New theoretical constraint on the non-decoupling physics with the mass dependent beta function

Yushi Mura (Osaka U.)

Collaborator: Prof. Shinya Kanemura (Osaka U.)

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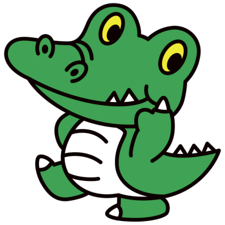


KEK-ph 2023 at Tsukuba 2023/11/08

Introduction

- **In 2012, the discovery of the Higgs particle (125 GeV)**

  The Standard Model (SM) well explains particle phenomenology.

ATLAS, Nature (2022);
CMS, CMS-PAS-HIG-19-005 (2020)

- **However, the Higgs sector is unknown**

  Property of the electroweak phase transition in the early universe?

- **Remaining questions**

  Dark matter, Neutrino mass, Baryon asymmetry of the Universe, etc.

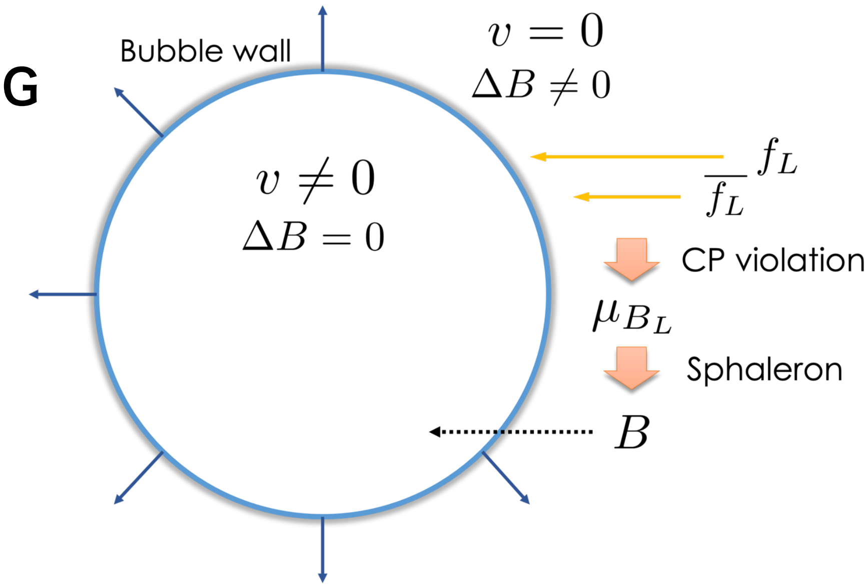
- **Those can be explained by the extension of the Higgs sector**

Ex) Weakly interacting massive particles — Steigman and Turner (1985)
Tree or radiative seesaw — T. Yanagida (1979); Zee (1980); and more
Electroweak baryogenesis (EWBG) — Kuzmin, Rubakov and Shaposhnikov (1985)

EWBG and non-decoupling effects

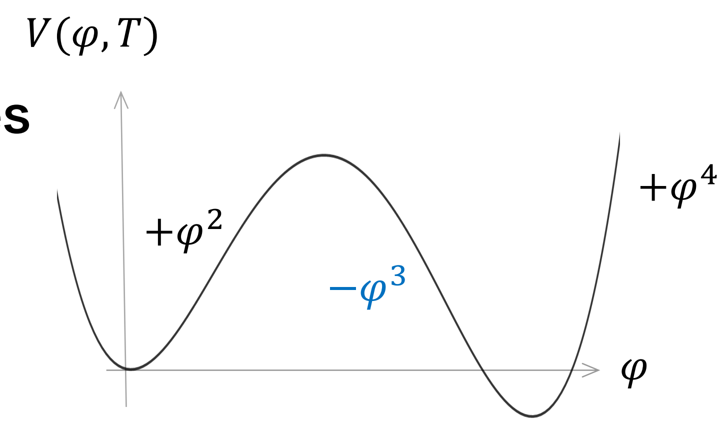
- **Sakharov third conditions and EWBG**

  ① Baryon # violation   Sakharov (1967)
  ② C and CP violation
  ③ Out of thermal equilibrium



- **Thermal effective potential**

$$V_{eff}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 - ET|\varphi|^3 + \frac{\lambda_T}{4}\varphi^4$$



- **Non-decoupling effects of heavy particles**

  Ex) Two Higgs Doublet Model

$$m_\Phi^2 = M^2 + \tilde{\lambda} v^2 \simeq \tilde{\lambda} v^2 \qquad (\tilde{\lambda} v^2 \gg M^2)$$

