offset of $\Delta x_w = F\sin\Delta v$ , $\Delta y_w = F\sinh\Delta u$ , and the witness particle locating at $(u_s,\ v_s) = (0, \frac{\pi}{2})$, the transverse dipole impedance per unit length are

$$Z_x^Q = \frac{1}{qF\omega\Delta x_w}\frac{\partial \tilde{E}_z}{\partial v}|_{(u_w,v_w)=(0,\frac{\pi}{2}-\Delta v)},$$

$$Z_y^Q = \frac{1}{qF\omega\Delta y_w}\frac{\partial c\tilde{B}_z}{\partial v}|_{(u_w,v_w)=(\Delta u,\pi/2)}.$$

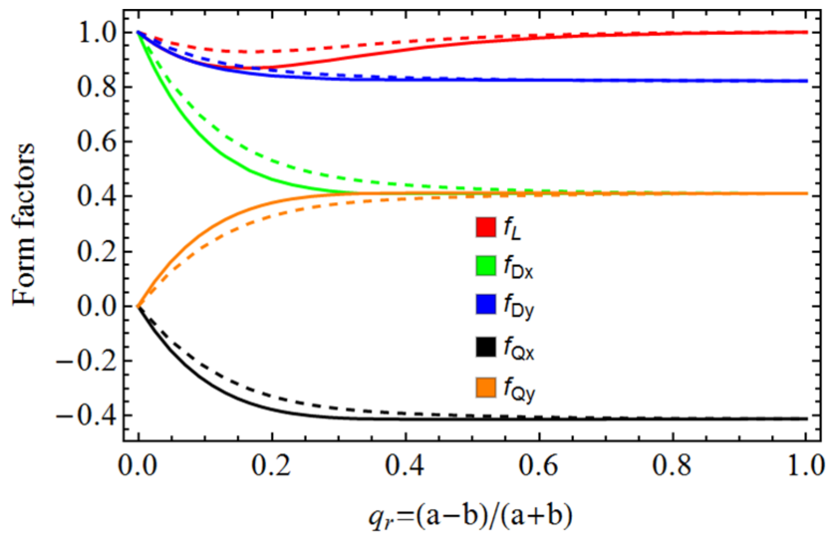


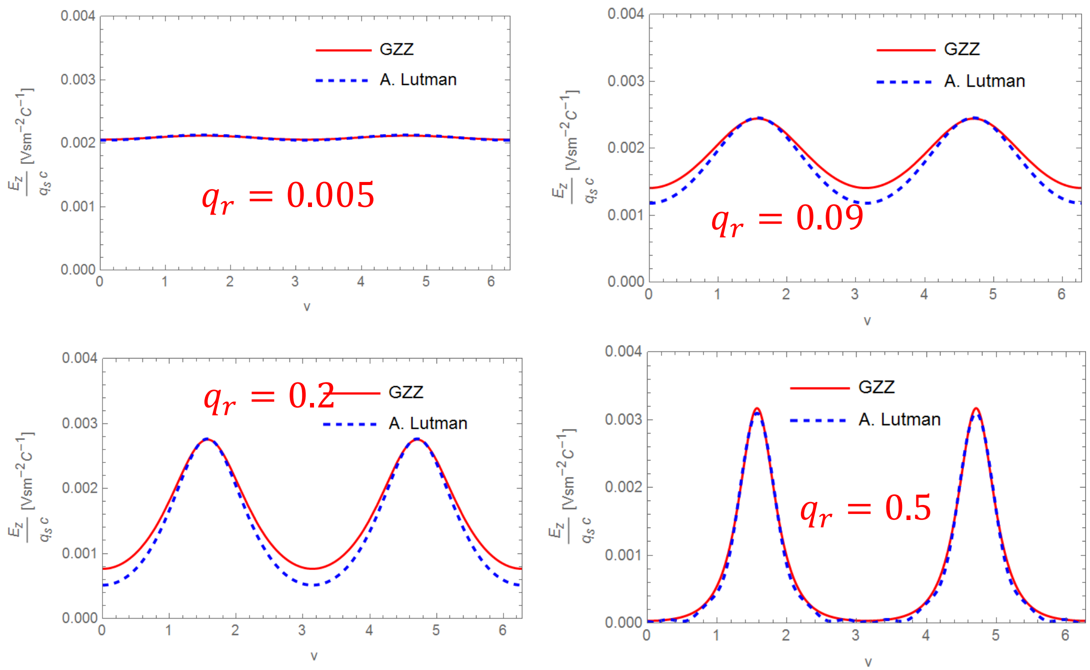

Fig.3 Form factors

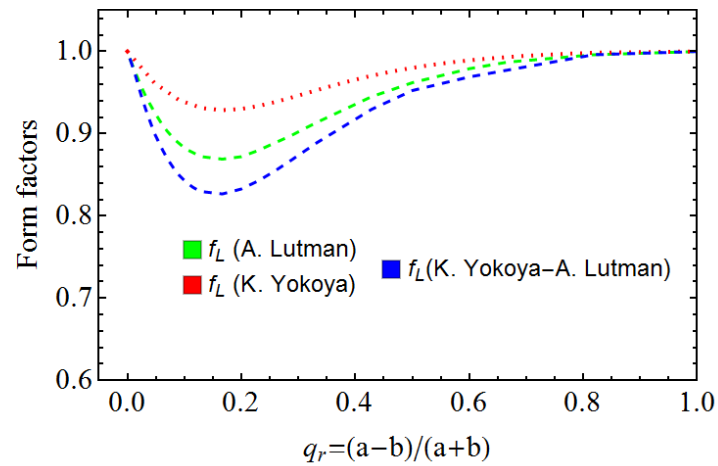
**Solid lines**: Based on A. Lutman's method, directly solving the Maxwell's equations in the elliptic coordinate system and the impedances were determined by field matching at the interface between the vacuum and the metal walls.
**Dashed lines**: Yokoya form factor, based on K.Yokoya's method that derived impedance formulae by introducing the surface impedance.

At lower $q_r$, small discrepancies for both longitudinal and transverse form factors are observed, which can be explained by the fact that the concept of surface impedance has been used in the K. Yokoya's method, instead of solving the Maxwell's equations directly as discussed in this report.

