W boson mass and grand unification via the type-Ⅱ seesaw-like mechanism

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Based on
Y. Shimizu and **ST,** Nucl. Phys. B **994** (2023), 116290, arXiv:2303.11070 [hep-ph].

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KEK-PH2023@KEK

Today’s talk

We extend the minimal SU(5) GUT model by adding the vector-like fermions.

Type-Ⅱ seesaw-like mechanism

Our motivation

- The W boson mass anomaly
- Proton decay
- Gauge unification

Constraints

- The second Higgs
- Vector-like fermions
- Proton decay

Contents

1. Minimal SU(5) GUT

2. W boson mass

3. Our model

4. Summary

Minimal SU(5) GUT

Minimal SU(5) GUT

The theory of embedding $SU(3)_C \times SU(2)_L \times U(1)_Y$ into SU(5) gauge group.

H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438-441

- Unify the SM gauge interactions

$$A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V_\mu^\dagger \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}$$

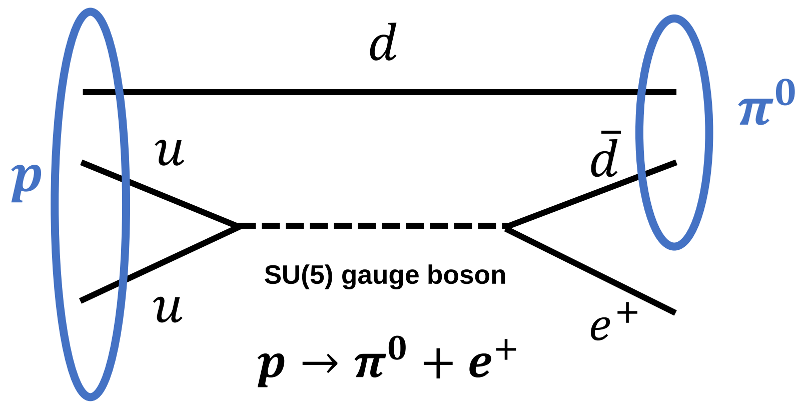
- Unification of quarks and leptons

$$\overline{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \quad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & e^c \\ d^1 & d^2 & d^3 & -e^c & 0 \end{pmatrix}_L$$

Proton decay

The GUT predicts the existence of proton decay.

The GUT can be tested
by proton decay search near future!



Current experimental results
Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years
A. Takenaka et al. Phys. Rev. D **102** (2020) no.11, 112011

The problems of GUT

- Inconsistency with experimental results

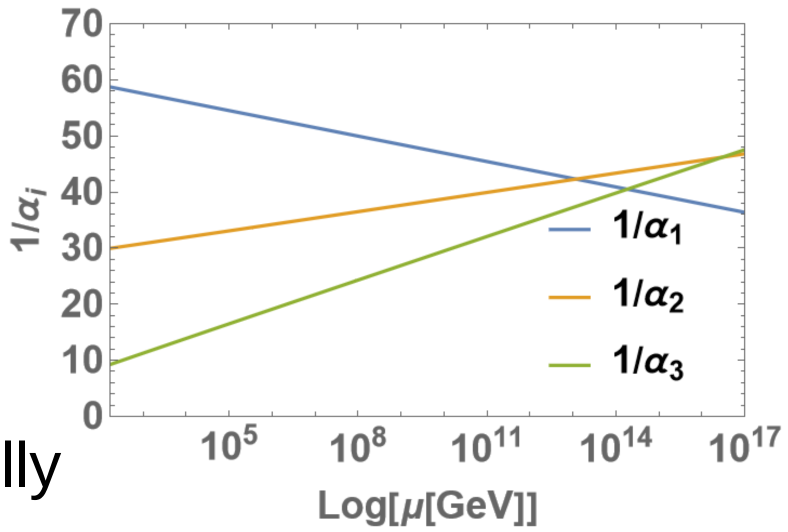
Minimal SU(5) GUT : $\tau_p(p \to \pi^0 e^+) \approx 10^{30}{\sim}10^{31}$ years

H. Georgi, H.R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33** (1974), 451-454

Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years

A. Takenaka et al. Phys. Rev. D **102** (2020) no.11, 112011

- No unification of the SM gauge couplings successfully



α_1 : U(1) gauge coupling
α_2 : SU(2) gauge coupling
α_3 : SU(3) gauge coupling

W boson mass

W boson mass anomaly

The CDF collaboration reported a new result of the W boson mass.

J. de Blas, M. Pierini, L. Reina and L. Silvestrini, Phys.Rev.Lett. **129** (2022) no.27, 271801

The SM prediction : $M_W^{\rm SM} = 80.3500 \pm 0.0056$ GeV

6.5 σ

The experimental value : $M_W^{\rm CDF} = 80.4133 \pm 0.0080$ GeV

[CDF Collaboration], Science **376** (2022), no.6589, 170-176

The new physics contribute to the W boson mass.

