# Dynamical realization of the small field inflation in the post supercooled universe

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Based on: H.Ishida and S.Matsuzaki, Phys.Lett.B 804 (2020) 135390,
**H.-X.Z.**, H.Ishida, and S.Matsuzaki, Phys.Lett.B 846 (2023) 138256

# Outline

Introduction: Cosmology and inflation

Model: The walking dilaton inflation

Analyses & results: Dynamical trapping mechanism

Conclusions

# Standard Big Bang Theory

## Advantages:

- Hubble's Law
- Abundance of light elements
- CMB temperature

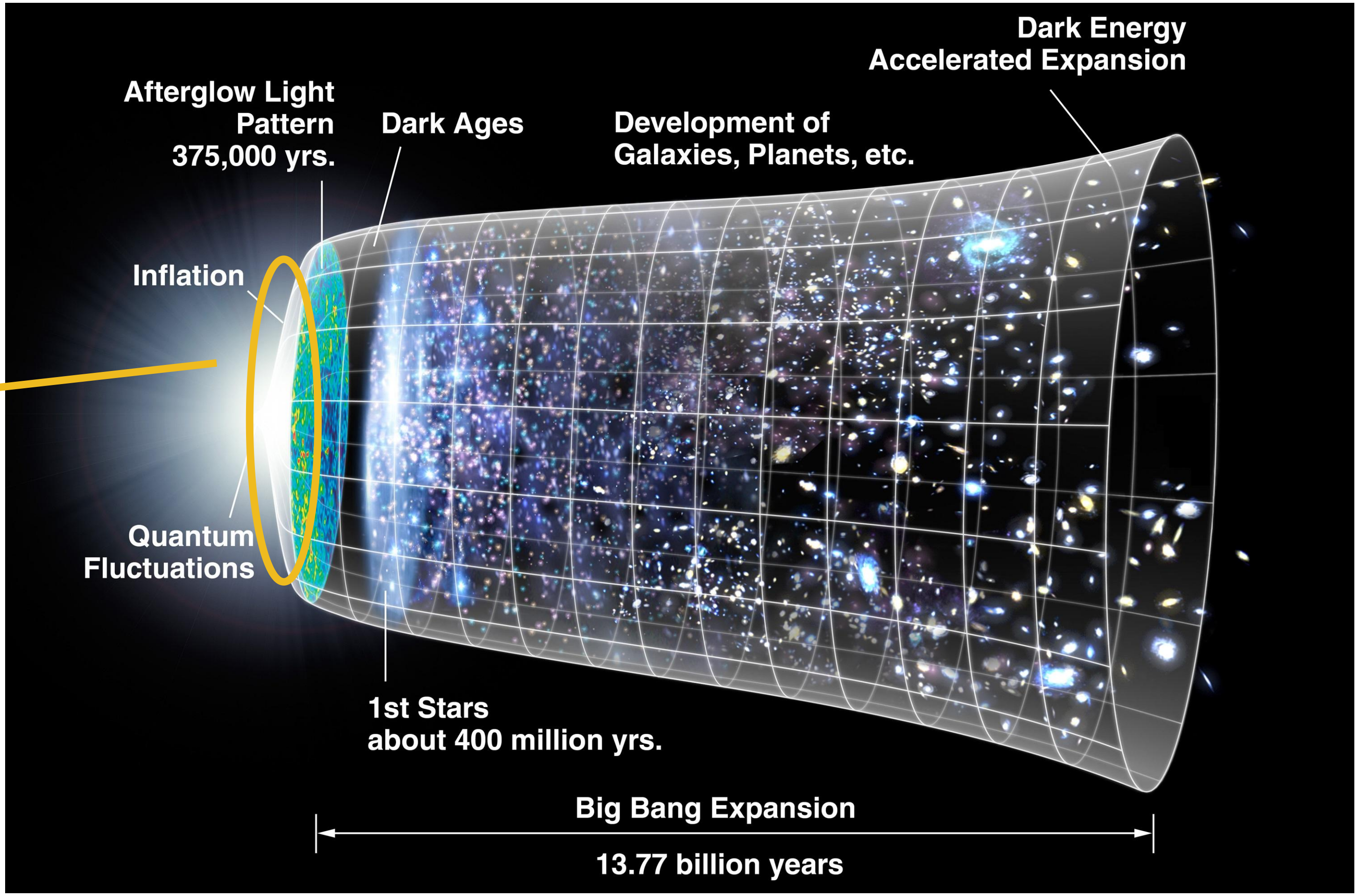
[XUANYU HAN/GETTY IMAGES]

## Disadvantages:

- Horizon problem
- Flatness problem
- Singularities
- ......

# Solution: Inflation

The universe experienced a period of exponential expansion after the Big Bang.




[NASA]

*Then, what kind of dynamics is needed?*

# Inflation

## Guth's old inflation:

[Guth(1981)]

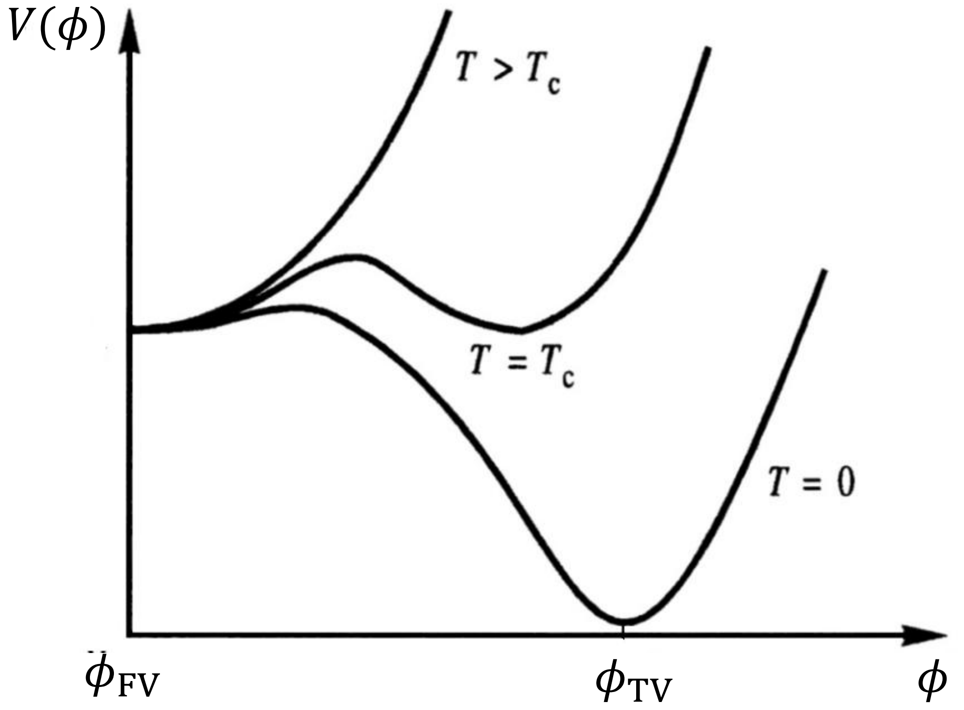
The universe was trapped in a metastable state (it was supercooled), and terminated by the bubble nucleation.

Exponential expansion means $H \approx constant$.

$$a \sim e^{H\Delta t}, \quad H^2 = \frac{V(0)}{3M_{\rm pl}^2}$$

☆ During the supercooling, the universe experienced a exponential expansion.

☆ Inflation was terminated by the quantum tunneling.




