Conformality and duality in neutron stars

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Refs: Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL 129, 252702 (2022).
Y. Fujimoto, T. Kojo, L. McLerran, to appear soon

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Outline

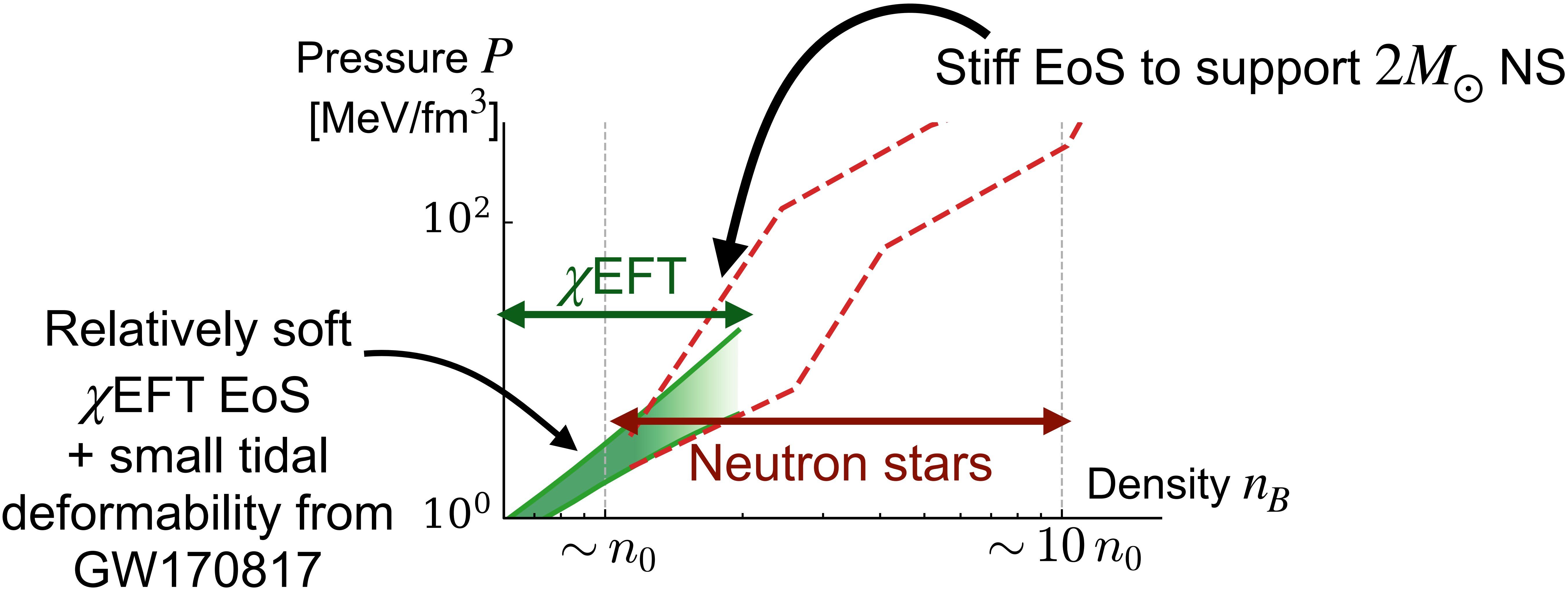
Conformality

Equation of state (EoS) at relatively high density is consistent with the conformal EoS

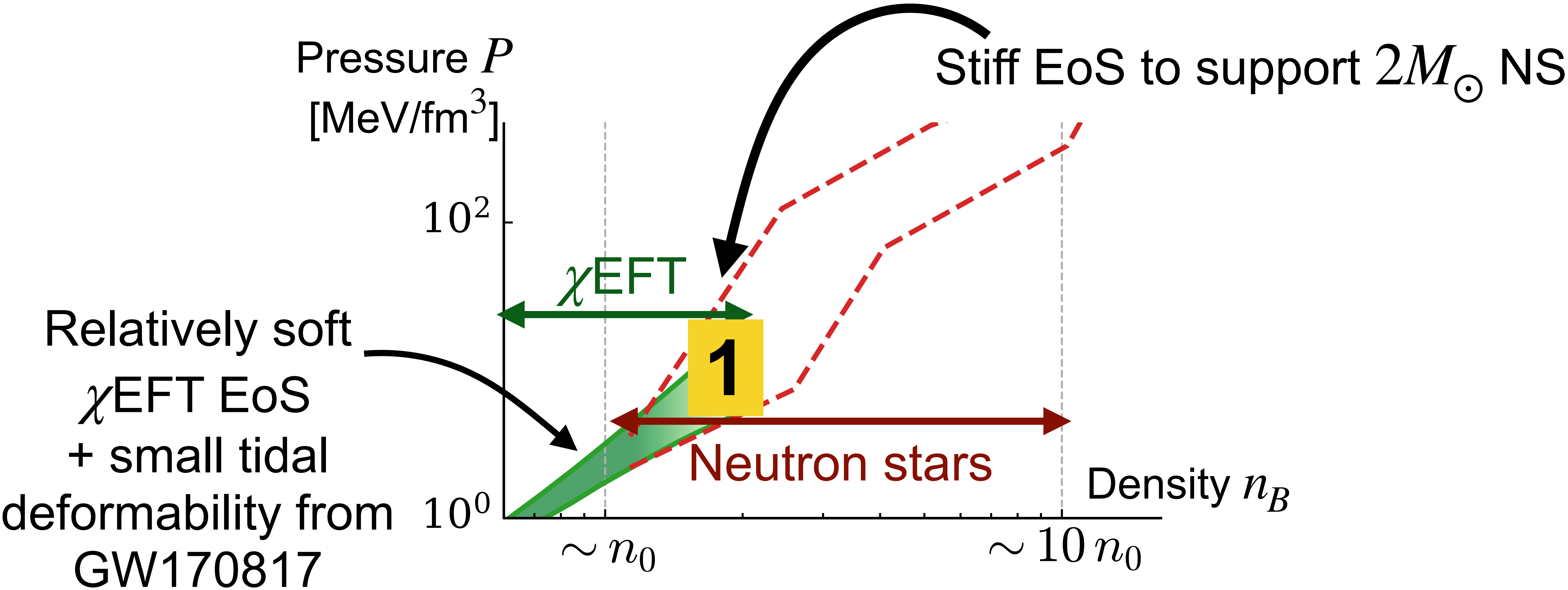
Duality

Quarkyonic duality seems to explain all the desired features of the EoS that are currently observed