Triplet scalar

We focus on a real $SU(2)_L$ triplet coming from 24_H.

$$24_H = \begin{pmatrix} H_8 - \frac{2}{\sqrt{30}} H_0 & H_{(\bar{3},2)} \\ H_{(3,2)} & T + \frac{3}{\sqrt{30}} H_0 \end{pmatrix}$$

This scalar breaks the SU(5) symmetry.

$$H_8 \sim (8,1,0), \quad T \sim (1,3,0), \quad H_0 \sim (1,1,0),$$
$$H_{(3,2)} \sim (3,2,-5/6), \quad H_{(\bar{3},2)} \sim (\bar{3},2,5/6).$$

Triplet scalar

model : The SM Higgs H + a real triplet with zero hypercharge T

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1/2) \qquad T = \frac{1}{2}\begin{pmatrix} T^0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T^0 \end{pmatrix} \sim (1, 3, 0)$$

Lagrangian : $\mathcal{L}_{scalar} \supset \left(D_\mu H\right)^\dagger (D^\mu H) + \mathrm{Tr}\left(D_\mu T\right)^\dagger (D^\mu T) - V(H, T)$

Covariant derivative : $D_\mu H = \partial_\mu H + \mathrm{i} g_1 \frac{B_\mu}{2} + \mathrm{i} g_2 W_\mu, \quad D_\mu T = \partial_\mu T + \mathrm{i}\, g_2 \left[W_\mu , T\right]$

R. S. Chivukula, N. D. Christensen, and E. H. Simmons, Phys. Rev. D **77** (2008), 035001

P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B **833** (2022), 137371

Triplet scalar

R. S. Chivukula, N. D. Christensen, and E. H. Simmons, Phys. Rev. D **77** (2008), 035001
P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B **833** (2022), 137371

Potential : $$V(H,T) = -m_h^2 H^\dagger H + \lambda_0 \left(H^\dagger H\right)^2 + M_T^2 \mathrm{Tr}[T^2] + \lambda_1 \mathrm{Tr}[T^4] + \lambda_2 (\mathrm{Tr}[T^2])^2 + \alpha\left(H^\dagger H\right)\mathrm{Tr}[T^2] + \beta H^\dagger T^2 H + \mu H^\dagger T H$$

The H and T acquire the VEVs at the minimum of the potential.

Fields : $$H = \begin{pmatrix} \phi^+ \\ (v_h + h^0 + \mathrm{i}G^0)/\sqrt{2} \end{pmatrix}, T = \frac{1}{2}\begin{pmatrix} v_T + t^0 & \sqrt{2}t^+ \\ \sqrt{2}t^- & -v_T - t^0 \end{pmatrix}.$$

W boson mass

The VEV of the $SU(2)_L$ triplet scalar contributes to the W boson mass.

$$M_W^2 = \left(M_W^{SM}\right)^2 + g_2^2 v_T^2$$

Tree level contribution

The CDF W boson mass anomaly can be explained!

$$v_T = 4.85 \text{ GeV}$$

Mass eigenvalues

In the limit $v_h \gg v_T$, the mixing angles are too small, $\theta_0 \ll 1$.

Mixing angles

$$\tan 2\theta_0 = \frac{4v_h v_T(-\mu + 2Av_T)}{8\lambda_0 v_h^2 v_T - 4Bv_T^3 - \mu v_h^2},$$

$$\tan 2\theta_+ = \frac{4v_h v_T}{4v_T^2 - v_h^2}.$$

$$M_h^2 = 2\lambda_0 v_h^2, \longrightarrow \text{SM-like Higgs}$$

$$M_H^2 = Bv_T^2 + \frac{\mu v_h^2}{4v_T},$$

$$M_{H^\pm}^2 = \mu v_T + \frac{\mu v_h^2}{4v_T}.$$

$\xrightarrow{v_h \gg v_T}$

$$M_H^2 = M_{H^\pm}^2 \approx \frac{\mu v_h^2}{4v_T} (= M_T^2)$$

Our model

Y. Shimizu and **ST,** Nucl. Phys. B **994** (2023), 116290,
arXiv:2303.11070 [hep-ph].

Our model

We extend the minimal SU(5) GUT by adding the vector-like fermions.