### Advantages:

- solve horizon problem
- solve flatness problem

### Disadvantages:

- graceful exit problem
- inhomogeneities problem

# Slow-Roll Inflation

Linde (1982)
Albrecht, Steinhardt (1982)

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi} + V(\phi)\right] \approx \frac{V(\phi)}{3M_{\rm pl}^2}$$

The EOM of the inflaton field:

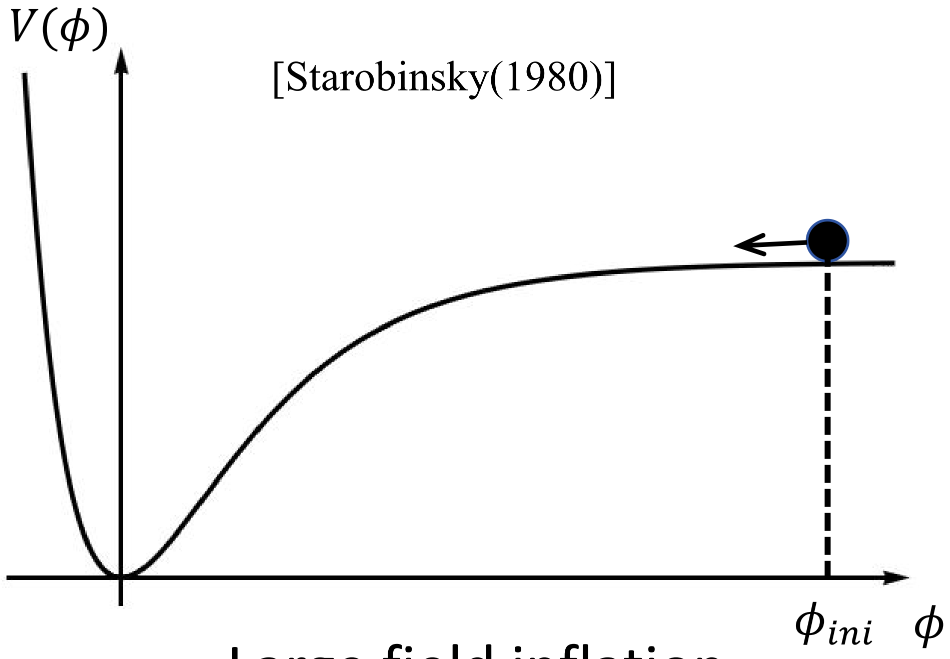
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

The conditions of the slow-roll approximation:

$$\dot{\phi}^2 \ll V(\phi), \quad \ddot{\phi} \ll H\dot{\phi}$$

Slow-roll parameters:

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1, \quad |\eta| = M_{\rm pl}^2\left(\frac{V'(\phi)}{V(\phi)}\right) \ll 1$$




Large field inflation

Beyond-Planck problem:

$$N > 60 \Rightarrow \phi_{\rm ini} > M_{\rm pl}$$

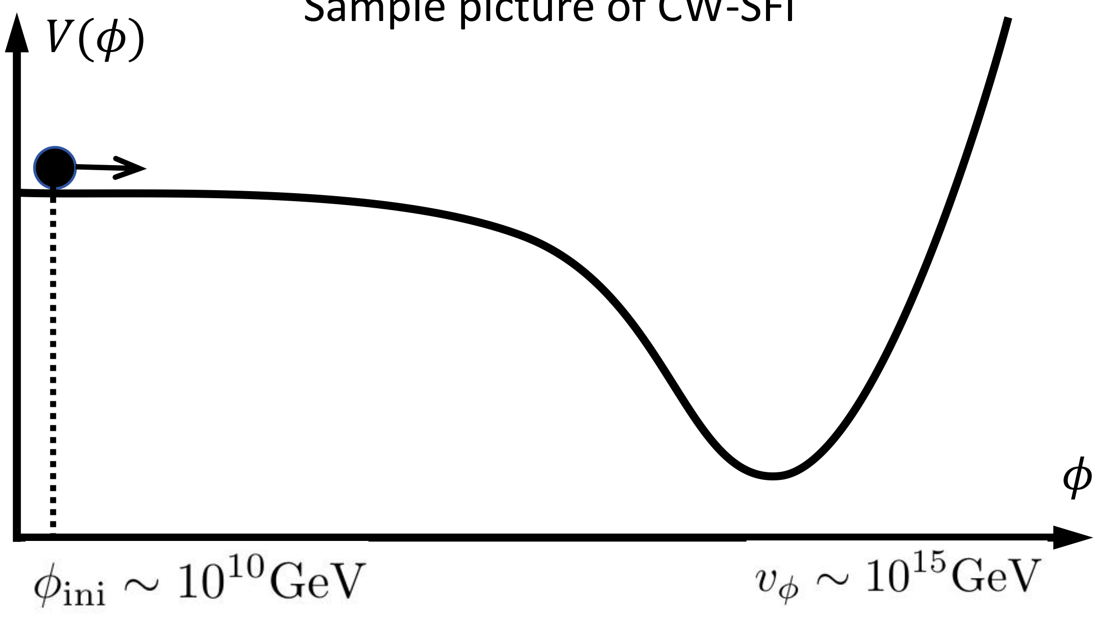
# Small Field Inflation of CW type

The potential of inflaton:

[Coleman, Weinberg(1973)]

$$V(\phi) = \frac{\lambda\phi^4}{4}\left(\ln\frac{\phi^2}{v_\phi^2} - \frac{1}{2}\right) + V_0$$

Sample picture of CW-SFI




## Fine-tuning problem: [Iso, Kohri, Shimada(2016)]

The inflation must start from the very small initial condition, essentially due to scale invariance

$$\phi_{\rm ini} \ll v_\phi$$

## Incompatibility between $N$ and $n_s$ [specialized to CW]:

$$n_s \simeq 0.968 \Rightarrow N = \frac{3}{1-n_s} - \frac{3}{2} = 73.5$$

## Extremely tiny coupling, i.e., large hierarchy between mass and VEV:

$$\Delta_R^2 \simeq 2.137 \times 10^{-9} \Rightarrow \lambda = \left(\frac{m_\phi}{v_\phi}\right)^2 \sim 10^{-15}$$

*Where the large hierarchy comes from?*

# Motivation

## Can we dynamically solve the fine-tuning problem?

Actually, one proposal is already present: [Iso, Kohri, Shimada(2016)]

This mechanism to trap the inflaton around the false vacuum has been proposed, in which the trapping dynamically works due to the particle number density (like plasma or a medium) created by the preheating.

Today our purpose and goal are:

Propose alternative trapping mechanism by supercooling, and construct a model for small field inflation consistent with the observations by solving all 3 intrinsic problems.

