

Interplay of beam-beam and impedance effects at e^+e^- colliders

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Contributors: [Mini-workshop on impedance modeling and impedance effects at SuperKEKB and future colliders](#)

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Outline

- Introduction
- Longitudinal Impedance
- Transverse Impedance
- Summary

Crab-waist collision

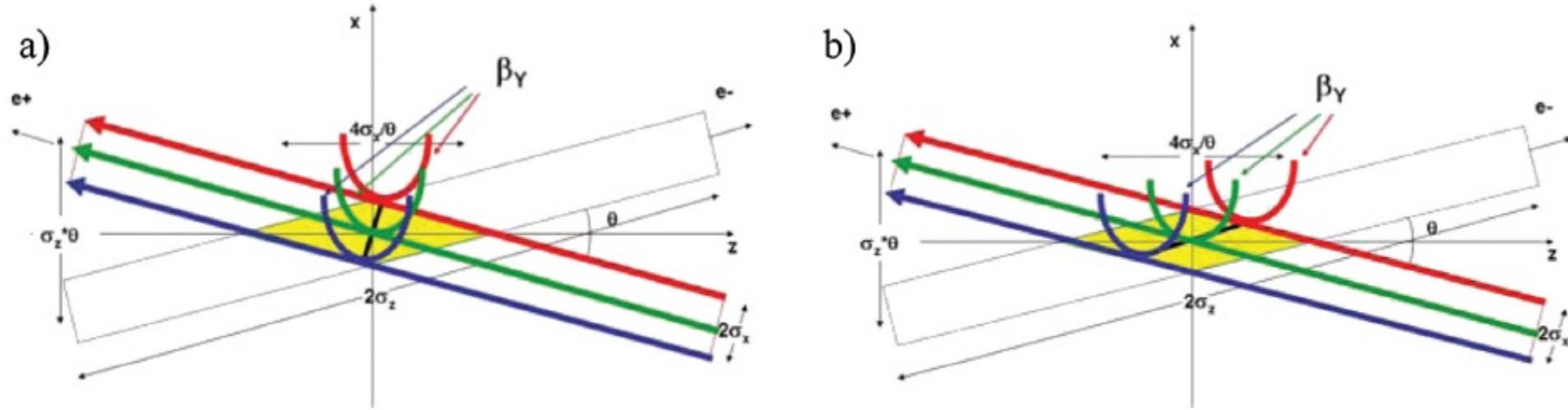


FIG. 1 (color). Crab-waist collision scheme. The color straight lines show directions of motion for particles with different horizontal deviations from the central orbit. The arrows indicate the corresponding β function variations along these trajectories.

$$L \propto \frac{N \xi_y}{\beta_y^*}; \quad \xi_y \propto \frac{N \sqrt{\beta_y^* / \epsilon_y}}{\sigma_z \theta}; \quad \xi_x \propto \frac{N}{(\sigma_z \theta)^2},$$

$$\varphi = \frac{\sigma_z}{\sigma_x} \tan\left(\frac{\theta}{2}\right) \approx \frac{\sigma_z}{\sigma_x} \frac{\theta}{2}.$$

$$\beta_y^* \approx \frac{\sigma_x}{\theta} \ll \sigma_z.$$

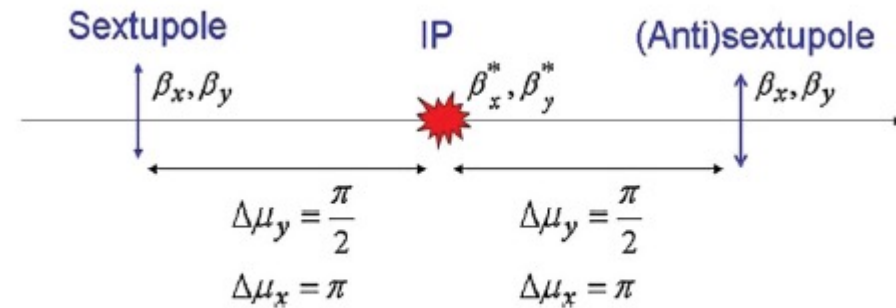
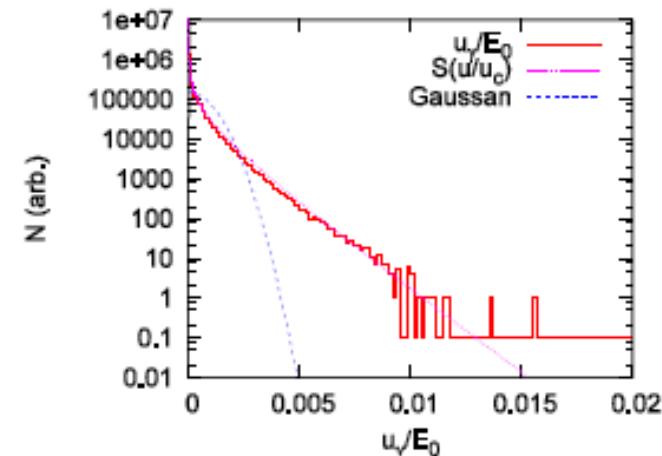
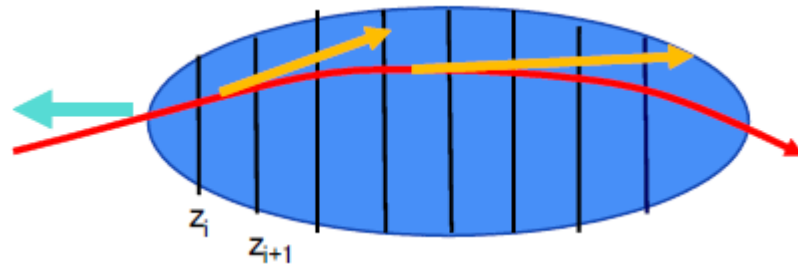


FIG. 2 (color). Crab sextupole locations.

Beamstrahlung Effect & 3D flip-flop

- Synchrotron radiation during beam-beam interaction
- High energy photon \rightarrow Momentum acceptance \rightarrow Lifetime
- Longer bunch length and Higher energy spread
- Beam blowup: 3D flip-flop



Coherent Beam-Beam Instability with a Large Crossing Angle

K. Ohmi, Int. J. Mod. Phys. A, 31, 1644014 (2016).

K. Ohmi and et al., PRL 119, 134801 (2017)

N. Kuroo et al, PHYS. REV. ACCEL. BEAMS 21, 031002 (2018)

K. Ohmi, eeFACT 2018

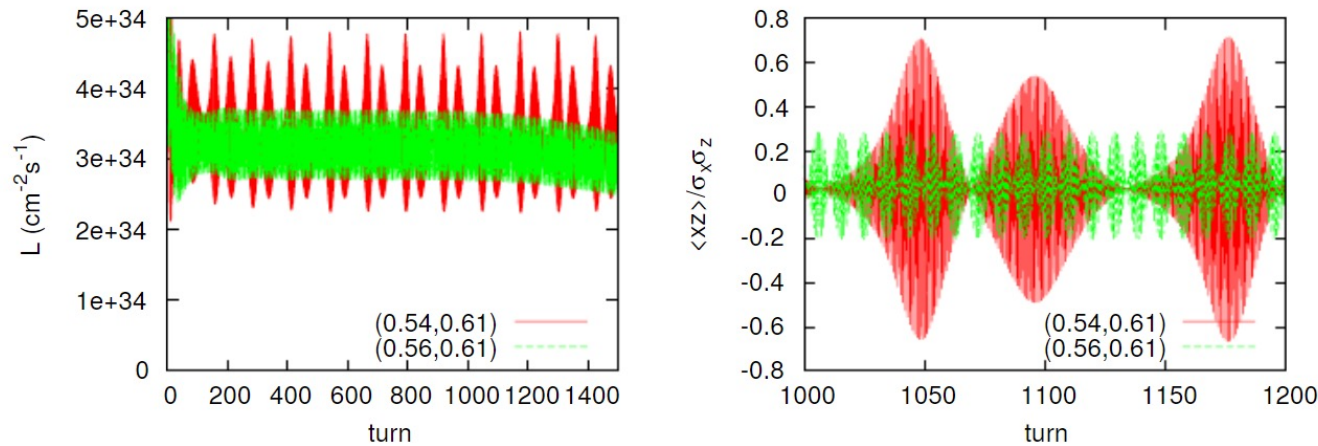


Fig. 5. Luminosity and $\langle xz \rangle$ evolutions given by a strong-strong simulation using BBSS code.

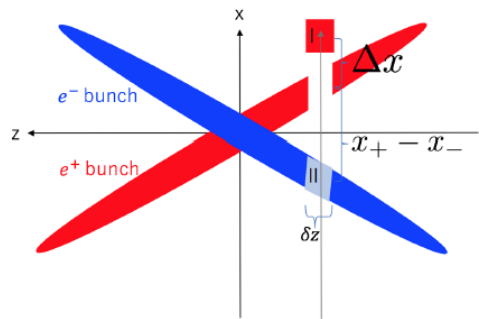


FIG. 1. Illustrative representation of the evaluation of the cross-wake force.

- Usual wake force gives correlation between bunch head to tail. Head-tail instability is induced by synchrotron motion

$$\Delta p_x(z) = - \int_z^\infty W(z - z') \rho_x(z') dz'$$

- **Cross wake field** gives correlation of two colliding beam by convolution of each dipole moment.

$$\Delta p_{x,\mp}(z_\mp) = - \int_{-\infty}^\infty W_x^{(\mp)}(z_\mp - z'_\pm) \rho_x^{(\pm)}(z'_\pm) dz'_\pm$$

- Cross wake force induced by the beam-beam interaction is localized at IP.

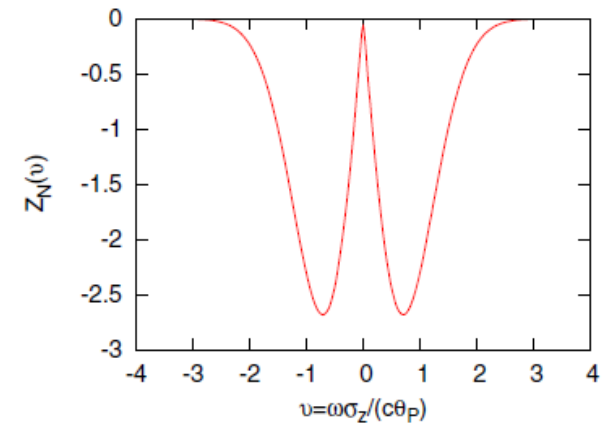
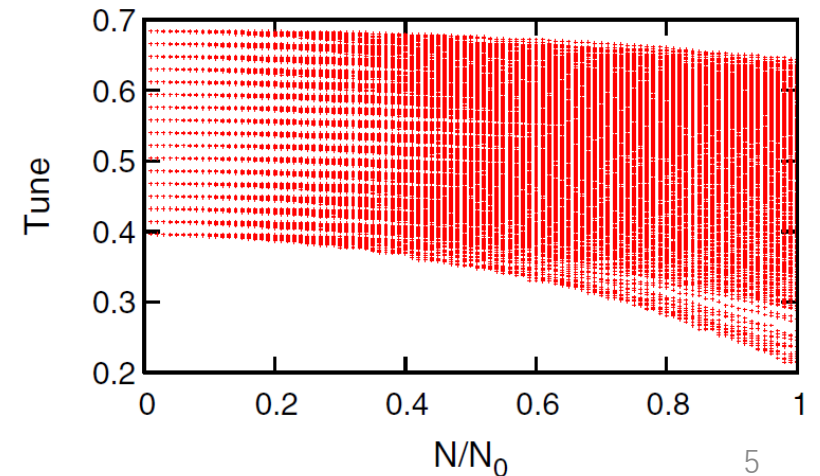


FIG. 3. Imaginary part of normalized cross impedance, where $v = \omega \bar{\sigma}_z / (c \theta_P)$.



Why have we started with the longitudinal impedance and transverse impedance?

1. In the collision scheme with Crab Waist and Large Piwinski Angle the luminosity and tune shifts **strongly depend on the bunch length**

$$L \propto \frac{N\xi_y}{\beta_y^*}, \quad \xi_y \propto \frac{N\sqrt{\beta_y/\varepsilon_y}}{\sigma_z\theta}, \quad \xi_x \propto \frac{N}{(\sigma_z\theta)^2}$$

2. For the future circular colliders with extreme beam parameters in collision several **new effects** become important such as beamstrahlung, coherent X-Z instability and 3D flip-flop. **The longitudinal beam dynamics plays an essential role for these effects**

3. Considering transverse impedance is very natural:

- Why not transverse impedance, since longitudinal impedance is included
- Transverse impedance change the coherent betatron tune

Simulation

IBB

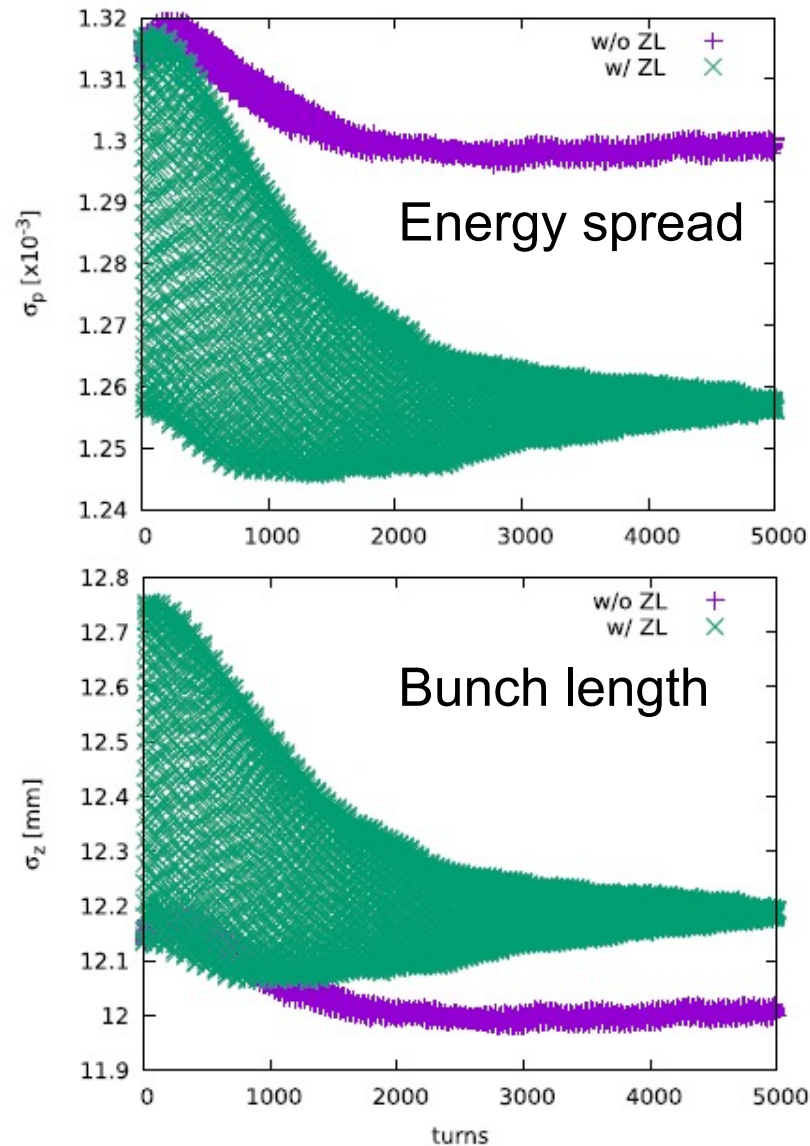
K. Hirata et al., PA 40, 205-228 (1993)
K. Hirata, PRL, 74, 2228 (1995)
Y. Zhang et al., PRST-AB, 8, 074402 (2005)
Y. Seimiya et al., PTP 127, 1099 (2012)
K. Ohmi, IPAC16
Y. Zhang et al., PRAB 23, 104402, (2020)

- Linear Arc Map with SR radiation
- One turn map including general chromaticity
- Horizontal crossing angle: Lorentz boost map
- Bunch slice number is about 10 times Piwinski angle
- Slice-Slice collision: Synchro-beam mapping method (or PIC)
- Synchrotron radiation during collision
- Longitudinal wake potential is calculated in frequency domain before IP each turn
- Transverse wake field kick may be applied once or many times each turn

Machine Parameter (CDR version)

	CEPC-Z	FCCee-Z
Beam Energy	45.5 GeV	45.6 GeV
Bunch Population	8e10	17e10
Arc Cell	90°/90°	60°/60°
$\beta_{x/y}^*$	0.2 m/ 1mm	0.15 m/0.8 mm
ϵ_x/ϵ_y	0.18 nm/1.6 pm	0.27 nm/1.0 pm
ν_s /superperiod	0.014	0.0125
σ_z [SR/BS]	2.42 / 8.5 mm	3.5 / 12.1 mm
σ_p [SR/BS]	3.80 / 8×10^{-4}	3.8 / 13.2×10^{-4}
ξ_x [BS]	0.004	0.004
ξ_y [BS]	0.079	0.133
Piwinski Angle [SR/BS]	6.6 / 23	8.2 / 28.5

Combined effect of beamstrahlung and longitudinal impedance in stable tune areas



Semi-analytical calculations are in reasonable agreement with numerical modeling

TABLE IV. The FCC-ee beam energy spread and length as well as the synchrotron tune parameter due to the combined effect of SR, BS, and PWD.