$10^4_{L,R} = Q(3,2,1/6) \oplus U^c(\bar{3},1,-2/3) \oplus E^c(1,1,1)$ $\bar{5}^i_L, 10^i_L, 5_H, 24_H, A_\mu$

New particles

The Lagrangian for the vector-like fermions:

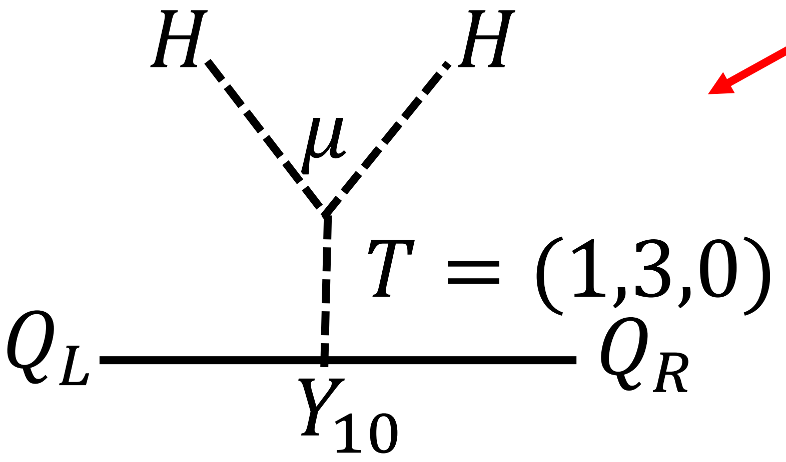
$$\mathcal{L}_{VL} \supset \overline{10}^4_L [Y^4_{10} 24_H + M^4_{10}] 10^4_R + \text{h.c.}$$

Yukawa interaction   Mass term

Type-Ⅱ seesaw-like mechanism

Vector-like quark doublet acquires the mass via the type-Ⅱ seesaw-like mechanism.

$Q(3,2,1/6)$



The mechanism for explaining the neutrino mass

Type-Ⅱ seesaw mechanism is that the triplet with hypercharge one is added.

In order to explain the W boson mass anomaly, the real triplet gets the VEV in our model.

$$\mathcal{L}_{VL-RT} \supset Y_{10}^{4} T \bar{Q}_L^4 Q_R^4 + \text{h.c.}$$

Mass eigenvalues

We assume that vector-like quark doublet acquires the mass only through the type-Ⅱ seesaw-like mechanism.

$$M_Q^4 = M_{10}^4 - \frac{Y_{10}^4 V_{24}}{4\sqrt{15}} + Y_{10}^4 \frac{\mu v_h^2}{8M_T^2},$$

$$M_U^4 = M_{10}^4 + \frac{Y_{10}^4 V_{24}}{\sqrt{15}},$$

$$M_E^4 = M_{10}^4 - \frac{3Y_{10}^4 V_{24}}{2\sqrt{15}}.$$

$$M_{10}^4 = \frac{Y_{10}^4 V_{24}}{4\sqrt{15}}$$

$$M_Q^4 = Y_{10}^4 \frac{\mu v_h^2}{8M_T^2},$$

$$M_U^4 = M_E^4 = \frac{5Y_{10}^4 V_{24}}{4\sqrt{15}}.$$

Constraints

- Perturbative unitarity of the WW scattering cross-section

R. S. Chivukula, N. D. Christensen and E. H. Simmons,
Phys. Rev. D **77** (2008), 035001

$$M_H, M_{H^\pm} \lesssim \frac{2\sqrt{\pi} v_h^2}{v_T}$$

$$M_H^2 = M_{H^\pm}^2 \approx \frac{\mu v_h^2}{4 v_T}$$

$$\mu < \frac{16\pi v_h^2}{v_T} \approx 6.28 \times 10^2 \text{ TeV}$$

$$v_h = \left(\sqrt{2} G_F\right)^{-1/2} \approx 246.22 \text{ GeV}$$

$$v_T \approx 4.85 \text{ GeV}$$

$$M_Q^4 = Y_{10}^4 \frac{\mu v_h^2}{8 M_T^2}$$

$$M_Q^4 \times (M_T)^2 < 4.76 \times Y_{10}^4 \ (\text{TeV})^3$$

Experimental constraints

- The bound for vector-like quark : $M_Q^4 > 1660$ GeV

A. M. Sirunyan et al. [CMS], Eur. Phys. J. C **79** (2019), 90

- The bound for heavy neutral Higgs : $M_H > 1400$ GeV

G. Aad et al. [ATLAS], JHEP **06** (2021), 145

- The bound for charged Higgs: $M_{H^\pm} > 1000$ GeV

G. Aad et al. [ATLAS], Phys. Rev. D **102** (2020) no.3, 032004

Allowed mass range

We assume that $Y_{10}^4 = 1.$

If Y_{10}^4 is small, the upper bound is inconsistent with the experimental constraints.