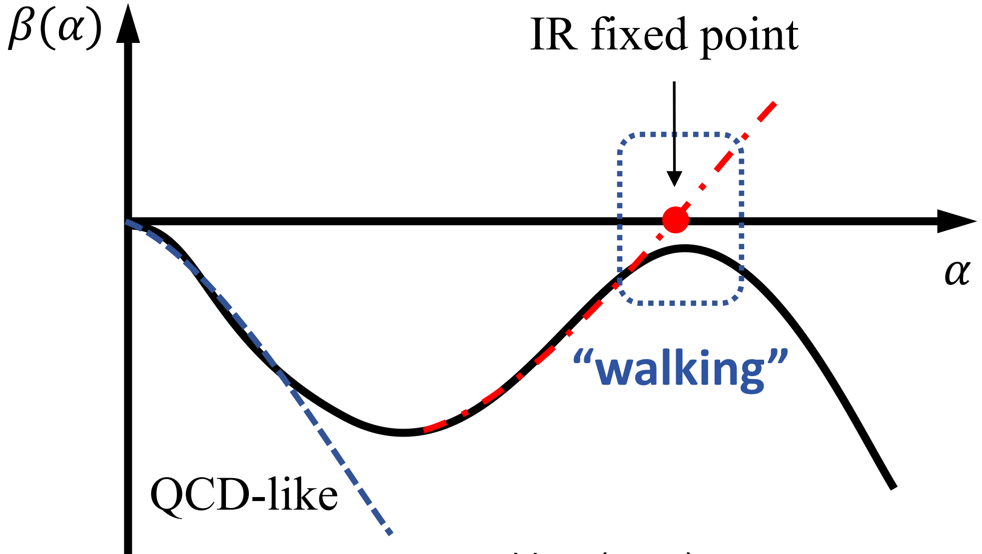
# Many-flavor QCD

E.g., S.Matsuzaki and K.Yamawaki, JHEP 12, 053 (2015)

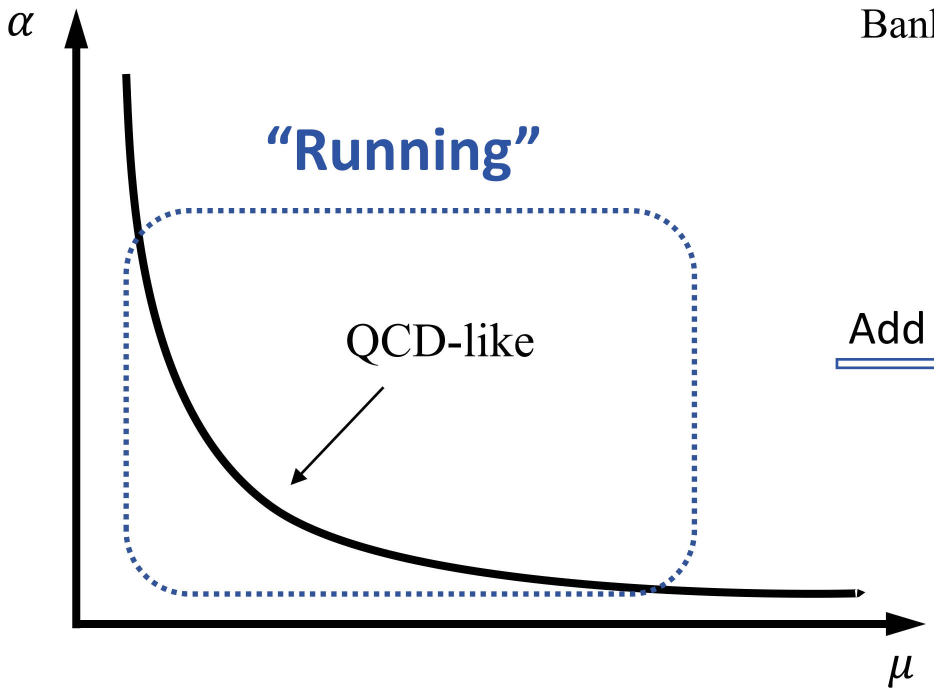
2-loop RGE: $$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\alpha(\mu) = -b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$$

| $(N_c = 3)$ | $N_f < 8.05$ | $8.05 < N_f < 16.5$ | $16.5 < N_f$ |
|---|---|---|---|
| $b = \frac{1}{64\pi}(33 - 2N_f)$ | $+$ | $+$ | $-$ |
| $c = \frac{1}{12\pi^2}(153 - 19N_f)$ | $+$ | $-$ | $-$ |

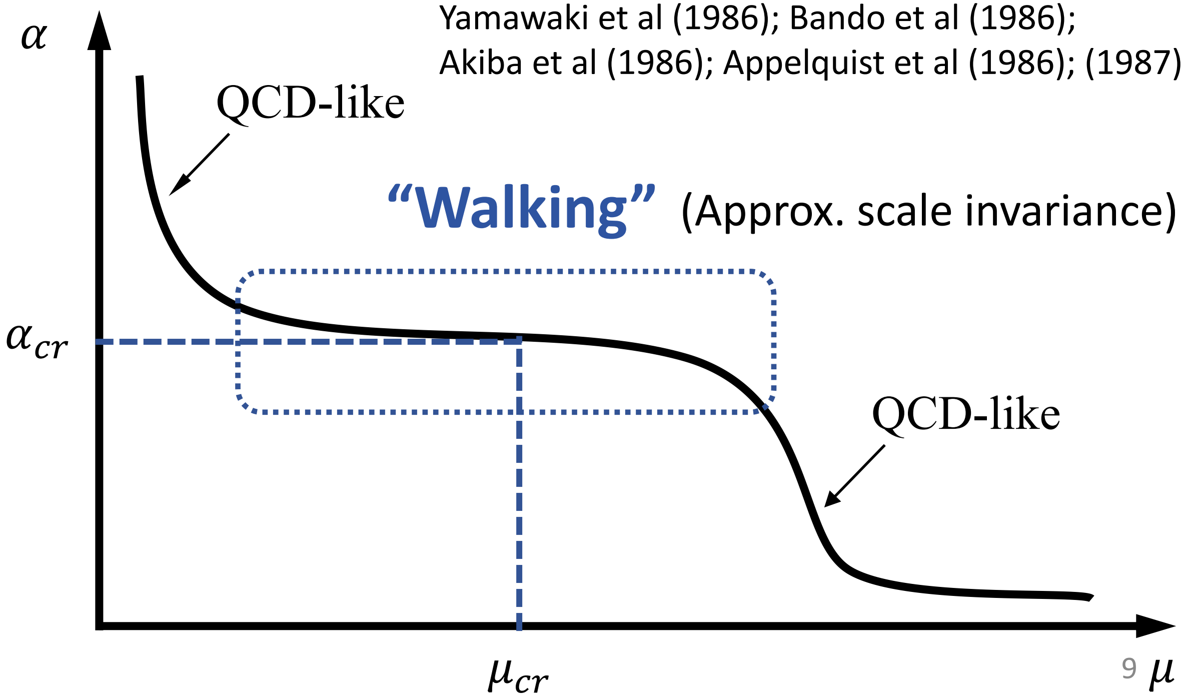
Caswell(1974)
Banks, Zaks(1982)




Holdom (1981);
Yamawaki et al (1986); Bando et al (1986);
Akiba et al (1986); Appelquist et al (1986); (1987)




Add many Fermions ⟹



# Many-flavor QCD

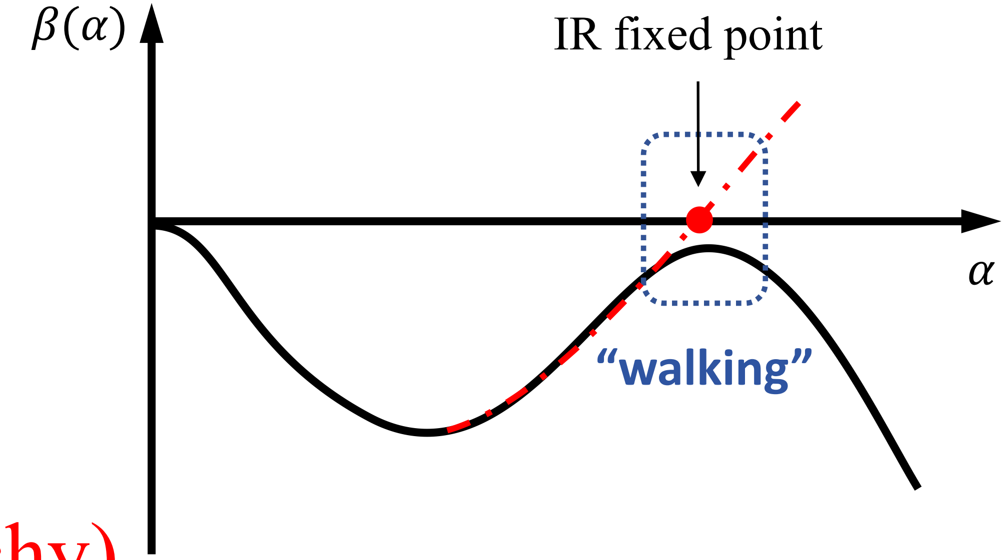
## Miransky scaling:

[Miransky(1985)]

$$m_F \sim \Lambda_{\rm UV}\, e^{-\frac{\pi}{\sqrt{\alpha/\alpha_{\rm cr}-1}}} \quad \text{for} \quad \alpha > \alpha_{\rm cr}$$

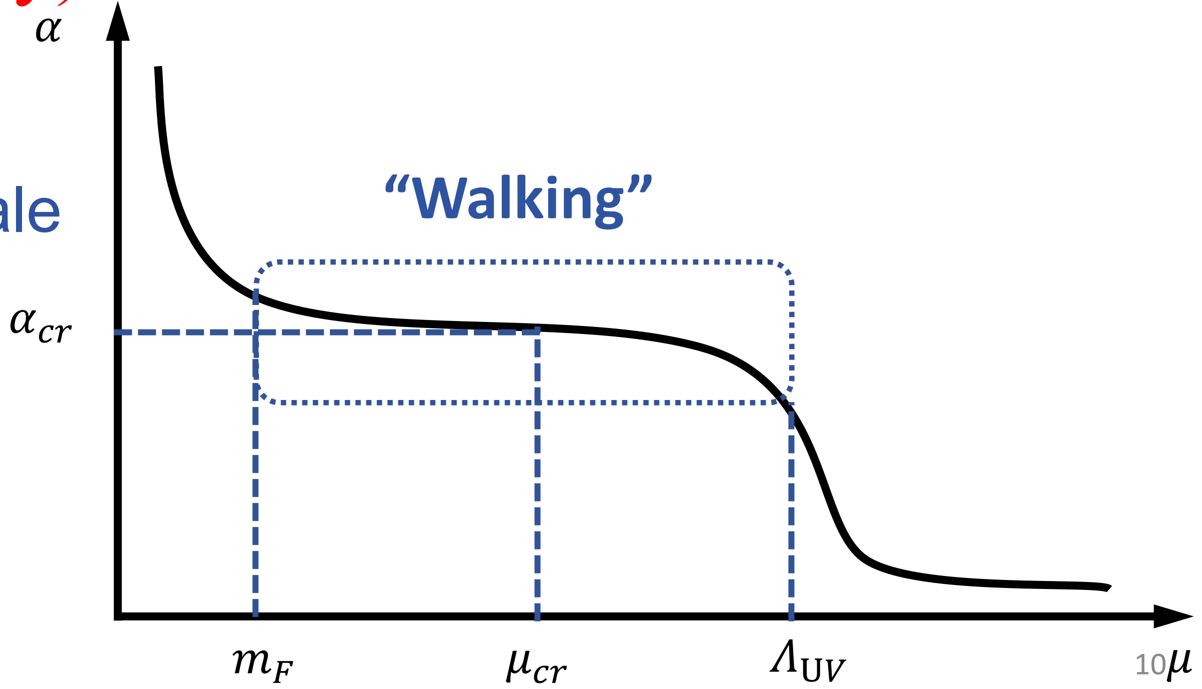
- $m_F$ - dynamical mass of the fermion
- $\Lambda_{\rm UV}$ - the UV scale
- $\alpha_{cr}$ - the critical coupling

$\rightarrow$ $m_F \ll \Lambda_{UV}$ when $\alpha \approx \alpha_{cr}$ (large hierarchy)



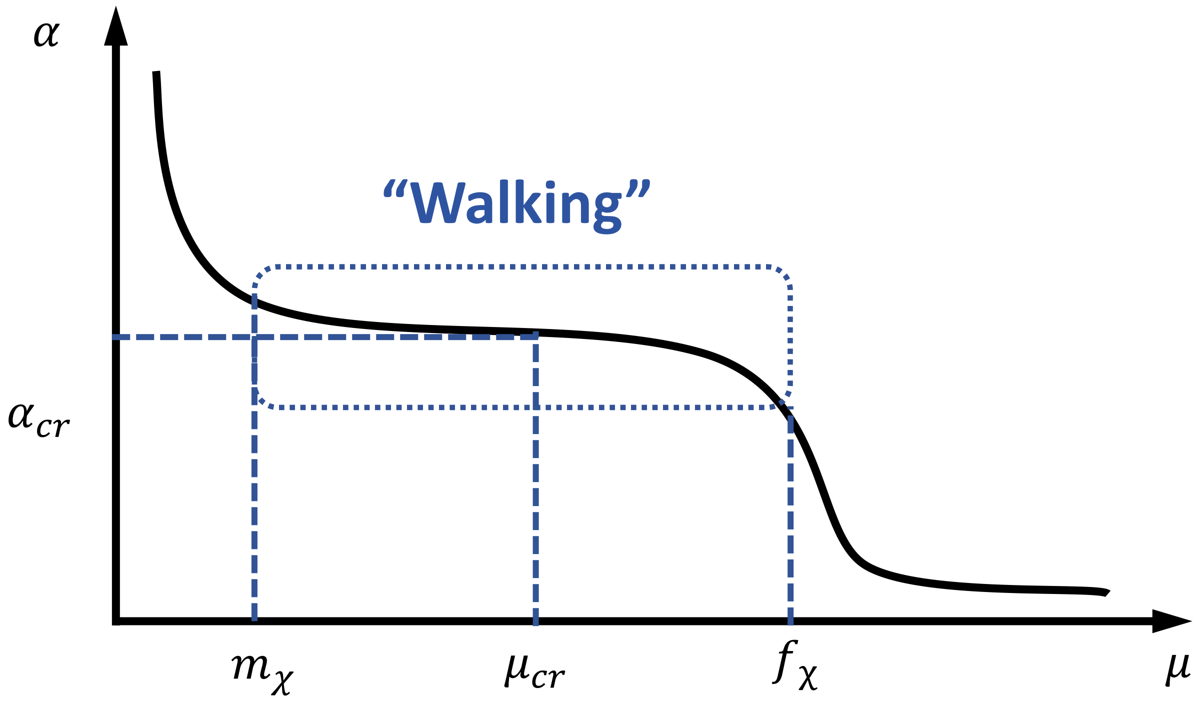

Approximate scale invariance $\beta(\alpha_{\rm cr}) \approx 0$

Spontaneously breaking of (approximate) scale symmetry $\rightarrow$ "pseudo-walking **dilaton**"


Dilaton gets mass due to the explict breaking induced by the scale anomaly.

# Many-flavor QCD




Dilaton mass (Milansky scaling):

$$m_\chi \sim f_\chi \, e^{-\frac{\pi}{\sqrt{\alpha/\alpha_{\rm cr}-1}}}$$

$f_\chi$: scale symmetry breaking scale

In our model, we regard this dilaton as the inflaton.