[1] **Y.T. Wang**, N. Wang, Q. Qin, G. Xu and S. Yue, Resistive-wall impedance of an elliptical beam pipe with multilayer, Nuclear Inst. and Methods A, 166928 (2022).

# Form factors




Approximate boundary are used in Ref.[1,2]

$$E_z \cong -k\delta(1+j)Z_0 H_{1s}^*/2$$

$$Z_{\parallel}(k) = \frac{1}{{I_0}^2}\oint ds\, E_z\left(\frac{2E_z}{k\delta(1+j)Z_0}\right)^*$$

[1] K. Yokya, Resistive wall impedance of beam pipes of general cross-section, Part. Accel. 41, 221(1993)
[2] R.L. Gluckstern, Johannes van Zeijts, B.Zotter, Couping impedance of beam pipe of general cross section, Phys. Rev .E 47 (1993) 656-663

To mitigate the impedance of the elliptical beam pipe with NEG coating, the influence of the nonuniform NEG coating on impedance is investigated.

Two models are introduced:

## Model X

Horizontal thickness of $t$

vertical thickness of $F(\sinh u_2 - \sinh u_1)$

where $u_1 = \cosh^{-1}\frac{a_1}{F}, \;\; u_2 = \sinh^{-1}\left(\frac{t}{F} + \cosh u_1\right)$.