Features of dense matter EoS



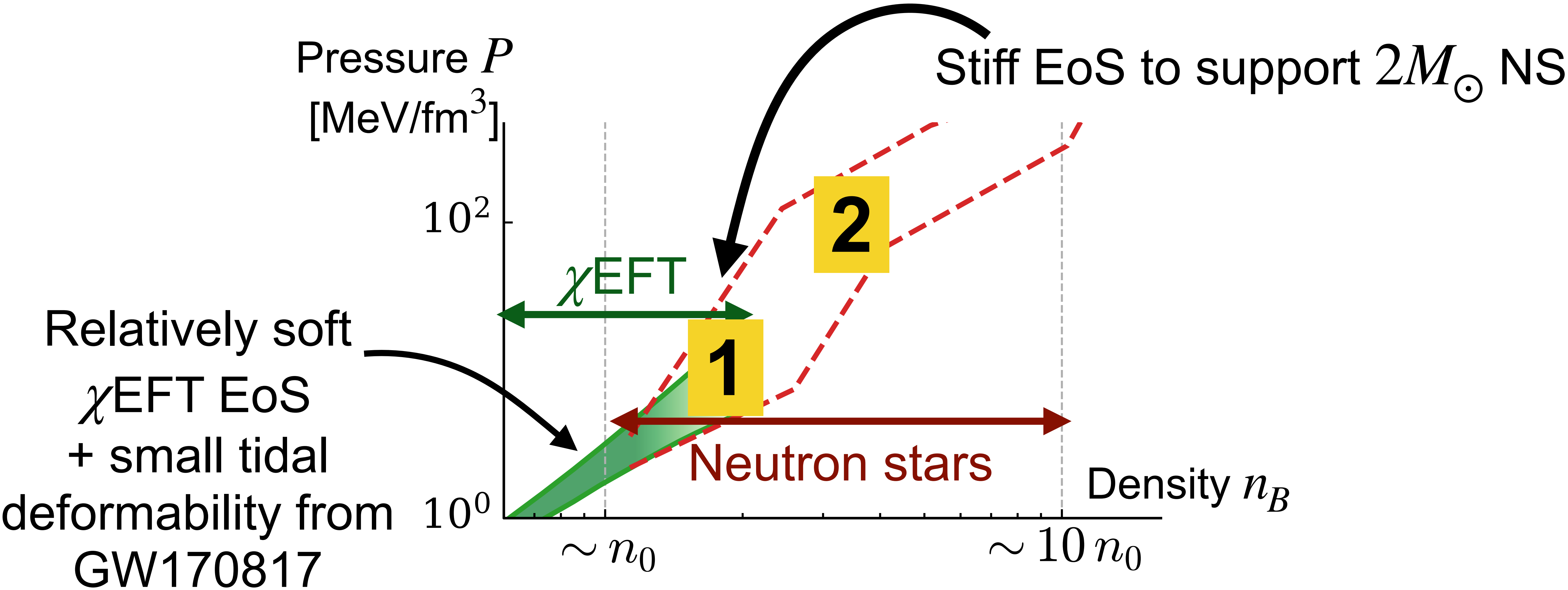
Features of dense matter EoS



1. Rapid stiffening at $n_B \gtrsim 1.5\, n_0$

Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020); etc…

Features of dense matter EoS



1. Rapid stiffening at $n_B \gtrsim 1.5\,n_0$

Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020); etc…

2. Conformality at a few n_0

Fujimoto,Fukushima,McLerran,Praszalowicz (2022)

Features of dense matter EoS

EoS may be characterized by two dimensionless quantities:

Speed of sound

Slope of the EoS

$$v_s^2 = \frac{dP}{d\varepsilon}$$

Trace anomaly (conformality measure)

Trace of the energy-momentum tensor normalized by the energy density

$$\Delta = \frac{\varepsilon - 3P}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