# Model: The walking dilaton inflation

The walking dilaton potential: [Ishida, Matsuzaki(2020)]

$$V(\chi) = \underbrace{-\frac{C}{2N_f}\chi \,\mathrm{Tr}\left[\langle U\rangle + \langle U^\dagger\rangle\right]}_{\text{“tadpole”}} + \underbrace{\frac{\lambda_\chi}{4}\chi^4\left(\ln\frac{\chi}{v_\chi} + A\right)}_{\text{“CW-type potential”}} + V_0$$

$\chi$ : dilaton $v_\chi$ : VEV of dilaton
$N_f$ : number of fermions
$V_0$ : vacuum energy with $V(v_\chi)$=0
A : fixed by $V'(v_\chi) = 0$

$$U = \mathrm{e}^{2i\pi/f_\pi}$$

$$C = N_f\frac{m_\pi^2 f_\pi^2}{2v_\chi}$$

☆ Tadpole by explicit chiral symmetry breaking

☆ CW-type potential by explicit scale anomaly

# Cosmological parameters

slow-roll parameters: $\begin{cases} \epsilon & = \frac{M_{\rm pl}^2}{2}\left(\frac{V'(\chi)}{V(\chi)}\right)^2 \\ \eta & = M_{\rm pl}^2\left(\frac{V''(\chi)}{V(\chi)}\right) \end{cases}$

e-folding number: $N = \dfrac{1}{M_{\rm pl}^2}\displaystyle\int_{\chi_{\rm end}}^{\chi_{\rm ini}} d\chi \left(\frac{V(\chi)}{V'(\chi)}\right)$

scalar perturbation: $\Delta_R^2 = \dfrac{V(\chi)}{24\pi^2 M_{\rm pl}^4 \epsilon(\chi)}$

spectral index $n_s = 1 - 6\epsilon + 2\eta \simeq 1 + 2\eta$

# Cosmological parameters in the present model

$$\eta \simeq 24\frac{M_{\rm pl}^2}{v_\chi^2}\frac{\chi^2}{v_\chi^2}\ln\frac{\chi^2}{v_\chi^2}\,,$$

$$\epsilon \simeq \frac{\pi^4}{2}\left(\frac{M_{\rm pl}}{v_\chi}\right)^2\left(\frac{m_\pi}{m_F}\right)^4\,,$$

$$\Delta_R^2 \simeq \frac{2}{\pi^{10}}\left(\frac{m_F}{v_\chi}\right)^4\cdot\left(\frac{v_\chi}{M_{\rm pl}}\right)^6\left(\frac{m_F}{m_\pi}\right)^4\,,$$

$$N \simeq \frac{(\chi_{\rm end}-\chi_{\rm ini})}{\sqrt{2\epsilon}M_{\rm pl}} \simeq \frac{(\chi_{\rm end}-\chi_{\rm ini})v_\chi}{6\pi^2 M_{\rm pl}^2}\left(\frac{m_F}{m_\pi}\right)^2$$

Constraints:

Planck Collaboration(2018):

$$\Delta_R^2 \simeq 2.137\times 10^{-9}$$

$$n_s \simeq 0.968$$

Incompatibility between $N$ and $n_s$ resolved!

# Cosmological parameters

At this bechmark point: [Ishida, Matsuzaki(2020)]

$$v_\chi \simeq 1.7 \times 10^{15}\mathrm{GeV}, \quad m_\chi \simeq 3.8 \times 10^{8}\mathrm{GeV}$$
$$m_F \simeq 4.1 \times 10^{11}\mathrm{GeV}, \quad m_\pi \simeq 6.7 \times 10^{4}\mathrm{GeV}$$

which is in agreement with the observations.

So far, we have solved

- ☑ Extremely tiny CW coupling
- ☑ Incompatibility between $N$ and $n_s$

But, the fine-tuning problem is still there...

$$\chi_{\rm ini} \simeq 6.7 \times 10^{9}\mathrm{GeV} \ll v_\chi .$$

# Model in the finite temperature

$\chi$ : SU($N_f$) singlet dilaton

$s^i$ : SU($N_f$) adjoint scalar mesons

Note $N_f^2-1$ pions ($\pi^i$) : negligible couplings, due to lightness enough

The effective potential with thermal corrections: [**H.-X.Z.**, H.Ishida, and S.Matsuzaki]

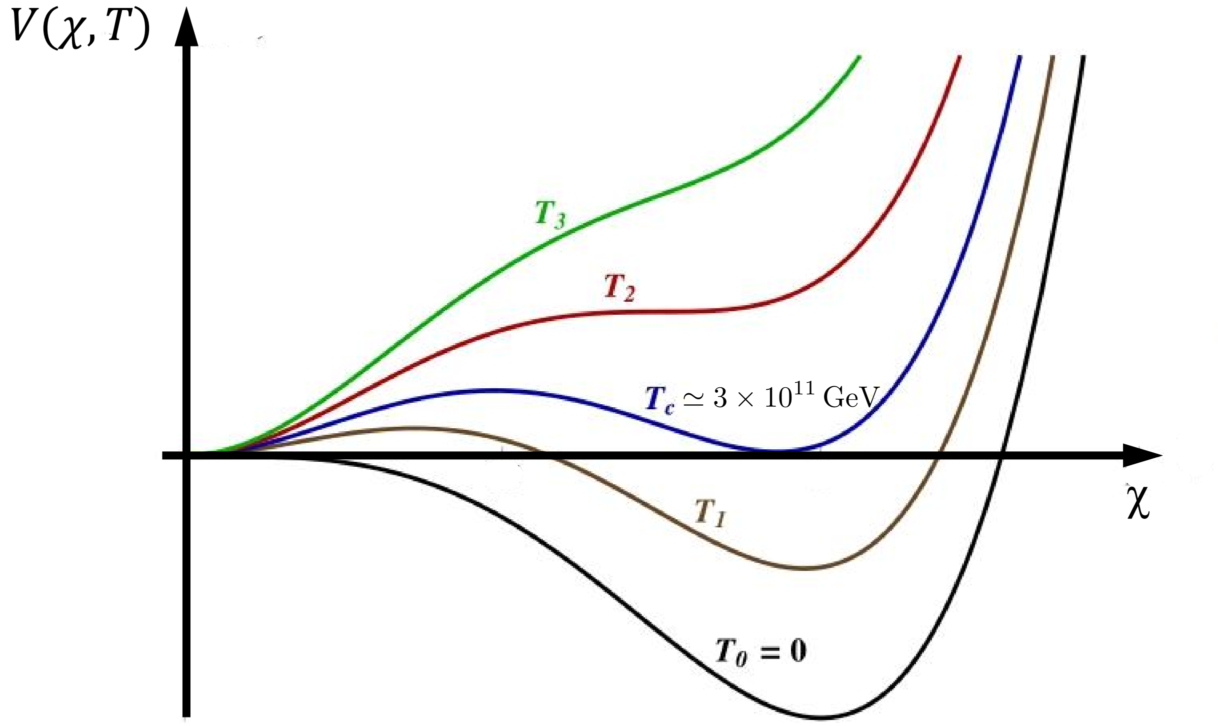
$$V_{\rm eff}(\chi,T) = -C\,\chi + \frac{N_f^2-1}{64\pi^2}\mathcal{M}_{s^i}^4(\chi,T)\left(\ln\frac{\mathcal{M}_{s^i}^2(\chi,T)}{\mu_{GW}^2}-\frac{3}{2}\right)+\frac{T^4}{2\pi^2}(N_f^2-1)J_B\left(\mathcal{M}_{s^i}^2(\chi,T)/T^2\right)+V_0\,,$$

