National Synchrotron Light Source II

Brookhaven National Laboratory

U.S. DEPARTMENT OF ENERGY

Impedance Computations and Beam-Based Measurements: a Problem of Discrepancy

Victor Smaluk

Mini-workshop on impedance modeling and impedance effects at SuperKEKB and future colliders

December 2022

Broadband Impedance Effects
What Can We Measure?

Single-bunch effects are dependent on integral parameters combining the impedance $Z_{||}, Z_{\perp}$ and the bunch power spectrum $h(\omega) = \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega)$; $\tilde{\lambda}(\omega)$ is a Fourier transform of the longitudinal bunch profile $\lambda(t)$

$h(\omega) = e^{-\omega^2\sigma_t^2}$ for a Gaussian bunch

Longitudinal effective impedance

$$\left(\frac{Z_{\parallel}}{n}\right)_{\text{eff}} = \frac{\int_{-\infty}^{\infty} Z_{\parallel}(\omega)\frac{\omega_0}{\omega}h(\omega)d\omega}{\int_{-\infty}^{\infty} h(\omega)d\omega}$$

Energy loss factor

$$k_{\parallel} = \int_{-\infty}^{\infty} V_{\parallel}(t)\lambda(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\parallel}(\omega)h(\omega)d\omega$$

Measurable effects

- bunch lengthening
- synchronous phase shift

Transverse effective impedance

$$(Z_{\perp})_{\text{eff}} = \frac{\int_{-\infty}^{\infty} Z_{\perp}(\omega)h(\omega-\omega_{\xi})d\omega}{\int_{-\infty}^{\infty} h(\omega-\omega_{\xi})d\omega}$$

$\omega_{\xi} \equiv \frac{\xi\omega_0}{\alpha}$ is the chromatic frequency

Transverse kick factor

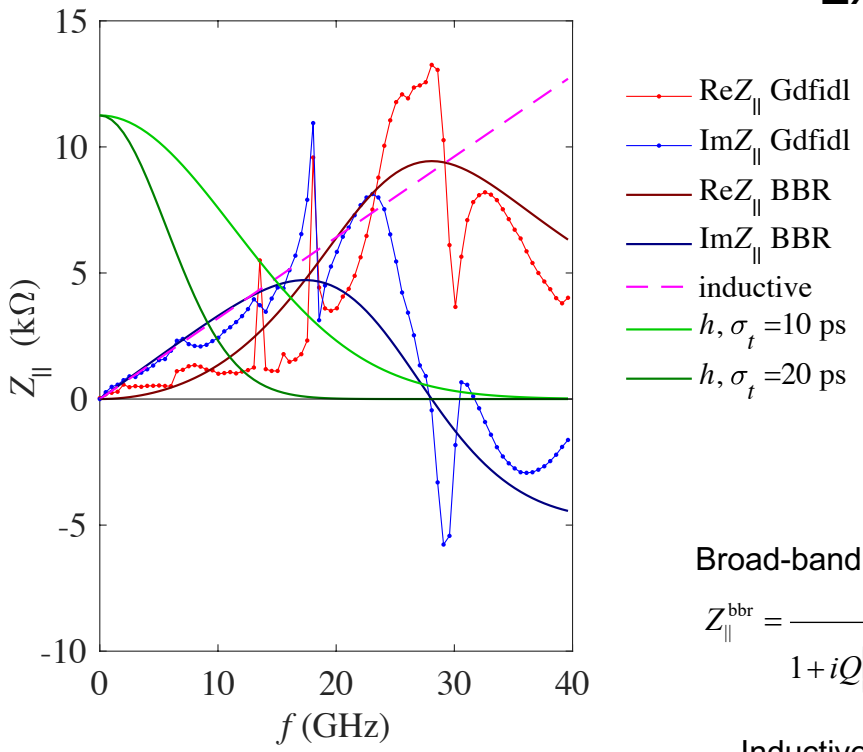
$$k_{\perp} = \int_{-\infty}^{\infty} V_{\perp}(t)\lambda(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\perp}(\omega)h(\omega)d\omega$$

Measurable effects

- coherent betatron tune shift
- chromatic head-tail damping
- orbit distortion by local impedance

Impedance Models

Example: NSLS-II

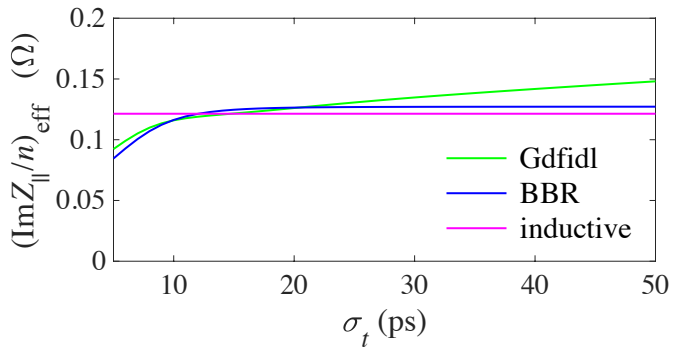


Broad-band resonator

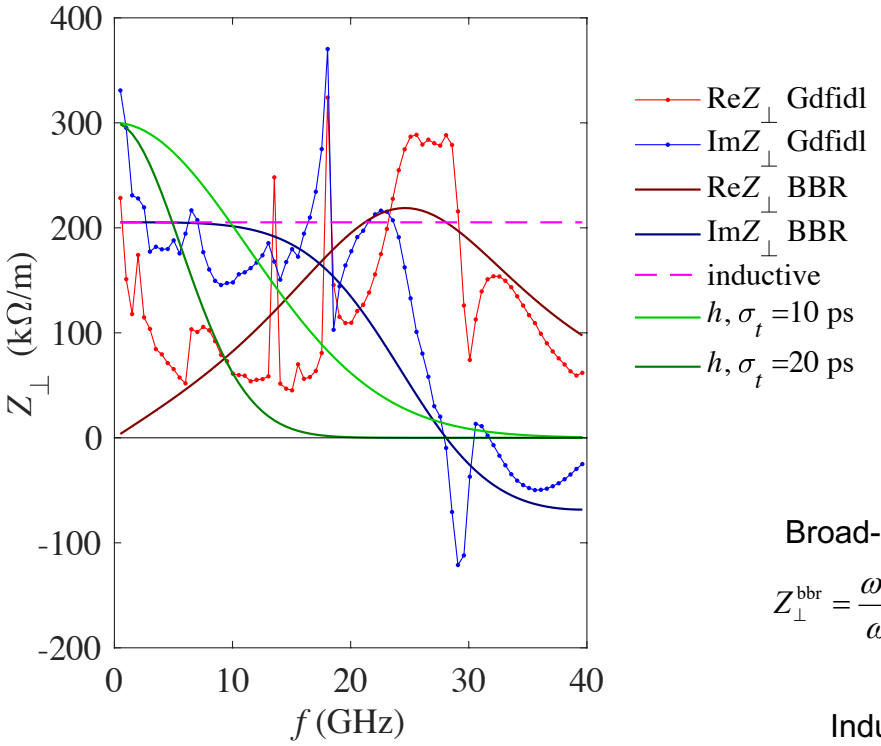
$$Z_{\parallel}^{\mathrm{bbr}} = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Inductive model

$$\frac{Z_{\parallel}}{n} = i\frac{R_s}{Q}\frac{\omega_0}{\omega_r}$$



$$Z_{\perp}(\omega) \approx \frac{2c}{b^2\omega} Z_{\parallel}(\omega)$$

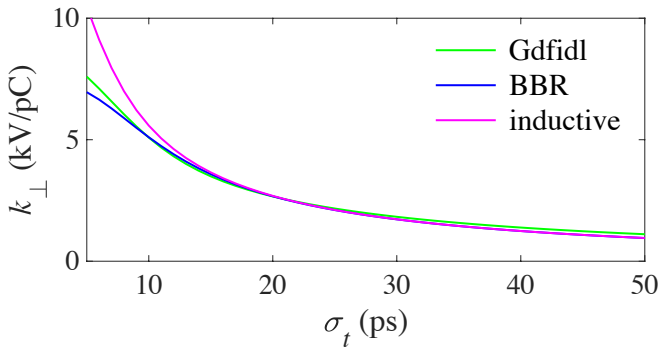


Broad-band resonator

$$Z_{\perp}^{\mathrm{bbr}} = \frac{\omega_r}{\omega}\frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Inductive model

$$Z_{\perp} = iR_s$$



Potential Well Distortion: Bunch Lengthening

Potential well distortion

Haissinski equation

J.Haissinski, Nuovo Cimento 18, 1 (1973)

$$\lambda(t) = K\lambda_0(t)\exp\left(\frac{I_b}{\omega_s\sigma_\delta\frac{E}{e}}\int_{-\infty}^{t}dt''\int_{-\infty}^{\infty}dt' W_\parallel(t''-t')\lambda(t')\right)$$