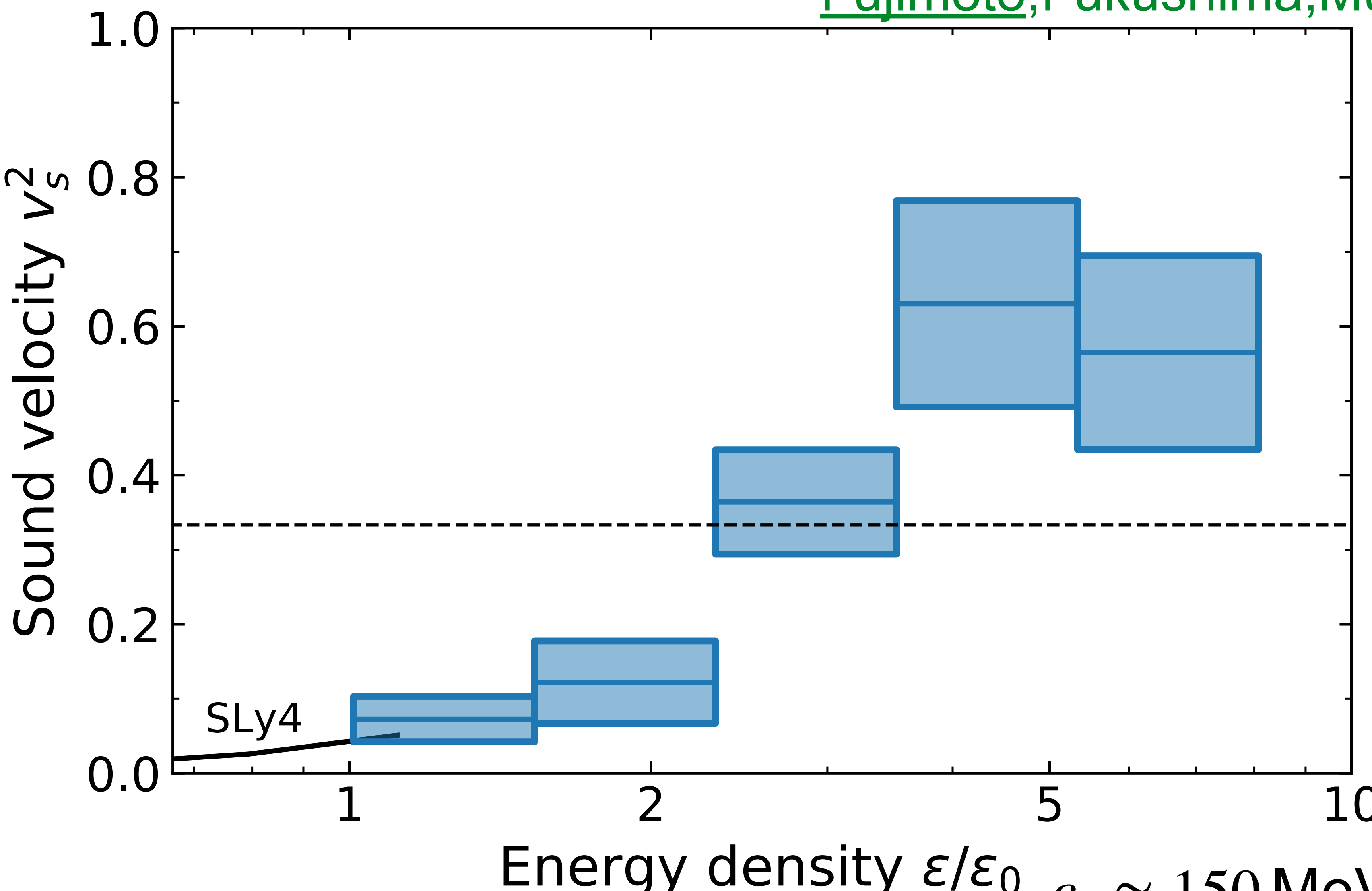
Rapid stiffening

NS data favors rapid increase in **speed of sound**, accompanied by a peak(?) structure

Bedaque,Steiner (2015); Tews,Carlson,Gandolfi,Reddy (2018); Altiparmak,Ecker,Rezzolla (2022); Gorda,Komoltsev,Kurkela (2022) & many others

Fujimoto,Fukushima,Murase (2019)

$$v_s^2 \equiv \frac{dP}{d\varepsilon}$$

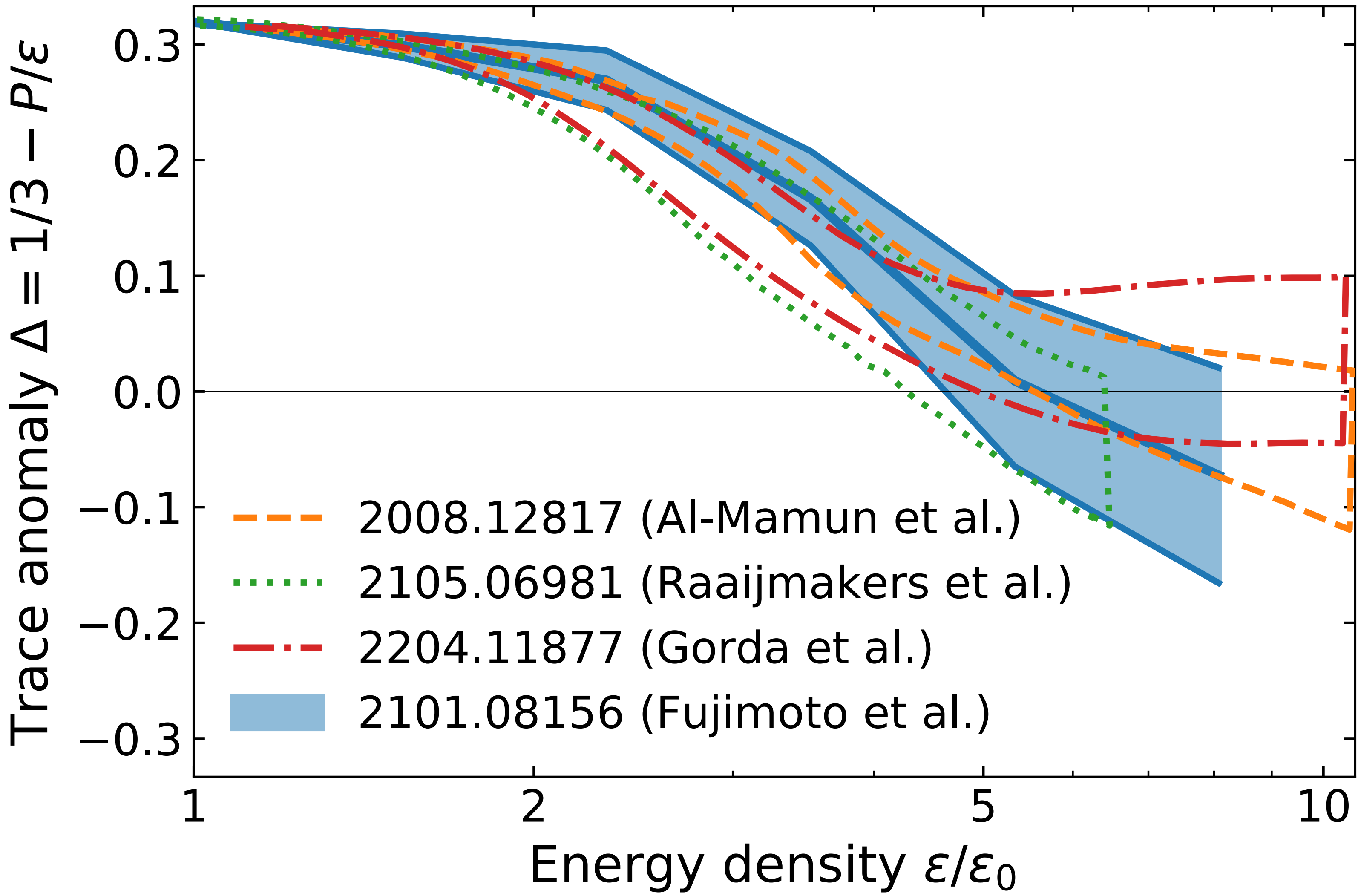


$\varepsilon_0 \approx 150\,\mathrm{MeV/fm^3}$

Conformality

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

$$\Delta \equiv \frac{\langle T^{\mu}{}_{\mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$



$\Delta \sim 0$ (i.e. $P \sim \varepsilon/3$)

already around $\sim 5\varepsilon_0$

rapid stiffening to conformality

Origin of the rapid stiffening & conformality? $\rightarrow$ **Quarkyonic matter has the properties necessary for this behavior**

Quarkyonic duality

Collins,Perry (1974)

Naive expectation: Free deconfined quarks at high density

Large Nc QCD implies:

McLerran,Pisarski (2008)

Duality between <u>quark</u> matter and bar<u>yonic</u> matter

$$\frac{1}{r_{\mathrm{Debye}}} \sim \frac{\lambda_{\mathrm{'t\ Hooft}}\,\mu^2}{N_{\mathrm{c}}}$$ … never screened at finite μ!!