E [GeV]	45.6
σ_E	0.00126 ^a
	0.00132 ^b
σ_z [mm]	12.2 ^a
	12.6 ^b
ν_s/ν_{s0}	0.964 ^b

^aBeam-beam simulation [21].

^bSemianalytical model (SR + BS + PWD).

Longitudinal Impedance induces

- Longer bunch length
- Lower energy spread
- Lower incoherent synchrotron tune

Review of CDR parameters of CEPC-Z Considering Impedance

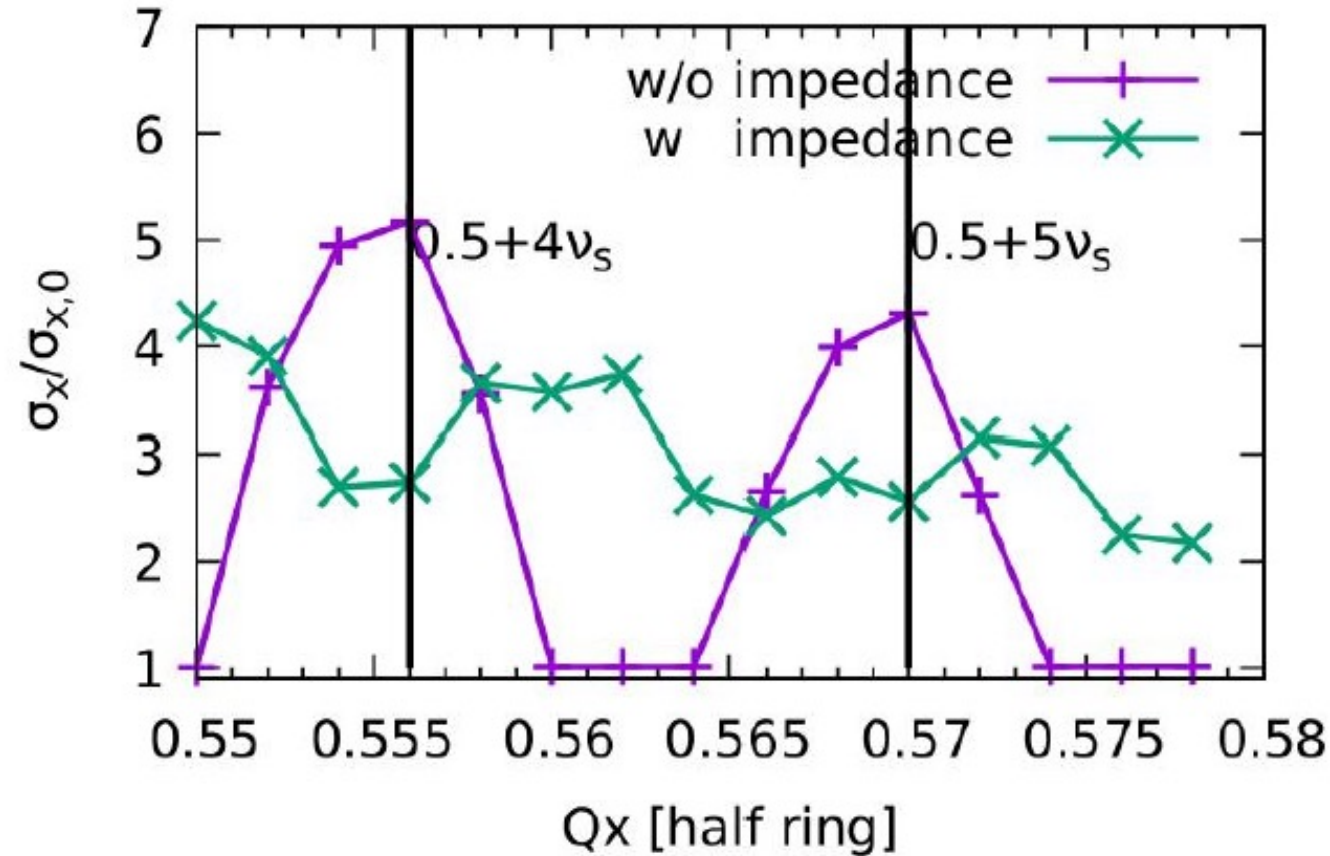


Figure 13: Horizontal beam size blow up in collision obtained by simulation with and without impedance.

X-Z instability tune scan with and without beam coupling impedance (CEPC)

After the horizontal beta function reduction from 0.2 m down to 0.15 m

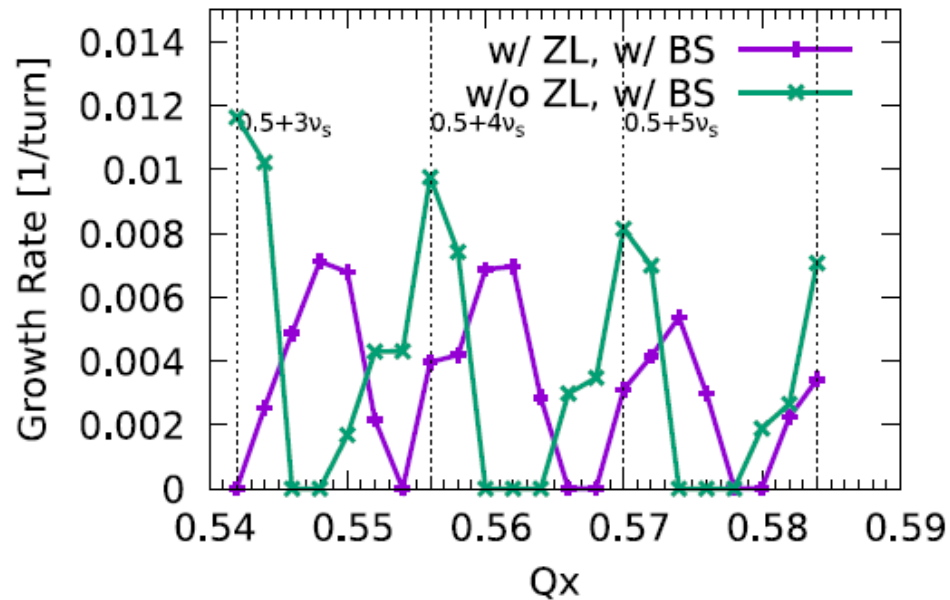
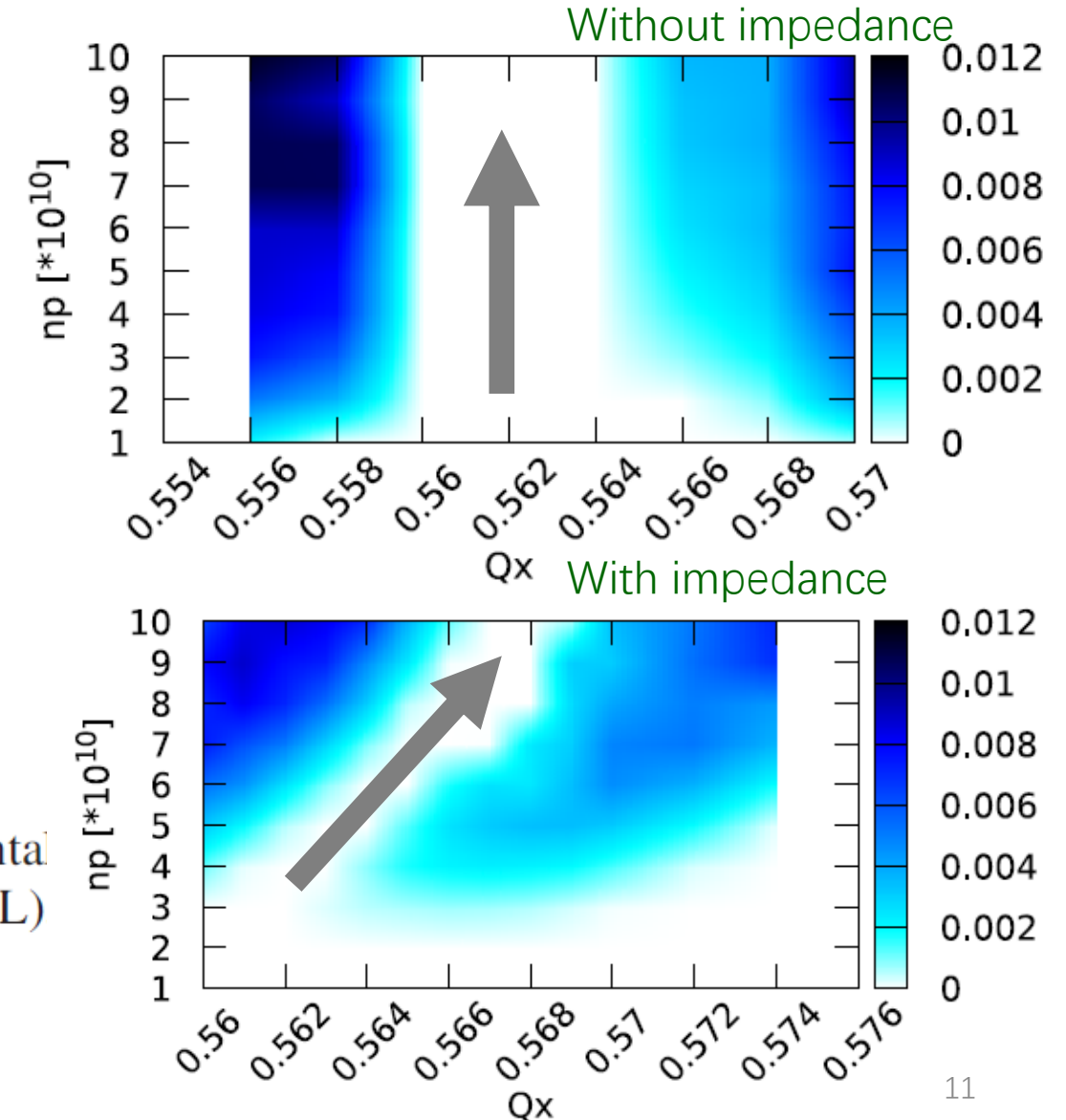


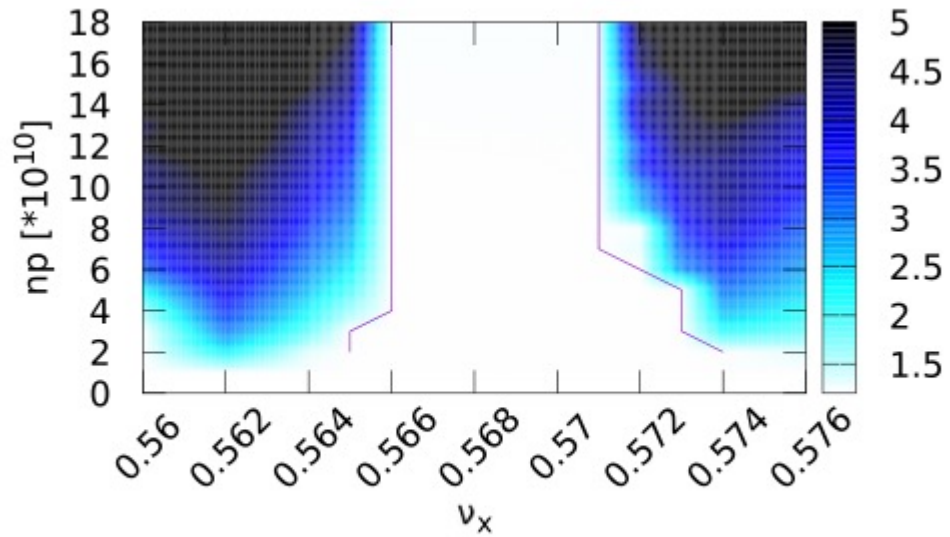
FIG. 3. The horizontal beam size growth rate versus horizontal tune with and without longitudinal coupling impedance (ZL) Beamstrahlung (BS) effect is turned on.

By including the impedance stable areas become narrower and are shifted

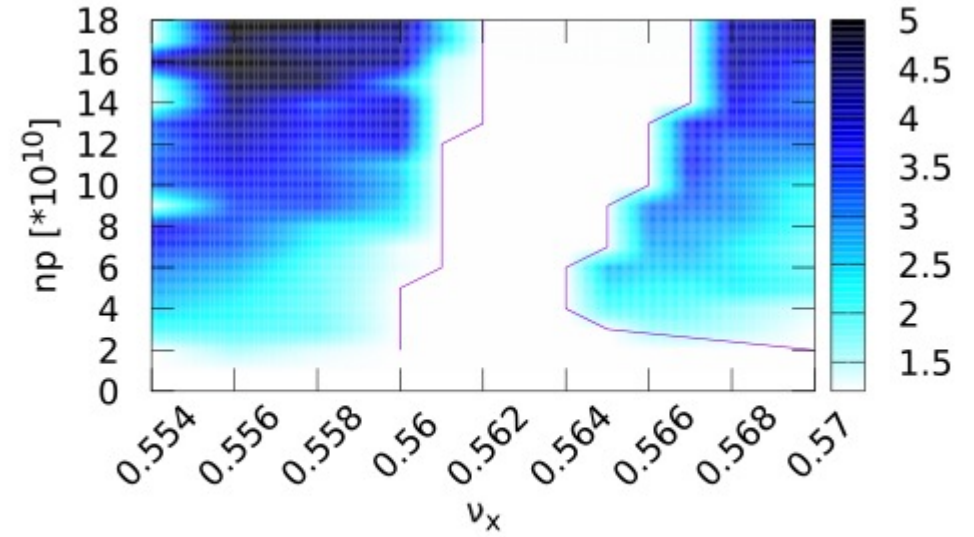


X-Z instability tune scan with and without coupling impedance (FCC-ee, CDR)

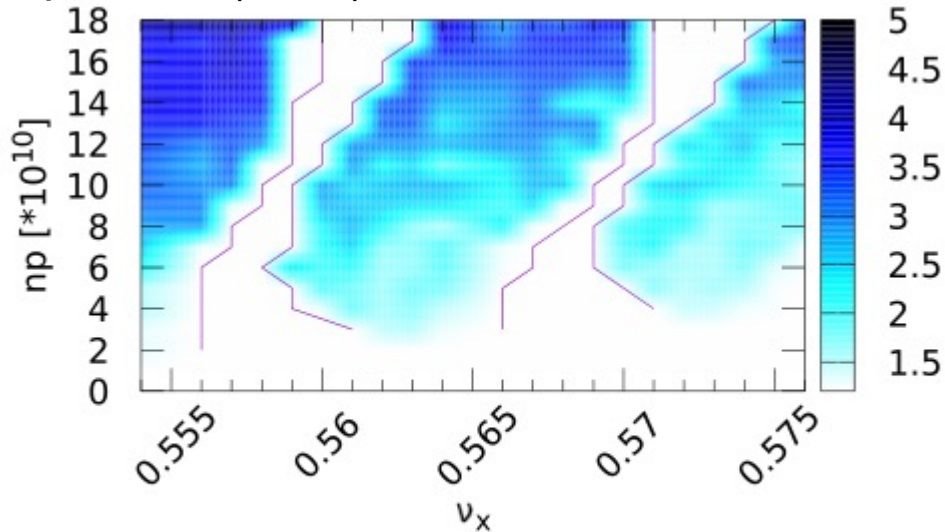
w/o ZL



Only with RW



with full impedance(2021)