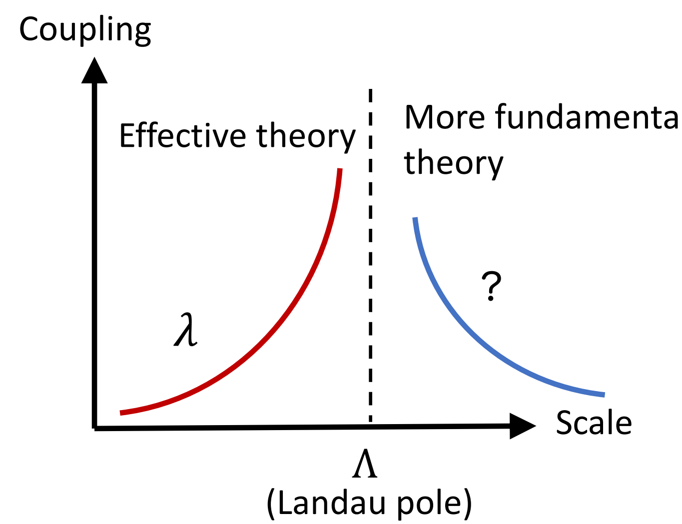
$$E \simeq \frac{1}{4\pi v^3}\left(m_W^3 + m_Z^3 + m_\Phi^3\right) \sim g^{3/2} + \tilde{\lambda}^{3/2}$$

The Landau pole

- **Relatively large scalar couplings for EWBG**

Cline, Kainulainen and Trott (2011); Dorsch, Huber, Konstandin and No (2017); and more

  - From RGE analysis, the Landau pole appears typically at 1-100TeV
  - Theoretical constraints on the models for EWBG



- **Threshold correction** Weinberg (1980)
  - Mass independent beta function ($\overline{\mathrm{MS}}$ scheme)
  - Matching the low energy effective theory (SM) and the UV theory (new physics)

$$\beta(\lambda, Q) = \beta_{\mathrm{IR}}(\lambda) + \theta_{\mathrm{step}}(Q/Q_m)\,\beta_{\mathrm{UV}}(\lambda)$$

Q: Energy scale
Q_m: Matching scale

- **In this talk**
  - Natural treatment for running couplings
  - Precise evaluation of the Landau pole

For the situation that

$$m_\Phi^2 = M^2 + \tilde{\lambda} v^2 \simeq \tilde{\lambda} v^2$$

Mass dependent beta function

- **$\lambda\phi^4$ theory** $$\mathcal{L} = (\partial_\mu \phi)^2 - m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

On-shell mass: $\Gamma^{(2)}(k^2 = m^2, m^2, \lambda) = 0,$

Wave function: $\frac{\partial}{\partial k^2}\Gamma^{(2)}(k^2, m^2, \lambda)|_{k^2=-Q^2} = 0,$

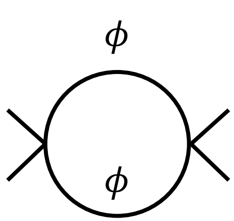
Momentum subtraction: $\Gamma^{(4)}(k_i, m^2, \lambda)|_{k_i\cdot k_j = -Q^2\delta_{ij}+\frac{1}{3}Q^2(1-\delta_{ij})} = -\lambda,$

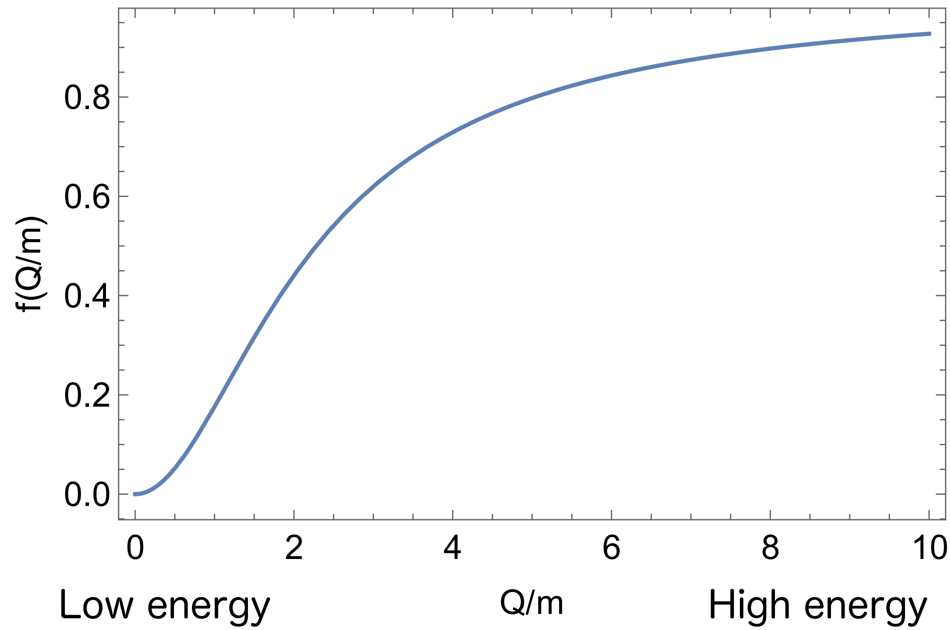
- **Mass dependent beta function (1 loop level)**

$$\beta\left(\lambda, \frac{Q}{m}\right) = \frac{3\lambda^2}{16\pi^2} f\left(\frac{Q}{m}\right),$$