Zotter cubic equation for the r.m.s. bunch length

B.Zotter, CERN SPS/81-14 (DI), 1981

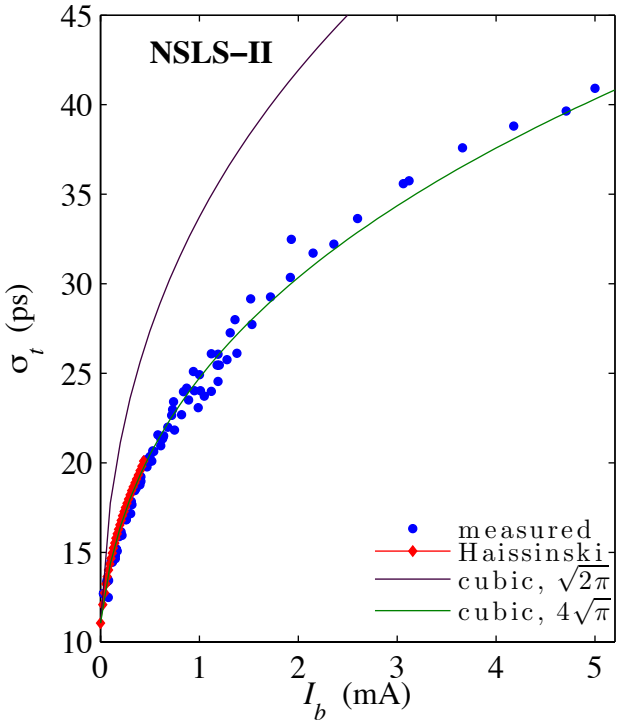
$$\left(\frac{\sigma_z}{\sigma_{z0}}\right)^3 - \frac{\sigma_z}{\sigma_{z0}} = \frac{\alpha I_b \,\mathrm{Im}(Z/n)_{eff}}{\pi\nu_s^2 E/e}\left(\frac{R}{\sigma_{z0}}\right)^3 \qquad \nu_s = \frac{\alpha\sigma_\delta}{\sigma_{t0}\omega_0}$$

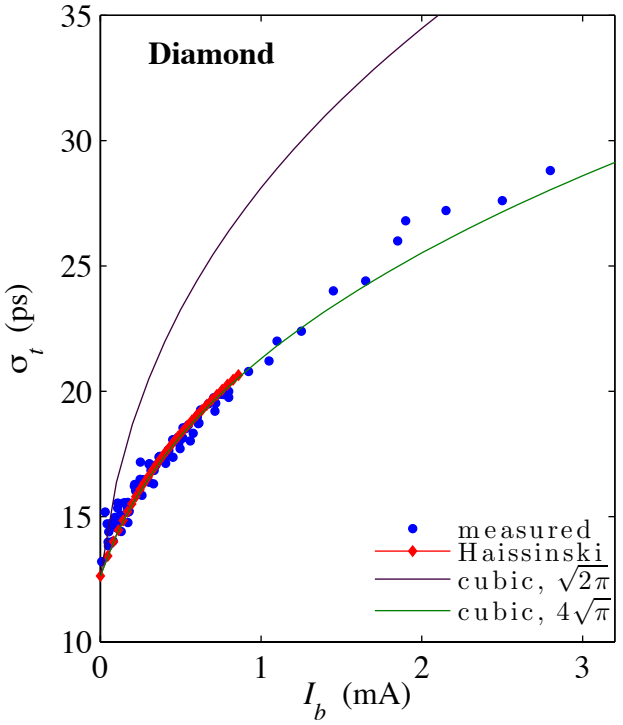
Modified Zotter equation

R.Nagaoka, HBSLS Workshop, 2017

$$\left(\frac{\sigma_t}{\sigma_{t0}}\right)^3 - \frac{\sigma_t}{\sigma_{t0}} = \frac{I_b}{4\sqrt{\pi}\,\alpha\,\omega_0\,\sigma_{t0}\,\sigma_\delta^2\,E/e}\,\mathrm{Im}\left(\frac{Z_\parallel}{n}\right)_{\mathrm{eff}} \qquad \sqrt{2\pi} \rightarrow 4\sqrt{\pi}$$