doi:10.18429/JACoW-IPAC2021-MOXC01
Eur. Phys. J. Plus (2021) 136:1190

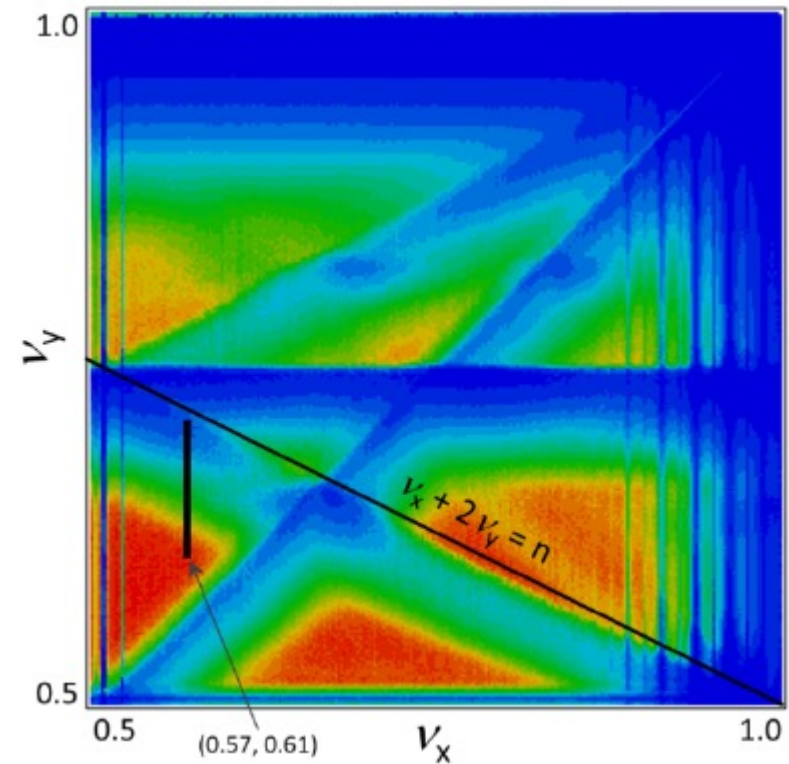
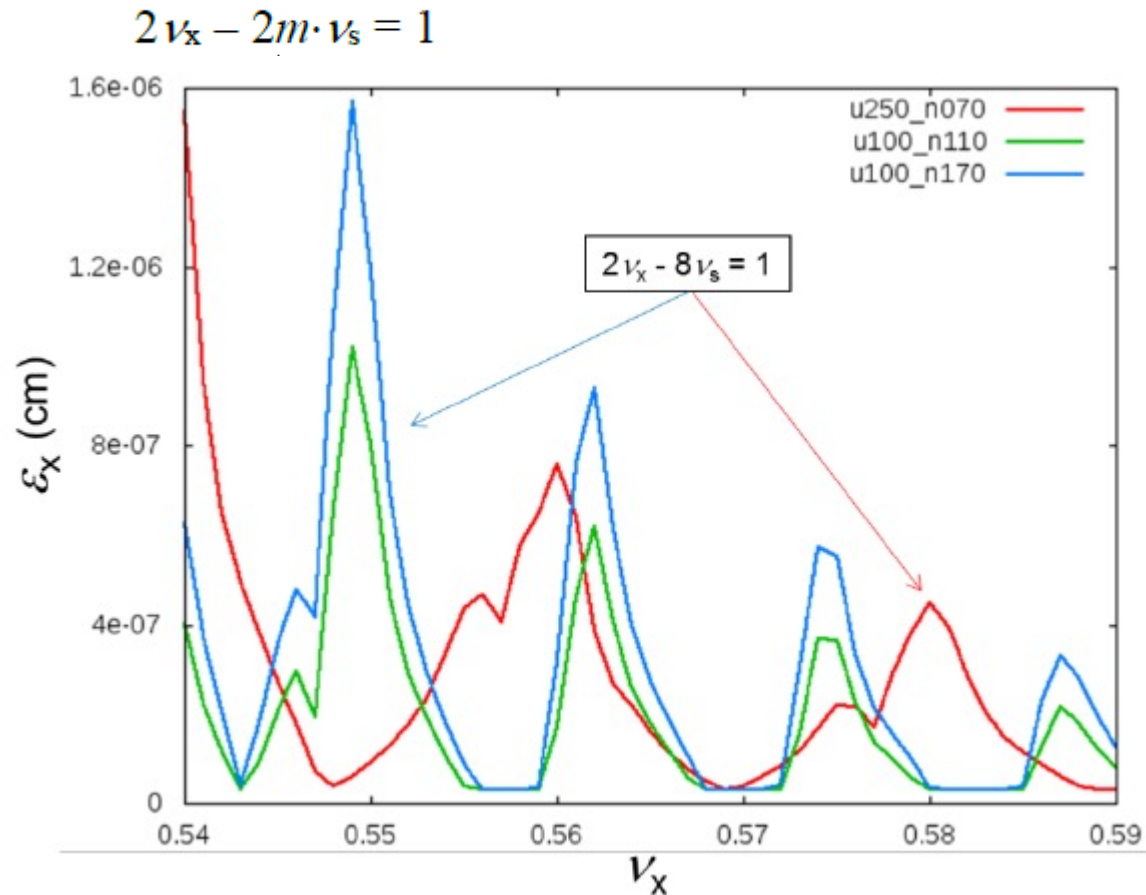
Parameter Optimization

$$N_{th} \propto \frac{\alpha_p \sigma_\delta \sigma_z}{\beta_x^*},$$

$$\alpha_p \sigma_\delta \propto v_s \sigma_z$$

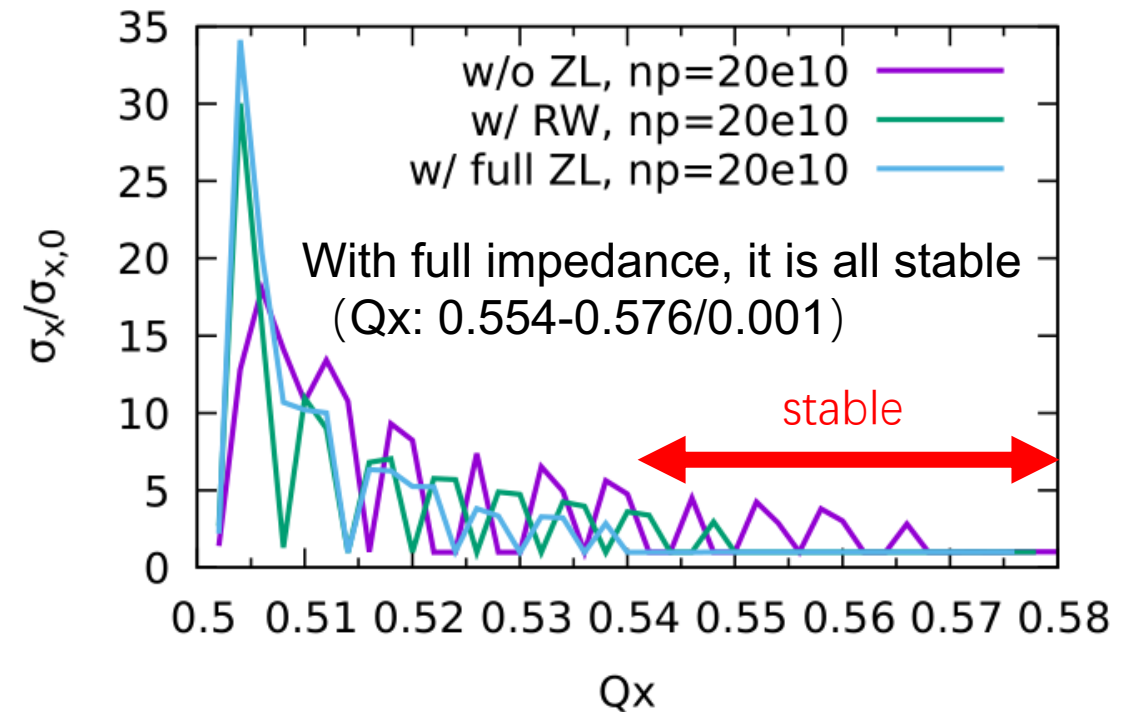
$$\xi_x \propto N_p \beta_x^* / \sigma_z^2$$

Larger v_s / ξ_x is preferred!



Idea of using harmonic cavities

- Lower synchrotron tune
- Higher order X-Z resonances
- Landau damping due to higher synchrotron frequency spread
- Longer bunches reduce the horizontal tune shift, which helps in suppressing the X-Z instability.
- Longer bunches in collision result in a smaller energy spread due to beamstrahlung.



$$\sigma_z = 18 \text{ mm (CDR: 12 mm)}$$
$$\sigma_p = 9.2\text{e-}4 \text{ (CDR: } 13\text{e-}4\text{)}$$

Higher Momentum Compaction

- CEPC-Z: $90^\circ/90^\circ$ (CDR) to $60^\circ/60^\circ$
- FCC-ee-Z CDR: $60^\circ/60^\circ$ FODO cell
- Switching from $60^\circ/60^\circ$ to $45^\circ/45^\circ$ arc cell lattice has been proposed for FCC-ee Z. The lattice for $45^\circ/45^\circ$ does not exist yet*.
- To restore the luminosity of CDR, higher bunch population ($28e10$) has been proposed.

FCCee

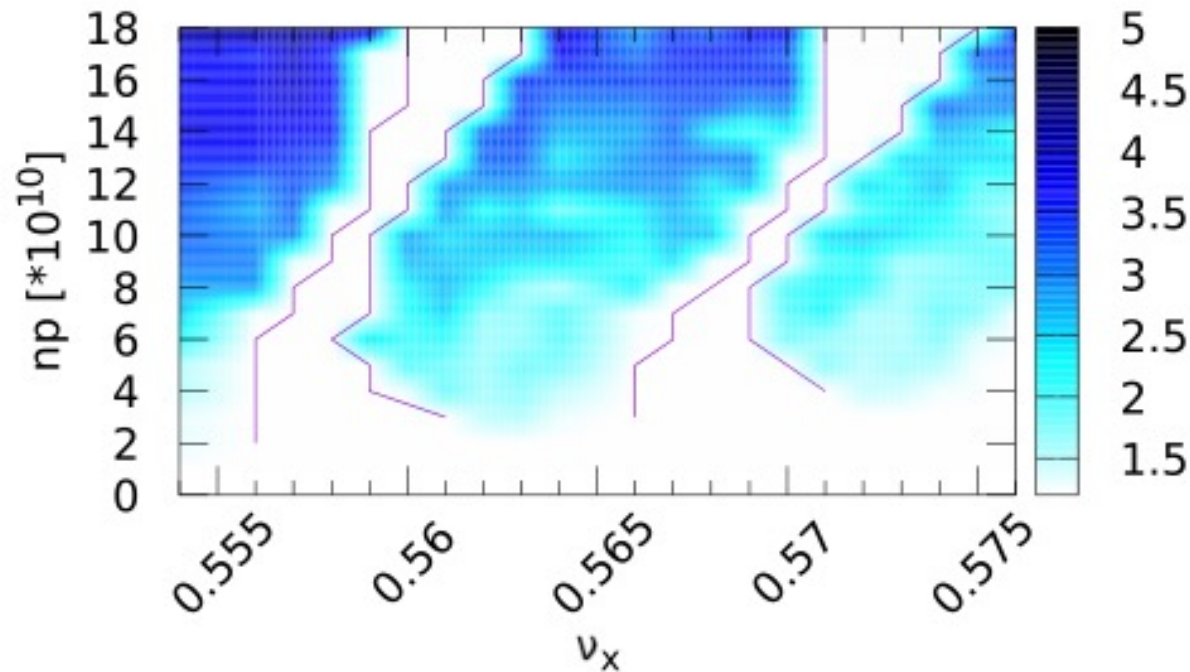
Arc Cell	α_p [10^{-5}]	ϵ_x [nm]	ϵ_y [pm]	ν_s	σ_{z0} [mm]	σ_z [mm]	σ_p [10^{-4}]	L/IP 10^{36}	ϕ	ξ_x
45°	2.5	0.6	1.5	0.0163	4.5	11.5	9.7	1.9	18.2	0.004
60°	1.48	0.27	1.0	0.0125	3.5	12	13	2.3	28.5	0.004

* This is info in 2021, not updated here. The parameters are also old ones estimated by D. Shatilov.

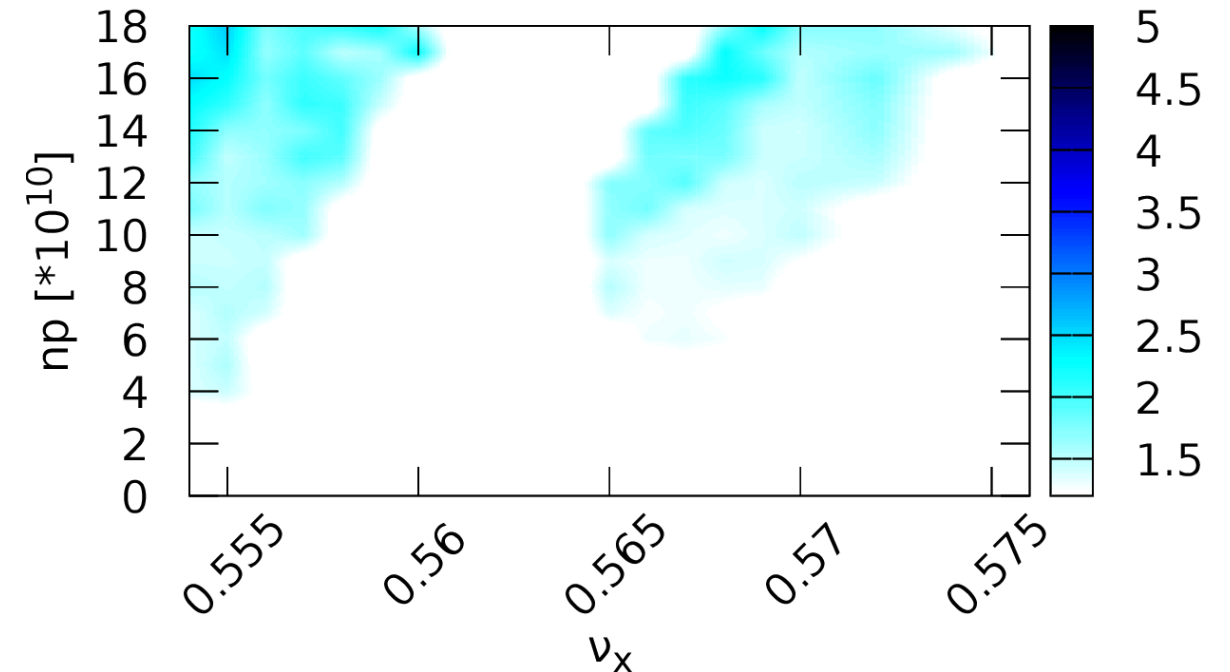
Bootstrapping Injection

- Instability growth rate during injection

- Good tune area
- Higher synchrotron frequency is better for the energy calibration

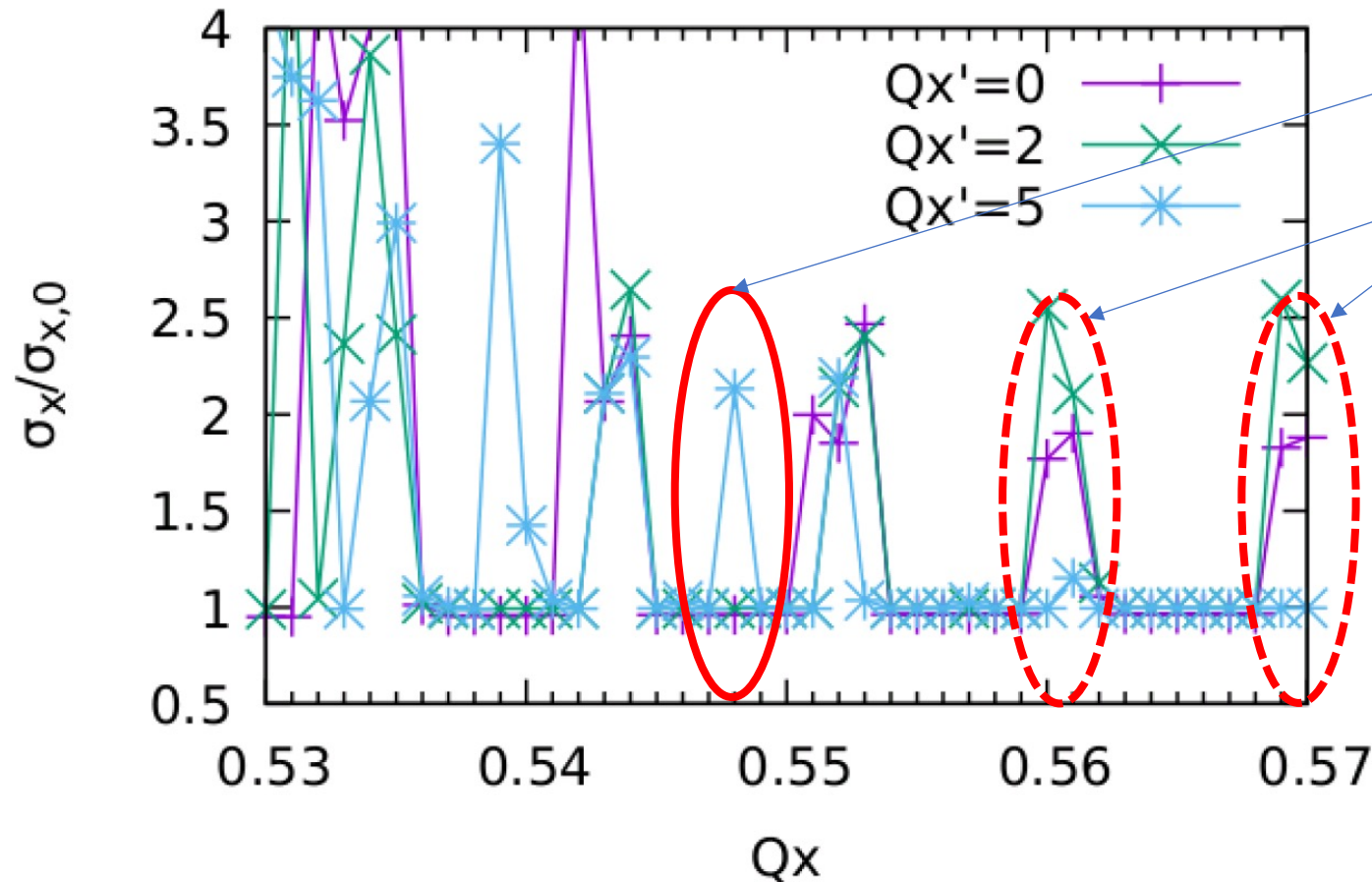


FODO CELL: 60°/60°



FODO CELL: 45°/45°

Effect of Chromaticity on X-Z instability (FCCee)



- Non-zero tune chromaticity bring new resonance
- In the high order resonance region ($0.5+n*\nu_s$), some resonance may be suppressed or weakened

Future work :

- Analysis work considering linear tune chromaticity
- Simulation work considering realistic chromaticity (from lattice model)

Analysis with PWD

- Azimuthal Mode Expansion

$$x(J, \phi) = \sum_{l=-\infty}^{\infty} x_l(J) e^{il\phi}, \quad p_x(J, \phi) = \sum_{l=-\infty}^{\infty} p_l(J) e^{il\phi}$$

- Synchro-betatron motion in Arc

$$\begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} = e^{-2\pi i l \nu_s(J)} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix}$$

- Action Discretization

we truncate l at $\pm l_{\max}$, and discretize J at J_1, J_2, \dots, J_{n_J} .