$$M_Q^4 \times (M_T)^2 < 4.76\ (\mathrm{TeV})^3$$

$M_Q^4 > 1660$ GeV → $M_T < 1693$GeV

$M_H > 1400$ GeV ($M_{H^\pm} > 1000$ GeV) → $M_Q^4 < 2428$ GeV ($M_Q^4 < 4759$ GeV)

Heavy neutral Higgs : $1660 < M_Q^4 < 2428$ GeV, $1400 < M_T < 1693$ GeV

Charged Higgs : $1660 < M_Q^4 < 4759$ GeV, $1000 < M_T < 1693$ GeV

One generation case

The candidates for contributing to the RGE

$$24_{\mathrm{H}} = \begin{pmatrix} H_8 - \frac{2}{\sqrt{30}} H_0 & H_{(\bar{3},2)} \\ H_{(3,2)} & T + \frac{3}{\sqrt{30}} H_0 \end{pmatrix}$$

$$Q^4, U^4, E^4, H_8, T$$

↓

In the case that $Y_{10}^4 = 1$, the vector-like right-handed up quark and electron have the same GUT scale mass, $M_U^4 = M_E^4 = \frac{5V_{24}}{4\sqrt{15}}$.

The candidates for contributing to the RGE

$1660 < M_Q^4 < 2428$ GeV,

$1400 < M_T < 1693$ GeV

H_8 : only experimental constraints

One generation case

- Only one generation for the 10 representation vector-like fermions

The candidates for contributing to the RGE

$1660 < M_Q^4 < 2428$ GeV,
$1400 < M_T < 1693$ GeV
H_8 : only experimental constraints

The SM gauge couplings do not unify successfully!

If we do not consider the constraints from the type-Ⅱ seesaw-like mechanism, the SM gauge couplings unify successfully by adding only one generation.

J. L. Evans, T. T. Yanagida and N. Yokozaki, Phys. Lett. B **833** (2022), 137306

Our model

The candidates for contributing to the RGE

We add two generations.

$$Q^4, U^4, E^4, Q^5, U^5, E^5, H_8, T$$

$$24_H = \begin{pmatrix} H_8 - \frac{2}{\sqrt{30}} H_0 & H_{(\bar{3},2)} \\ H_{(3,2)} & T + \frac{3}{\sqrt{30}} H_0 \end{pmatrix}$$

In the case that $Y_{10}^4 = 1$, the vector-like right-handed up quark and electron have the same GUT scale mass, $M_U^4 = M_E^4 = \frac{5V_{24}}{4\sqrt{15}}$.

The candidates for contributing to the RGE

$1660 < M_Q^4 < 2428$ GeV,

$1400 < M_T < 1693$ GeV

Q^5, U^5, E^5, H_8 : only experimental constraints

Numerical analysis

Q^5, U^5, E^5, H_8

We investigate the mass ranges for the new particles that achieve the unification of the SM gauge couplings.

We evaluate the contributions of the SM particles to the RGE at the 2-loop level and the new contributions at the 1-loop level.