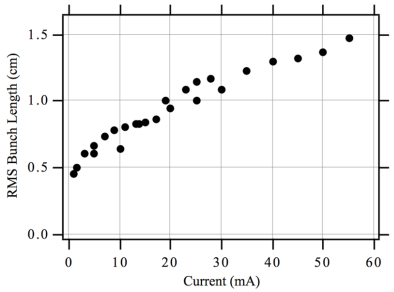
Examples



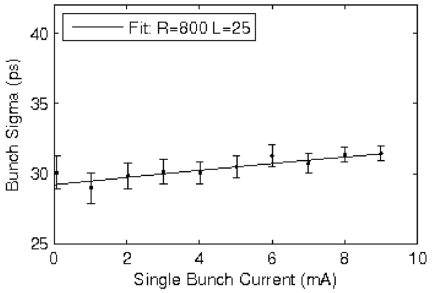


Measured Bunch Lengthening

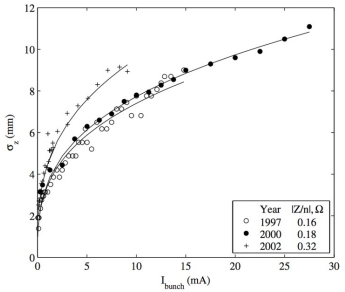
ALS PAC-1995, WXE06



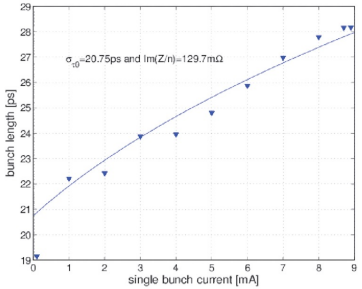
AS EPAC-2008, TUPC010



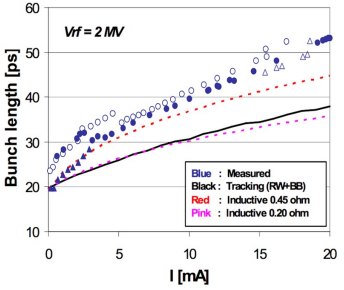
ELETTRA PRSTAB 6.030703



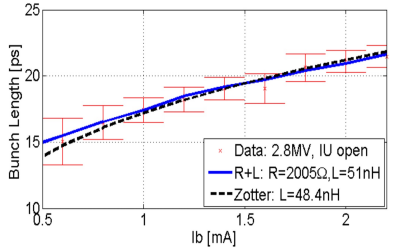
ALBA IPAC-2014, TUPRI052



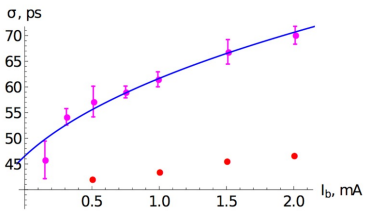
SOLEIL PAC-2007, WEOBC01



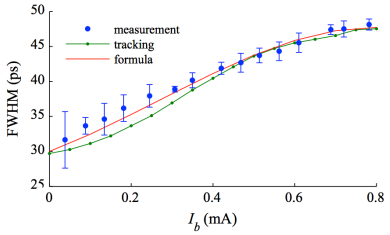
TPS IPAC-2016, MOPOR016



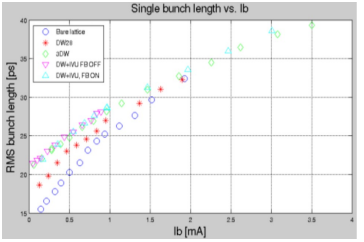
MAX-IV NAPAC-2016, WEA3CO04



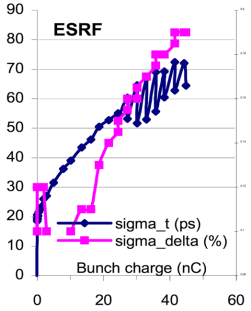
DLS PRSTAB 18.064401



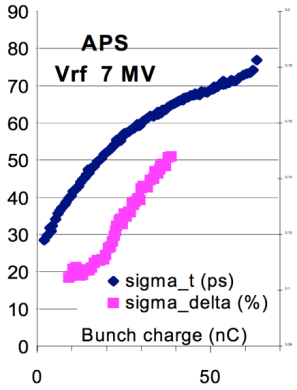
NSLS-II IPAC-2015, TUAB2



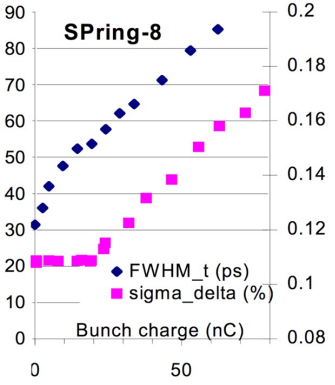
ESRF EPAC-2002, WEPRI013



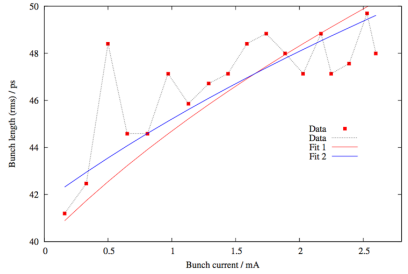
APS EPAC-2002, WEPRI013



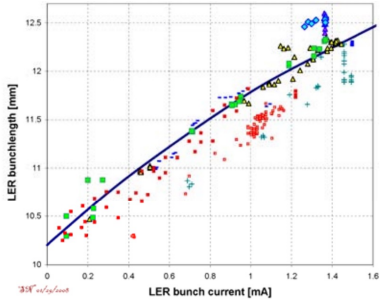
SPRING8 EPAC-2002, WEPRI013



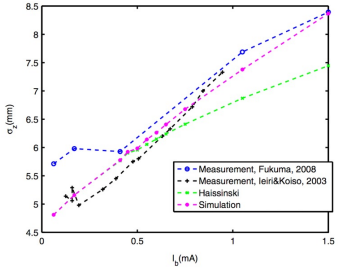
PETRA-III IPAC-2011, MOPS055



PEP-II EPAC-2008, TUPP055



KEKB PRSTAB 12.061002



Head-Tail Mode Coupling: Betatron Tune Shift

Eigenvalue problem for head-tail modes:

A.Chao, Physics of Collective Beam Instabilities. Wiley, NY, 1993

$$\det\left[\left(\frac{\Delta\Omega_\beta}{\omega_s}-l\right)\mathbf{I}-\mathbf{M}\right]=0$$

$$M_{kk'}=\frac{I_b}{2\omega_s\,E/e}\sum_i\beta_i\int_{-\infty}^{\infty}Z_{\perp i}(\omega)g_{lk}(\omega-\omega_\xi)g_{lk'}(\omega-\omega_\xi)d\omega$$

$$g_{lk}(\omega)=\frac{(-1)^{|l|}}{\sqrt{2\pi k!(|l|+k)!}}\left(\frac{\omega\sigma_t}{\sqrt{2}}\right)^{|l|+2k}e^{-\frac{\omega^2\sigma_t^2}{2}}$$ (for Gaussian bunch)

Linear approximation:

if $\Delta\nu_\beta \ll \nu_s$

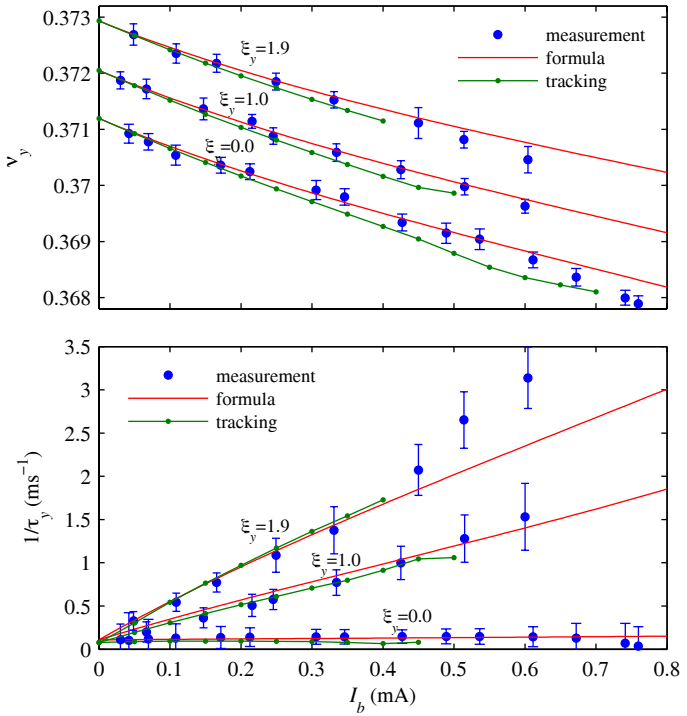
$$\frac{d\Omega_\beta}{dI_b}=i\frac{1}{4\sqrt{\pi}\sigma_t\,E/e}\sum_i\beta_i Z_{\perp i}^{\text{eff}}$$