For example,

$$\begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} = e^{-2\pi i l \nu_s(J_i)} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} \quad n_J = 40, \quad l_{\max} = 8$$

- Momentum Kick due to localized Wake force (beam-beam)

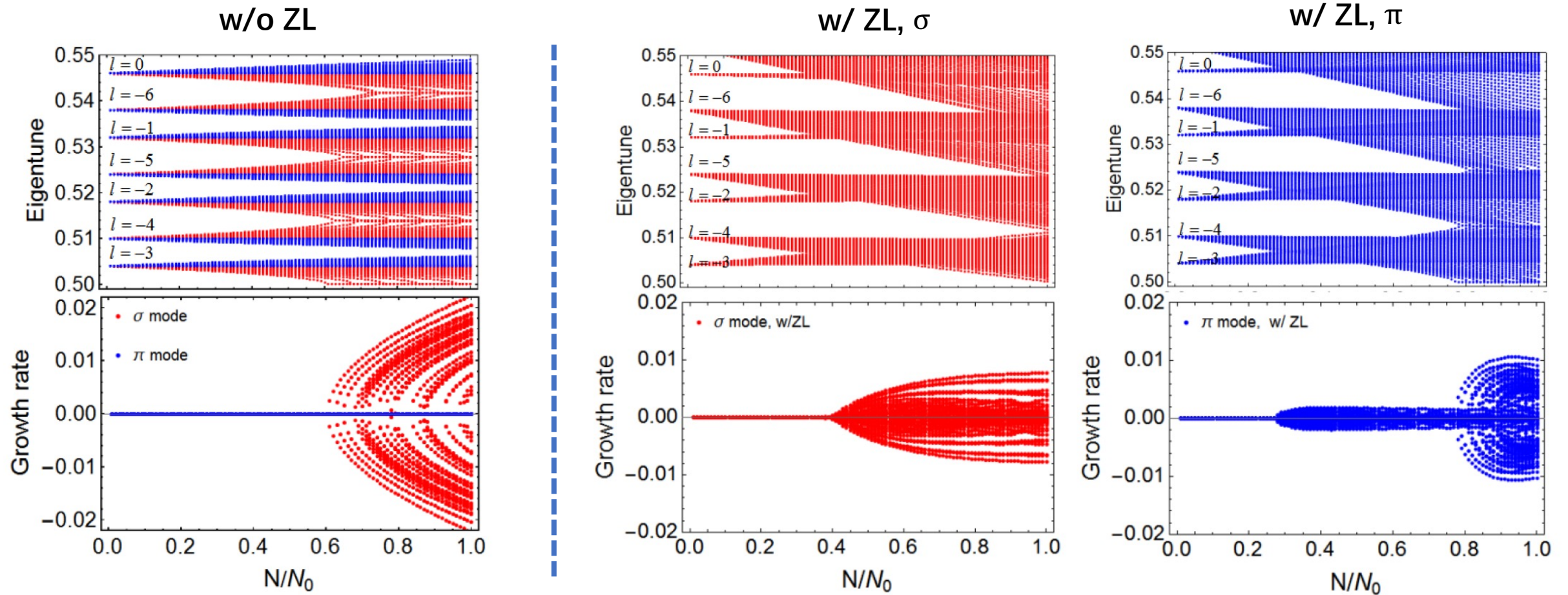
$$M_W = \begin{pmatrix} 1 & 0 \\ \beta_x M_{lil'i'} & 1 \end{pmatrix}.$$

$$\begin{aligned} \Delta p_l(J_i) &= \mp \frac{\beta_x}{2\pi} \sum_{l'} \sum_{i'} \Delta J_{i'} W_{ll'}(J_i, J_{i'}) \psi(J_{i'}) x_{l'}(J_{i'}) \\ &\equiv \beta_x M_{lil'i'} x_{l'}(J_{i'}). \end{aligned}$$

- Stability analysis of $M_0 M_W$

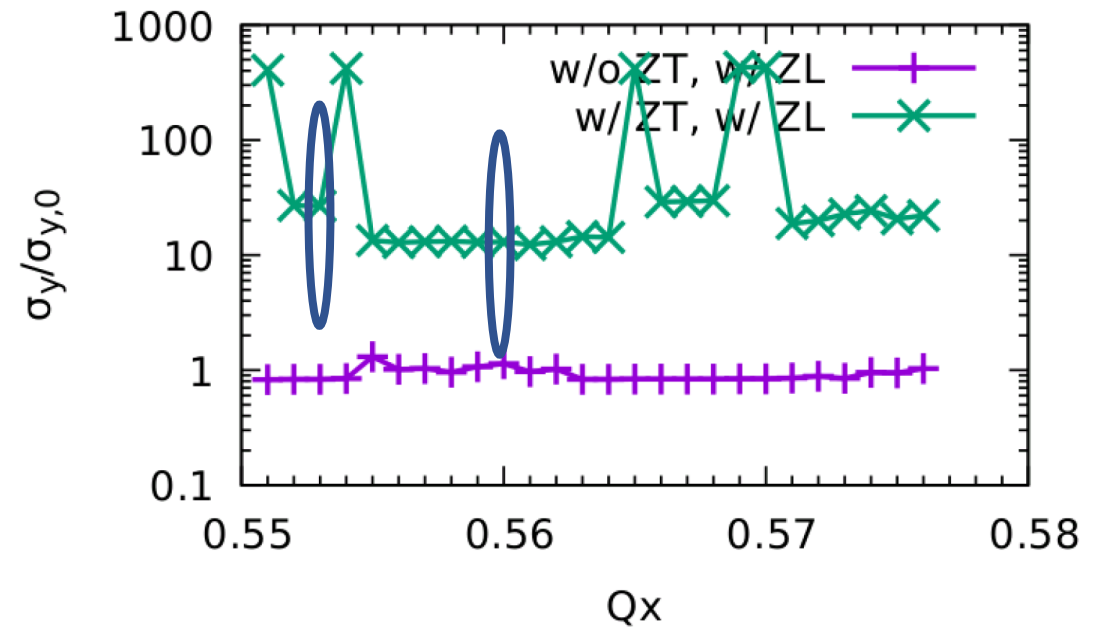
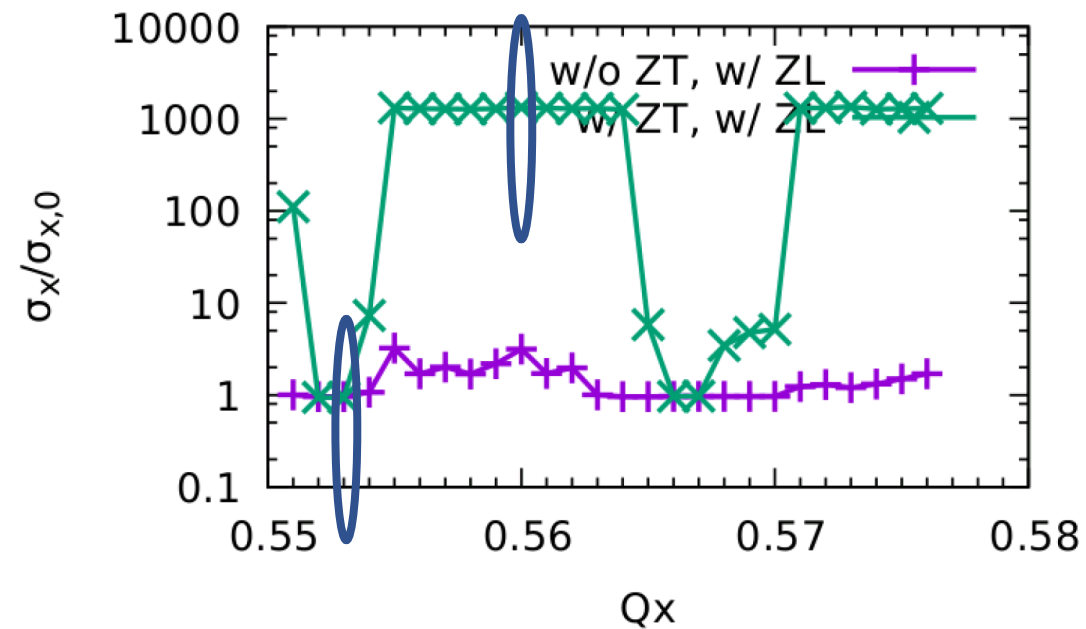
$$W_{ll'}(J_i, J_{i'}) = \sum_j \sum_{j'} e^{-il\phi_j + il'\phi_{j'}} W_x(z(J_i, \phi_j) - z(J_{i'}, \phi_{j'})) \Delta\phi_j \Delta\phi_{j'}.$$

Eigen-Mode Analysis w/o and w/ ZL



CEPC-Z, w/ ZT

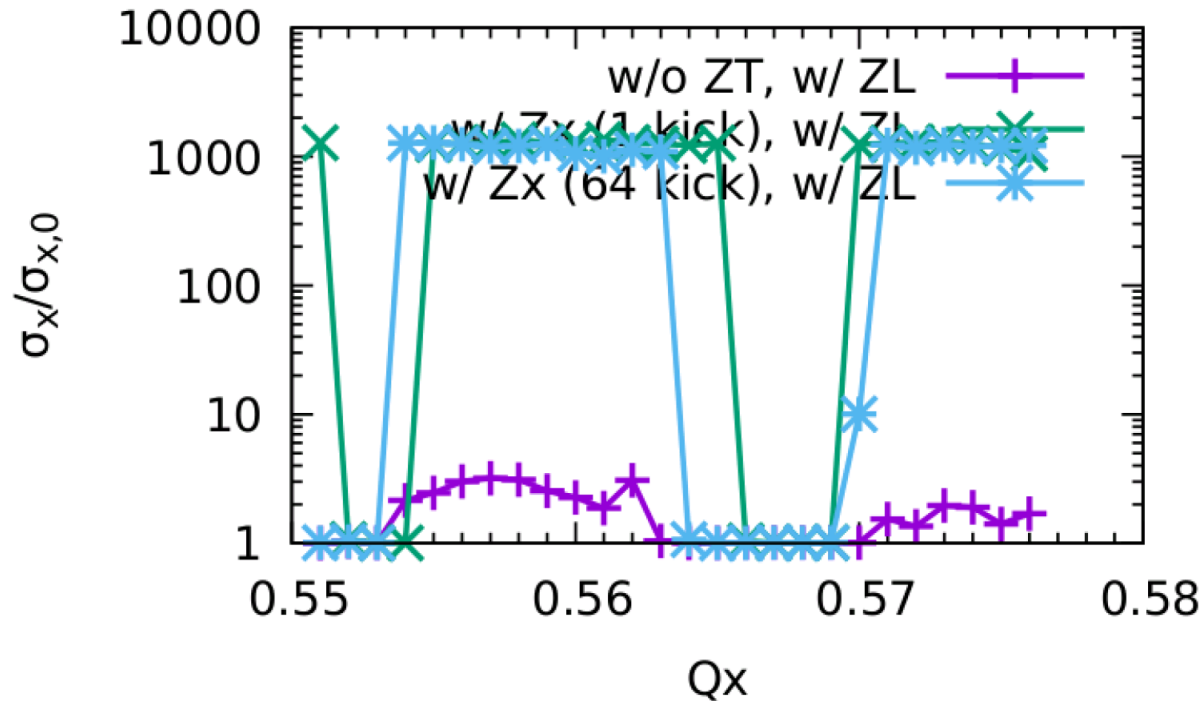
- No stable working points
- There exist very strong blowup in both X/Y direction



CEPC Only Zx(+ZL)

- Stable tune area is large enough (w/ZT)
- Simulation and analysis agrees qualitatively.

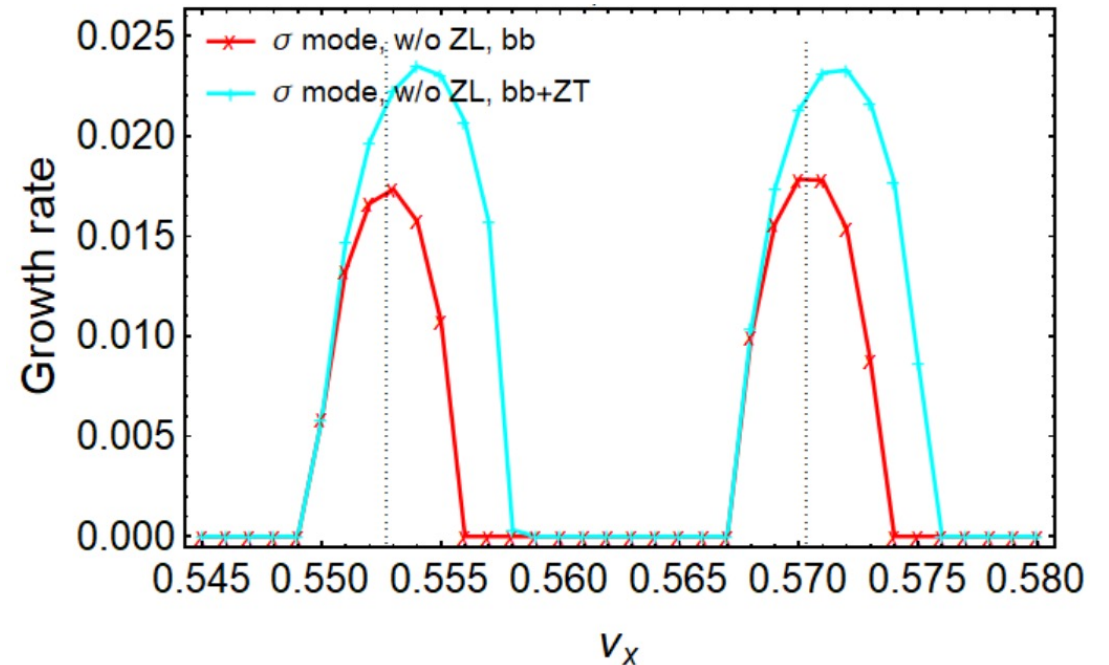
Simulation



Kick number of wake field affect the result

- In horizontal direction, smooth distributed impedance nearly does not squeeze the stable tune area serious
- A very local impedance may squeeze the stable area.