The candidates for contributing to the RGE

$1660 < M_Q^4 < 2428$ GeV,

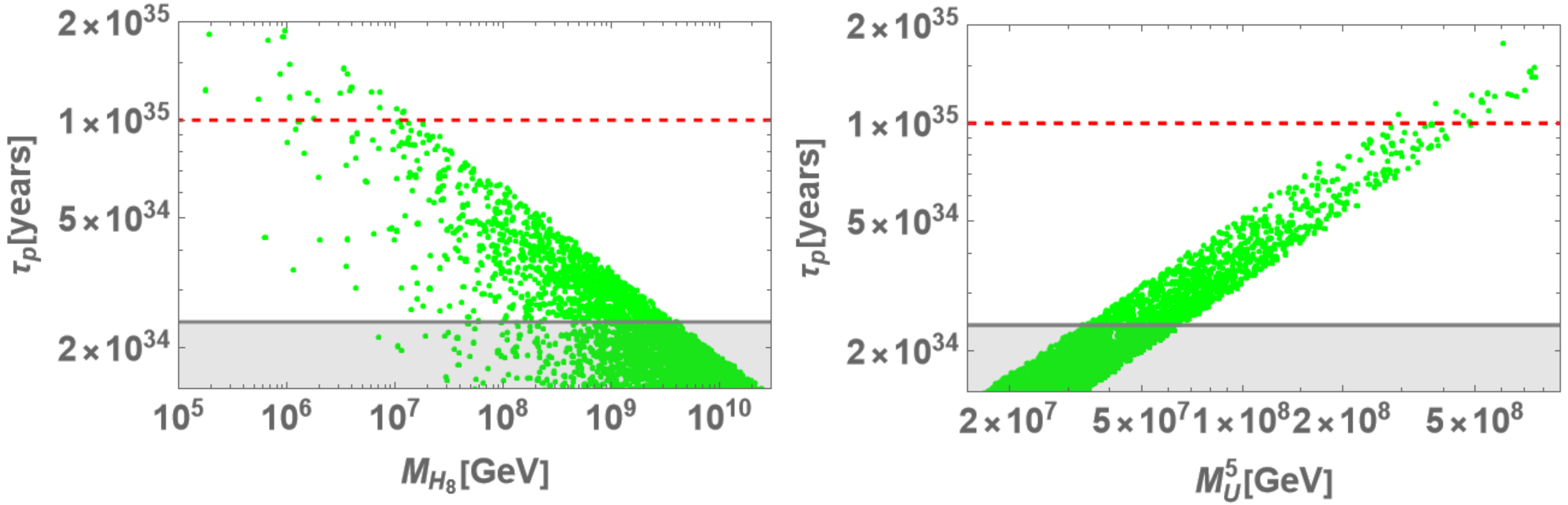
$1400 < M_T < 1693$ GeV

Q^5, U^5, E^5, H_8 : only experimental constraints

Results

Gauge unification
within an accuracy of 1% or less

M_{H_8}, M_U^5: an intermediate scale
M_Q^5, M_E^5 : GUT scale



Red dashed line :
Hyper-Kamioknade
$\tau_p(p \to \pi^0 e^+) < 1.0 \times 10^{35}$ years

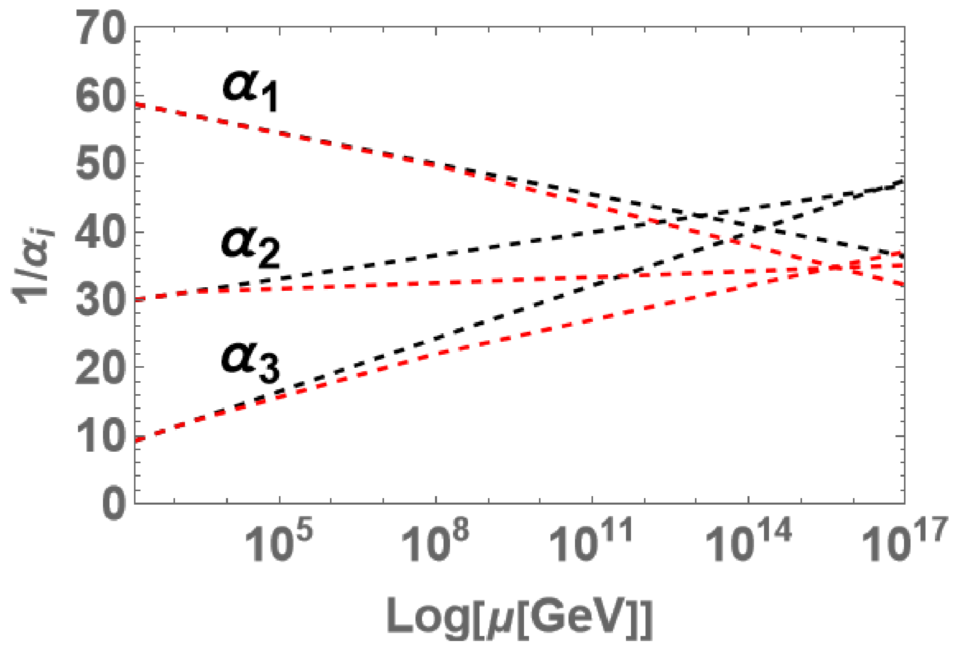
Gray shaded region :
Super-Kamiokande
$\tau_p(p \to \pi^0 e^+) > 2.4 \times 10^{34}$ years

Gauge unification

Benchmark

$M_Q^4 = 2000$ GeV, $M_T = 1500$ GeV,
$M_U^5 = M_{H_8} = 10^8$ GeV,
The other particles: GUT scale

$$M_{GUT} \approx 5.1 \times 10^{15} \text{ GeV},$$
$$\alpha_{GUT} = \alpha_1 = \alpha_2 = \alpha_3 \approx 1/34.7,$$
$$\tau_p(p \to \pi^0 e^+) \approx 4.12 \times 10^{34} \text{ years.}$$