$$\Delta\nu_\beta=-\frac{I_b}{2\omega_0\,E/e}\sum_i\beta_i k_{\perp i}$$

Examples

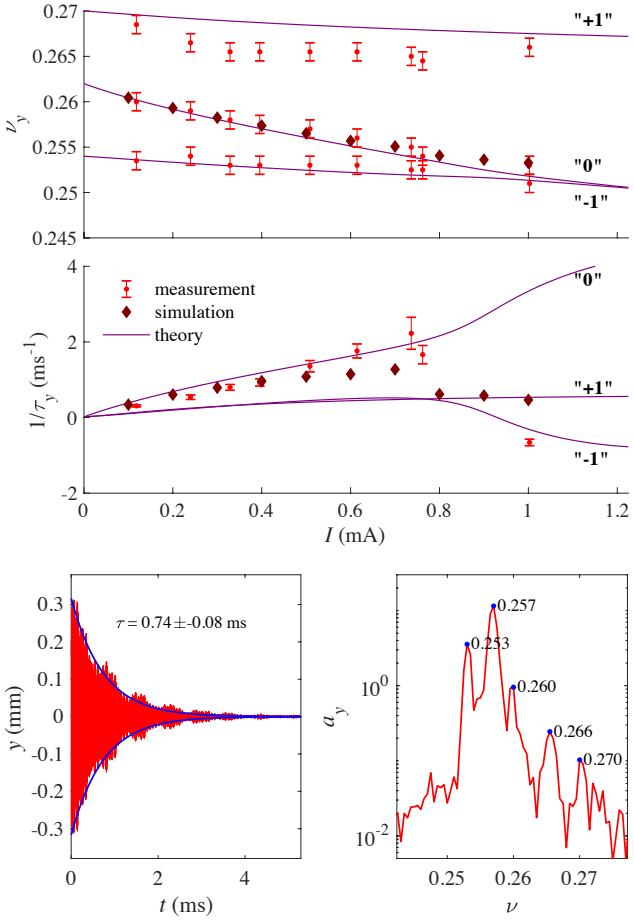
Diamond Light Source

PRSTAB 18 (2015) 064401



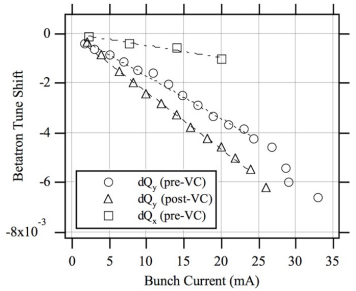
NSLS-II

PRAB 24 (2021) 054401

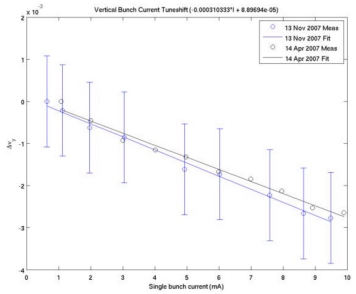


Measured Betatron Tune Shift

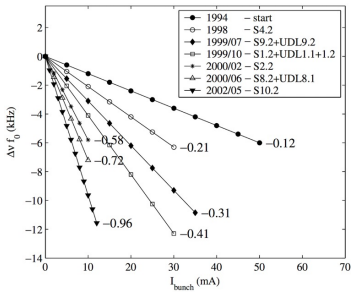
ALS PAC-1995, WXE06



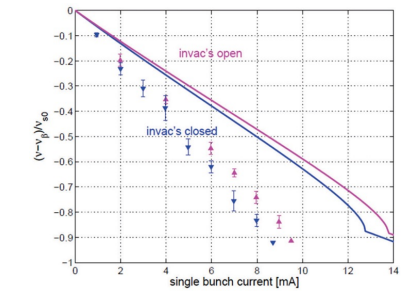
AS EPAC-2008, TUPC010



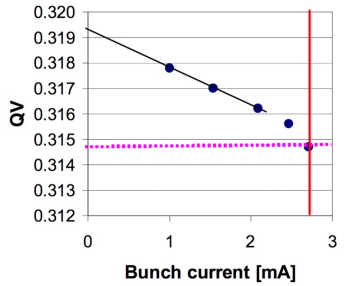
ELETTRA PRSTAB 6.030703



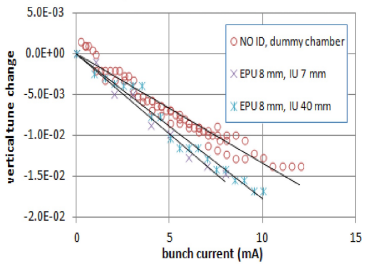
ALBA IPAC-2014, TUPRI052



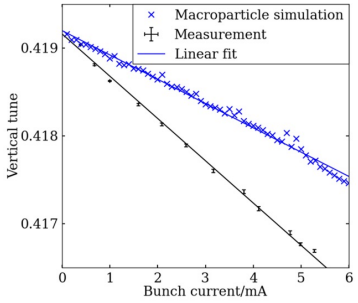
SOLEIL PAC-2007, WEOBC01



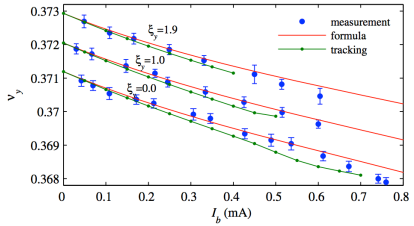
TPS IPAC-2016, MOPOR016



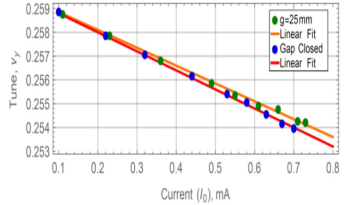
MAX-IV NAPAC-2016, WEA3CO04



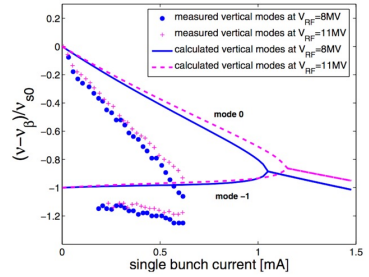
DLS PRSTAB 18.064401



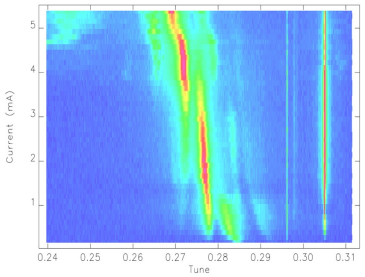
NSLS-II IPAC-2015, TUAB2



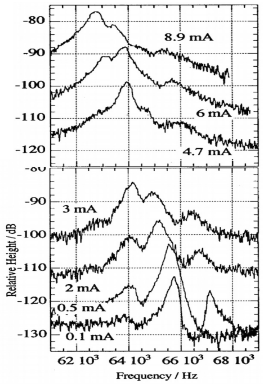
ESRF PRSTAB 9.114402



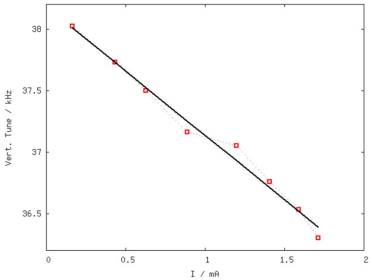
APS PAC-1999, TUP72



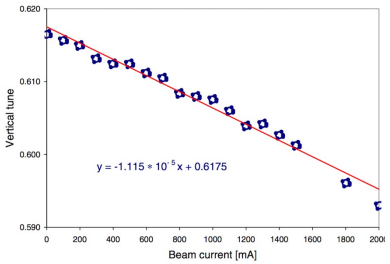
SPRING8 BIW?