Courtesy of Chuntao Lin and Na Wang
Analysis, ZT kick applied at IP

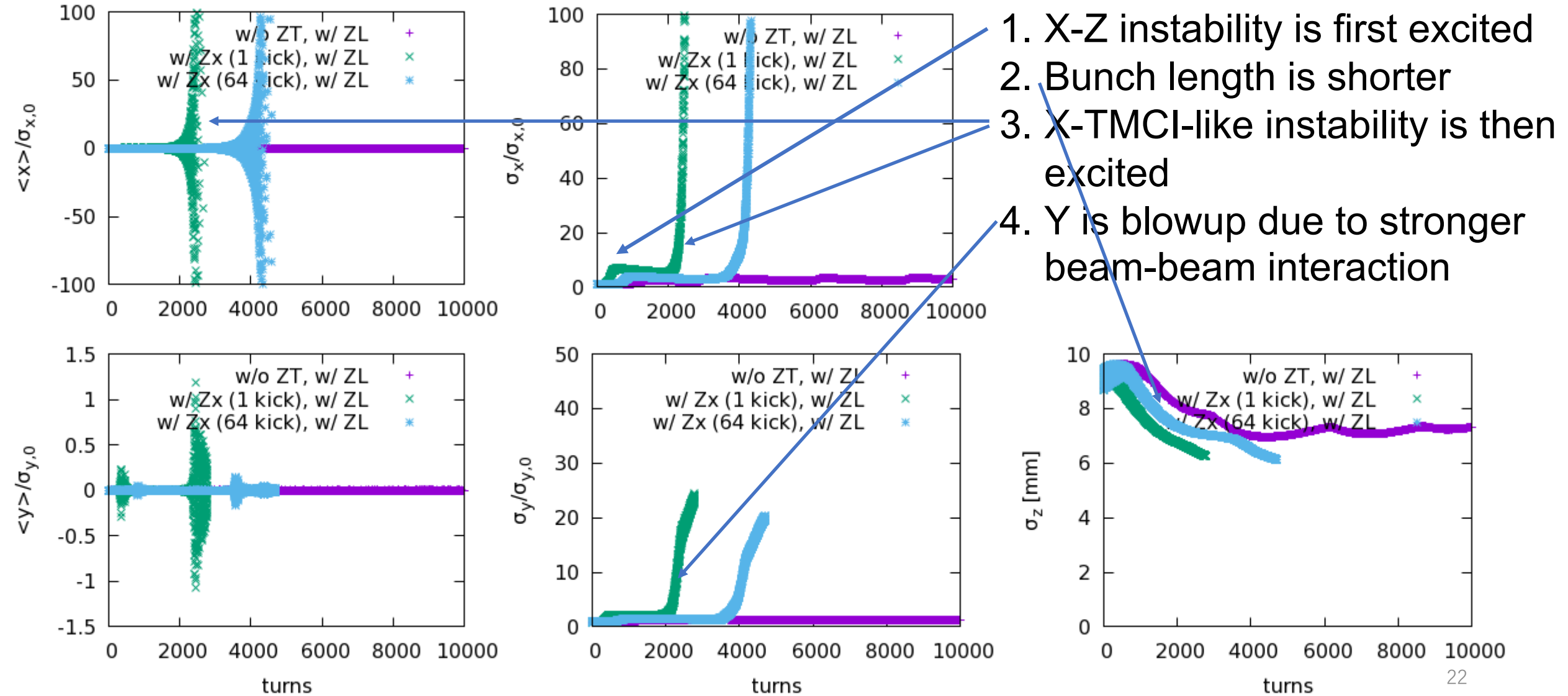


In horizontal direction, considering ZX

- the instability growth rate is faster,
- unstable tune area increases

CEPC Only Zx(+ZL) @ $Q_x=0.562$

It has been simulated that w/o BS (but keep same bunch length), the TMCI-like instability would not appear.

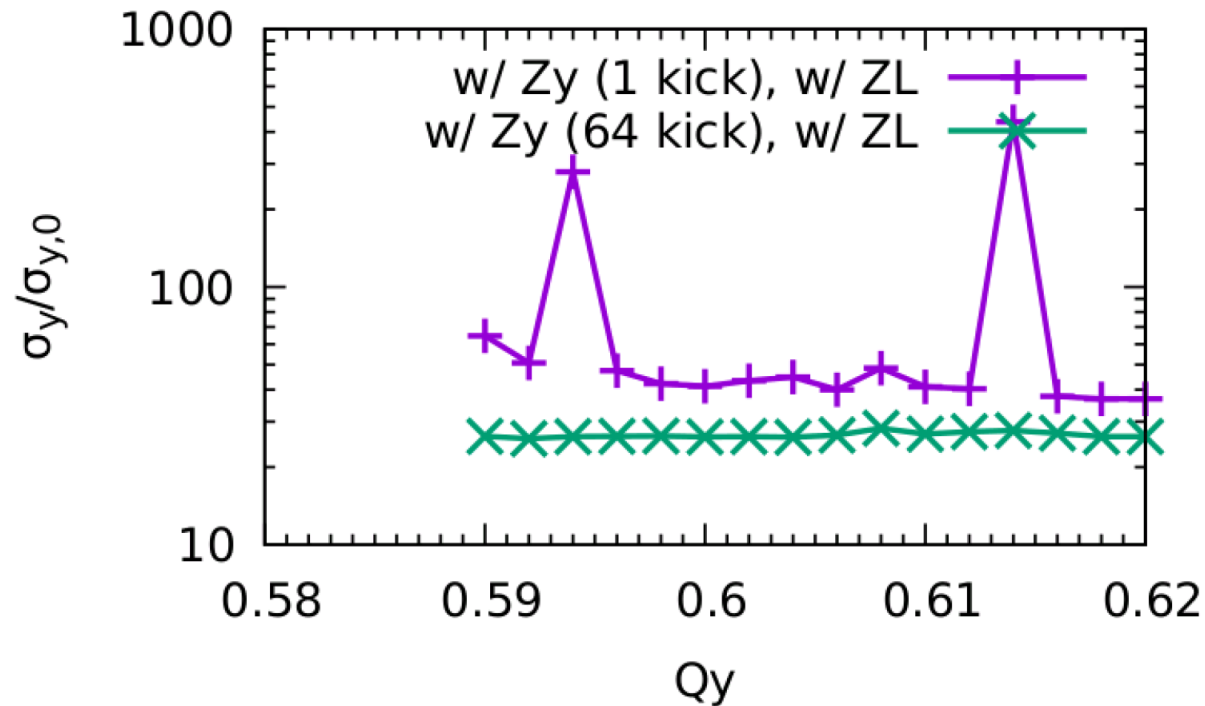


- No stable working point (w/ZT)
- Simulation and analysis agrees qualitatively.

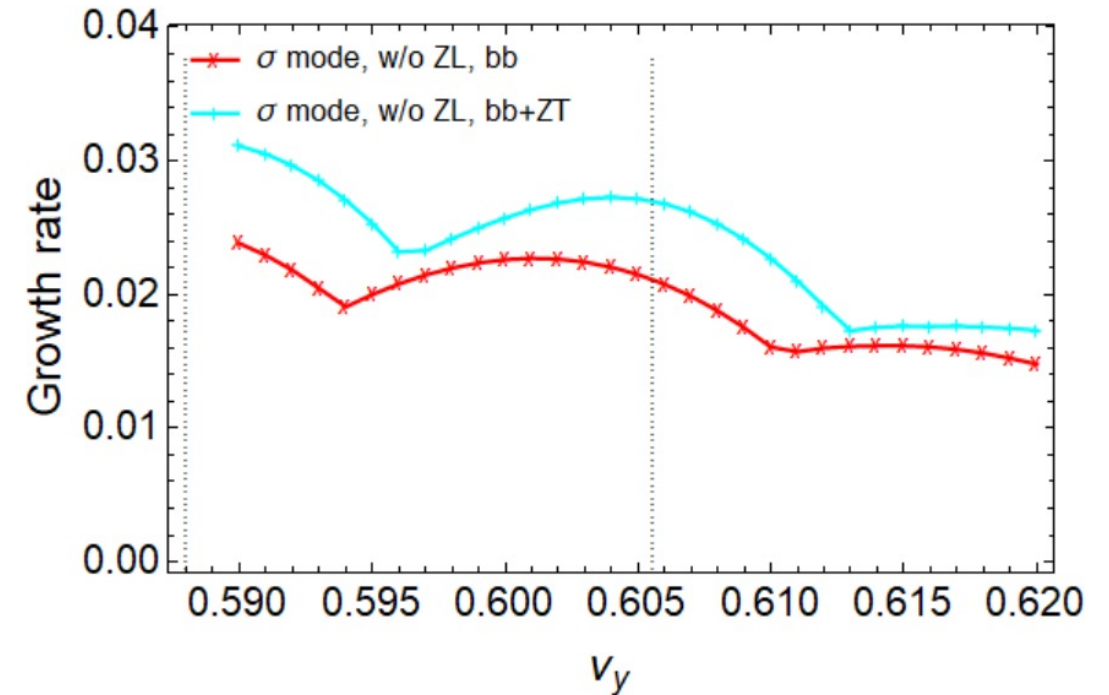
CEPC Only Zy(+ZL) Qx=0.567

Courtesy of Chuntao Lin and Na Wang
Analysis, ZT kick applied at IP

Simulation



- Kick number of wake field affect the result
- No stable tune area



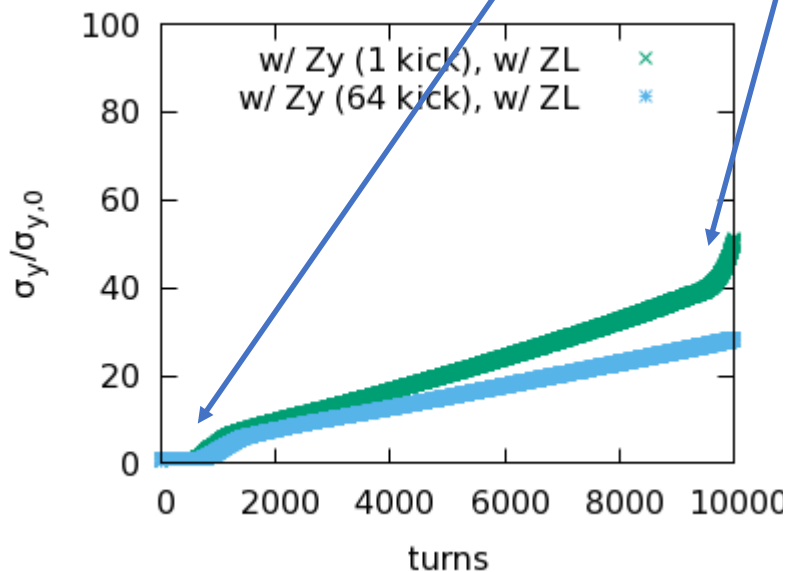
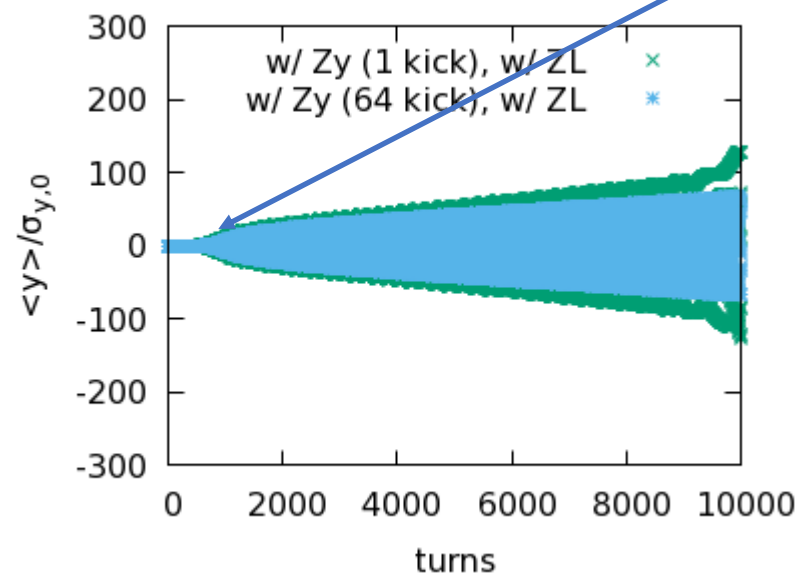
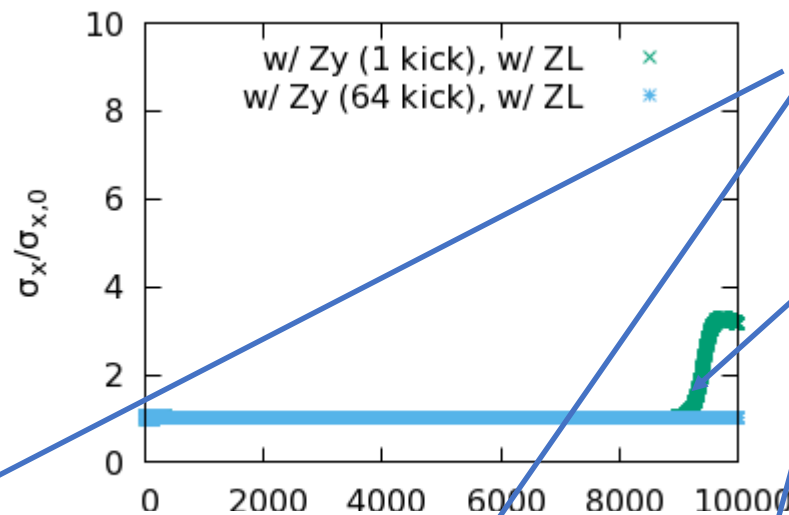
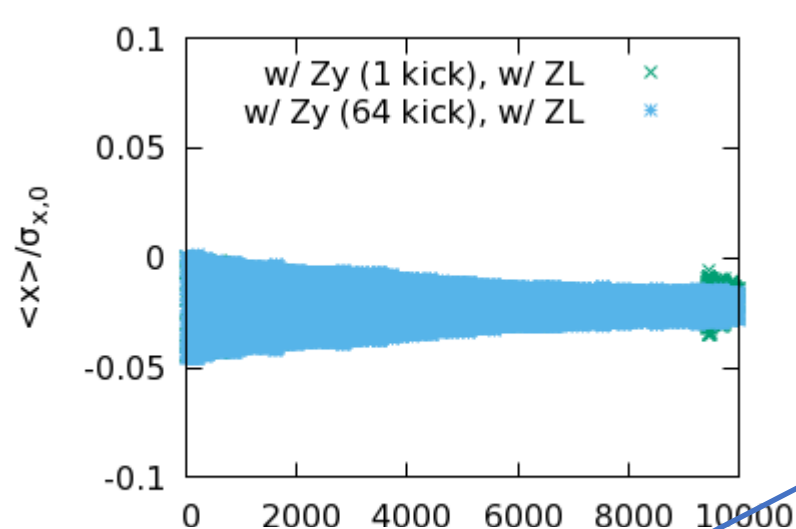
- pure beam-beam is unstable due to ignorance of strong nonlinearity
- enhancement of instability when considering ZY

Behavior at unstable working point

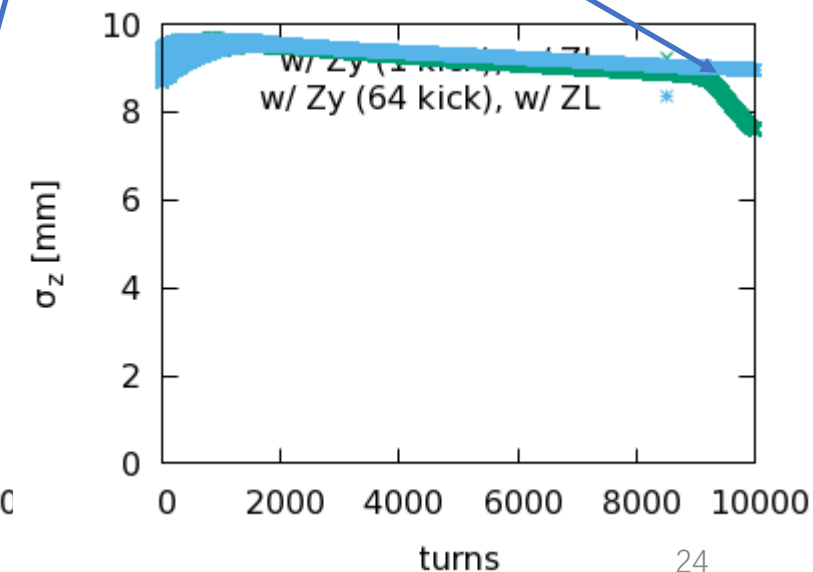
σ mode

It has been simulated that w/o BS (but keep same bunch length), the X-Z instability would not appear.

