Fast envelope tracking for high intensity low energy electrons

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Outline

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3. Examples
4. Conclusion

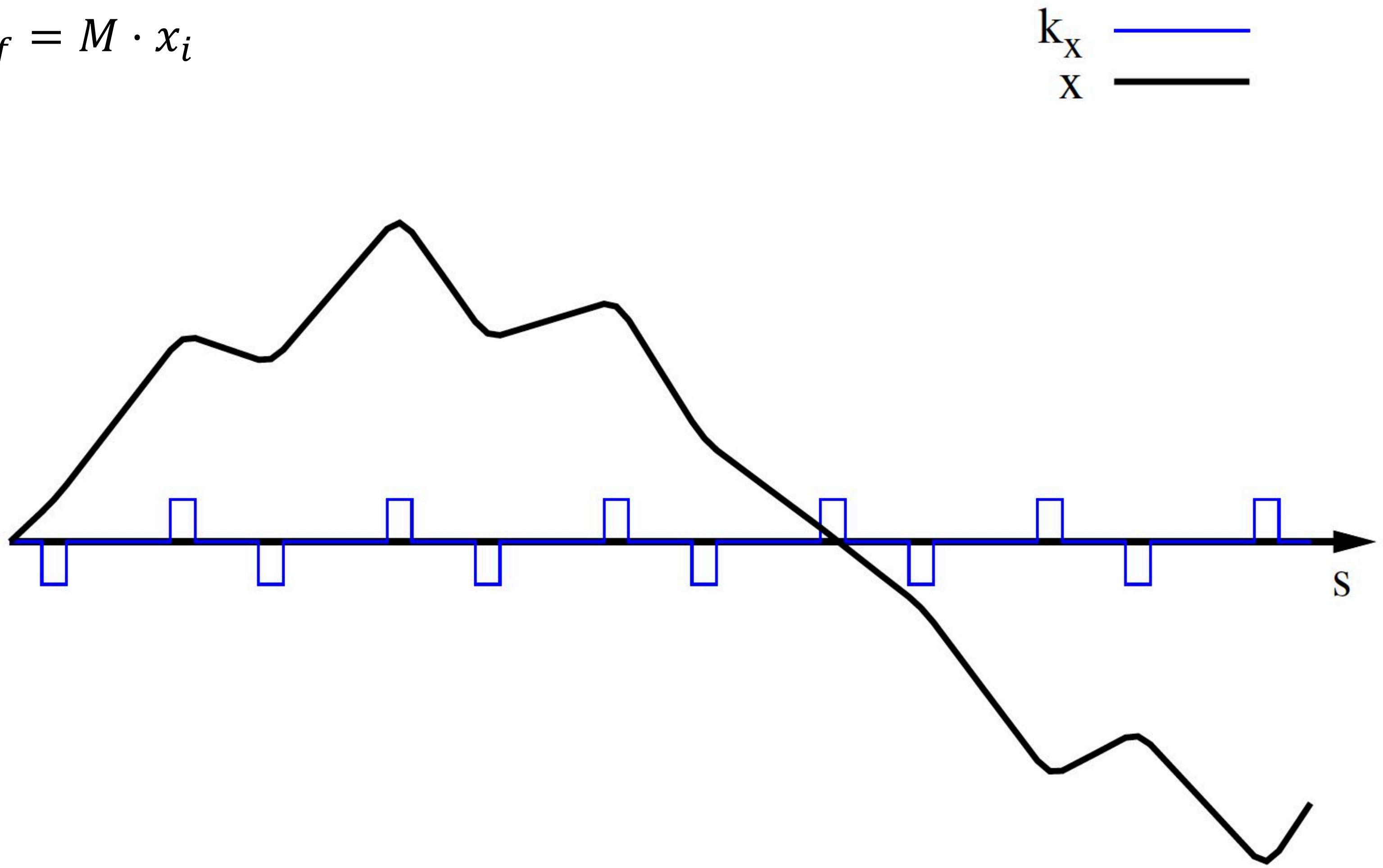
Introduction

Type of simulations	Envelope	Multi-particle (PIC)
Beam representation	Statistical moments	Individual macro particles
Computing speed	Fast	Slow
Optics model	Non-linearity are usually excluded	Realistic beamline modelling that includes non-linearity
Main usage	• Machine design • Real-time tuning	• Detailed beam dynamic study • Track particle loss
Examples	TRACE3D, TRANSPORT, MAD-X, TRANSOPTR	Warp, ASTRA, GPT, Elegant