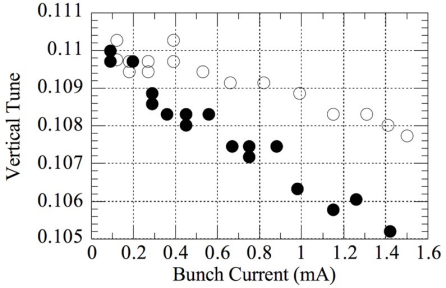
PETRA-III IPAC-2010, WEPEA018



PEP-II PRSTAB.10.011001



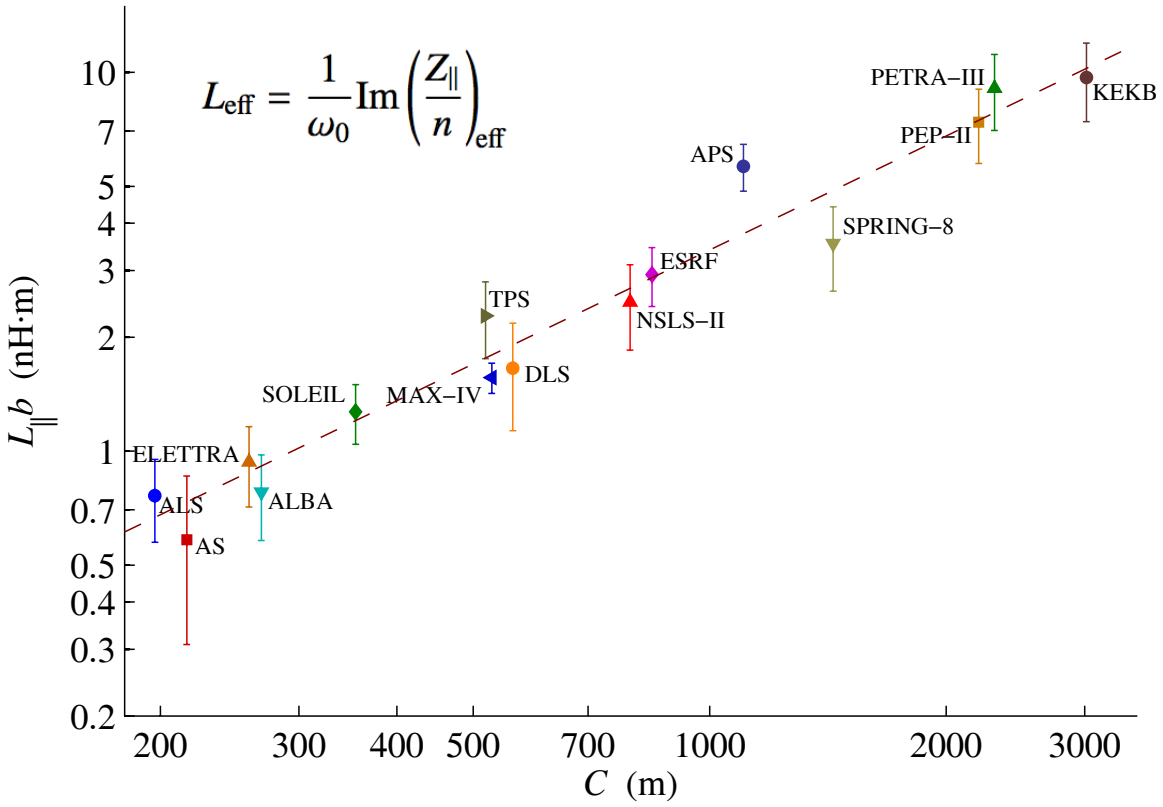
KEKB EPAC-2000, TUP4A12



Impedance from the Measurements

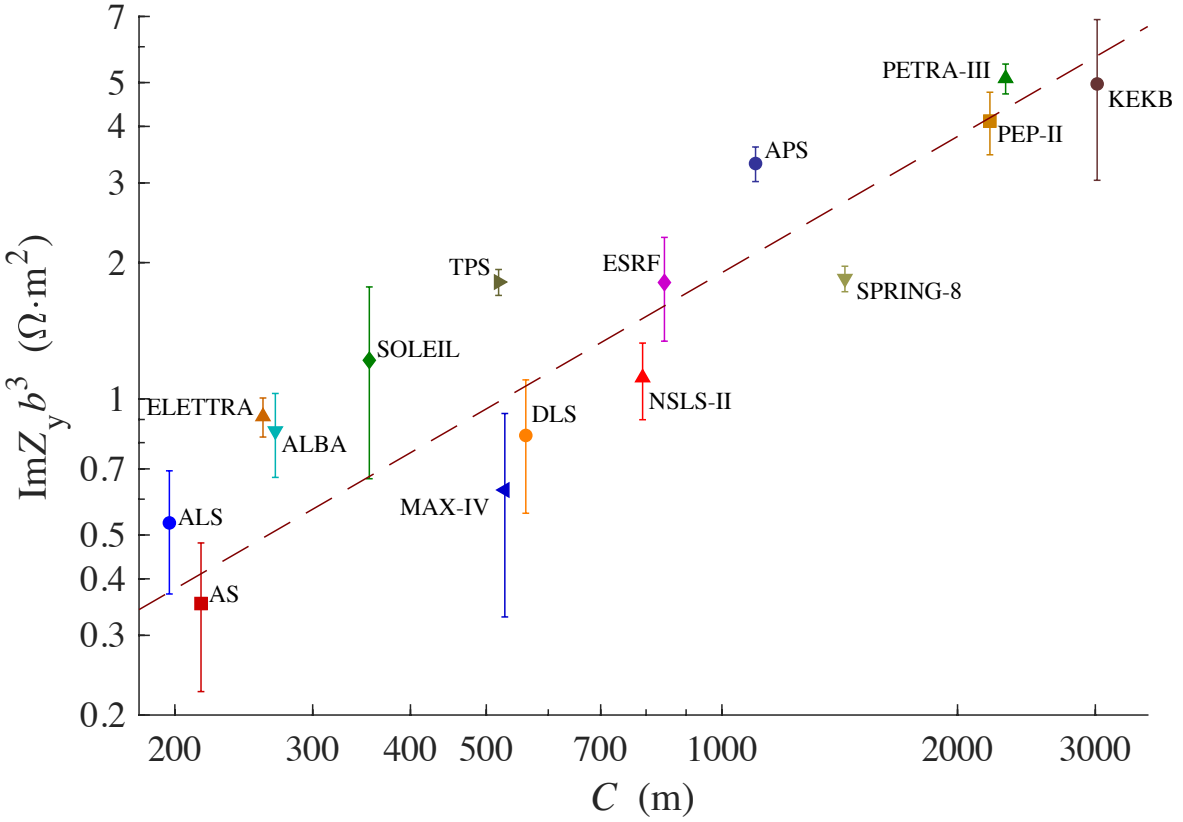
Fit measured bunch lengthening with the modified Zotter equation

$$\left(\frac{\sigma_t}{\sigma_{t0}}\right)^3 - \frac{\sigma_t}{\sigma_{t0}} = \frac{I_b}{4\sqrt{\pi}\alpha\omega_0\sigma_{t0}\sigma_\delta^2 E/e}\,\mathrm{Im}\left(\frac{Z_\parallel}{n}\right)_{\mathrm{eff}}$$



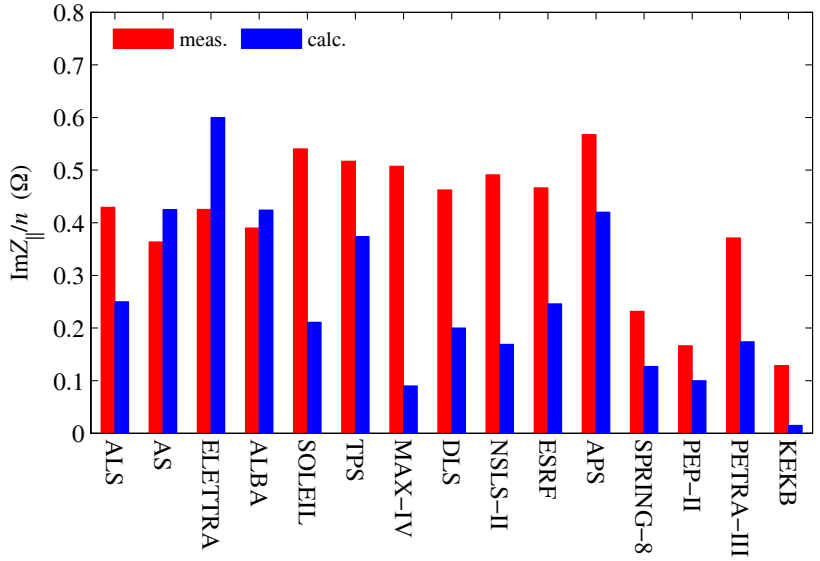
Fit measured betatron tune shift with the linear formula valid for $\Delta\nu_\beta \ll \nu_s$