where

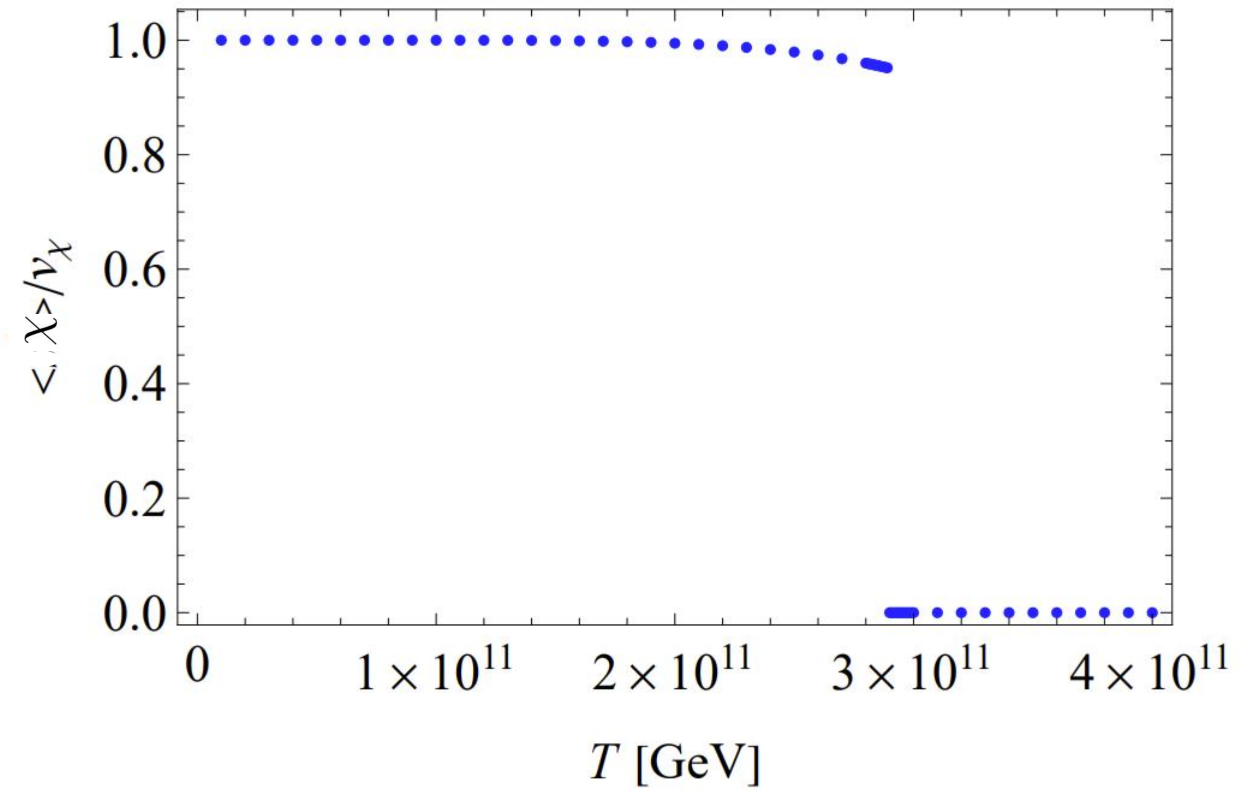
$$J_B(X^2) \equiv \sum_{a=0}^{N_f^2-1}\int_0^\infty x^2\ln\left(1-e^{-\sqrt{x^2+X^2}}\right)\mathrm{d}x,$$

$$\mathcal{M}_{s^i}^2(\chi,T) = m_{s^i}^2(\chi)+\frac{T^2}{6}\left((N_f^2+1)\lambda_1+2N_f\lambda_2\right)\Big|_{\lambda_1=-\lambda_2/N_f}$$

# Thermal phase transition




C.f. Fig. Schematic potential deformation for 1st order phase transition




Ultra-supercooling 1st order phase transition

$$\frac{v_\chi(T_c)}{T_c} \simeq 5400 \gg 1$$

# Trapping mechanism

The inflaton keeps being trapped but gets shifted with the false vacuum, depending on T !

The probability of bubble nucleation rate per unit volume per unit time is:

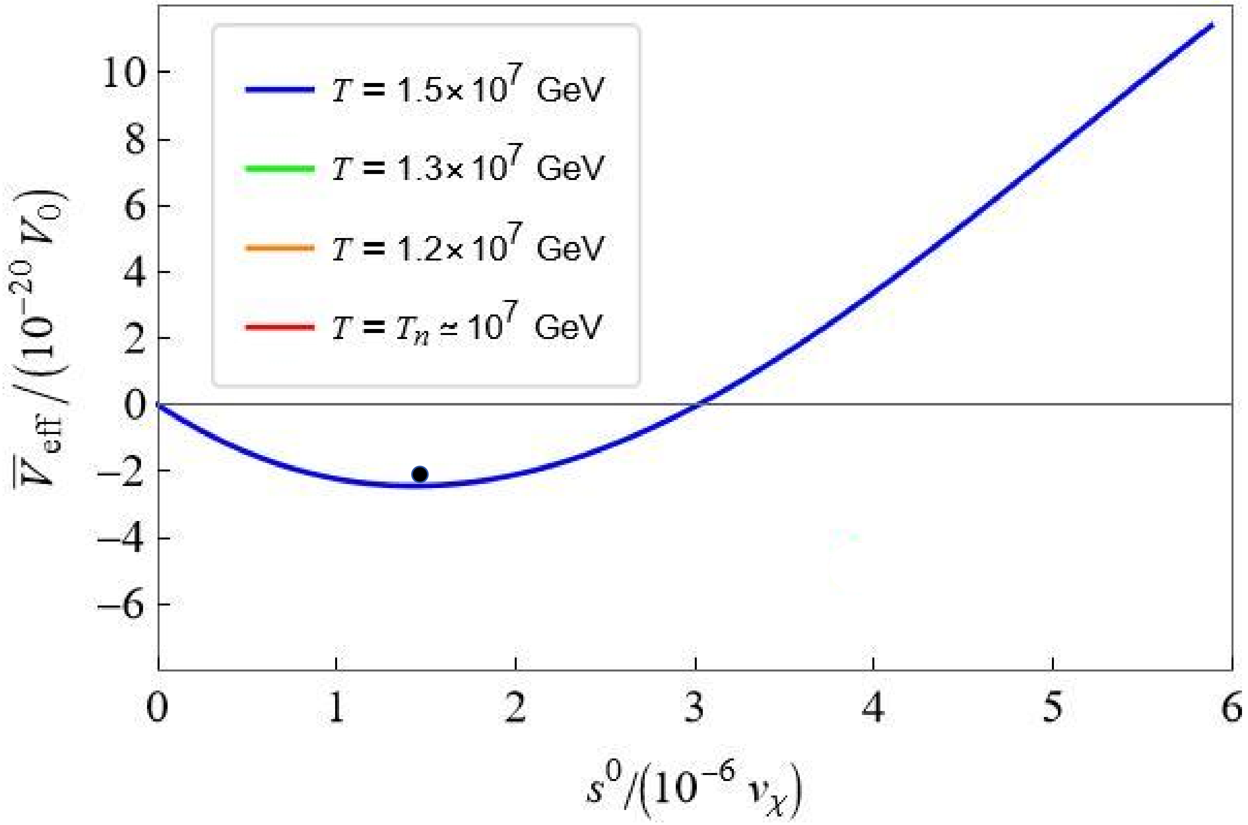
$$\Gamma(T) \simeq T^4 \left(\frac{S_3/T}{2\pi}\right)^{3/2} \exp\left(-\frac{S_3(T)}{T}\right)$$

The nucleation temperature is defined as:

$$\frac{\Gamma(T_n)}{H(T_n)^4} \sim 1 \Rightarrow \frac{S_3(T_n)}{T_n} \sim 100$$

Analytic approximations for bubble action:

$$\frac{S_3(T_n)}{T_n} \simeq \frac{37.794\pi^2}{\sqrt{6}} \frac{N_f^{3/2}}{\lambda_2^{3/2}(N_f^2-1)^{1/2}} \frac{1}{\ln(\mu_{\rm GW}/T_n)}$$




So, the nucleation temperature is exponentially suppressed by the extremely tiny coupling. Further numerical analysis shows that

the tunneling never happens and the inflaton *keeps being trapped* until the barrier almost vanishes!