- Coincides with $\overline{\text{MS}}$ scheme for UV limit
- Becomes 0 for (Energy) << (Mass)
- How ϕ contributes is controlled by $f(Q/m)$

(Ex. of divergent diagrams)





Application for non-decoupling case

- **Toy model for the non-decoupling situation**

U(1) symmetry for ϕ_1, ϕ_2

$$\phi_{1,2} \to \phi_{1,2}\, e^{i\theta_{1,2}}$$

$$\mathcal{L} = (\partial_\mu \phi_1)^\dagger (\partial_\mu \phi_1) + (\partial_\mu \phi_2)^\dagger (\partial_\mu \phi_2) - \mu_1^2 |\phi_1|^2 - \mu_2^2 |\phi_2|^2 \\ - \frac{\lambda_1}{2} |\phi_1|^4 - \frac{\lambda_2}{2} |\phi_2|^4 - \lambda_3 |\phi_1|^2 |\phi_2|^2 \, .$$

- $\boldsymbol{\mu_1^2 < 0}$ ⇨ **SSB of U(1) symmetry for $\boldsymbol{\phi_1}$**

$$\phi_1 = (v + \rho + i\eta)/\sqrt{2}$$

Mass spectrum

$$m_\rho^2 = \lambda_1 v^2, \quad m_\eta^2 = 0,$$

$$m_{\phi_2}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2,$$

ρ : SM like Higgs field

ϕ_2 : Additional scalar field

$m_{\phi_2}^2 \simeq \mu_2^2$ ⇨ ϕ_2 can decouple

$m_{\phi_2}^2 \simeq \frac{1}{2}\lambda_3 v^2$ ⇨ ϕ_2 is fixed at EW scale (Non-decoupling)

Application for non-decoupling case

- **Seven renormalization conditions**
  - On-shell condition for $m_\rho, m_\eta, m_{\phi_2}$
  - Two wave function renormalization
  - Two momentum subtraction $\Gamma^{(4)}_{\rho\rho\phi_2\phi_2^\dagger}|_{k=\mathrm{symmetric}} = -\lambda_3,\ \Gamma^{(4)}_{\phi_2\phi_2^\dagger\phi_2\phi_2^\dagger}\Big|_{k=\mathrm{symmetric}} = -2\lambda_2.$

- **Beta function for non-decoupling parameter $\boldsymbol{\lambda_3}$ (1 loop level)**

$$16\pi^2\,\beta_{\lambda_3} \simeq \lambda_1\lambda_3 + 3\lambda_1\lambda_3\, f\left(Q/m_\rho\right) + 4\lambda_3^2\, f\left(Q/m_{\phi_2}\right) + 4\lambda_2\lambda_3\, f\left(Q/m_{\phi_2}\right)$$

- **For $\boldsymbol{Q/m \to \infty}$, they coincide with the ones of the $\overline{\mathbf{MS}}$ scheme**

- **Calculate running couplings**

- **Inputs** $(v,\ m_\rho, m_{\phi_2}) = (246, 125, 400)$ GeV $\lambda_3(Q_0) = 5.3,\ \lambda_2(Q_0) = 0.01,$