Quarkyonic regime: $\Lambda_{\mathrm{QCD}} < \mu < \sqrt{N_{\mathrm{c}}}\Lambda_{\mathrm{QCD}}$

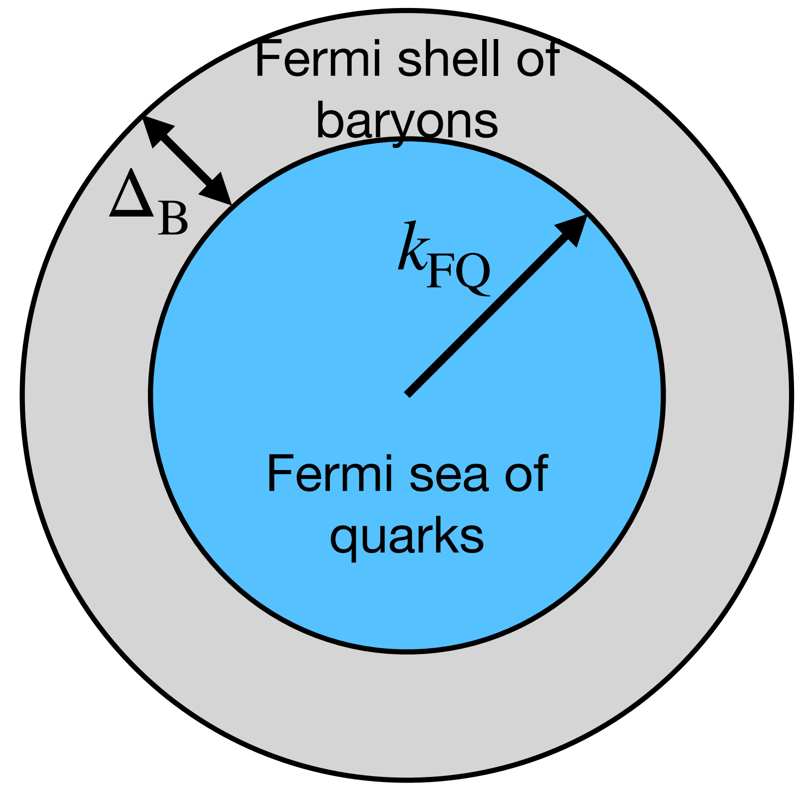
High-density yet confined matter, where we naively expect the weakly-coupled pQCD is valid [$P \sim O(N_{\mathrm{c}})$]

Quarkyonic Fermi "shell" picture

McLerran,Pisarski (2008)

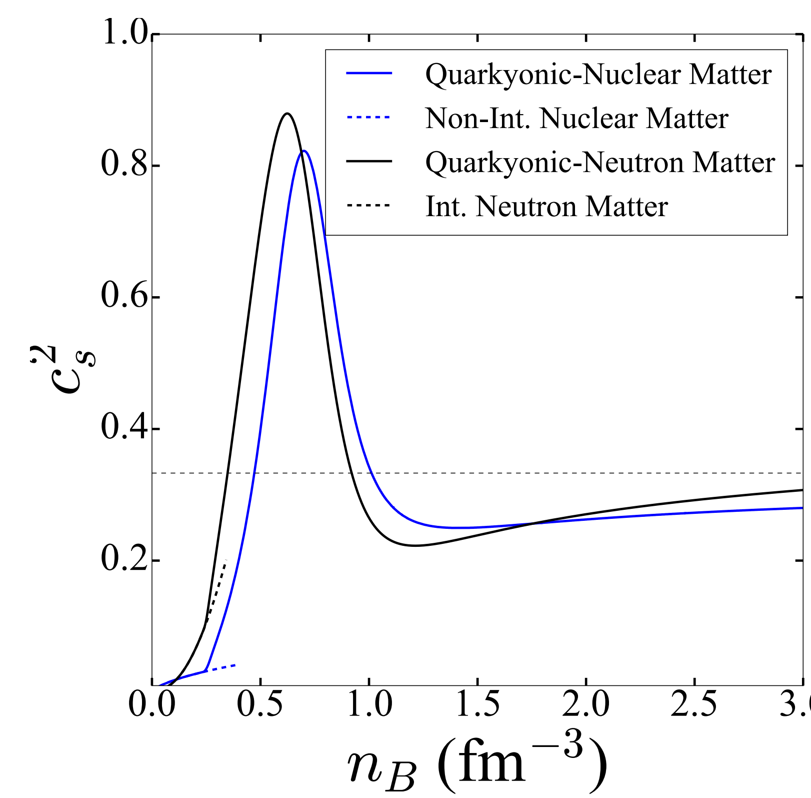
Fermi shell: interaction sensitive to IR d.o.f. → baryons, mesons, glues…

Fermi sea: dominated by interaction that is less sensitive to IR → quarks



This has a desired feature for NS EoS

McLerran,Reddy (2018)



In this work, we give a refined view on the shell structure

Theory w. explicit Quarkyonic duality

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Occupation of baryons and quarks in momentum space:

$$0 \le f_{\mathrm{B}}(k) \le 1, \quad 0 \le f_{\mathrm{Q}}(q) \le 1$$

Energy and baryon density functional with an explicit duality

$$\varepsilon = \varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)] = \varepsilon_{\mathrm{Q}}[f_{\mathrm{Q}}(q)]$$

$$n_{\mathrm{B}} = \int \frac{d^d \boldsymbol{k}}{(2\pi)^d} f_{\mathrm{B}}(k) = \int \frac{d^d \boldsymbol{q}}{(2\pi)^d} f_{\mathrm{Q}}(q)$$

We posit the duality relation between f_{B} and f_{Q}

$$f_{\mathrm{Q}}(q) = \int \frac{d^d \boldsymbol{k}}{(2\pi)^d} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$

Goal: Minimize ε w.r.t. f_{B} or f_{Q} $\rightarrow$ Variational problem

Theory w. explicit Quarkyonic duality

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Energy density functional:

$$\varepsilon = \varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)] = \varepsilon_{\mathrm{Q}}[f_{\mathrm{Q}}(q)]$$

The duality relation between f_{B} and f_{Q}:

$$f_{\mathrm{Q}}(q) = \int \frac{d^d \boldsymbol{k}}{(2\pi)^d} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$

We use the specific model for $\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)]$ and φ:

