(Towards) Generalized Symmetry in Particle Phenomenology

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Symmetry in Phenomenology

Symmetry has been a pillar in theoretical physics!!
Particle pheno isn't an exception.

☆ e.x. classification of hadrons wrt. $SU(3)_F$

$\Longrightarrow$ Quark model!

☆ Central in many ongoing ideas/problems
e.x. Strong CP, GUT, discrete flavor sym....

☆ Potentially responsible for Big Mysteries

- (Higgs)Naturalness?
- Cosmological constant???

Symmetry, Scales, Generalizations

- Symmetry $\Longleftrightarrow$ Conservation Law

  $\xRightarrow[\text{Violation}]{}$ Scales (Spontaneous/explicit breaking)

- What about other scales?

  e.g. confinement?

- Generalized Symmetry

  $\rightsquigarrow$ $\left\{ \begin{array}{l} \cdot \text{ Larger variety of scales} \\ \cdot \text{ More precise treatment of those scales} \end{array} \right.$

Generalized Symmetry

☆ Generalizing symmetry $\Longrightarrow$ broader applicability

☆ Two types:
- Higher-form symmetry

  [Gaiotto, Kapustin, Seiberg, Willet '14]
- Non-invertible symmetry

  Old (in 1+1d) and new (in 3+1d)

☆ Both exists in Standard Model!!

Applications?

☆ Generalized symmetry has refined existent understandings on various phenomena.

☆ New result?

- Higher-form: 't Hooft anomaly constraint on pure YM @ $\theta = \pi$

  [Gaiotto, Kapustin, Komargodski, Seiberg '17]

- Non-invertible: attempts...

  Opportunity!

Today: Review on generalized symmetries
in Standard Model & SM + axion

Outline:

- Intro (done)
- One-form Symmetry in Gauge Theory
  [Gaiotto, Kapustin, Komargodski, Seiberg '17]
- Non-invertible Chiral symmetry
  [Choi, Lam, Shao '22], [Córdova, KO '22]
- Standard Model + axion
  [Choi, Forslund, Lam, Shao '23], [Reece '23], [Agrawal, Platschorre '23], [Cordova, Hong, Wang '23]

Higher-form Symmetry [Gaiotto, Kapustin, Seiberg, Willet '14]

☆ Ordinary symmetry e.g. $\phi(x) \mapsto e^{i\alpha}\phi(x)$

Acts on local operators: $\mathcal{O}_q(x) \mapsto e^{iq\alpha}\mathcal{O}(x)$

☆ One-form symmetry

p-form

$e^{iq\alpha}$

Acts on line operators: $W_q(L) \mapsto \cancel{e^{iq\alpha}} W_q(L)$

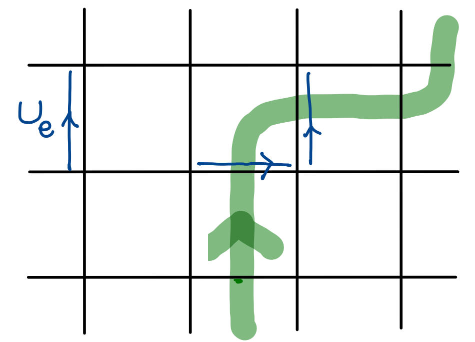
p-dim'el

Reformulates "center symmetry"

$\Longrightarrow$ Electric one-form symmetry

Electric One-form Symmetry

Codim 1 Σ'



★ Yang-Mills has $Z(G)$ one-form symmetry

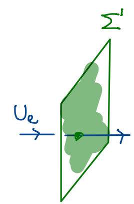
Center of G

★ Wilson action on lattice:

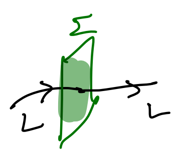
$$S \propto \sum_{\square} \mathrm{Tr}\left(1 - \prod_{\vec{e} \in \square} U_{\vec{e}}\right)$$

$\mathrm{Pexp}\left(\int_e A\right)$

★ One-form transformation

$U_e \longmapsto \gamma U_e$, $\gamma \in Z(G)$ if 

, $S \longmapsto S$;

★ $W_\square[L] \longmapsto \frac{\mathrm{Tr}\,\gamma}{\dim \square} W_\square[L]$ if 