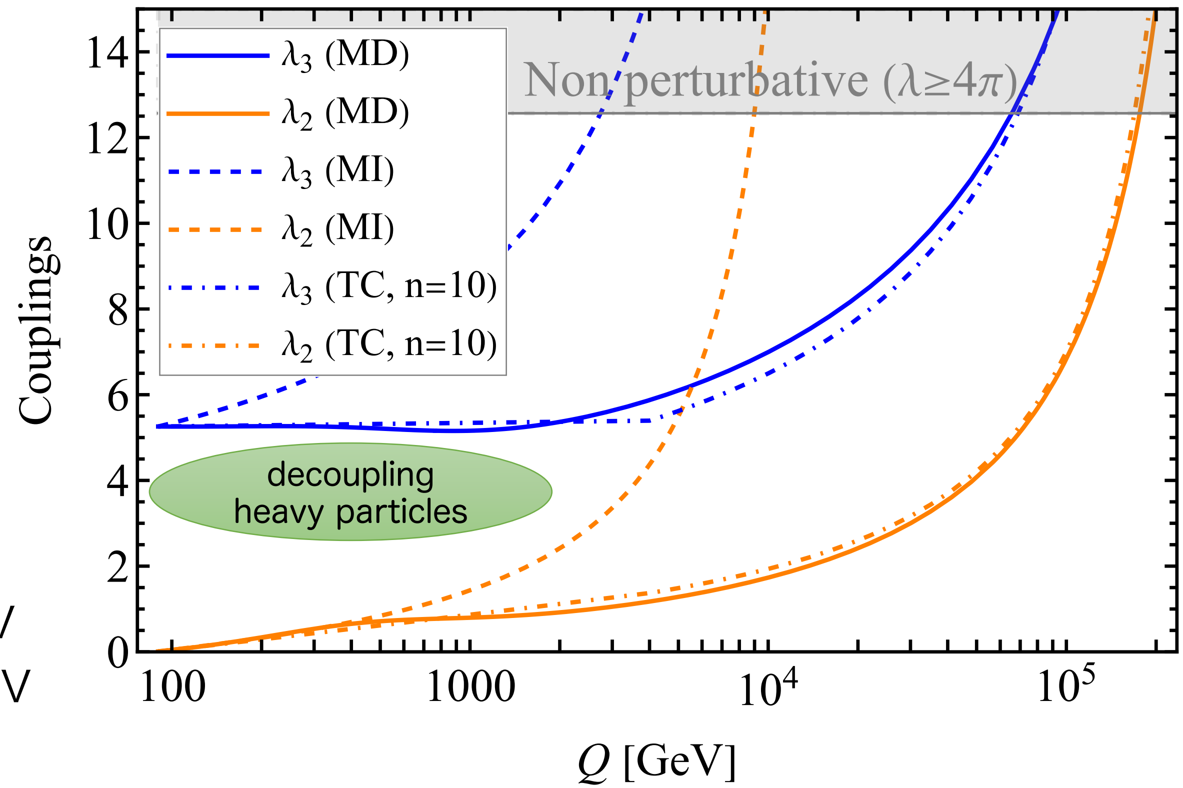
- **Mass dependent (solid) and mass independent (dashed)**

- $\boldsymbol{\Lambda_{4\pi}}$ **: The largest coupling to be $\boldsymbol{4\pi}$**

- **Running delay for $\boldsymbol{Q \lesssim m_{\phi_2}}$**

Mass independent: $\Lambda_{4\pi} \simeq 3$ TeV

Mass dependent: $\Lambda_{4\pi} \simeq 60$ TeV



- **Similarity to running with threshold correction at $\boldsymbol{Q = 10m}$ (dotdash)**

Application to extended Higgs models

- **Inert doublet model**

$$V = \mu_1^2|\Phi_1|^2 + \mu_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}\left|\Phi_1^\dagger\Phi_1\right|^2 + \frac{\lambda_2}{2}\left|\Phi_2^\dagger\Phi_2\right|^2$$
$$+\lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4\left|\Phi_1^\dagger\Phi_2\right|^2 + \lambda_5\,\mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right)^2$$

Unbroken Z_2 symmetry $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$

$$\Phi_1 = \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} H^\pm \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

- **Mass spectrum**

$$m_h^2 = \lambda_1 v^2, \quad m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2, \quad m_H^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 + \lambda_5)v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5)v^2.$$

- **Renormalization condition**

On-shell for mass parameters（scalar, fermion and gauge fields)
+ Wave function for Φ_1 and Φ_2 + Momentum subtraction for λ_2 and λ_3.

Gauge dependence ⇨ background field method

DeWitt (1967); Abbott (1981); Denner et. al. (1995)