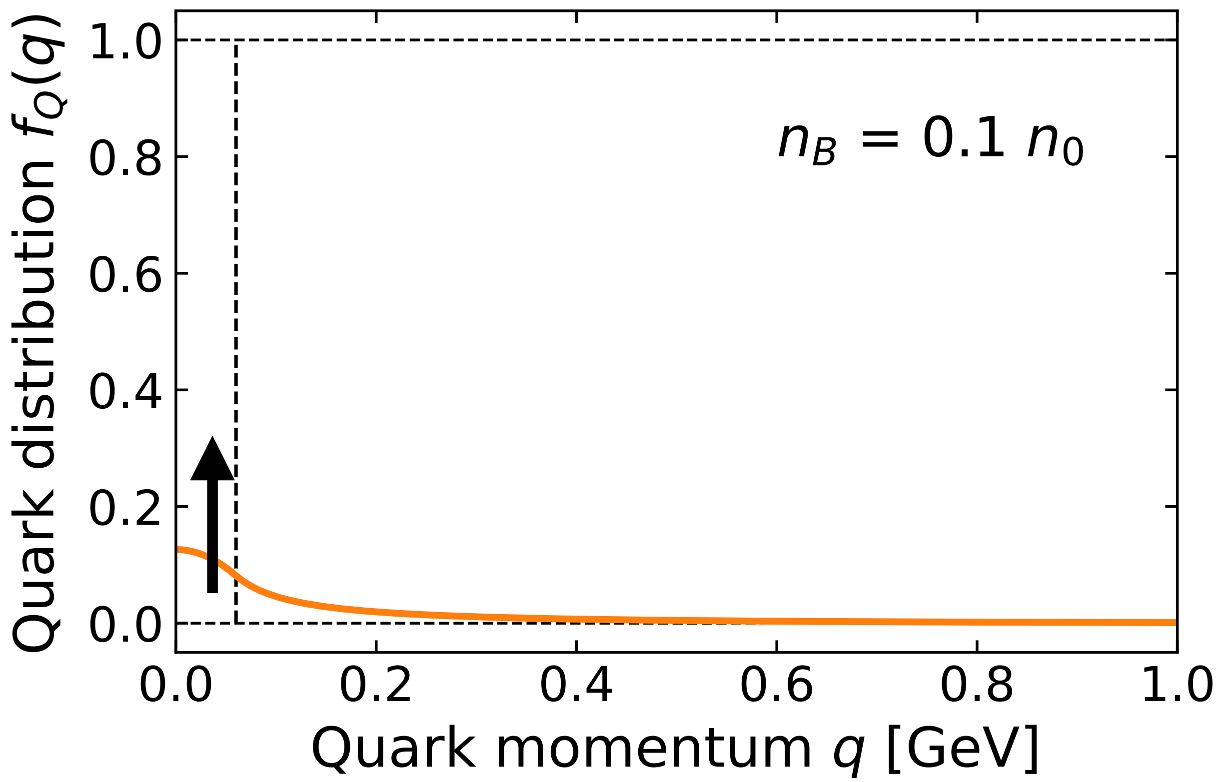
$$\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}(k)] = \int \frac{d^d \boldsymbol{k}}{(2\pi)^d} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k), \qquad \left(E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}\right)$$

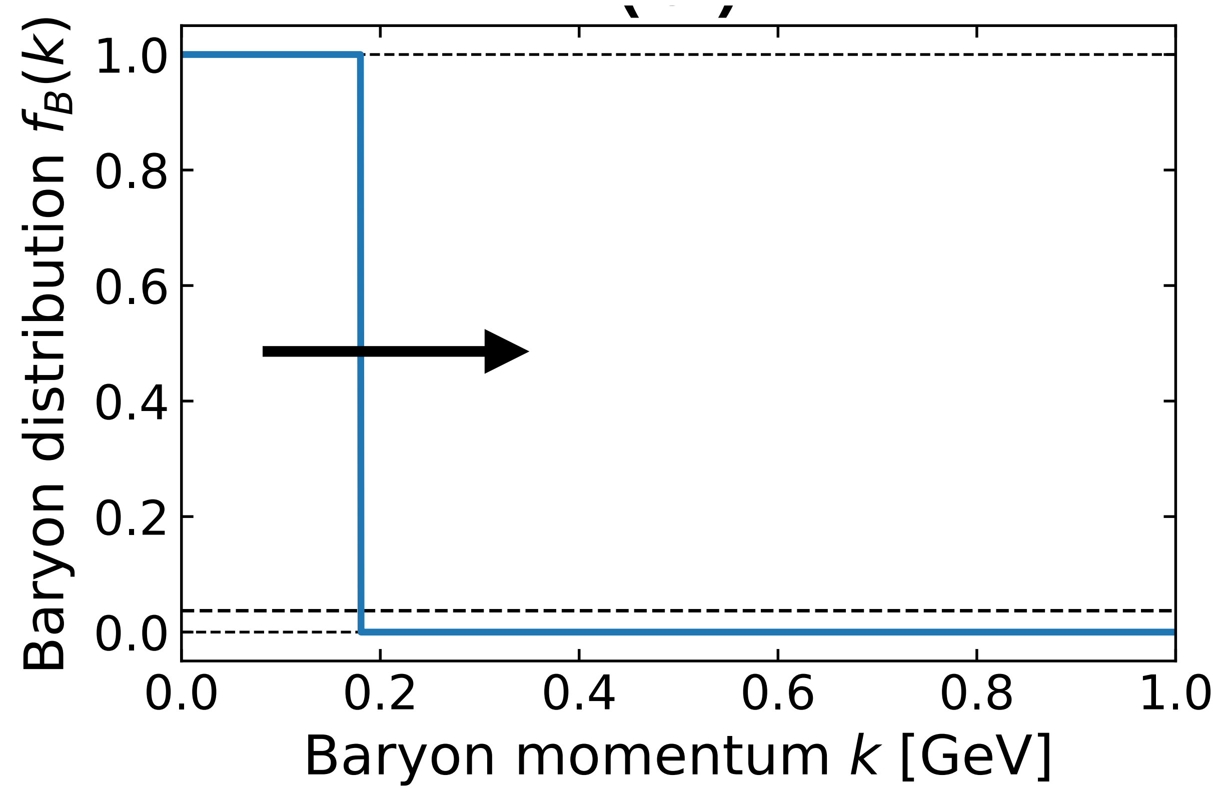
$$\varphi(q) = \frac{2\pi^2}{\Lambda^2} \frac{e^{-q/\Lambda}}{q}, \text{ with a normalization } \int_0^\infty \frac{d^3 \boldsymbol{q}}{(2\pi)^d} \varphi(\boldsymbol{q}) = 1$$

… this specific choice relates f_{B} and f_{Q} one-to-one

Solution at low density

Kojo (2021), Fujimoto,Kojo,McLerran (2023)