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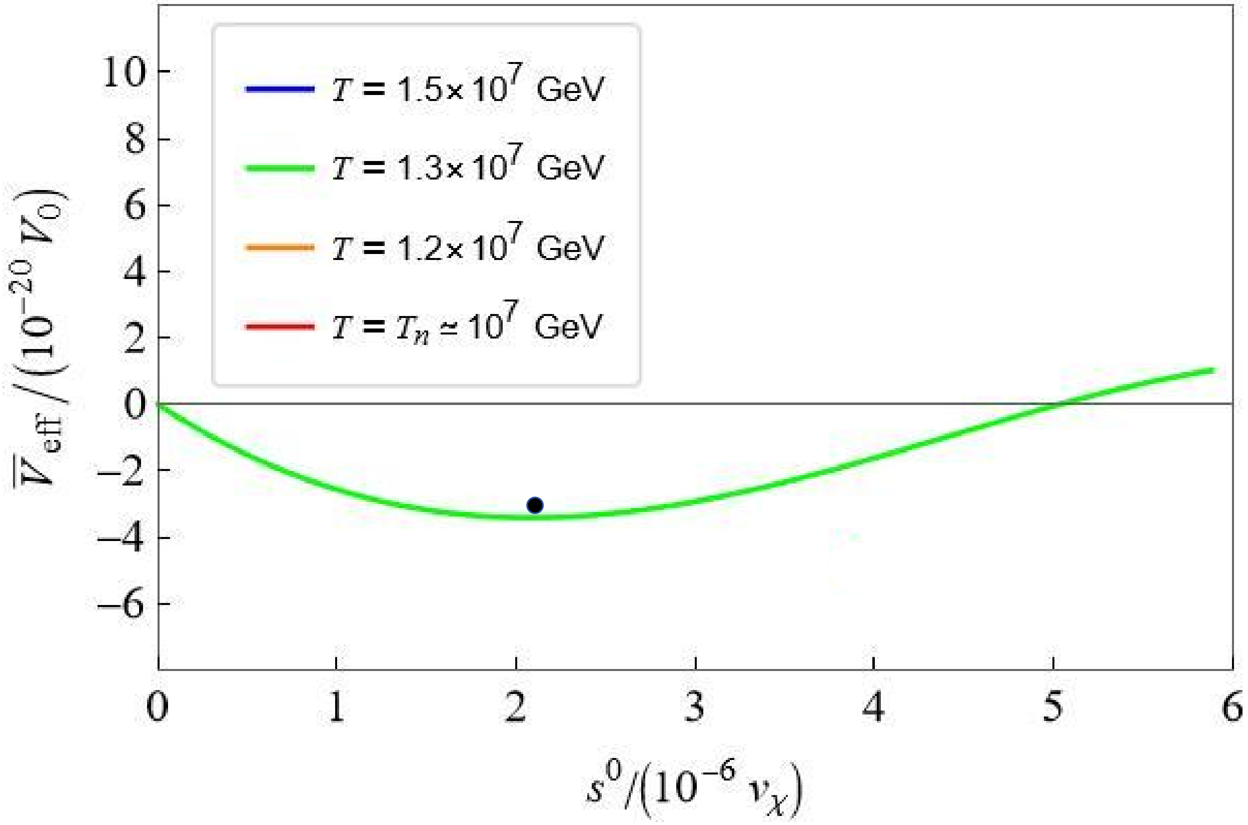


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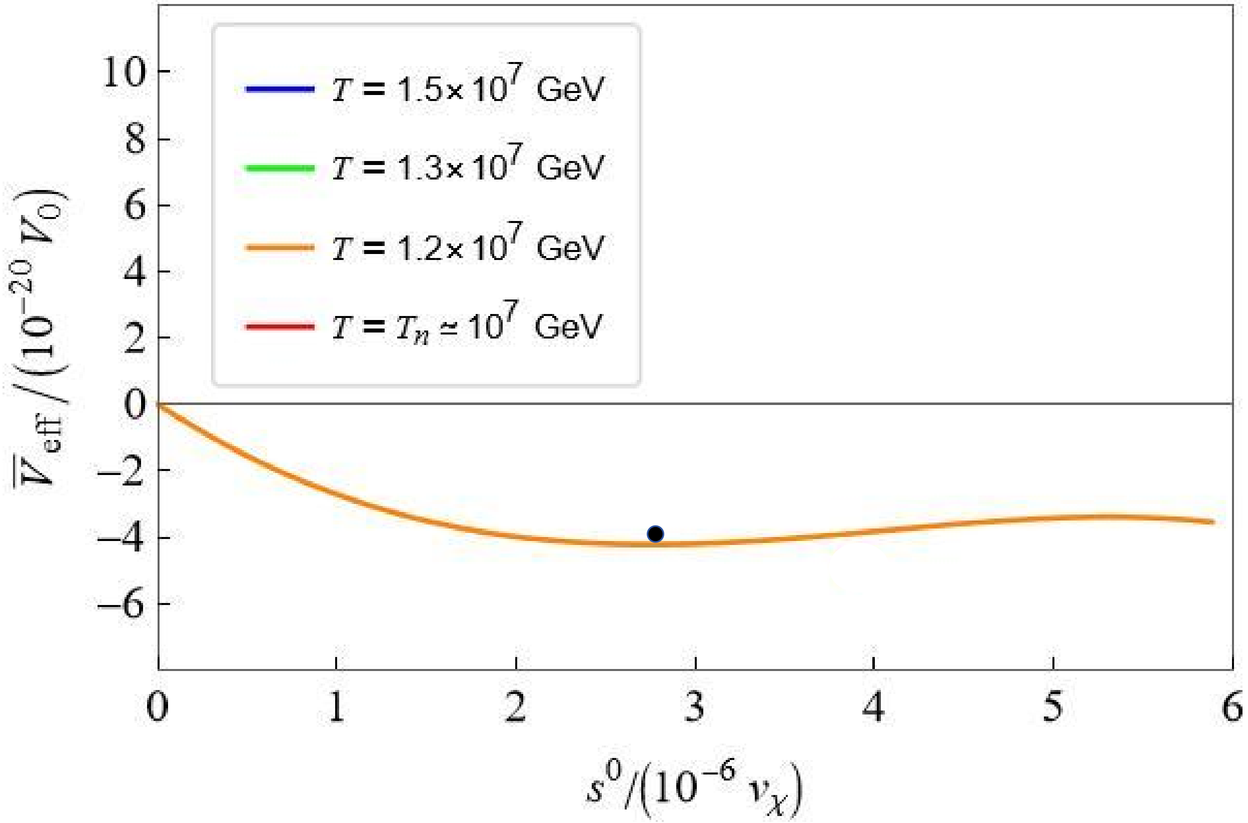