- **For vertex functions**

Mathematica packages `FeynCalc, FeynArts, FeynRules, LoopTools, FeynHelpers`

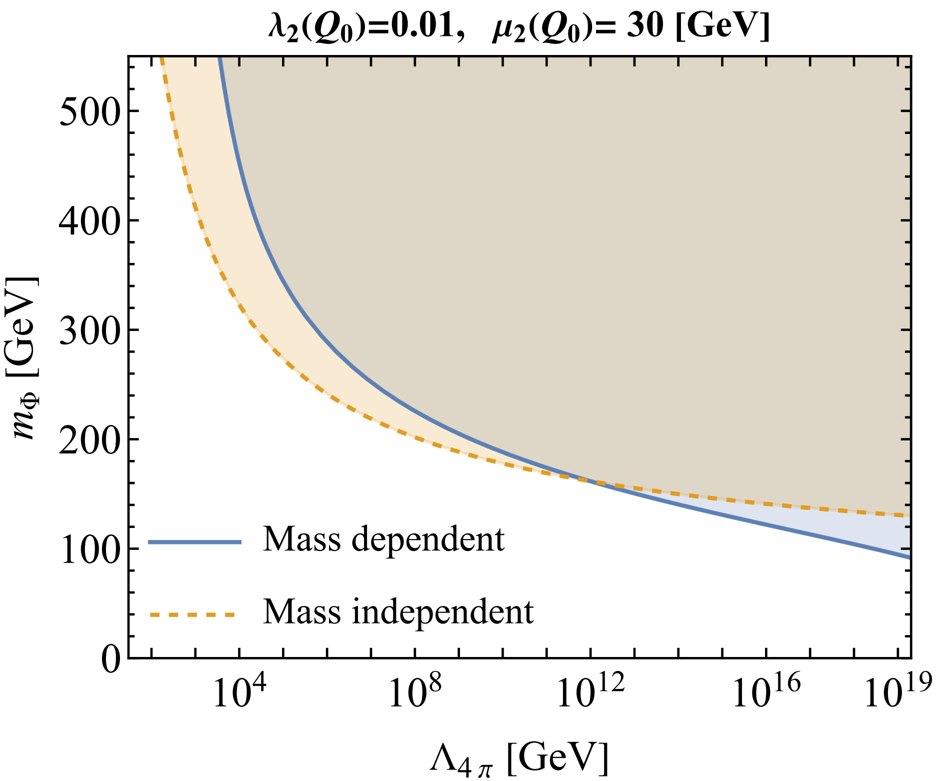
Mertig, Bohm and Denner (1991); Kublbeck, Bohm and Denner (1990); Christensen and Duhr (2009); Hahn (2000); Shtabovenko (2017)

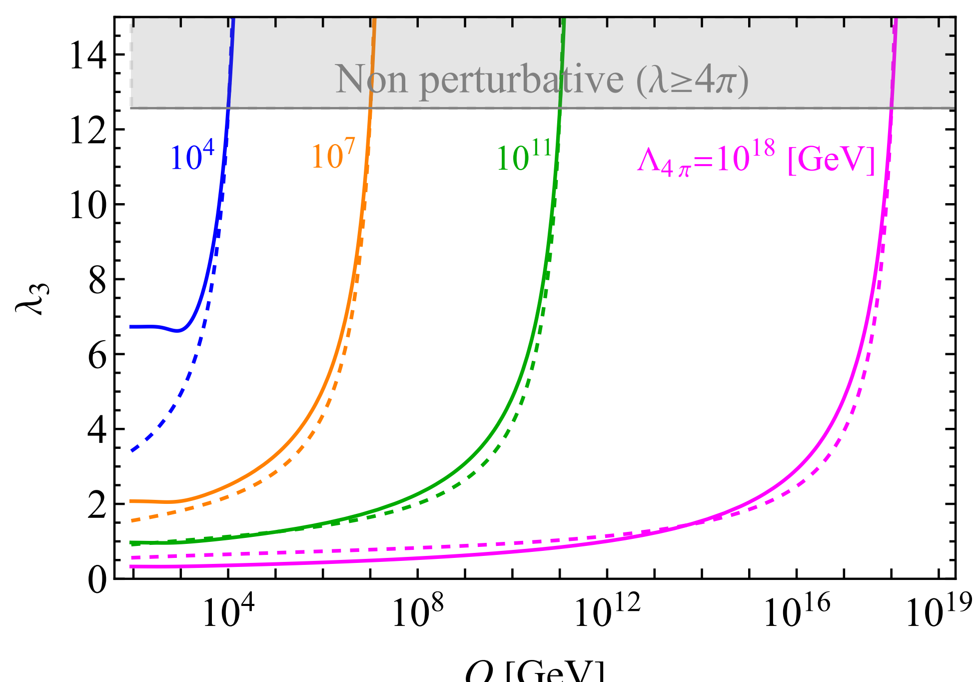
- Degenerated mass of additional scalars

$$m_\Phi^2 \equiv m_{H^\pm}^2 = m_H^2 = m_A^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2$$