; **Charged under one-form sym.**

One-form Symmetry and Confinement

☆ Electric one-form symmetry: reformulation of "center symmetry"

↳ Confinement

☆ Area/Perimeter law

$$W_q[\longrightarrow] \xrightarrow{\square \to \infty} W_q[\square] = \begin{cases} e^{-(\text{area})} \xrightarrow{\text{long range}} 0 \\ e^{-c(\text{perimeter})} \end{cases}$$

screened

One-form Changed

$$W_q \cdot e^{c\oint_{\square} dl} \longrightarrow \neq 0$$

☆ Deconfinement = one-form SSB!

One-form Symmetry & Scales

☆ Gauge group G + matter ϕ in representation R

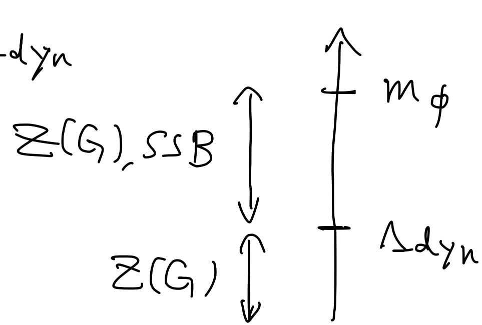
$\Longrightarrow$ Electric $Z_R(G) = \{\gamma \in Z(G) | \rho_R(\gamma) = 1\}$

W_R : **Screened**

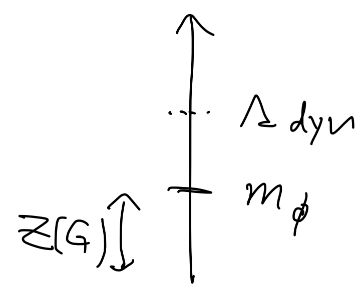
☆ **Emergence** $G + \phi_{\square} \overset{m_\phi}{\rightsquigarrow}$ pure G

Electric one-form: $Z_{\square} = \{1\} \rightsquigarrow Z(G)$:

$m_\phi > \Lambda_{dyn}$



$m_\phi < \Lambda_{dyn}$



Magnetic one-form symmetry

- Electric one- form counts Wilson lines

- Electro-magnetic dual: Another "magnetic" one-form sym. $U(1)$

- Counts 't Hooft lines (worldline of probe monopole)

- Non-abelian G: $\pi_1(G)$ magnetic symmetry

  ↳ Unscreened monopoles

- Emergence ; $SU(2) \overset{\text{Higgs}}{\rightsquigarrow} U(1)$

  Mag. One-form $\{1\} \rightsquigarrow U(1)$;

  ↳ Screened by dynamical monopole

One-form Symmetry in Standard Model

☆ $G_{SM} = \dfrac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_p}$ $\qquad p = 1, 2, 3, 6$

☆ **One-form** : $\underbrace{\mathbb{Z}_{6/p}}_{\text{Ele.}} \times \underbrace{U(1)}_{\text{Mag.}}$

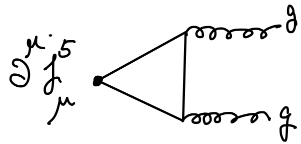
☆ **GUT** : $SU(5) \longrightarrow G_{SM,\, p=6}$

$SU(4) \times SU(2)^2 \longrightarrow G_{SM,\, p=3}$

$\Lambda_{mag,\,emerge} = \Lambda_{GUT}$

Non-invertible Chiral Symmetry

[Choi, Lam, Shao '22]
[Cordova, KO '22]

☆ **ABJ anomaly:** 

$\Longrightarrow$ $\partial_\mu j_5^\mu \propto \mathrm{tr} F_{\mu\nu}\widetilde{F}^{\mu\nu}$

instanton

☆ Abelian gauge group (e.g. massless QED) $\Longrightarrow$ **No instantan on** $\mathbb{R}^4$

$\Longrightarrow$ No violation on $\mathbb{R}^4$

☆ **Locally Conserved operator:** $[T_{\mu\nu}, D_\alpha] = 0$, for $\alpha = \frac{p}{q} 2\pi$

chiral rotation angle

Non-invertible Chiral Symmetry (cont'ed)