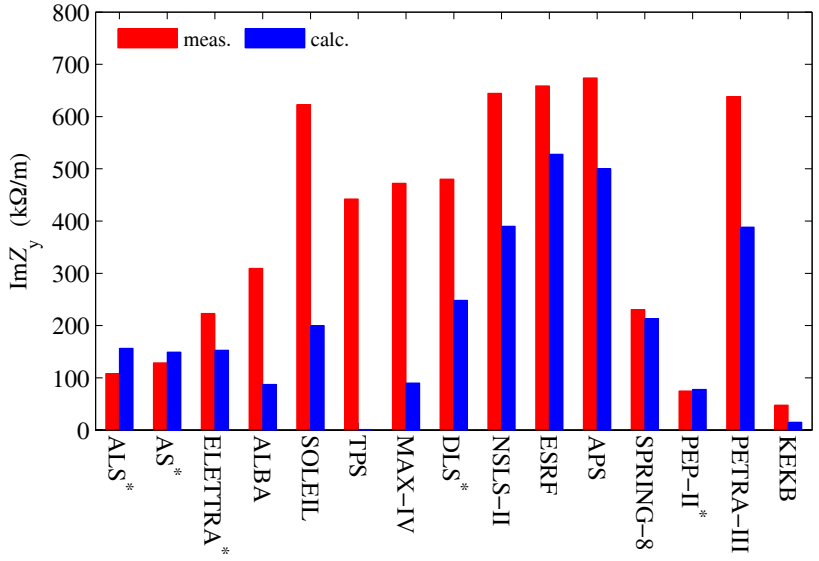
$$\Delta\nu_y = -\frac{I_b}{2\omega_0 E/e}\sum_j \beta_{yj}k_{yj} \approx -\frac{I_b}{4\sqrt{\pi}\sigma_t\omega_0 E/e}\,\beta_y^{\mathrm{aver}}\,\mathrm{Im}Z_y(0)$$



NIM A 888 (2018) 22-30

Measured Impedance vs Computed Budgets





	C (m)	E (GeV)*	σ_{t0} (ps)**	σ_E/E 10^{-3}	α 10^{-3}	ν_x	ν_y	$\beta_{x\,\mathrm{aver}}$ (m)	$\beta_{y\,\mathrm{aver}}$ (m)	$2a$ (mm)	$2b$ (mm)	
ALS	196.8	1.52	14	0.71	1.59	14.25	9.2	5	5.4	96	34	PAC-1995 WXE06
AS	216	3	29	1.02	2.11	13.3	5.2	8	14.1	70	28	NIM A 521 (2004) 306
ELETTRA	259.2	0.9	8	0.36	1.55	14.3	8.2	7.8	6.4	80	32	PRSTAB 6 (2003) 030703
ALBA	268.8	3	21	1.05	0.89	18.18	8.37	6.6	9.2	72	28	PAC-2005 MPPE069
SOLEIL	354.4	2.75	21	1.02	0.44	18.2	10.23	9	8.4	80	25	EPAC-2006 THPLS009
TPS	518.4	3	10	0.89	0.24	16.18	13.28	8.9	9	70	32	PAC-2009 WE5RFP006
MAX-IV	528	3	49	0.78	0.31	42.2	14.28	3.8	7	22	22	IPAC-2011 MOPS066
DLS	561.6	3	13	0.96	0.17	27.2	13.37	9.6	12.5	80	24	EPAC-2000 WEP7B03
NSLS-II	791.9	3	11	0.51	0.36	33.22	16.26	12.5	13.7	64	24	IPAC-2015 TUAB2
ESRF	844.4	6	20	1	0.19	36.44	14.39	19	22.7	76	28	EPAC-2002 WEPRI013
APS	1104	7	24	0.96	0.23	36.2	19.27	13.5	16	84	34	EPAC-2002 WEPRI013
SPRING-8	1436	8	12	1.09	0.15	40.14	18.35	17	18.1	70	40	EPAC-2002 WEPRI013
PEP-II	2200	3.1	34	0.77	1.23	24.51	13.61	15.9	12.1	110	76	PDG-2015
PETRA-III	2304	6	43	1.27	1.2	37.26	33.2	15.7	20.8	80	40	IPAC-2011 MOPS055
KEKB	3016	3.5	13	0.73	0.32	45.51	43.58	13.1	14.2	94	94	PDG-2015

* Impedance was measured at this energy
** Extrapolated from the measured data

Possible Reasons for the Discrepancy

- Use of approximate analytical formulae
- Missing contributions
- Computation mesh size
- Bandwidth of model impedance (bunch length for simulations)
- Interference of wakefields

Missing Contributions

Example: NSLS-II impedance budget

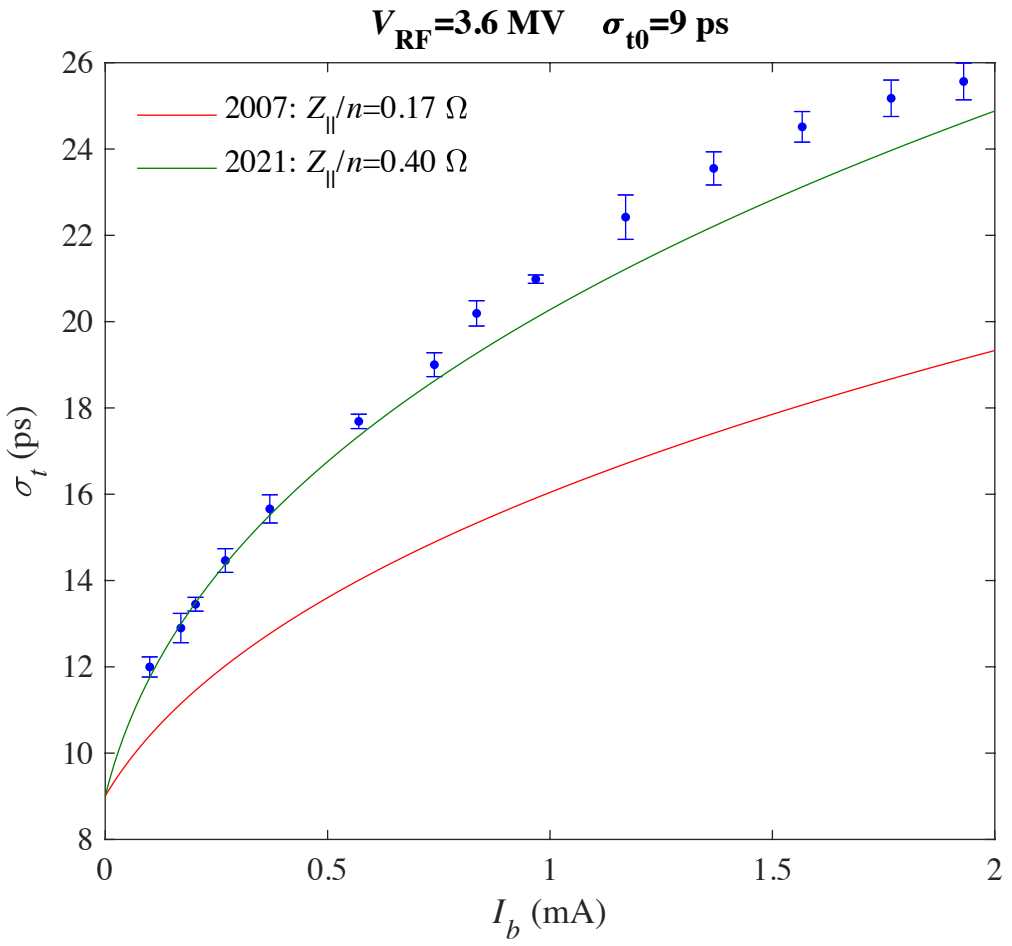
A.Blednykh, S.Krinsky, PAC'2007 FRPMS102

Object	Number of occurrences
Absorber	180
Bellows [1]	180
BPM	270
Cavity transitions/straight	2
500 MHz CESR-B cavity	4
1500 MHz CESR-B cavity	4
Dipole chamber	60
Quad. and Sext. chamber	90
Flange [1]	300
Injection Region	1
SCU chamber geometric	TBD
SCU chamber ease* (2.5m)	TBD
IR chamber [2]	4
CPMU geometric	TBD
CPMU resistive wall (3.5m)	TBD
720m Al resistive wall	1
Scraper (Horizontal)	2
Scraper (Vertical)	2