- Triviality bound: Upper bound on m_Φ for fixed $\Lambda_{4\pi}$

- Scheme difference prominently appears in the non-decoupling case





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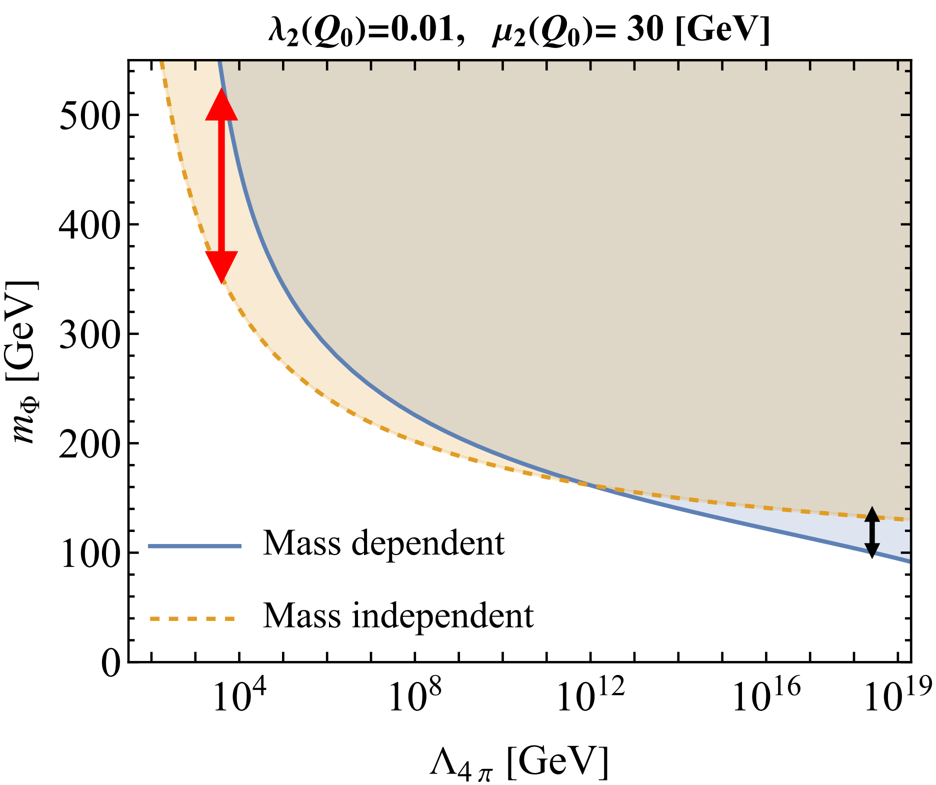