★ **Locally Conserved operator:** $[T_{\mu\nu}, D_\alpha] = 0$ $\alpha = \frac{p}{q} 2\pi$

$(P=1)$

$$D_{1,q}[M_3] = \underbrace{e^{\frac{2\pi i}{q}\int_{M_3} j_5^0 d^3x}}_{\text{Naive neither charge}} \; \underbrace{\mathcal{A}_q[M_3, A]}$$

↑ Spatial manifold

EM field

Fractional Hall State effective partition function $\nu = \frac{1}{q}$

$e^{\frac{i}{4\pi q}\int A dA}$ $\leftarrow \int \mathcal{D}a \, e^{i\left[q\int a da + \int A da\right]}$

★ **Non-invertible:**

$$D_{p,q}^\dagger D_{p,q} = \left(\textbf{Projection s.t. } \int_{\Sigma_2} \frac{F}{2\pi} \in q\mathbb{Z}\right) \neq 1$$

$D_{p,q}|\text{monopole}\rangle = 0 \rightsquigarrow$ **Monopole catalysis**

Standard Model + axion

[Choi, Forslund, Lam, Shao '23], [Reece '23], [Agrawal, Platschorre '23], [Cordova, Hong, Wang '23]

☆ $a \sim a + 2\pi f$, $\quad N_{DW} \frac{a}{f} \operatorname{tr} G\tilde{G} + E \frac{a}{f} F_{EM} \tilde{F}_{EM}$

☆ Two-form symmetry counting axion string

☆ $\mathbb{Z}_{N_{DW}}$ Shift symmetry $\Longrightarrow$ Domain Wall prob.

$G_{SM} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_p}$

$p < 6$, minimal E $\Longrightarrow$ Non-invertible!

☆ GUT: $G_{GUT} \to G_{SM,p}$, $p < 6$ $\quad$ (e.g. $G_{GUT} = SU(4) \times SU(2)_L$)

$\cancel{\mathbb{Z}_{N_{DW}}}$ by GUT monopole

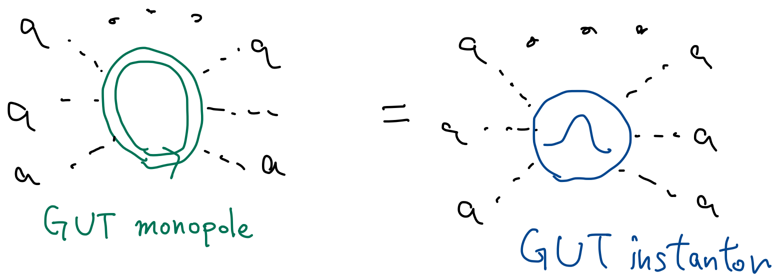
$$\Delta V_{vac} \sim e^{-M_{mon} R_{mon}} \Lambda_{GUT} \sim e^{-\frac{1}{g_{GUT}^2}} \Lambda_{GUT}$$

☆ $p < 6$, $\mathbb{Z}_{N_{DW}}$: Non-invertible Shift symmetry

☆ GUT: $G_{GUT} \rightarrow G_{SM,p}$, $p < 6$ (e.g. $G_{GUT} = SU(9) \times SU(2)_L$)

$$SU(9) \supset SU(3)^2$$

$\cancel{\mathbb{Z}_{N_{DW}}}$ by GUT monopole



[Fan, Fraser, Reece, Stout '21]
[Cordova, KO, '22]

☆ $\Delta V_{vac} \sim e^{-M_{mon} R_{mon}} \Lambda_{GUT} \sim e^{-\frac{1}{g_{GUT}^2}} \Lambda_{GUT}$

☆ Non-invertibility implies suppressed breaking of a symmety

IR property

Summary

- Standard Model has one-form & non-inv. symmetries!
- BSM models also has its own.
- Generalized Symmetry dictates various scales & relations among them.

  Inequalities among scales
  [Brennan Cordova '20], [Choi, Forslund,Lam,Shao '23]

  Hierarchy
  [Cordova KO '20], [Cordova, Hong, Koren, KO '22], [Forslund,Lam,Shao '23]

- Can provide bottom-up way of model-building

Thank You!!