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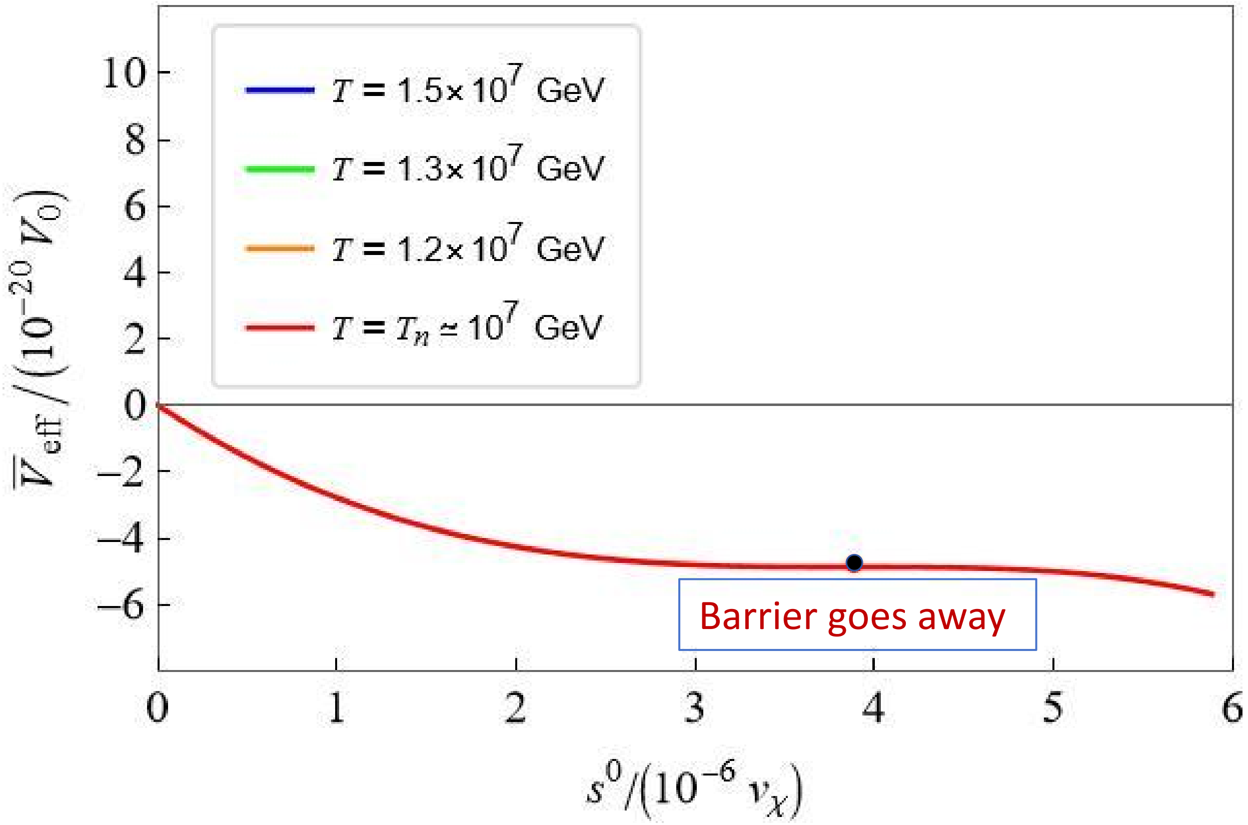


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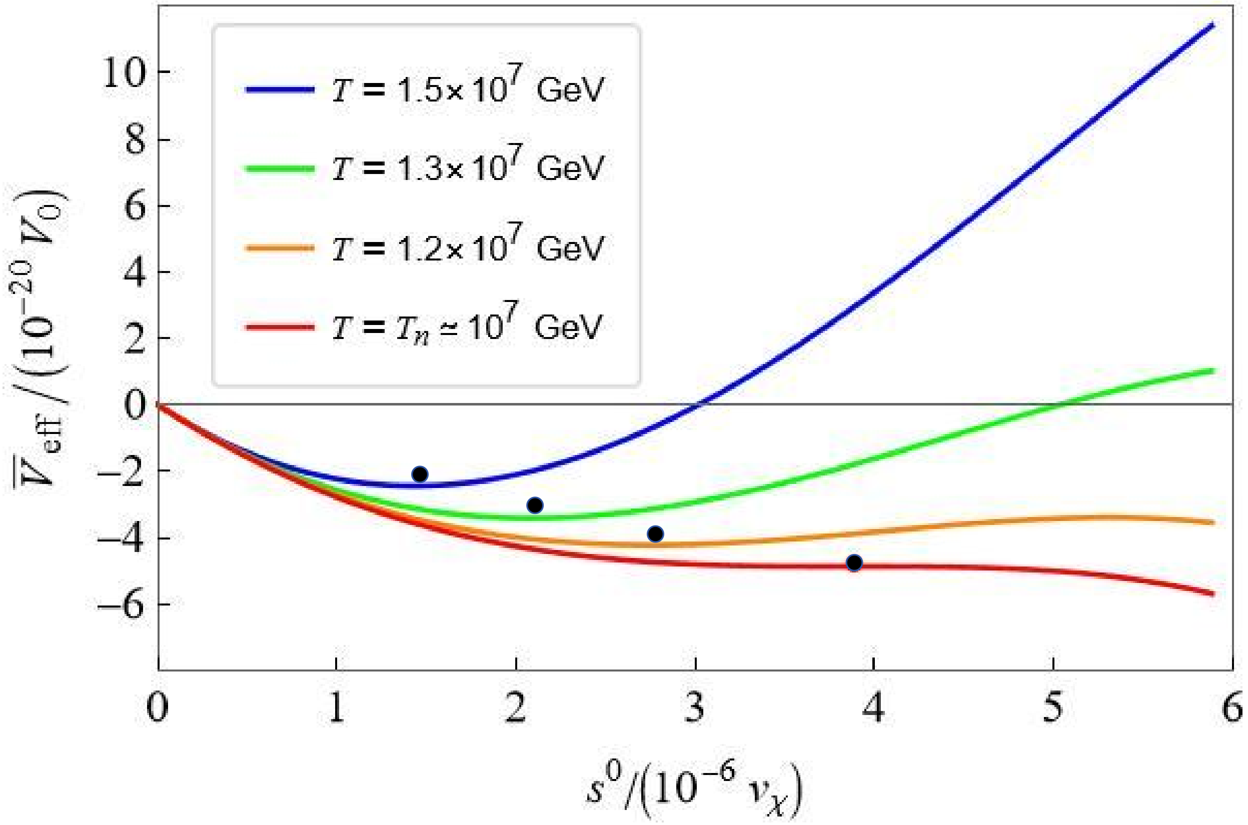


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# Analyses & results

## The inflationary history goes like:

- Initial thermal corrections traps dilaton in the false vacuum.
- When $T < T_c$ , the universe undergoes a supercooling period until the barrier vanishes at $T = T_n$.
- The inflaton rolls down the potential and the universe gets into the slow-roll phase.

Based on this bechmark parameters (compatible with [Ishida, Matsuzaki(2020)]):

$$N_c = 3\,, \qquad N_f = 8\,, \qquad v_\chi = 1.7 \times 10^{15}\,\mathrm{GeV}\,,$$
$$m_F = 6.2 \times 10^{11}\,\mathrm{GeV}\,, \qquad m_\pi = 1.1 \times 10^{5}\,\mathrm{GeV}\,,$$

we *dynamically* fixed the initial conditions of the inflaton $\chi_{\rm ini} \sim 7 \times 10^9\mathrm{GeV}\,.$

# Conclusions

- Large hierarchy is explained by the walking behavior in large $N_F$ QCD.
- Dilaton arising from the spontaneous breaking of (approximate) scale symmetry is regarded as inflaton.
- Thermal corrections traps the inflaton in the false vacuum and thus dynamically solved the fine-tuning problem in small field inflation.
- We gave the bechmark parameters consistent with observation.
- Similar supercooling dynamical trapping mechanism could be applicable to other types of small field inflation or initial condition for preheating models.

# Thanks for your attention!