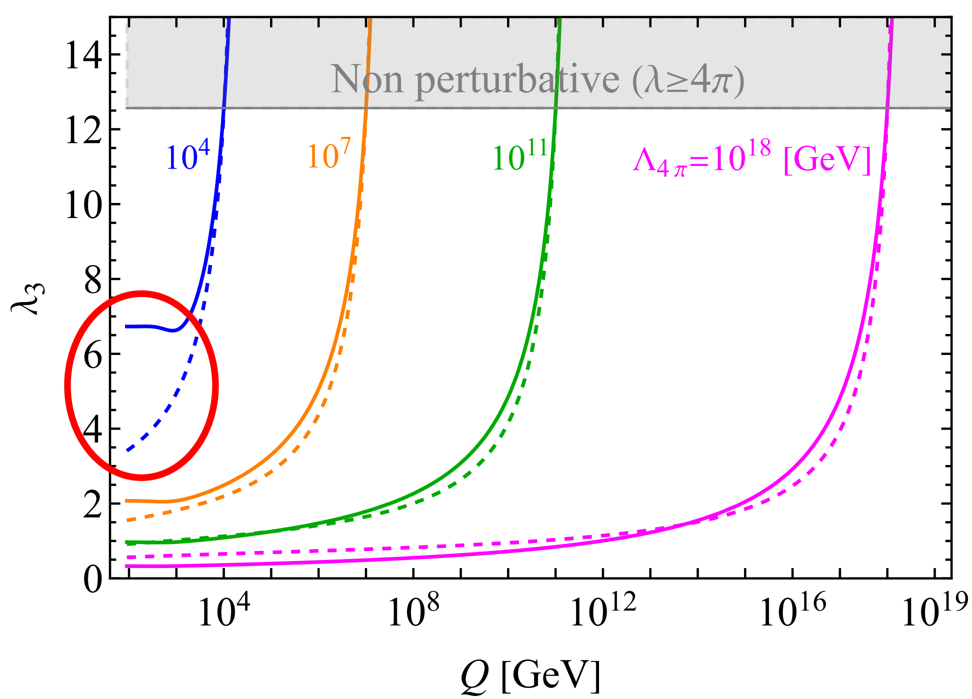

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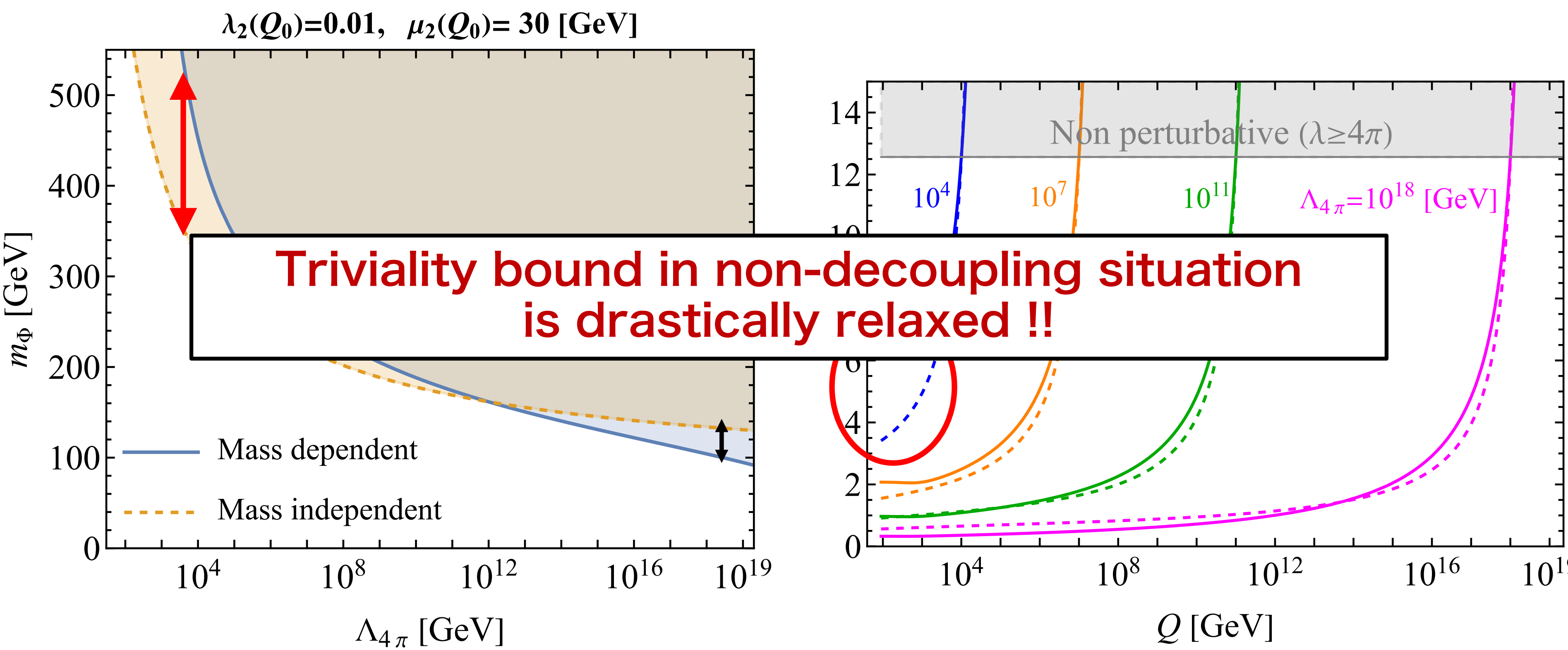


Summary

- **Electroweak baryogenesis and Landau pole**
  - Relatively large scalar coupling for EWBG (non-decoupling situation)
  - Landau pole appears at relatively low scale (1-100TeV)
  - Arbitrariness in threshold correction

- **Mass dependent beta function**
  - Decoupling mechanism for effects of heavy particles
  - Due to the running delay, Landau pole goes far away

- **Triviality bound**
  - Consider inert doublet model
  - Upper bound on the mass for fixed cutoff scale (triviality bound)
  - Triviality bound for EWBG is more relaxed than that has been evaluated