A.Blednykh, et al., NIM A 1005 (2021) 165349

Object	Abbreviation	Number of components
Bellows	BLW	218
Large aperture BPM	LABPM	237
Small aperture BPM (11.5 mm × 60 mm)	SABPMDW	10
Small aperture BPM (8 mm × 55 mm)	SABPMEPU	3
Damping wiggler chamber (11.5 mm × 60 mm)	DW	3
Elliptically polarized undulator chamber (11.5 mm × 60 mm)	EPU1	2
Elliptically polarized undulator chamber (8 mm × 55 mm)	EPU2	2
Gate valve (standard)	GV	61
Flange absorber (21 mm × 64 mm)	FABS	67
Flange absorber S4 (21 mm × 64 mm)	FABSS4	39
Flange absorber (Rest)	FABS2	7
Stripline (BBF), L = 300 mm	SL300	2
Standard RF sealed flanges	FLNG	739
EPU RF sealed flanges	EPUFLNG	4
DW RF sealed flanges	DWFLNG	13
Direct-current current transformer	DCCT	1
Kickers Ti-coated ceramics chambers	CCHM	5
RF HOM damper	HOMD	2
500 MHz RF cavity[a]	CAV	2
RF tapered transition	TPRDRF	1
RF flange absorber (21 mm × 64 mm)	FABSRF	1
Stripline (TMS), L = 150 mm	SL150	2
In-vacuum undulator	IVU	9
Dipole vacuum chamber	DCHM	60
Multipole vacuum chamber	MCHM	90
Resistive wall	RW	1

Buch lengthening measured in July 2021



Computation Mesh Size

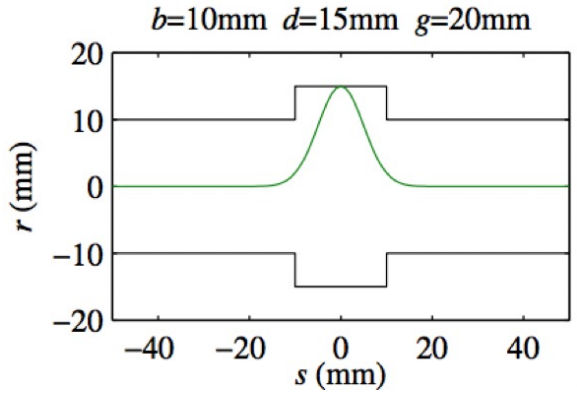
Example:

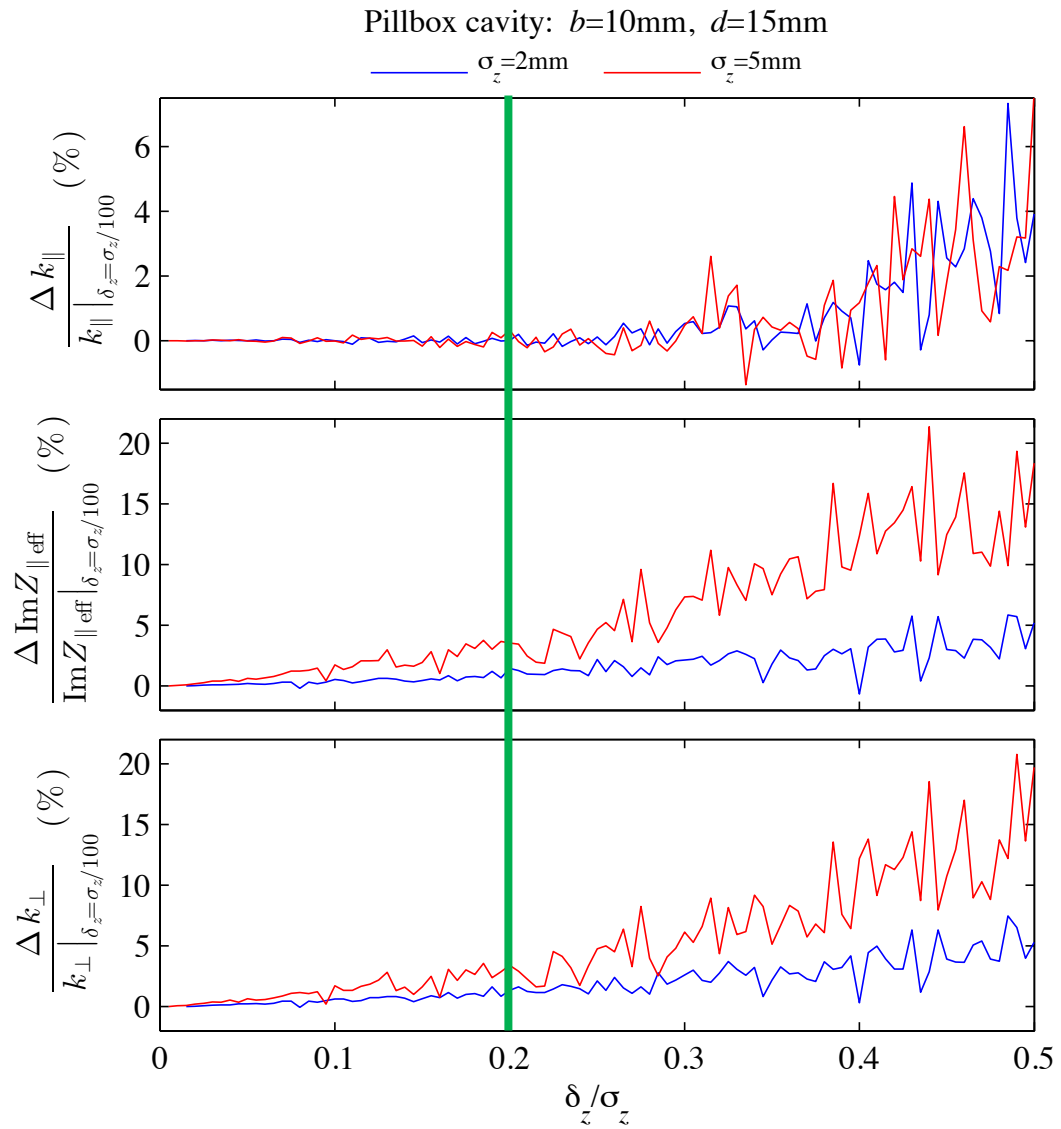
Pill-box cavity

ECHO simulations:

Relative errors of the integral parameters vs. the computation mesh size

The reference mesh size is $\sigma_z/100$





***ECHO code:** I.Zagorodnov, NAPAC2016 WEA1IO02*

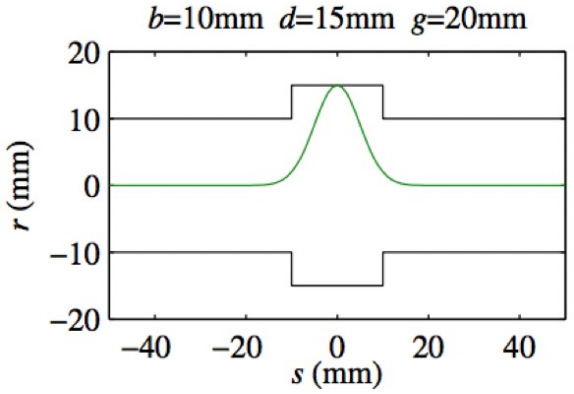
Bandwidth of Model Impedance (Bunch Length for Simulations)