CEPC Only Zy(+ZL) $Q_x=0.567$

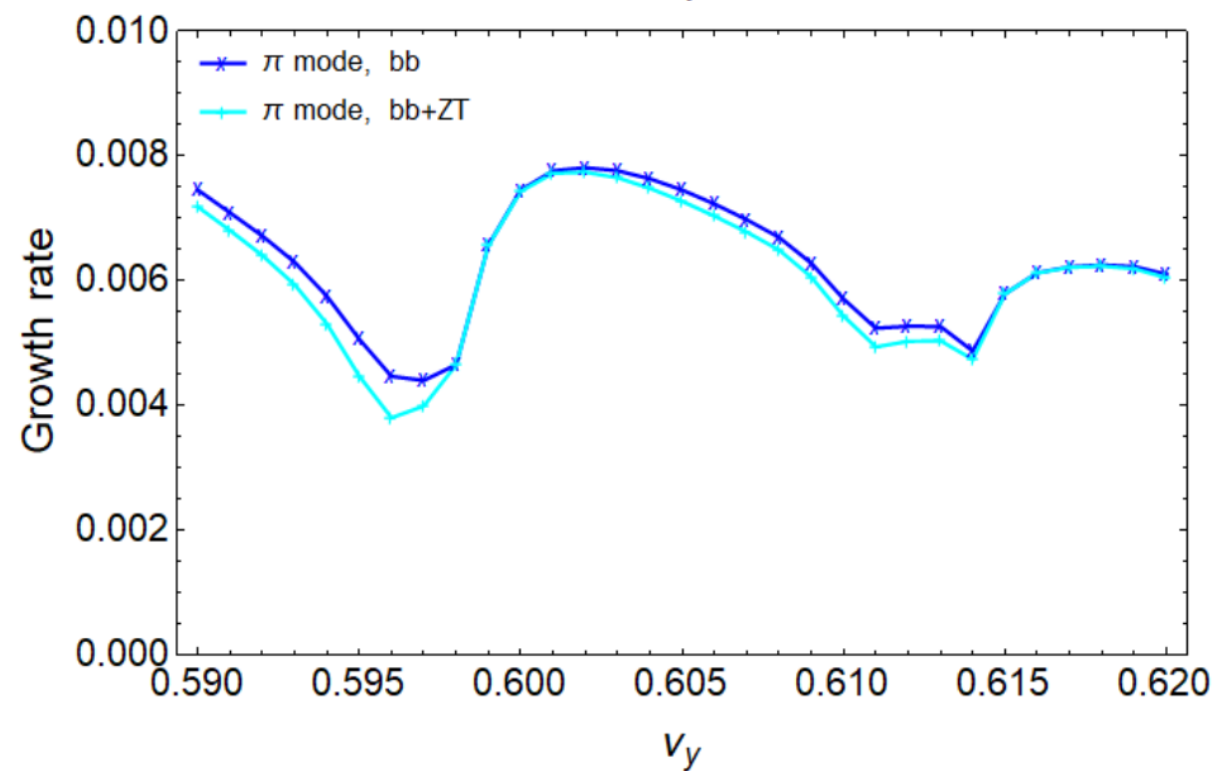
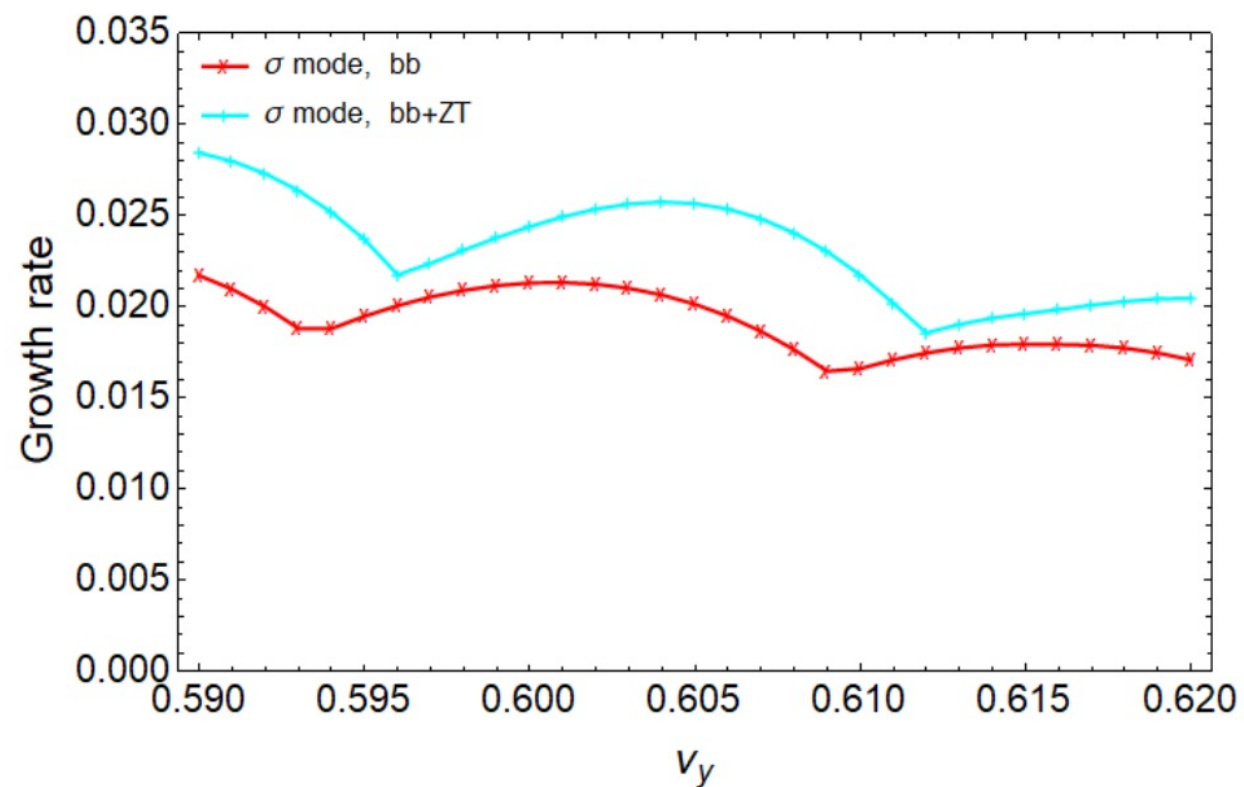
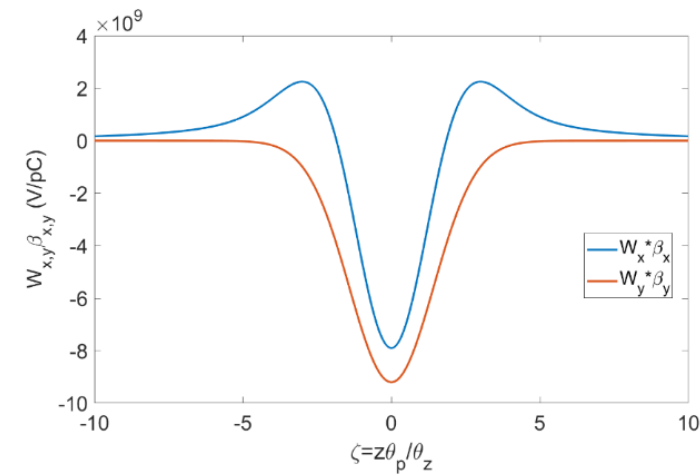


1. Y-TMCI-like instability is first excited
2. Bunch length is shorter
3. X-Z instability is excited
4. Stronger Y blowup due to strong beam-beam



π -mode is not sensitive to ZT

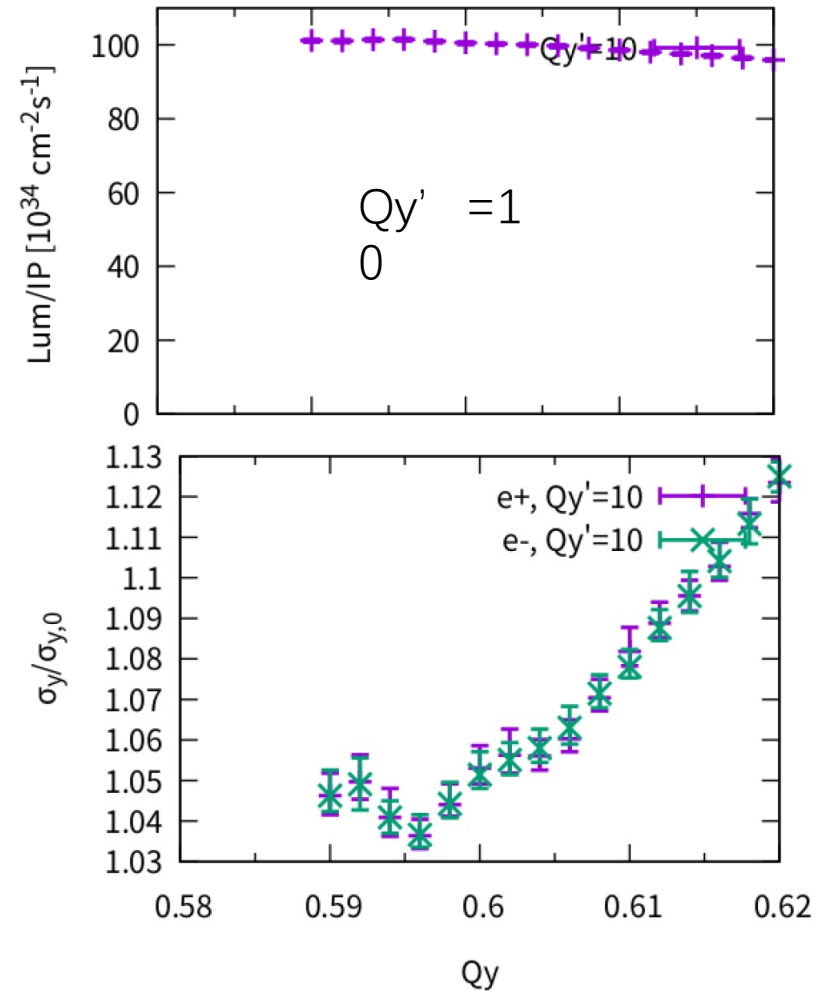
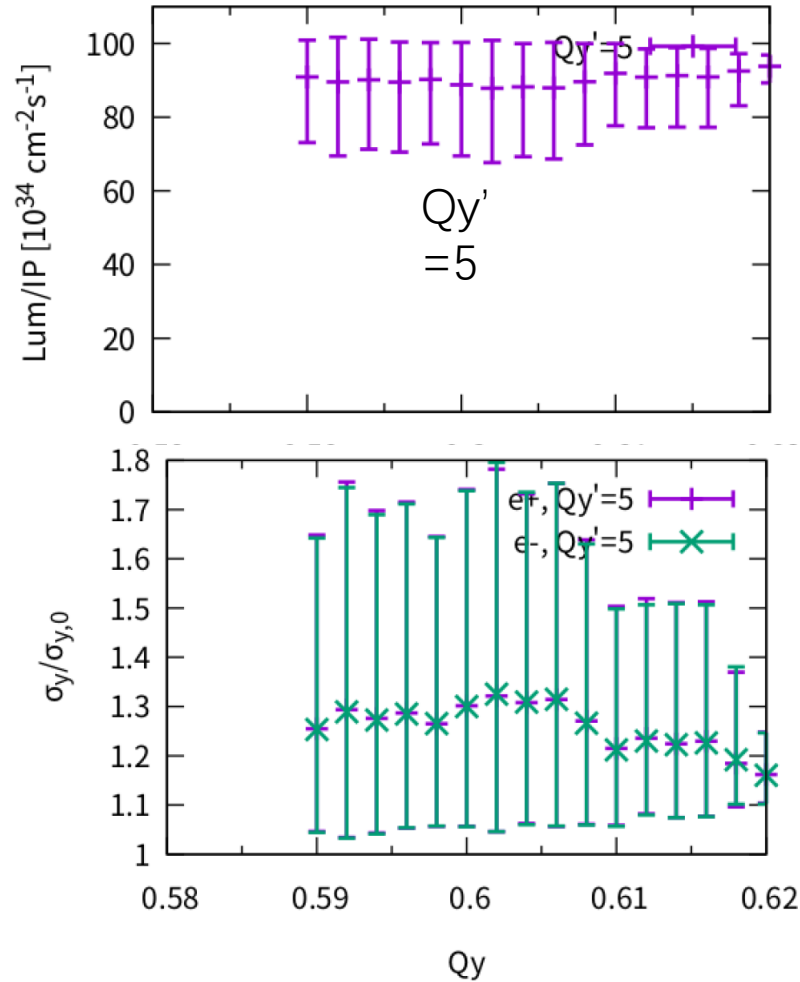
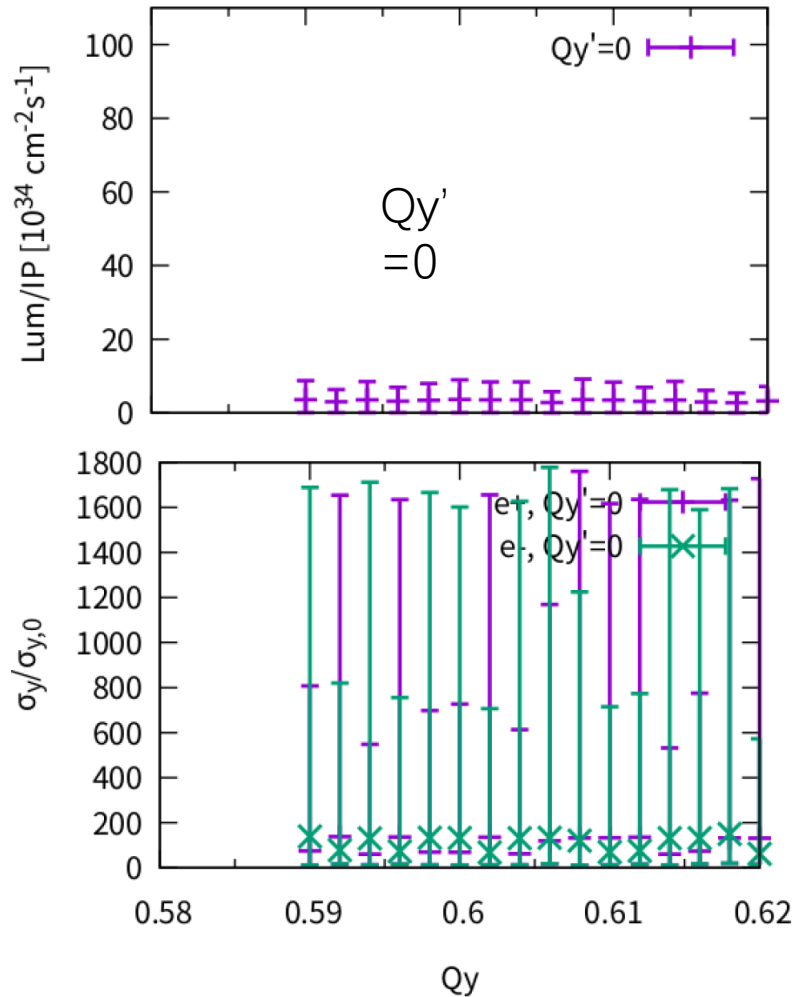
$$\Delta p_y(z) = \mp \int_{-\infty}^{\infty} W_y(z - z') \rho_y(z') dz.$$



$Q_{y'}$ ~10 could help suppress the strong TMCI-like instability induced by BB+ZT

Effect of Vertical Chromaticity (Lum & σ_y)