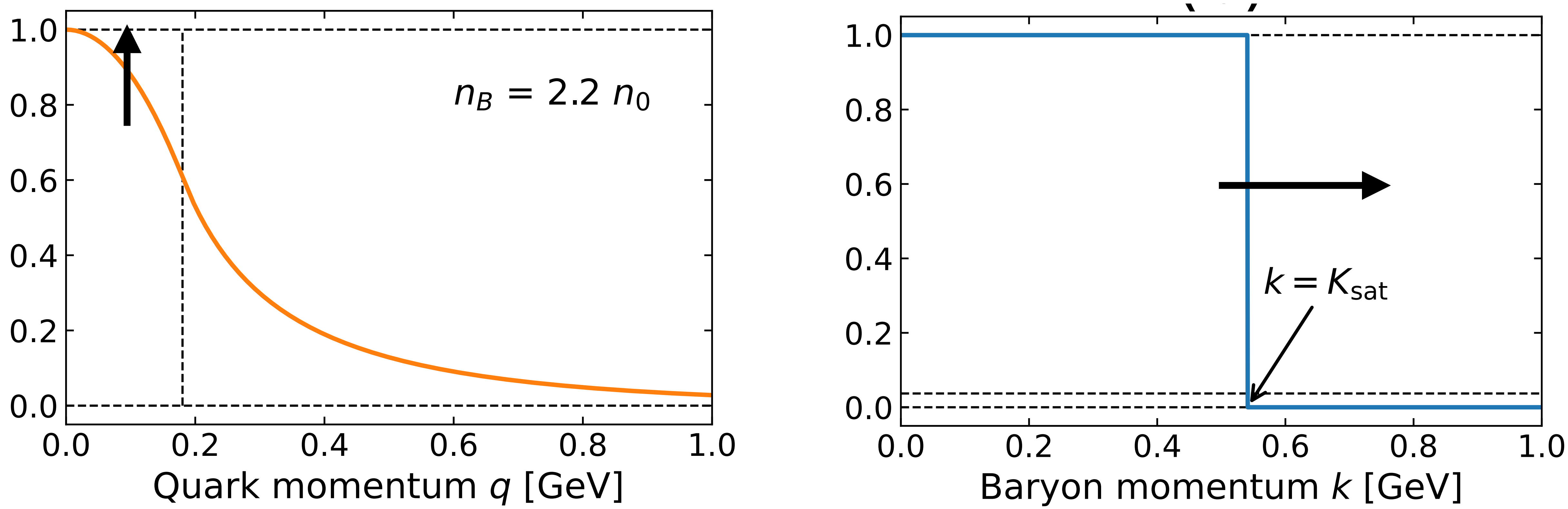


Fermi gas of baryons is formed.

Baryonic Fermi momentum k_{FB} grows until f_{Q} reaches 1

Saturation of the quark distribution

Kojo (2021), Fujimoto,Kojo,McLerran (2023)