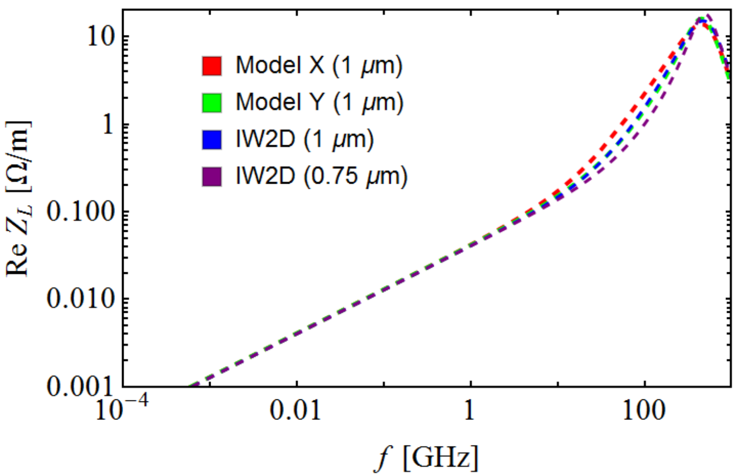
## Model Y

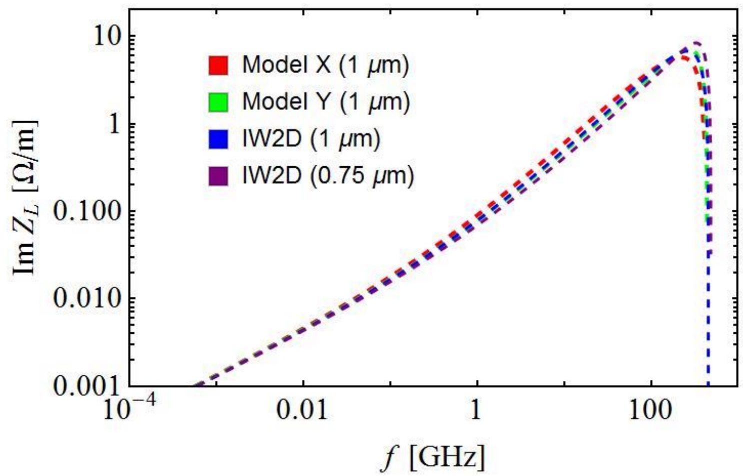
Vertical thickness of $t$

Horizontal thickness of $F(\cosh u_2 - \cosh u_1)$

where $u_1 = \sinh^{-1}\frac{b_1}{F}, \;\; u_2 = \sinh^{-1}\left(\frac{t}{F} + \sinh u_1\right)$.

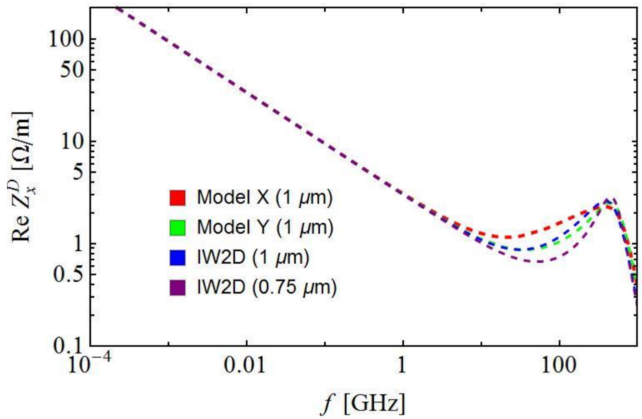
Parameter: Copper beam pipe treated with NEG coating. Model X with horizontal thickness of 1 μm in red lines; Model Y with vertical thickness of 1 μm in green lines. IW2D with thickness of 1 μm and 0.75 μm in blue and purple lines.