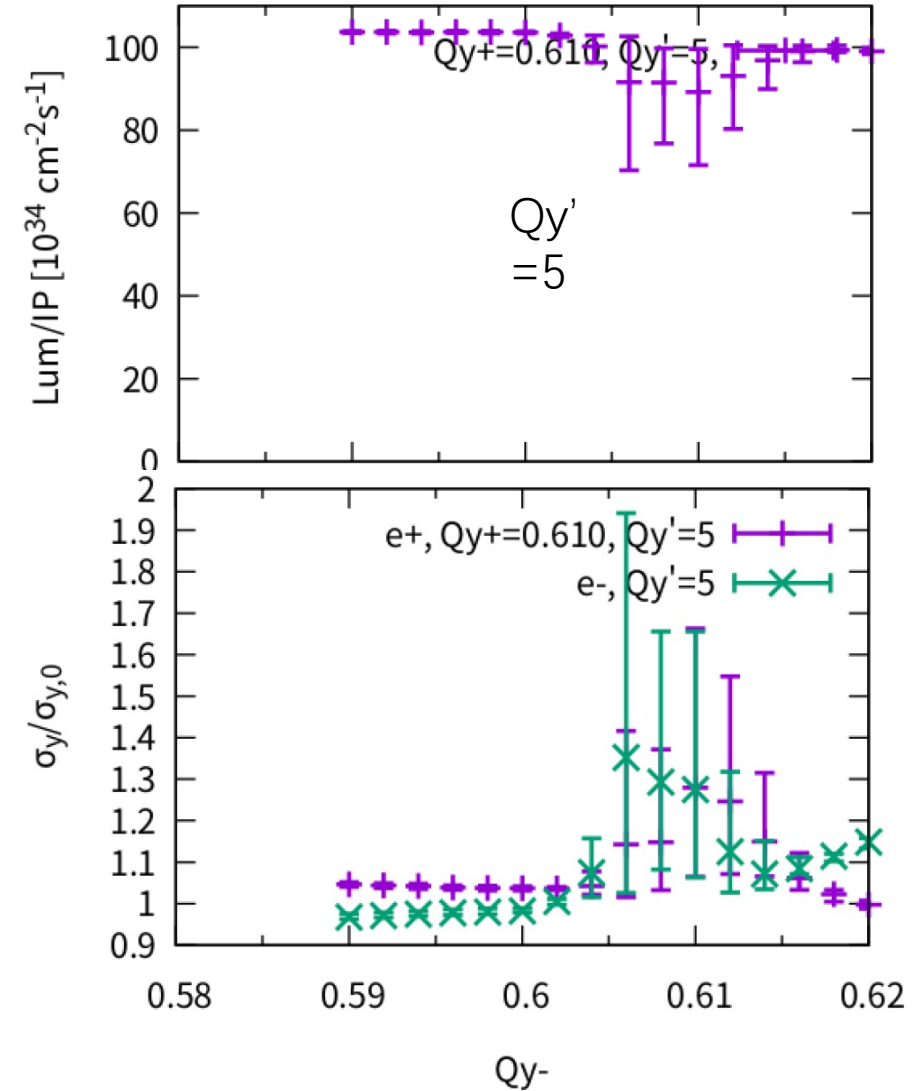
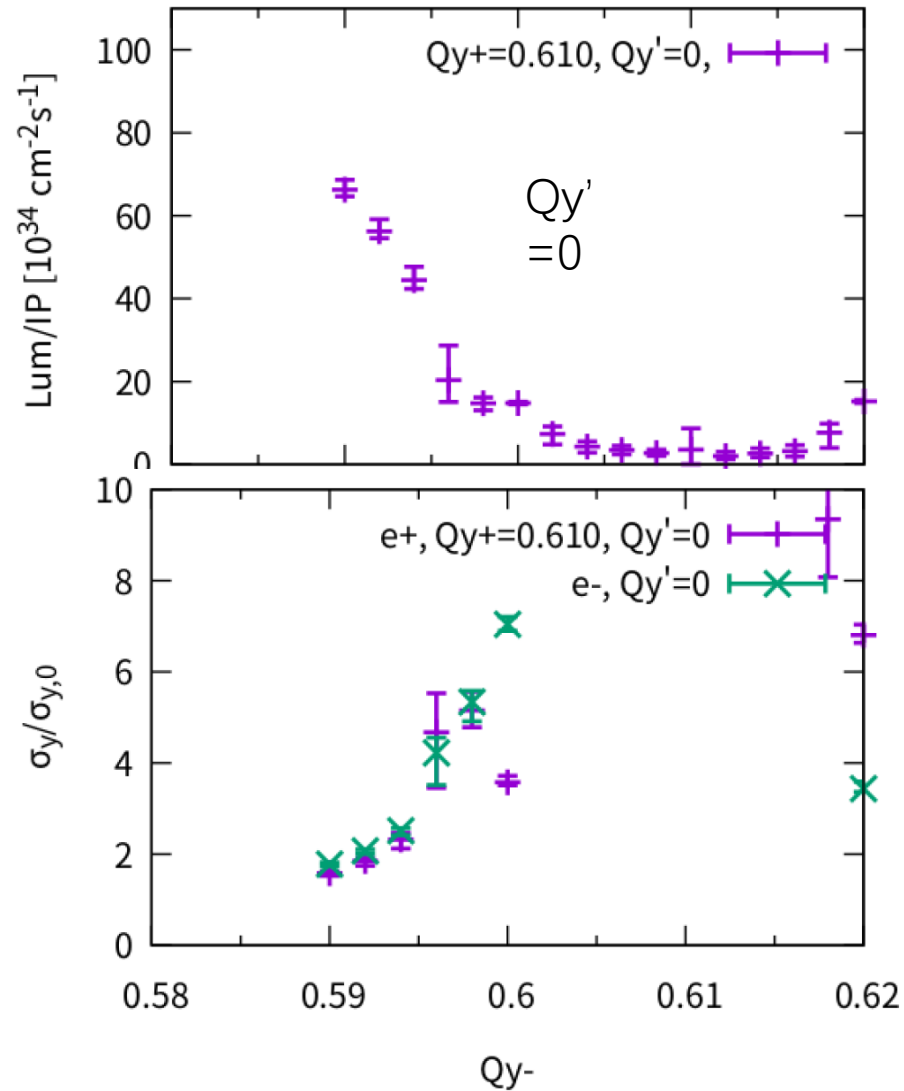
ZX+ZY+ZL, $Q_x=0.567$



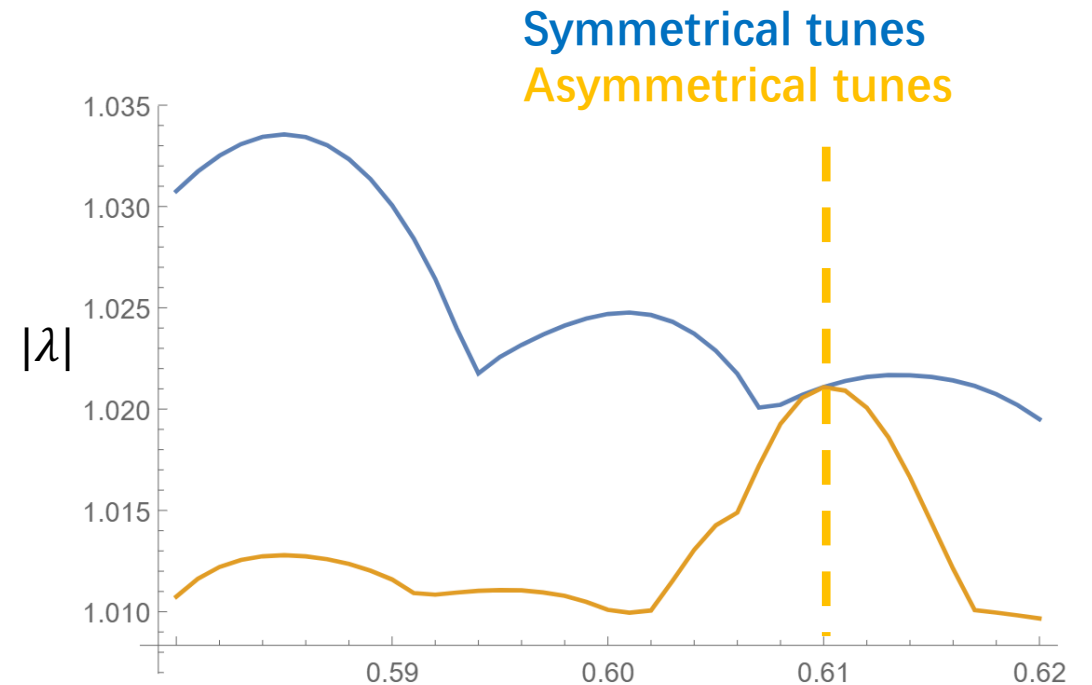
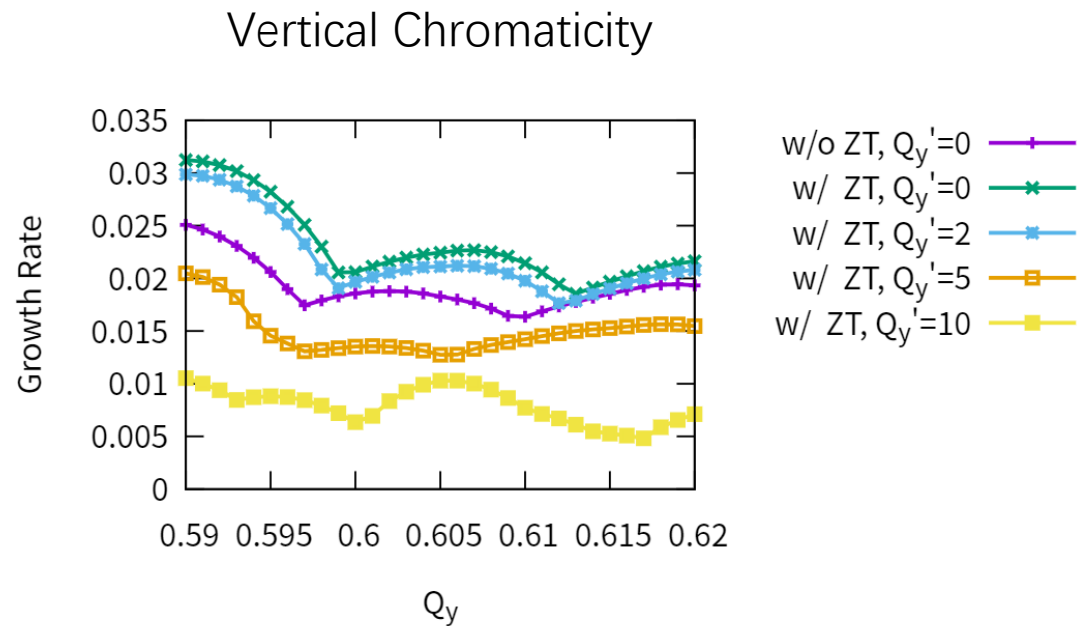
Qy Difference > 0.01 could help suppress instability with $Qy' = 5$

Thanks: K. Oide, K. Ohmi

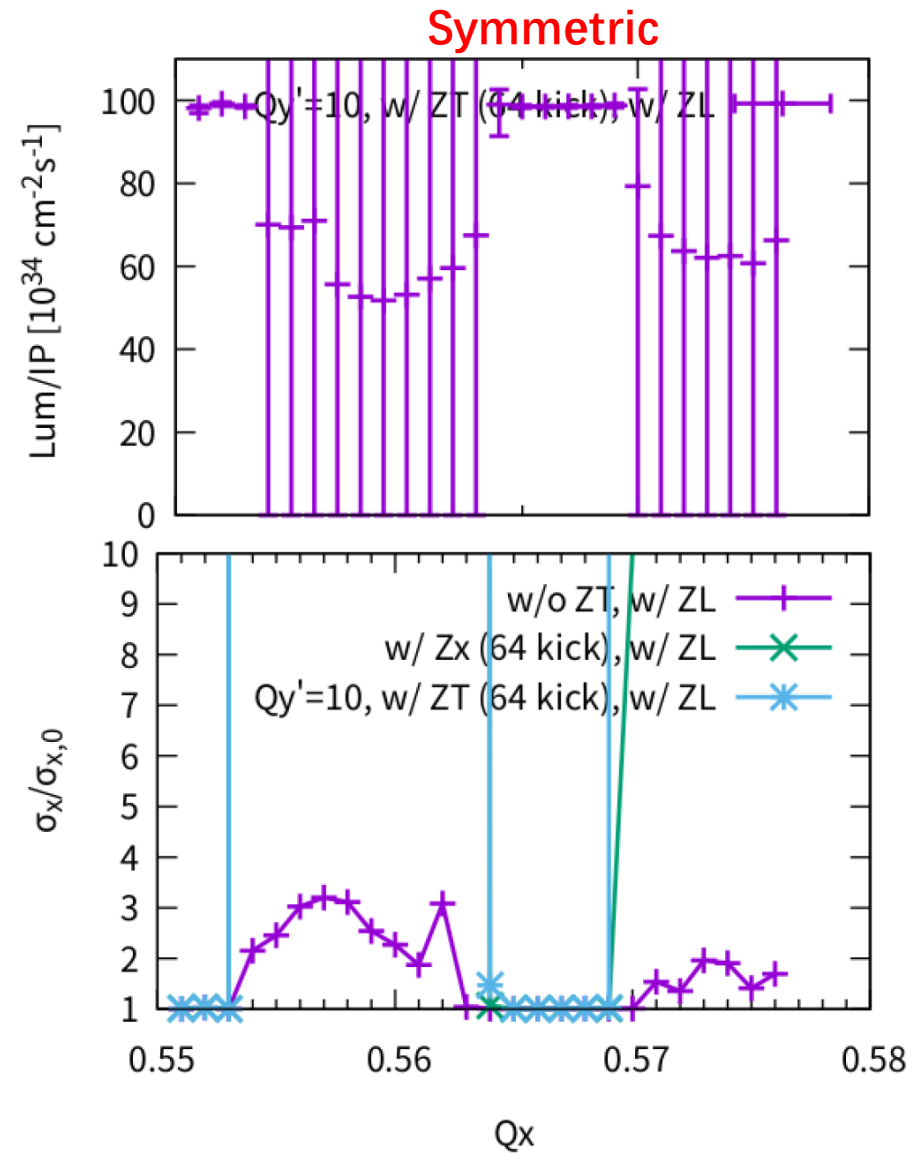
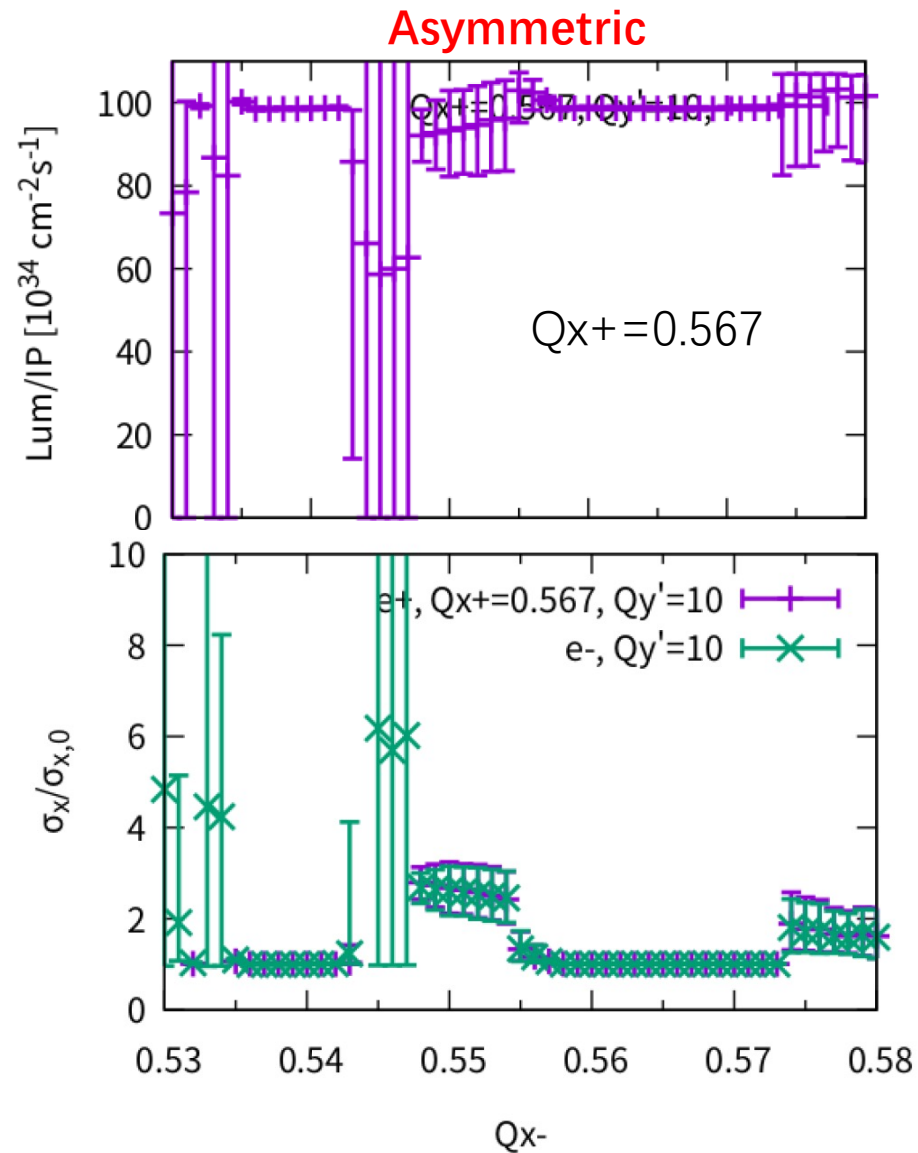
Effect of different vertical tune ($Qy += 0.610$)