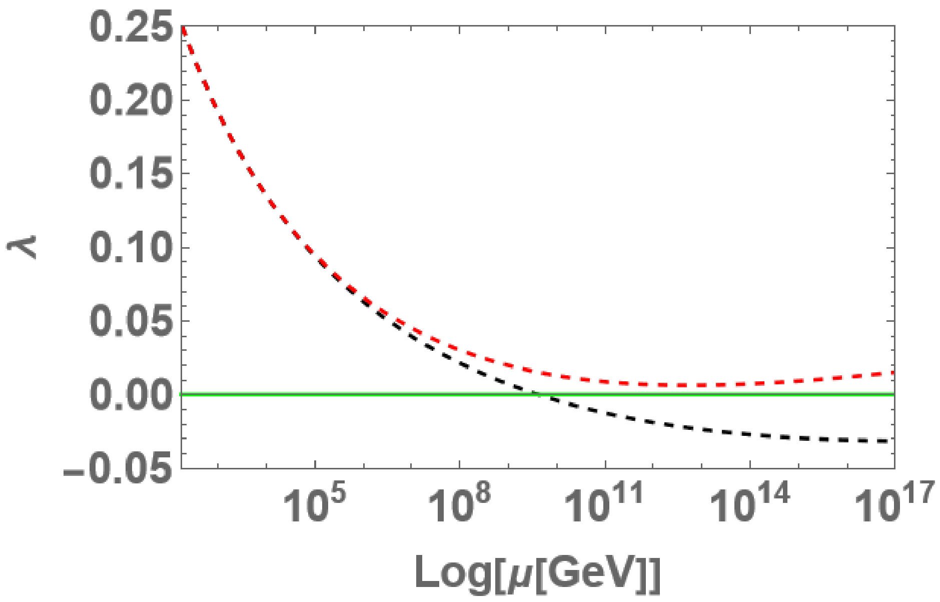
Black line: the SM
Red line: Our model

Higgs quartic couplings

Benchmark

$M_Q^4 = 2000$ GeV, $M_T = 1500$ GeV,
$M_U^5 = M_{H_8} = 10^8$ GeV,
The other particles: GUT scale

The SM Higgs potential is stabilized.



Black line: the SM
Red line: Our model

Summary

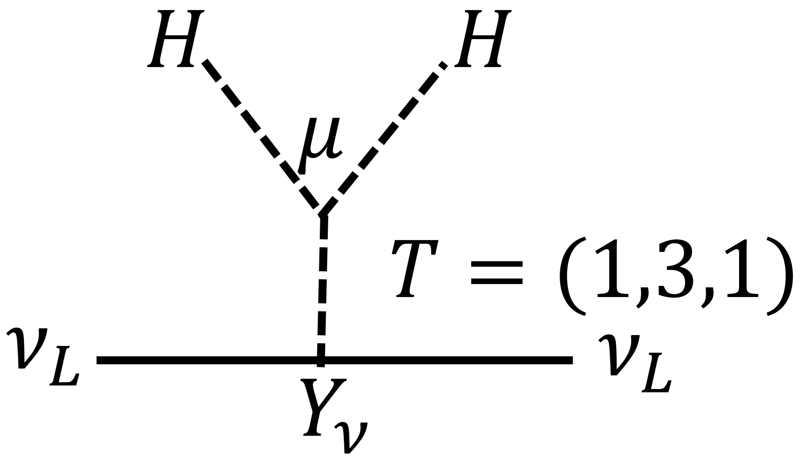
Summary

- We explain the W boson mass anomaly in the SU(5) GUT model.
- If we assume that vector-like quark doublet acquires the mass only through the type-Ⅱ seesaw-like mechanism, the mass ranges for the vector-like quark doublet and the real triplet are very restricted.
- Under these restrictions, we need two additional pairs of 10 representation vector-like fermions to unify the SM gauge couplings successfully.

Back up

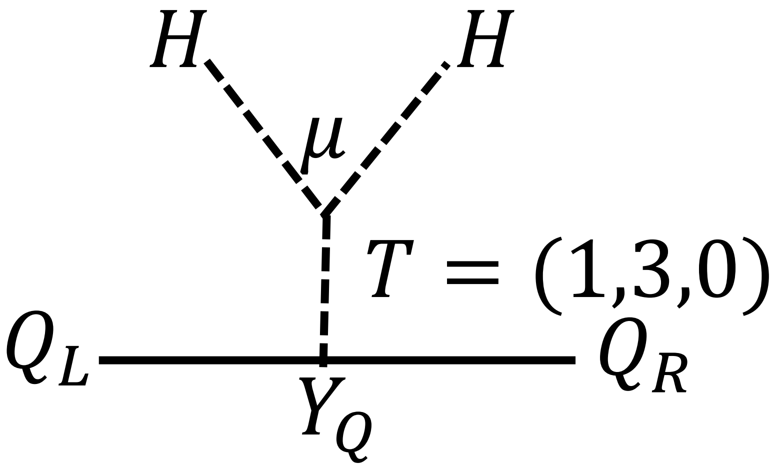
Type-Ⅱ seesaw mechanism

- Type-Ⅱ seesaw mechanism



$$M_\nu \approx Y_\nu \frac{\mu v_h^2}{8M_T^2}$$

- Type-Ⅱ seesaw-like mechanism



$$M_Q \approx Y_Q \frac{\mu v_h^2}{8M_T^2}$$

The minimization conditions

The minimization conditions :

$$m_h^2 - \lambda_0 v_h^2 - \frac{A}{2} v_h^2 + \frac{\mu}{2} v_T = 0,$$

$$M_T^2 - \frac{\mu v_h^2}{4 v_T} + \frac{A}{2} v_h^2 + \frac{B}{2} v_T^2 = 0.$$

$$\left(\boldsymbol{A = \alpha + \frac{\beta}{2},\ B = \lambda_1 + 2\lambda_2}\right)$$