Back up

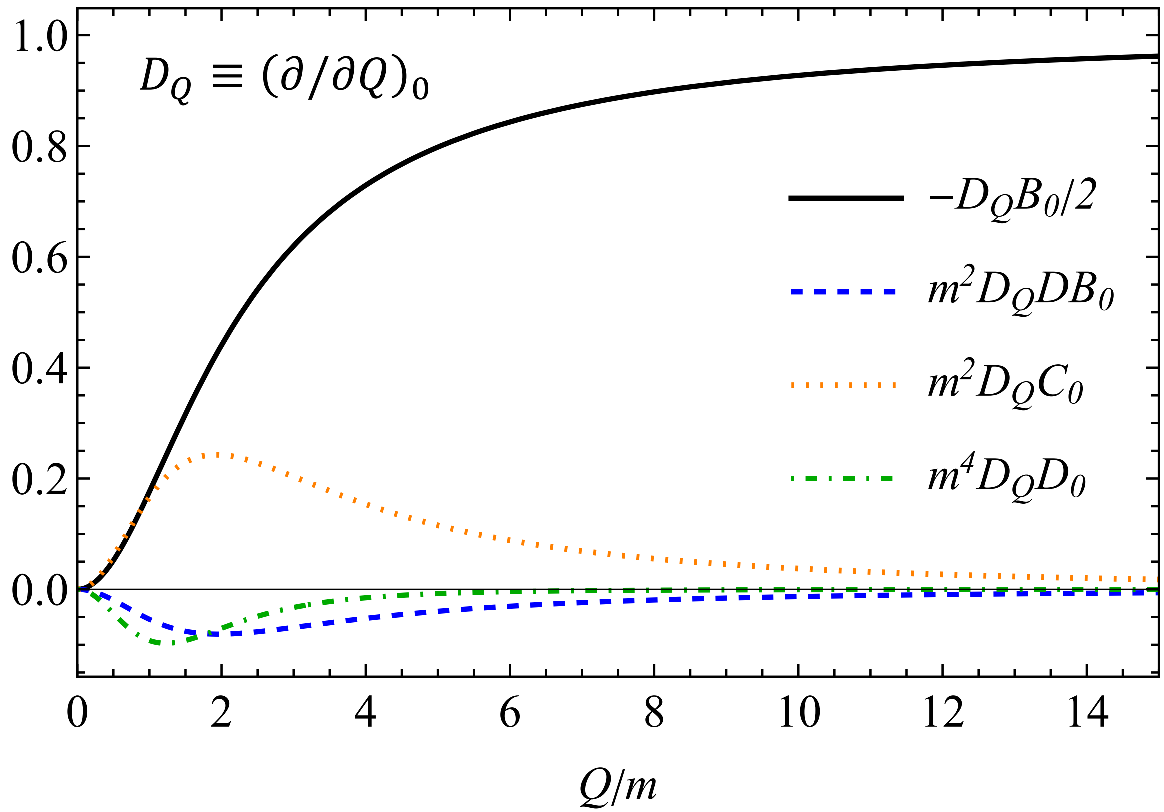
Loop functions

- Only the divergent diagrams are relevant for the beta function

- Derivative of the Passarino-Veltman functions respect with the scale

Passarino and Veltman (1979)

- Subscript 0 means fixed bare parameters



Analytical formulae of beta functions

- Toy model

$$16\pi^2\,\beta_{\lambda_3} = \lambda_1\lambda_3 + 3\lambda_1\lambda_3\, f^Q_{m_\rho,m_\rho} + 4\lambda_3^2 f^Q_{m_\rho,m_{\phi_2}}$$
$$+4\lambda_2\lambda_3 f^Q_{m_{\phi_2}m_{\phi_2}} + D_Q(DB_0, C_0, D_0\ \mathrm{terms})$$

$$16\pi^2\,\beta_{\lambda_2} = \lambda_3^2 + \lambda_3^2\, f^Q_{m_\rho,m_\rho} + 10\lambda_2^2 f^Q_{m_{\phi_2},m_{\phi_2}} + D_Q(DB_0, C_0, D_0\ \mathrm{terms})$$

$$f^Q_{m,m} \equiv -\frac{1}{2} D_Q B_0\left(-\frac{3}{4}Q^2, m^2, m^2\right)$$

- β_{λ_1} is zero for $Q/m \to \infty$, because of on-shell condition for m_ρ and m_η.

- But $\Gamma^{(4)}_{\rho\rho\rho\rho}$, which is relevant to λ_1, has scale dependence at the one loop level.

- Such corrections are $O\left(\frac{\lambda}{16\pi^2}\log(Q^2/m^2)\right)$.

Beta function with threshold correction

- Toy model

$$16\pi^2\,\beta_{\lambda_1} = 10\lambda_1^2 + 2\lambda_3^2\,\theta^n\left(Q/m_{\phi_2}\right)$$

$$16\pi^2\,\beta_{\lambda_2} = 10\lambda_2^2\,\theta^n\left(Q/m_{\phi_2}\right) + 2\lambda_3^2$$

$$\theta^n(Q/m_{\phi_2}) \equiv \begin{cases} 1 & (Q \geq n\, m_{\phi_2}) \\ 0 & (Q < n\, m_{\phi_2}) \end{cases}$$

$$16\pi^2\,\beta_{\lambda_3} = 4\lambda_1\lambda_3 + 4(\lambda_2\lambda_3 + \lambda_3^2)\,\theta^n(Q/m_{\phi_2})$$

- Thresholds are included for the diagrams involved by the heavy particle ϕ_2.

- Appropriate value of n depends on the input parameters.

μ_2 dependence