Some analysis results of mitigation method



Different Horizontal tune

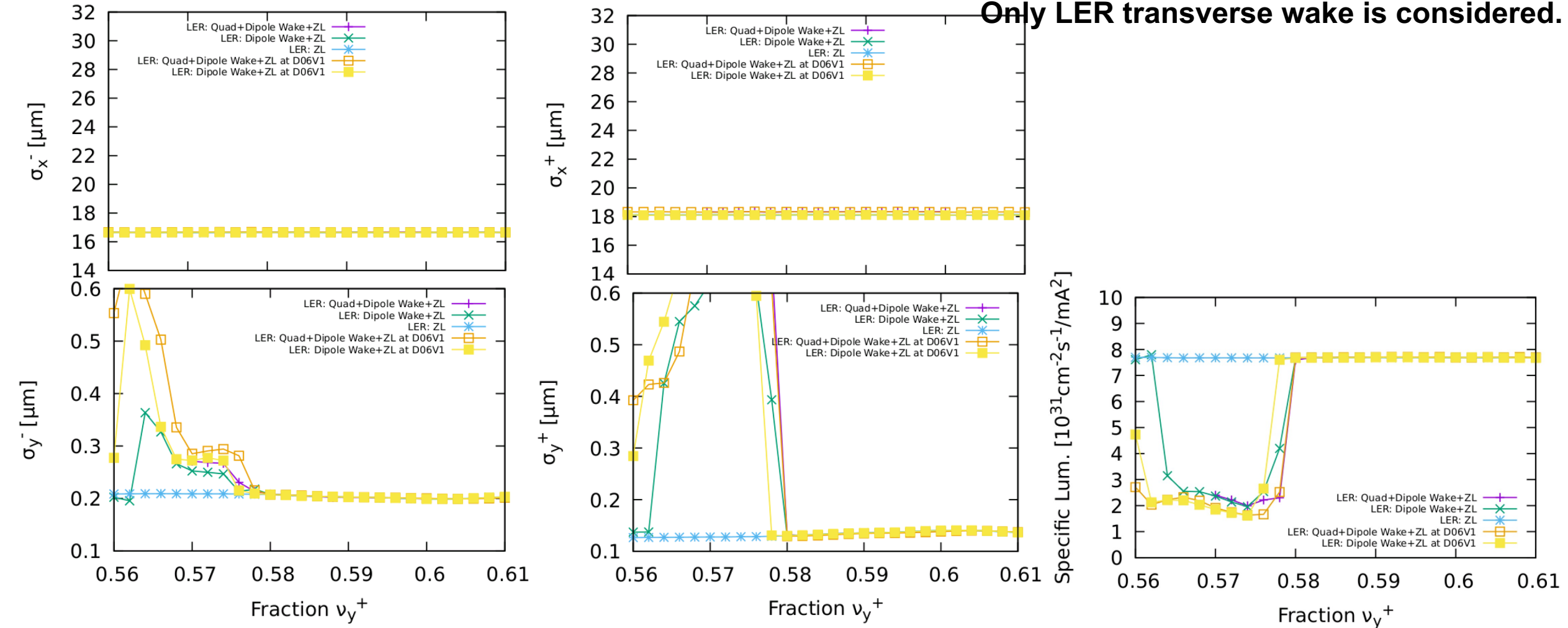


$$v_{x,0}^+ = 0.525$$

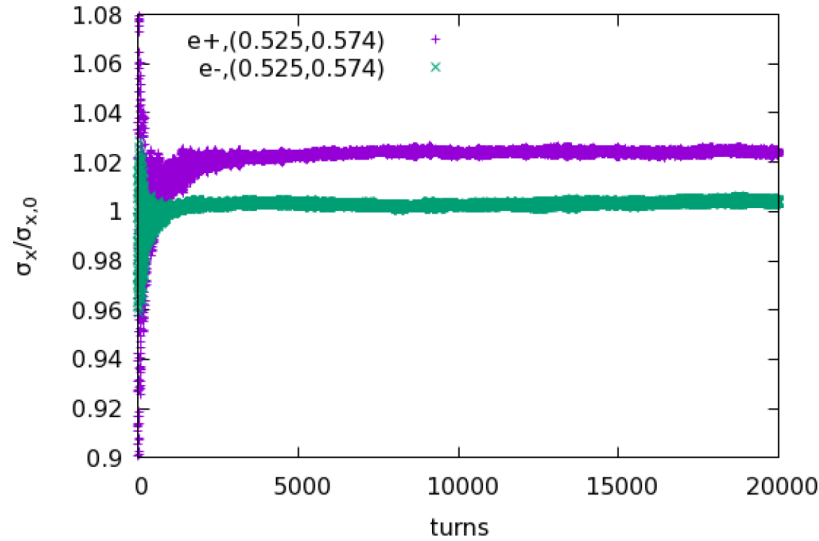
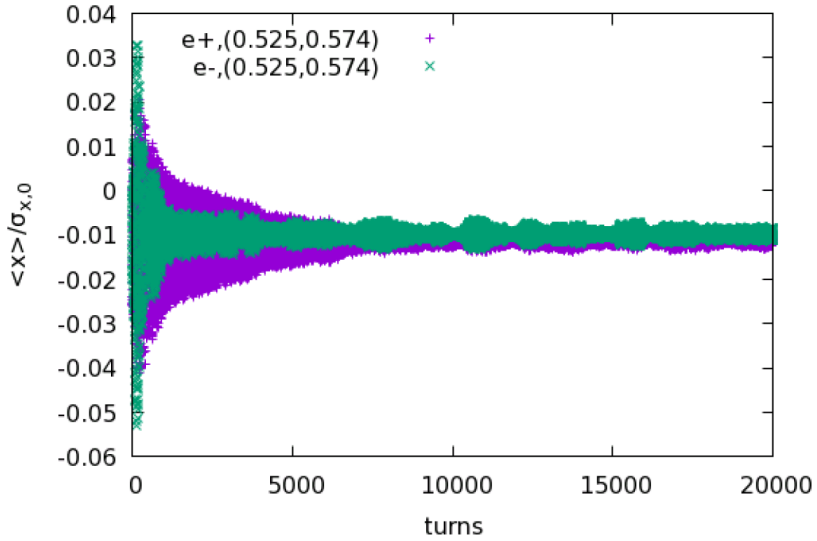
SKEKB Collision versus $v_{y,0}^+$

- There exist vertical instability when we consider transverse wake.
- It does not matter that the local transverse is put at IP or D06V1.

Only LER transverse wake is considered.

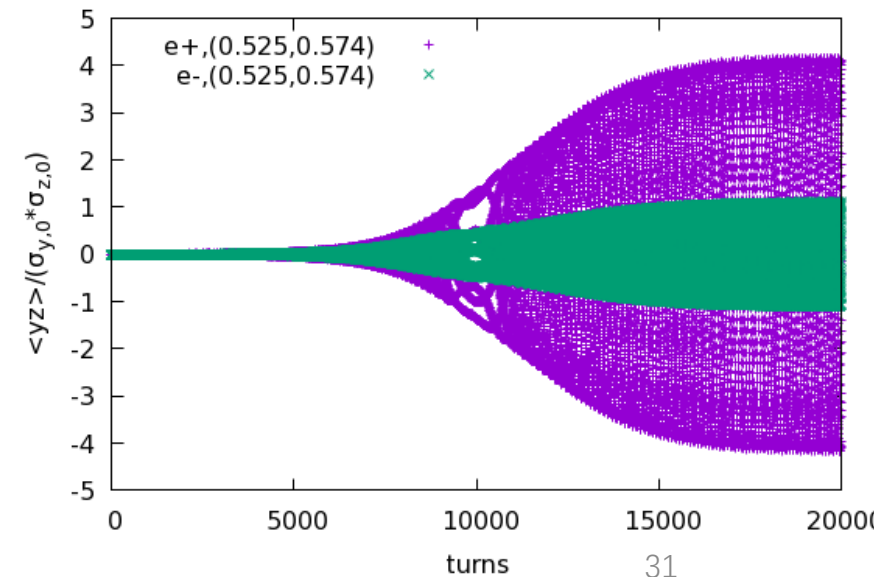
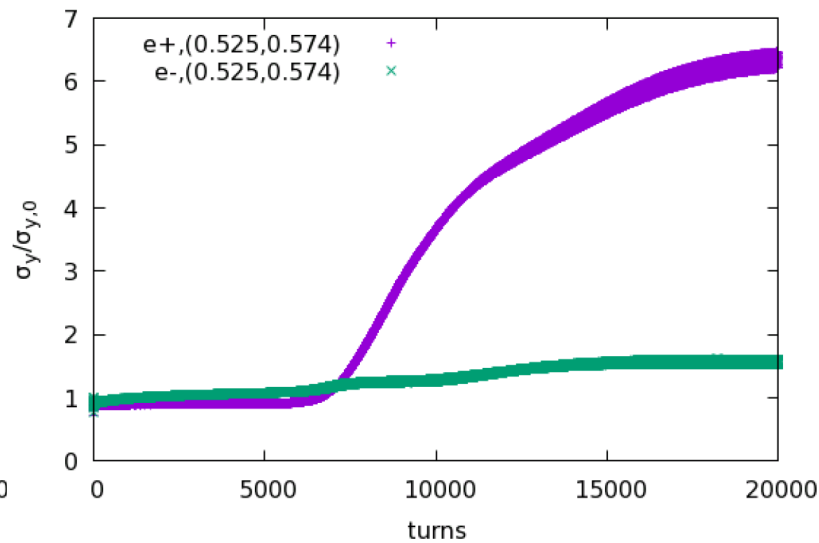
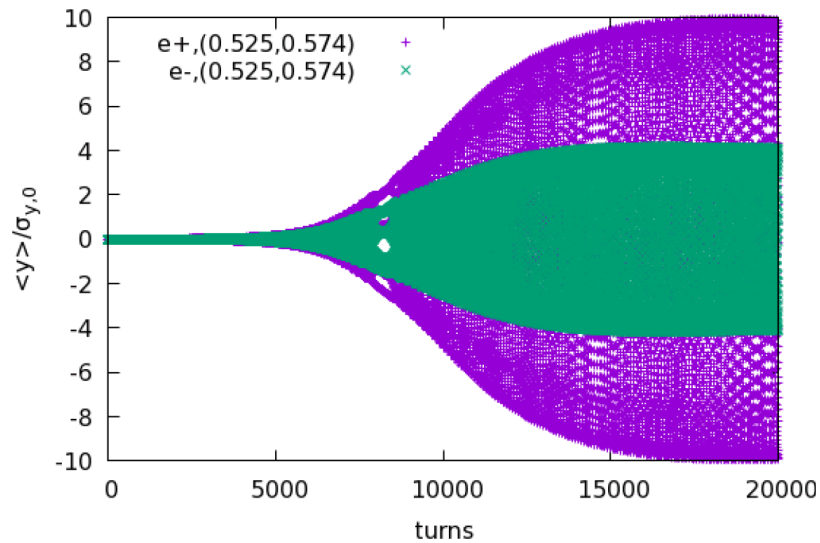


Moment evolution : LER: dipole+quad wake + ZL, HER: ZL



There exist :

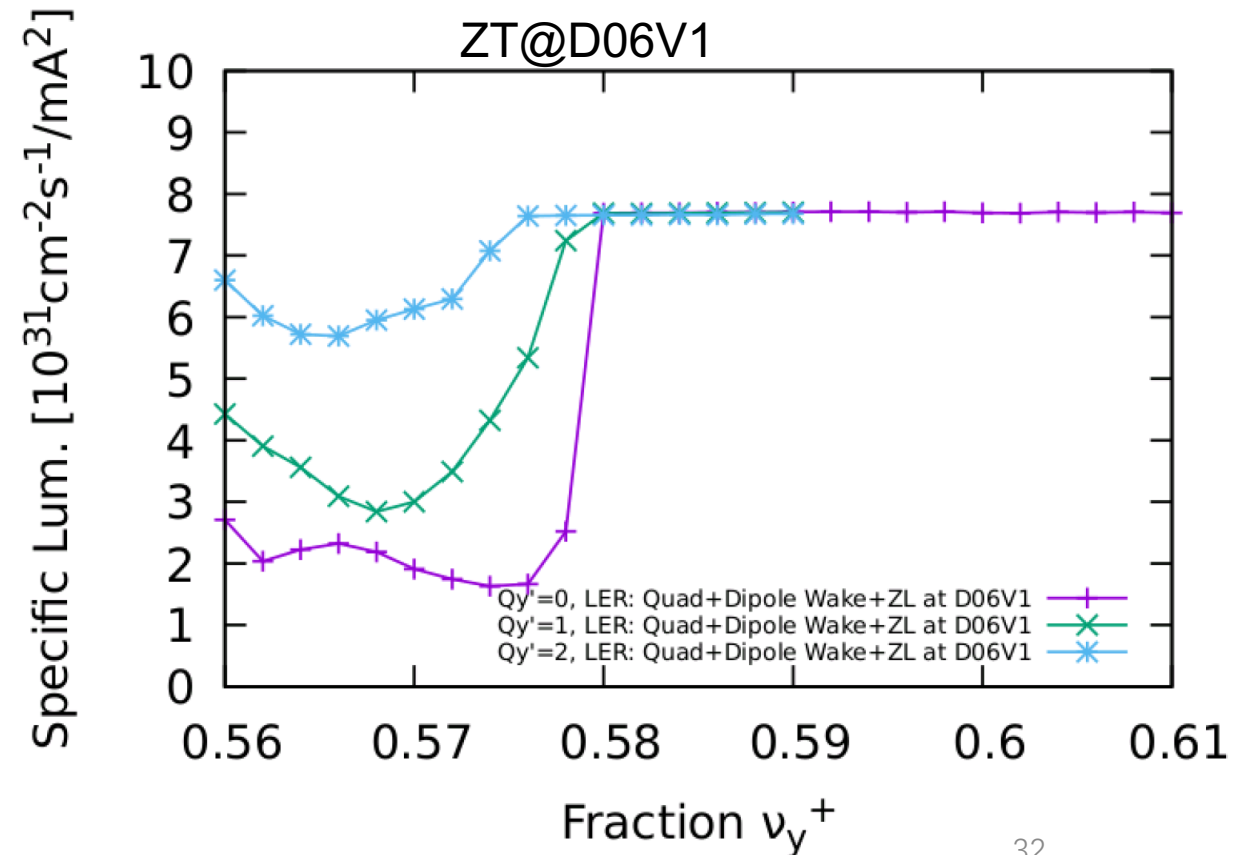
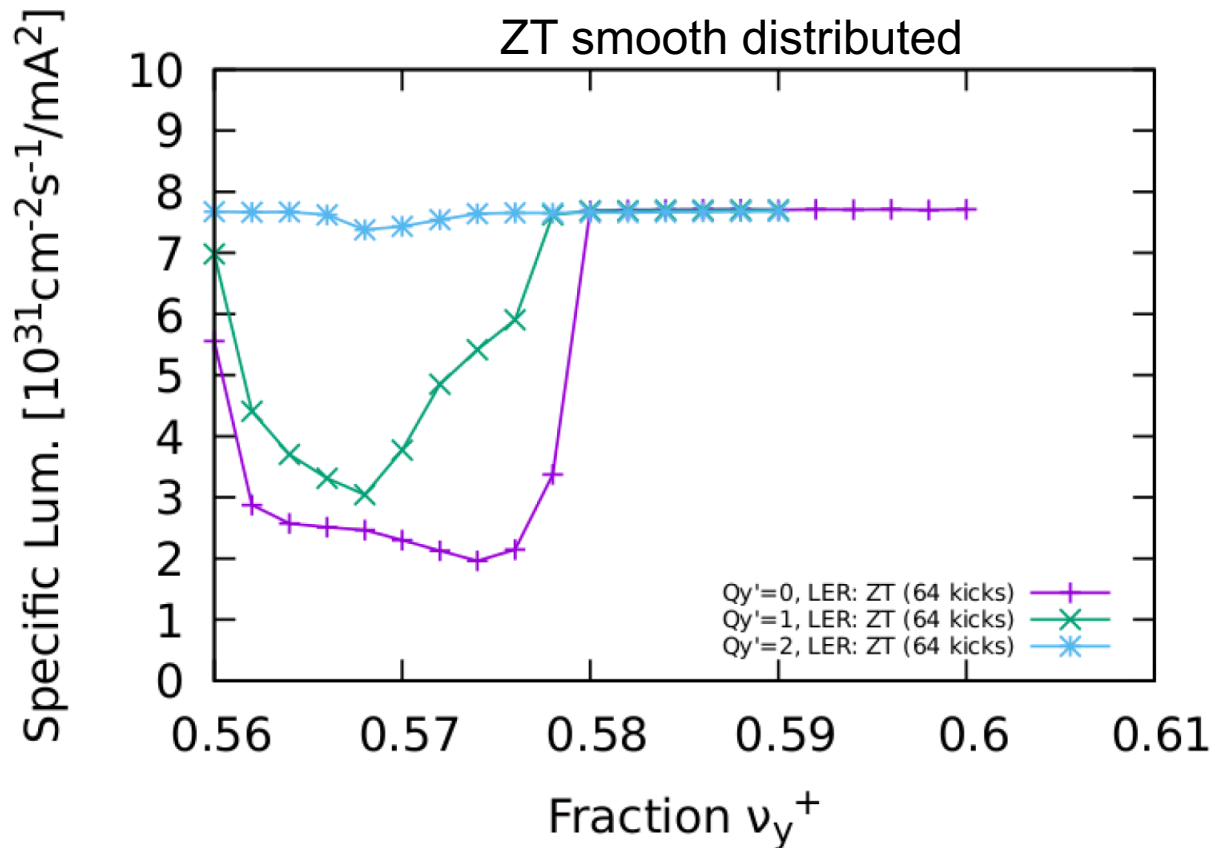
- Vertical Dipole oscillation
- Vertical Beam Blowup
- $\langle yz \rangle$ head-tail oscillation



SKEKB: Effect of vertical tune chromaticity

it is assumed same between LER/HER

- Q_y' could help suppress the instability



Summary

- Interplay between longitudinal coupling impedance and beam-beam interaction has a great influence on the behavior of X-Z instability in both CEPC/FCCee
- Combined effect of beam-beam interaction and transverse impedance may induce strong head-tail instability in vertical direction
- In the presence of Beamstrahlung, X-Z instability may induce strong TMCI instability in horizontal direction with conventional impedance. TMCI instability in vertical direction could induce X-Z instability
- Finite chromaticity and asymmetric tunes help mitigate instability
- **The interplay between beam-beam and other dynamics effect must be considered in future high performance e⁺e⁻ colliders**