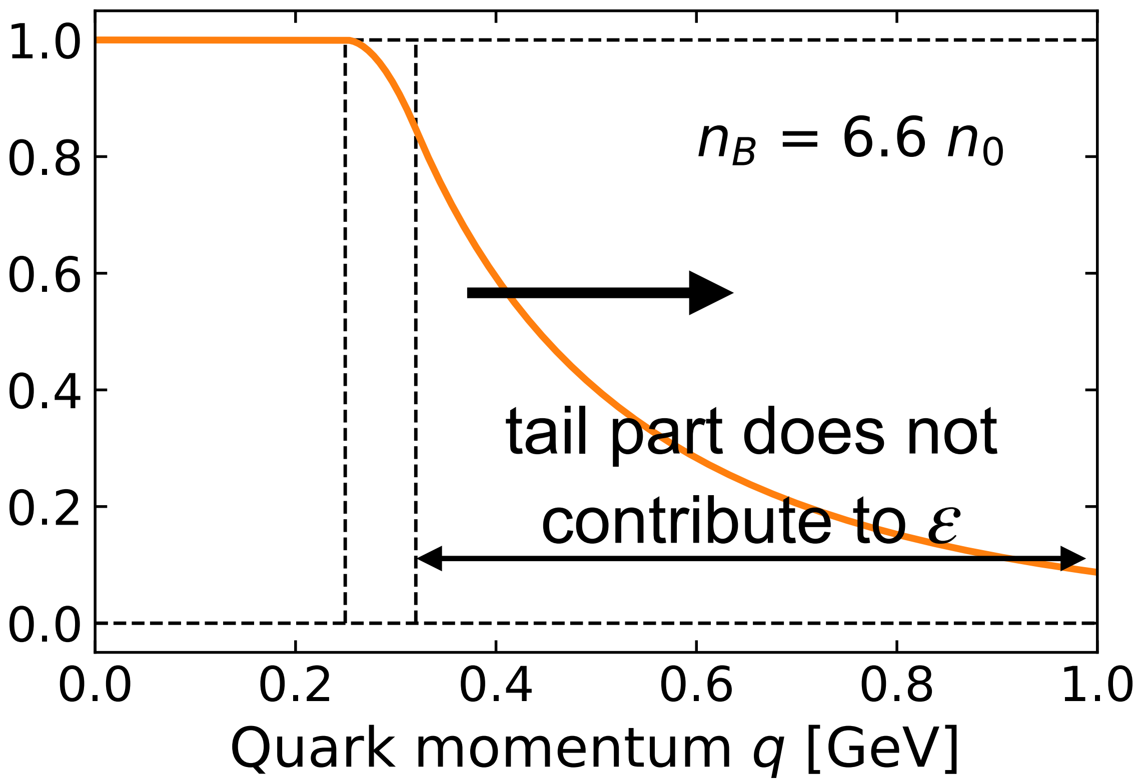


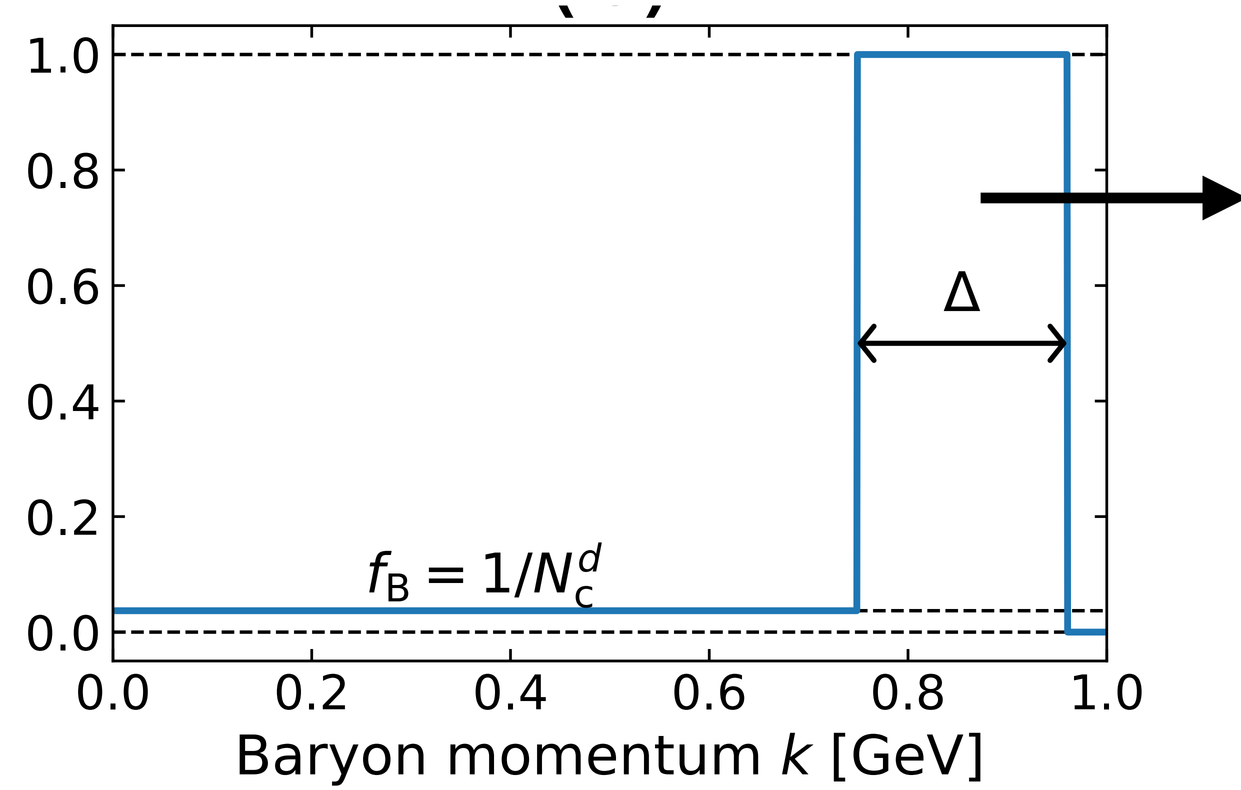
At this point, $f_{\rm Q}$ "saturates" and Pauli blocking constraint becomes essential.

In this model, saturation occurs at the density $\dfrac{n_{\rm B}}{n_0} = 1.1 \left(\dfrac{\Lambda}{0.3\ \text{GeV}} \right)^3$

Solution at high density

Fujimoto,Kojo,McLerran (2023)





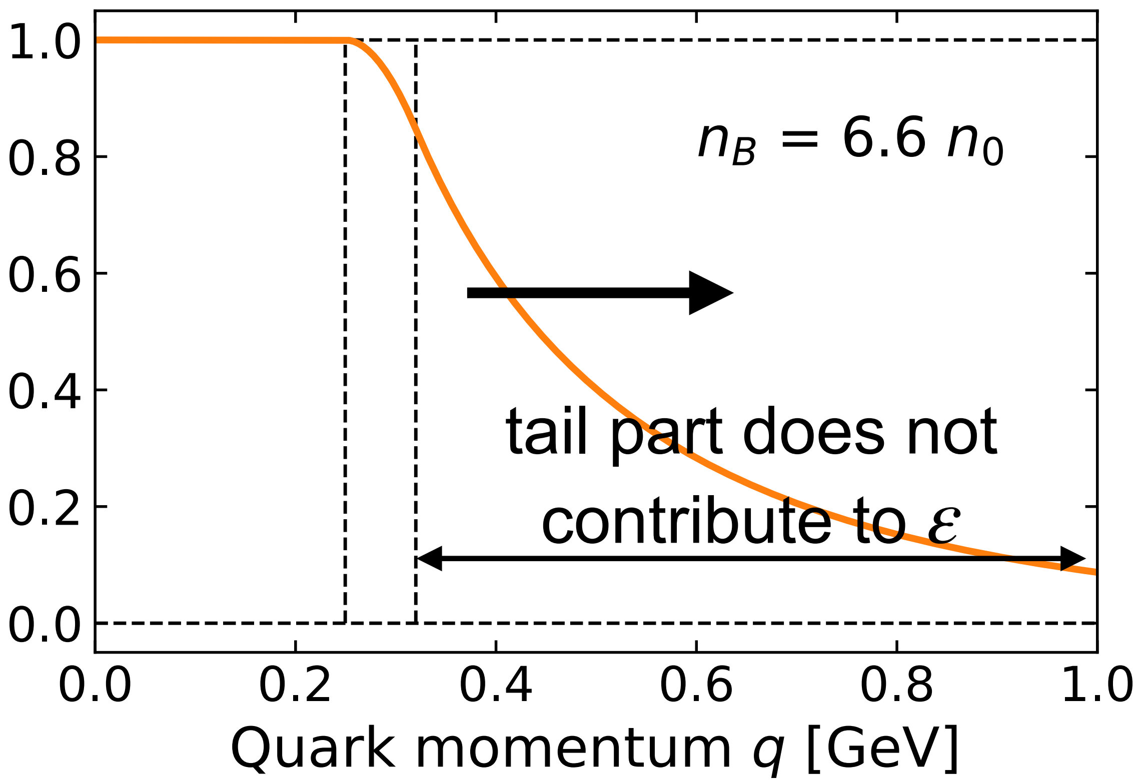
Saturation in low-momentum f_Q → Depletion in $f_B \sim 1/N_c^3$ (in 3d)