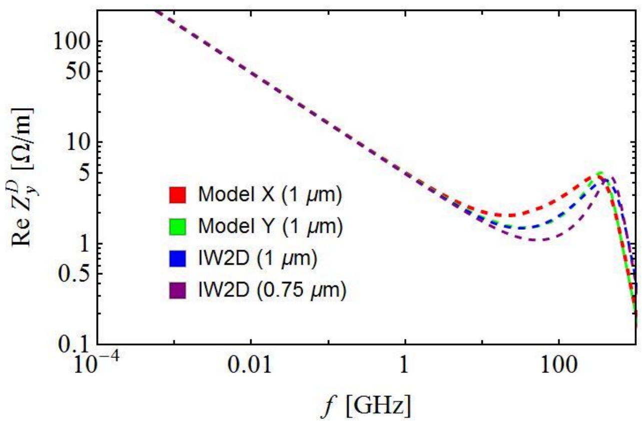


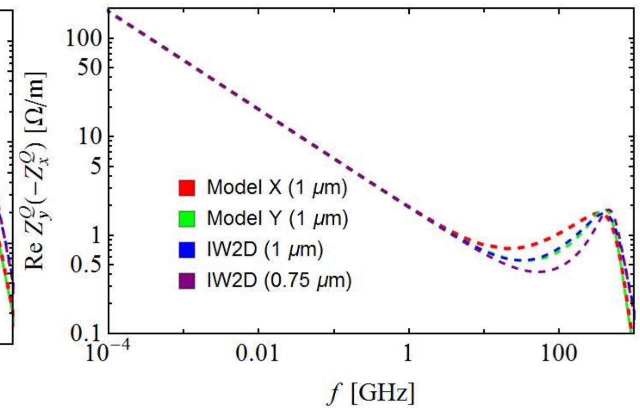



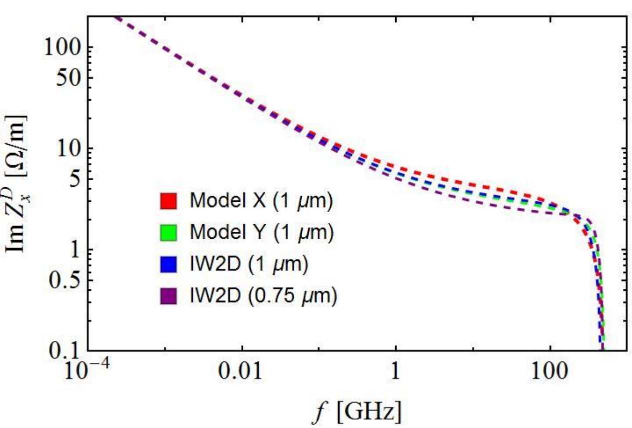

From the results we can conclude that the impedances are more sensitive to the vertical thickness of the NEG coating, other than the horizontal thickness. Therefore, in order to reach low impedance and high reliability at the same time concerning the NEG coating chambers, we can potentially increase the thickness of coating in the horizontal plane while keep the thin layer in the vertical plane.