$v_h \gg v_T$ →

$$M_T^2 \approx \frac{\mu v_h^2}{4 v_T} \left(v_T \approx \frac{\mu v_h^2}{4 M_T^2} \right)$$

Mass matrix

- Mass matrix

$$M_0^2 = \begin{pmatrix} 2\lambda_0 v_h^2 & -\frac{\mu v_h}{2} + A v_h v_T \\ -\frac{\mu v_h}{2} + A v_h v_T & B v_T^2 + \frac{\mu v_h^2}{4 v_T} \end{pmatrix}, \qquad M_\pm^2 = \begin{pmatrix} \mu v_T & \frac{\mu v_h^2}{2} \\ \frac{\mu v_h^2}{2} & \frac{\mu v_h^2}{4 v_T} \end{pmatrix}.$$

- Mass eigenstates

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} h^0 \\ t^0 \end{pmatrix}, \qquad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \begin{pmatrix} -\sin\theta_+ & \cos\theta_+ \\ \cos\theta_+ & \sin\theta_+ \end{pmatrix} \begin{pmatrix} \phi^\pm \\ t^\pm \end{pmatrix}.$$

Feynman Rule

P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B 833 (2022), 137371

Interaction	Feynman Rule
$hf\bar{f}$	$i(M_f/v_0)$
$H^+\bar{\nu}_i e_i$	$-i\frac{\sqrt{2}}{v_0}M_e^i \sin\theta_+ P_R$
$H^+\bar{u}d$	$-i\frac{\sqrt{2}}{v_0}\sin\theta_+ \left(-M_u V_{\rm CKM} P_L + V_{\rm CKM} M_d P_R\right)$
ZZh	$(2iM_Z^2/v_0)g^{\mu\nu}$
$ZW^\pm H^\mp$	$ig_2\left(-g_2 x_0 c_+ c_w + \frac{1}{2}g_Y v_0 s_+ s_w\right)g^{\mu\nu}$
W^+W^-h	$ig_2^2\left(\frac{1}{2}v_0\right)g^{\mu\nu}$
W^+W^-H	$ig_2^2\left(2x_0\right)g^{\mu\nu}$
γH^+H^-	$ie\,(p'-p)^\mu$
ZH^+H^-	$i\left(g_2 c_w - \frac{M_Z}{v_0}s_+^2\right)(p'-p)^\mu$
$W^\pm hH^\mp$	$\pm ig_2\left(\frac{1}{2}s_+\right)(p'-p)^\mu$
$W^\pm HH^\mp$	$\pm ig_2 c_+ (p'-p)^\mu$

TABLE I: Feynman Rules in the limit when h is SM-like $(\theta_0 \to 0)$

Decay mode

P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B 833 (2022), 137371

- The heavy neutral Higgs($\theta_0 \to 0$)

Main decay mode: $H \to WW$

- The charged Higgs($\theta_0 \to 0$)

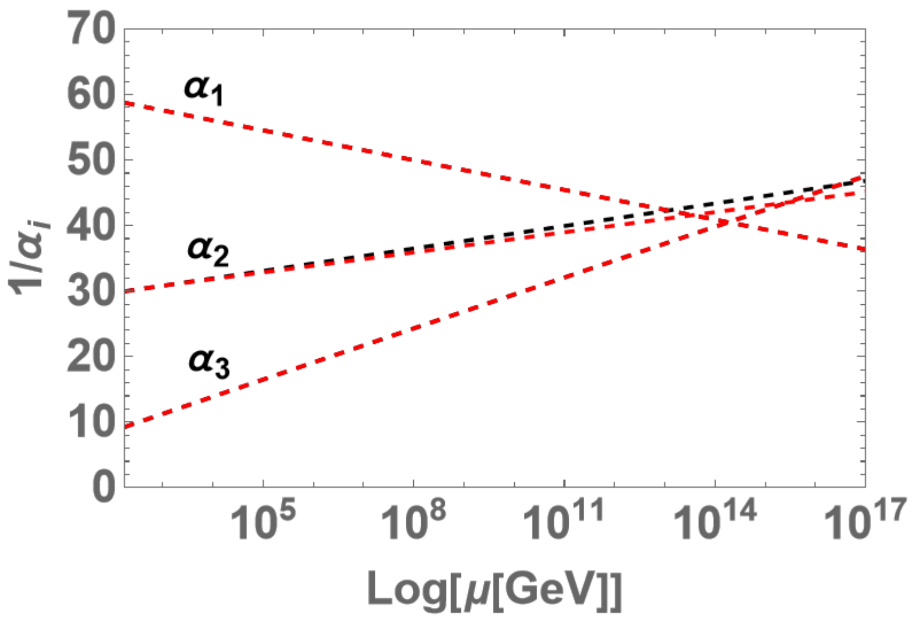
Main decay mode: $H^+ \to \tau^+ \nu_\tau$
$H^+ \to t\bar{b}$
$H^+ \to W^+ Z$
$H^+ \to h W^+$