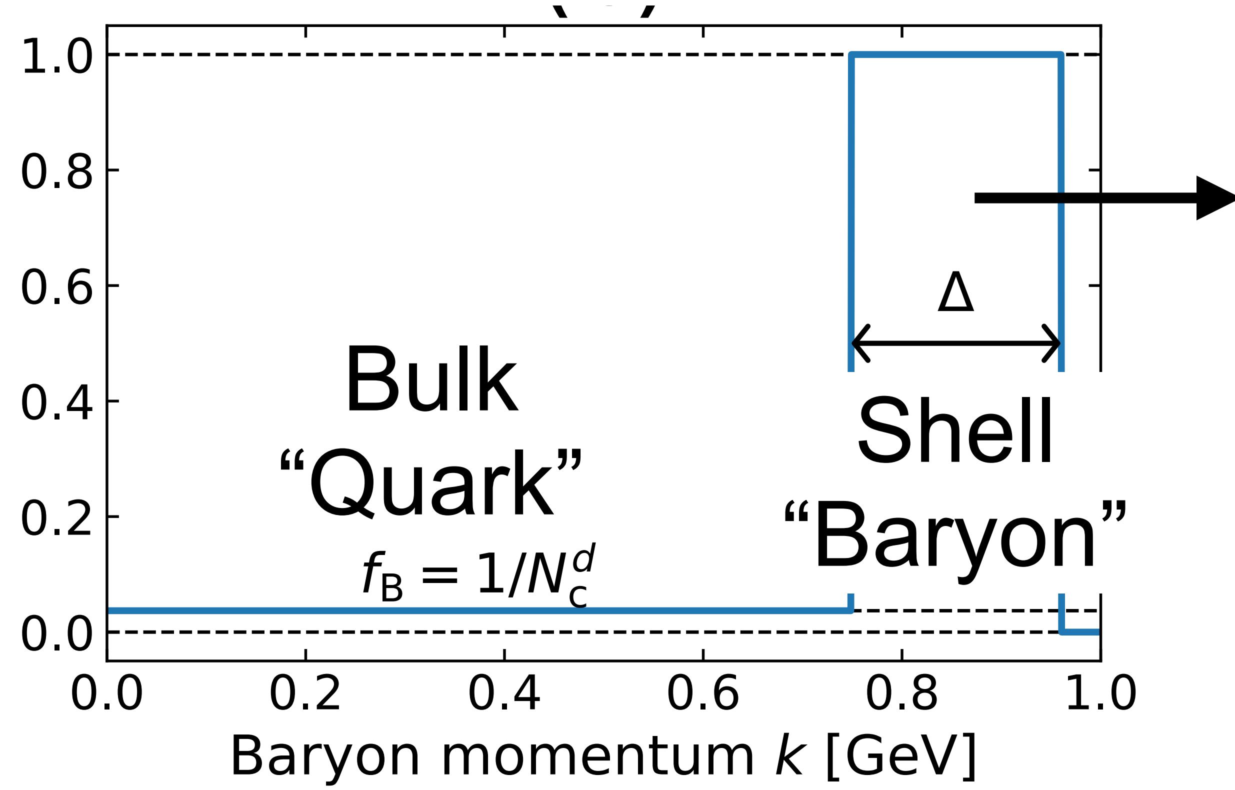
Tail in f_Q w/ a width $\sim \Lambda$

→ Shell formation in high-momentum f_B w/ $\Delta_B \sim \Lambda/N_c^2$

Solution at high density

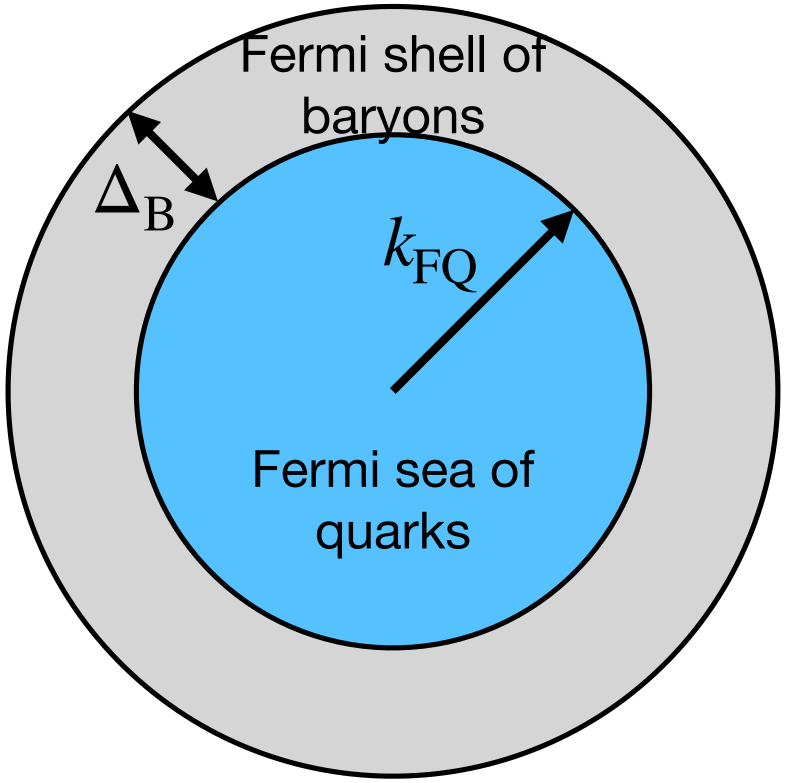
Fujimoto,Kojo,McLerran (2023)





Fermi shell structure arises in f_B

Integral is equivalent to McLerran-Reddy model apart from the behavior of Δ_B



Underoccupied f_B and occupied f_Q

Baryon number in the bulk “quark” region in the quark language:

$$n_\mathrm{B} = \int_0^{k_\mathrm{FQ}} \frac{d^3q}{(2\pi)^3} f_\mathrm{Q}(q) \sim k_\mathrm{FQ}^3 f_\mathrm{Q}$$

In the baryon language:

$$n_\mathrm{B} = \int_0^{k_\mathrm{FB}} \frac{d^3k}{(2\pi)^3} f_\mathrm{B}(k) \sim k_\mathrm{FB}^3 f_\mathrm{B} \sim N_\mathrm{c}^3 k_\mathrm{FQ}^3 f_\mathrm{B}$$

where the Fermi momenta are related as $k_\mathrm{FB} \sim N_\mathrm{c} k_\mathrm{FQ}$.

Because $f_\mathrm{Q} \le 1, \quad f_\mathrm{B} \sim 1/N_\mathrm{c}^3$ … composite baryon states are underoccupied

Rapid stiffening in the EoS

A partial occupation of available baryon phase space leads to the **large speed of sound**.

$$v_s^2 = \frac{n_{\mathrm{B}}}{\mu_{\mathrm{B}} dn_{\mathrm{B}}/d\mu_{\mathrm{B}}} \to \frac{\delta\mu_{\mathrm{B}}}{\mu_{\mathrm{B}}} \sim v_s^2 \frac{\delta n_{\mathrm{B}}}{n_{\mathrm{B}}}$$

If baryon is underoccupied, the density is not changing much, but the Fermi energy changes a lot.

Summary

- Currently observed features of the EoS: rapid rise in sound speed and vanishing conformality measure

- We formulate the quantum-mechanical theory of Quarkyonic matter with an explicit duality. We solved this theory for the ideal baryonic dispersion relation with an exactly solvable kernel.

- Previously proposed Fermi “shell” structure of Quarkyonic matter naturally arises in the baryon distribution.

- Rapid stiffening in the EoS is observed.