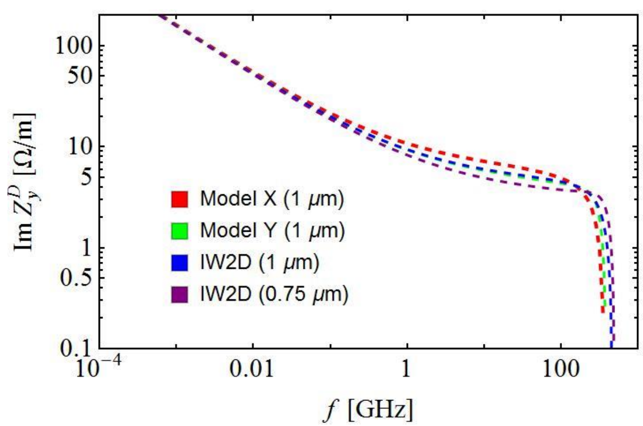
# Application

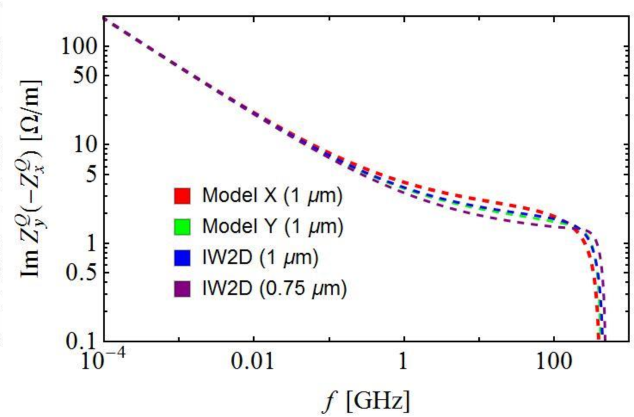


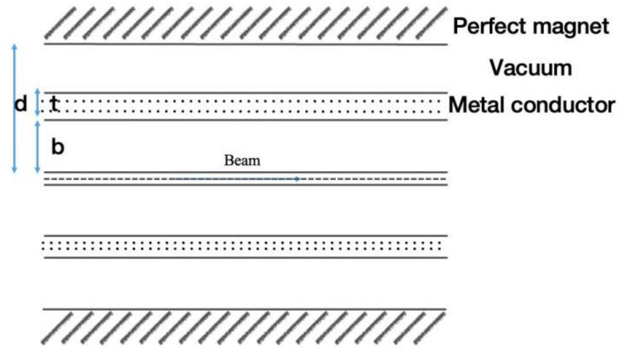



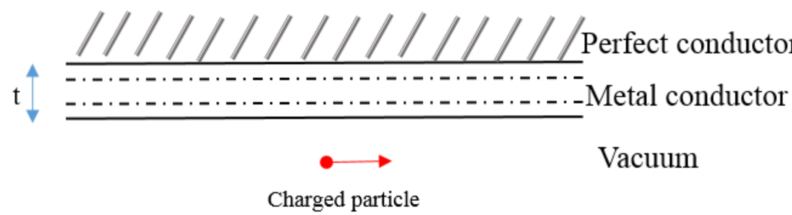
# 2. Tune shift due to quadrupolar impedance

1. Quadrupolar tune shift is widely observed. Two theoretical models developed by A.Chao and Y.Shobuda to reproduce the measurements.

2. A.Chao's model




3. Y.Shobuda's model




4. Comprehensive studies on SOLEIL shows, in the long range regime dealing with multibunch beams, the measurement result lies somewhere between the two models [3].

[1] A.Chao, S.Heifets, and B.Zotter, Tune shifts of bunch trains due to resistive vacuum chambers without circular symmetry, Phys. Rev. Accel. Beams 8, 042801 (2005)
[2] Y. Shobuda and K.Yokoya, Resistive wall impedance and tune shift for a chamber with a finite thickness, Phys. Rev. E, 66, 056501 (2002)
[3] P. Brunelle, R. Nagaoka, R. Sreedharan, Measurement and analysis of the impact of transverse incoherent wakefields in a light source storage ring, Phys. Rev. Accel. Beams 19 (4) (2016) 044401.

# Incoherent tune shift due to the quadrupolar impedance is derived from beam oscillation equations

$$\ddot{y}_n(t)+\omega_\beta^2 y_n(t)=-\frac{Nr_0c}{\gamma T_0}\sum_{m=0}^{M-1}\sum_{k=0}^{\infty}[y_m(t-(k+\frac{m-n}{M})T_0)W_1^d(-(k+\frac{m-n}{M})C)$$
$$+y_n(t)W_1^q(-(k+\frac{m-n}{M})C)],\quad (n=0,\ 1,\ \ldots,\ M-1),$$

$$\Omega-\omega_\beta=-i\frac{Nr_0c}{2\gamma\omega_\beta T_0^2}\sum_{\mu=0}^{M-1}\sum_{p=-\infty}^{\infty}[Z_1^d((\mu+pM)\omega_0+\omega_\beta)+Z_1^q(pM\omega_0)]$$

DC value of the quadrupolar impedance dominates the tune shift.

Incoherent tune slope: $$\frac{d\Delta\nu_\beta}{dI}=\frac{C}{8\pi^2(E/e)\cdot\nu_\beta}\sum_{p=-\infty}^{\infty}\mathrm{Im}Z_1^q(pM\omega_0)$$

With the impedance model, both the conductivity and permeability of different layers with finite thickness can be considered.

[1] **Y.T. Wang**, N. Wang, H.S. Xu and G. Xu, Tune shift due to the quadrupolar resistive-wall impedance of an elliptical beam pipe, Nuclear Inst. and Methods A, 166414, 2022

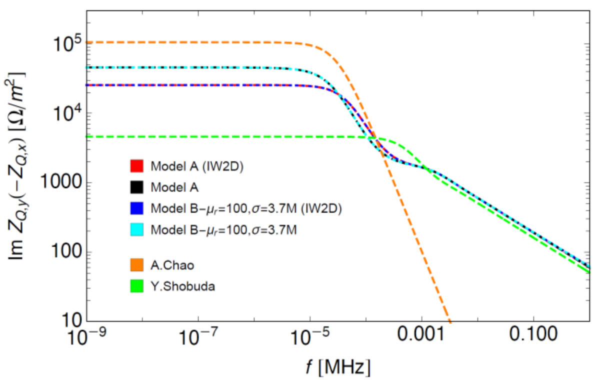
# Application in CEPC Z operation

**With frequency decreases, the skin depth increases and the background material outside the beam pipe should be considered.**