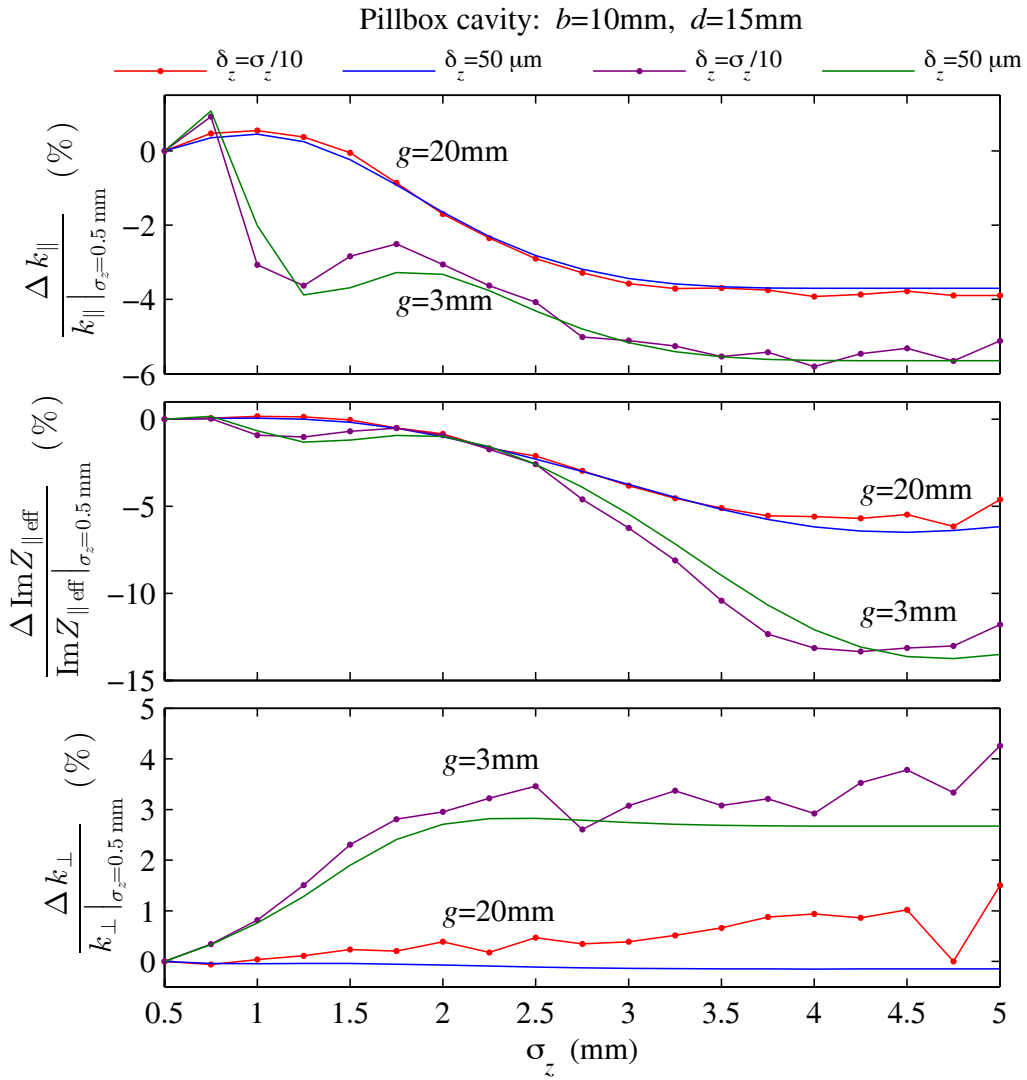
Example:

Pill-box cavity

ECHO simulations:

Relative errors of the integral parameters vs. the bunch length used for the wakefield computations





Interference of Wakefields

Analytical estimates:

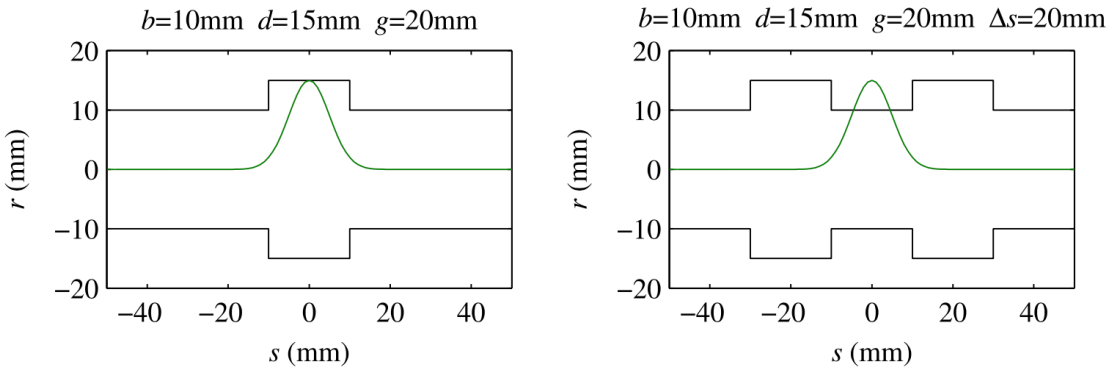
- Periodic array of cavities of small obstacles: R. Gluckstern, Phys. Rev. D. Vol. 39, (1989), 2780-2783
- Longitudinal impedance of one and two pillbox cavities: R.L. Gluckstern, F. Neri, PAC-1987, 1069-1071

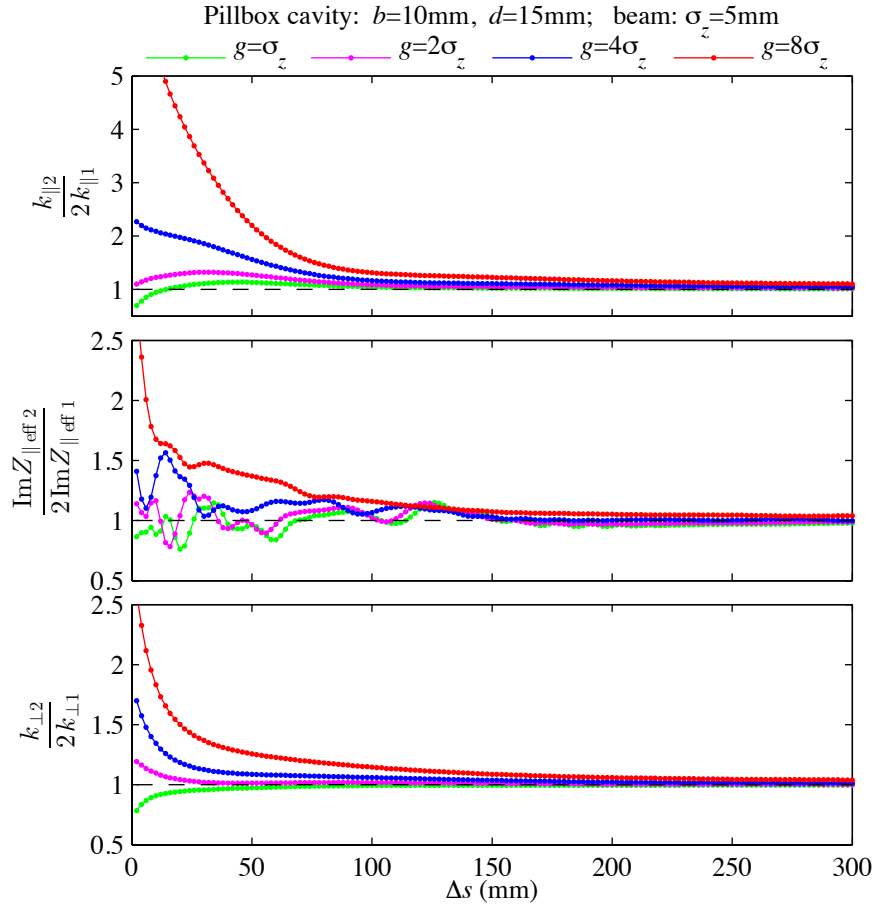
Example:

A structure with two pill-box cavities

ECHO simulations: 1 cavity vs 2 cavities

Relative errors of the integral parameters
vs. the distance between two cavities





What Can We Do Next?

- Cross-check the wakefield simulation codes
- Compare local impedance models with beam-based measurements
- Collect a database including both measurement and simulation results from as many machines as possible
- Implement advanced techniques for data analysis (machine learning?)

Thank you for your attention!