Gauge unification

J. L. Evans, T. T. Yanagida and N. Yokozaki,
Phys. Lett. B **833** (2022), 137306

The real $SU(2)_L$ triplet scalar contributes to only the $SU(2)_L$ gauge coupling.

$M_T = 1.5$ TeV



- No gauge unification
- Inconsistency with the experimental results

Black line: the SM
Red line: Including the triplet

Yukawa interactions

We impose Z_3 symmetry to forbid the mixing terms.

- The SM sector

$$\mathcal{L}_{SM} \supset \sum_{i.j=1}^{3} Y_1^{ij} 5_H 10_L^i 10_L^j + \sum_{i,j=1}^{3} Y_2^{ij} 5_H^* \bar{5}_L^i 10_L^j + \text{h.c.}$$

- The vector-like fermions sector

$$\mathcal{L}_{VL} \supset \overline{10}_L^4 [Y_{10}^4 24_H + M_{10}^4] 10_R^4 + \overline{10}_L^5 \left[Y_{10}^5 24_H + M_{10}^5\right] 10_R^5 + \text{h.c.}$$

The mass of the SM particles

▪The Yukawa interactions for the SM sector

$$\mathcal{L}_{SM} \supset \sum_{i.j=1}^{3} Y_1^{ij} 5_H 10_L^i 10_L^j + \sum_{i,j=1}^{3} Y_2^{ij} 5_H^* \bar{5}_L^i 10_L^j + \text{h.c.}$$

$$\longrightarrow \underline{m_{di} = m_{ei}}$$

By introducing the non-renormalizable operators, we correct the wrong mass relations.

Ex). $\frac{Y_u^{ij}}{\Lambda} 24_{\mathrm{H}} 5_{\mathrm{H}} 10_L^i 10_L^j, \quad \frac{Y_d^{ij}}{\Lambda} 24_{\mathrm{H}} 5_{\mathrm{H}}^* \bar{5}_L^i 10_L^j$

Accuracy of unification

We consider the ratio of each SM gauge couplings.

When we assume that $r_{12} = \frac{\alpha_2}{\alpha_1}, r_{23} = \frac{\alpha_3}{\alpha_2}$,

if $0.99 < \frac{r_{12}}{r_{23}} < 1.01$, accuracy of unification is 1% or less.
if $0.97 < \frac{r_{12}}{r_{23}} < 1.03$, accuracy of unification is 3% or less.

Proton decay

The full formula of the proton decay :

$$\Gamma\left(p \to \pi_0 e_\beta^+\right) = \frac{\pi m_p \alpha_{GUT}^2}{2M_{GUT}^4} A^2 [|c(e, d^c)\langle\pi^0|(ud)_R u_L|p\rangle|^2 + |c(e^c, d\ )\langle\pi^0|(ud)_L u_L|p\rangle|^2]$$

M_{GUT}: GUT scale
α_{GUT}: gauge coupling at M_{GUT}
$m_p = 0.938$ GeV
$A = A_{QCD} \times A_{AR} \approx 1.2 \times 1.5$

$c(e, d^c) = 1$
$c(e^c, d\ ) = 1 + |V_{ud}|^2 \approx 1.95$
$\langle\pi^0|(ud)_R u_L|p\rangle = -0.131(4)(13)$ GeV^2
$\langle\pi^0|(ud)_R u_L|p\rangle = 0.134(5)(16)$ GeV^2

The Yukawa couplings

- The heavy neutral Higgs: $Y_{10}^{4} > 0.785$

  1660 GeV< M_Q^4, 1400 GeV< M_T

$$M_Q^4 \times (M_T)^2 < 4.76 \times Y_{10}^4 \ (\text{TeV})^3$$

- The charged Higgs: $Y_{10}^{4} > 0.401$

  1660 GeV< M_Q^4, 1000 GeV< M_T