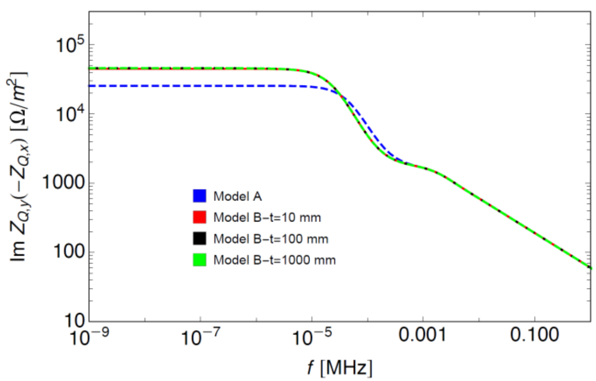
**Two impedance models:(the product of the form factors and the impedance for the circular chamber[1])**

**Model A**: Vacuum outside the chamber **Model B**: Magnet outside the chamber

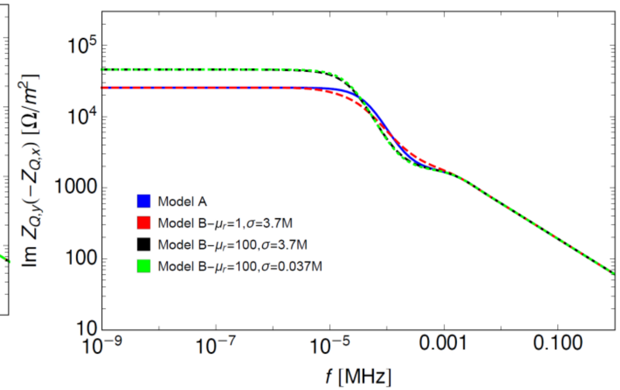
3mm Cu + Vacuum (Infinite) 3mm Cu + 100mm Magnet ($\mu$=100) + Vacuum (Infinite)




**Comparison between different impedance models**




**Dependence on the thickness of the magnets**




**Dependence on the permeability and conductivity of the magnets**

| Parameters | Values | Parameters | Values |
|---|---|---|---|
| Size | 28×37.8 mm | Conductivity of Magnet | 3.7 MS/m |
| Conductivity of Copper | 58 MS/m | Relative Permeability | 100 |
| Thickness of chamber | 3 mm | Thickness of Magnet | 100 mm |



[1] N. Wang and Q. Qin, Resistive-wall impedance of two-layer tube, Phys. Rev. ST Accel. Beams 10 (2007), 111003.

# Application in CEPC Z operation

Based on the conceptual design parameters, the incoherent tune shift for the Z operation mode, with beam energy of 45.5 GeV and beam current of 461 mA, are calculated and compared among different theoretical models.

- Consider that 80% of the collider is surrounded by magnets, the tune shifts are 0.26/-0.25 ($A^{-1}$)
- Main difference among different methods can be explained by the impedance model and the form factors used.

| Model | Tune slope (x/y) ($A^{-1}$) | Form factor $f_y^Q$ $\left(-f_x^Q\right)$ |
|---|---|---|
| A.Chao | 0.65/–0.64 | 0.41 |
| Y.Shobuda | 0.06/–0.06 | 0.28 |
| Model A | 0.16/–0.15 | 0.33 |
| Model B | 0.28/–0.28 | 0.33 |
| Model A (20%)+Model B (80%) | 0.26/–0.25 | 0.33 |

# Summary

- The general expressions of the longitudinal and transverse impedances for the multilayer elliptical beam pipe have been derived, by imposing field matching on adjacent confocal interfaces.

- The numerical results indicate that the impedances are more sensitive to the coating thickness in the vertical plane, other than the horizontal plane.

- Theoretical analysis indicates that the incoherent tune shift for multibunch operations only depends on the DC value of the quadrupolar impedance.

- The results show that the imaginary parts of impedances at low frequencies depend on the permeability of the magnet. The thickness and conductivity of the magnetic material have negligible influence on the quadrupolar impedances at low frequencies.

Thanks for your attention