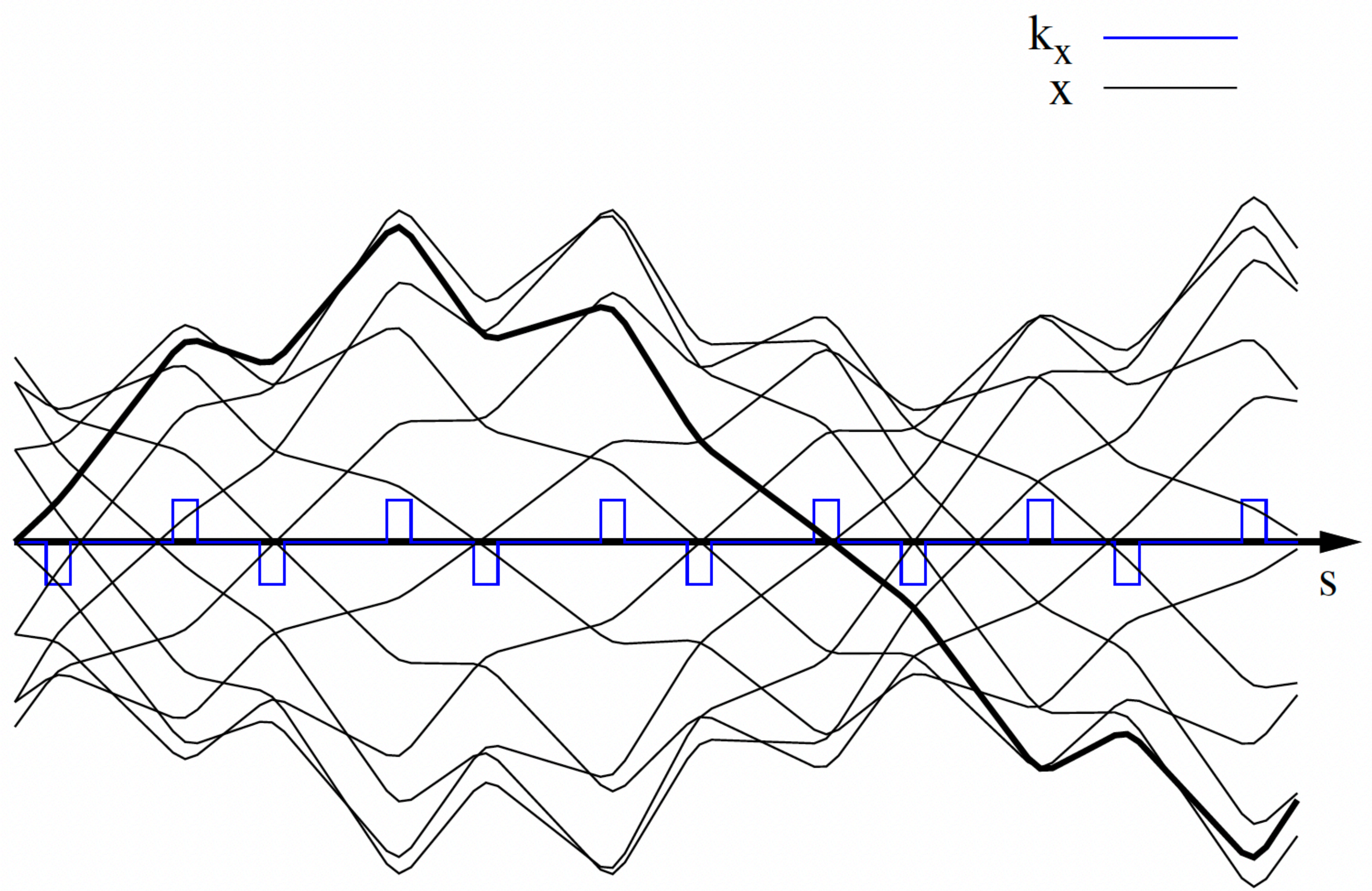
What is envelope tracking ?

Single particle tracking

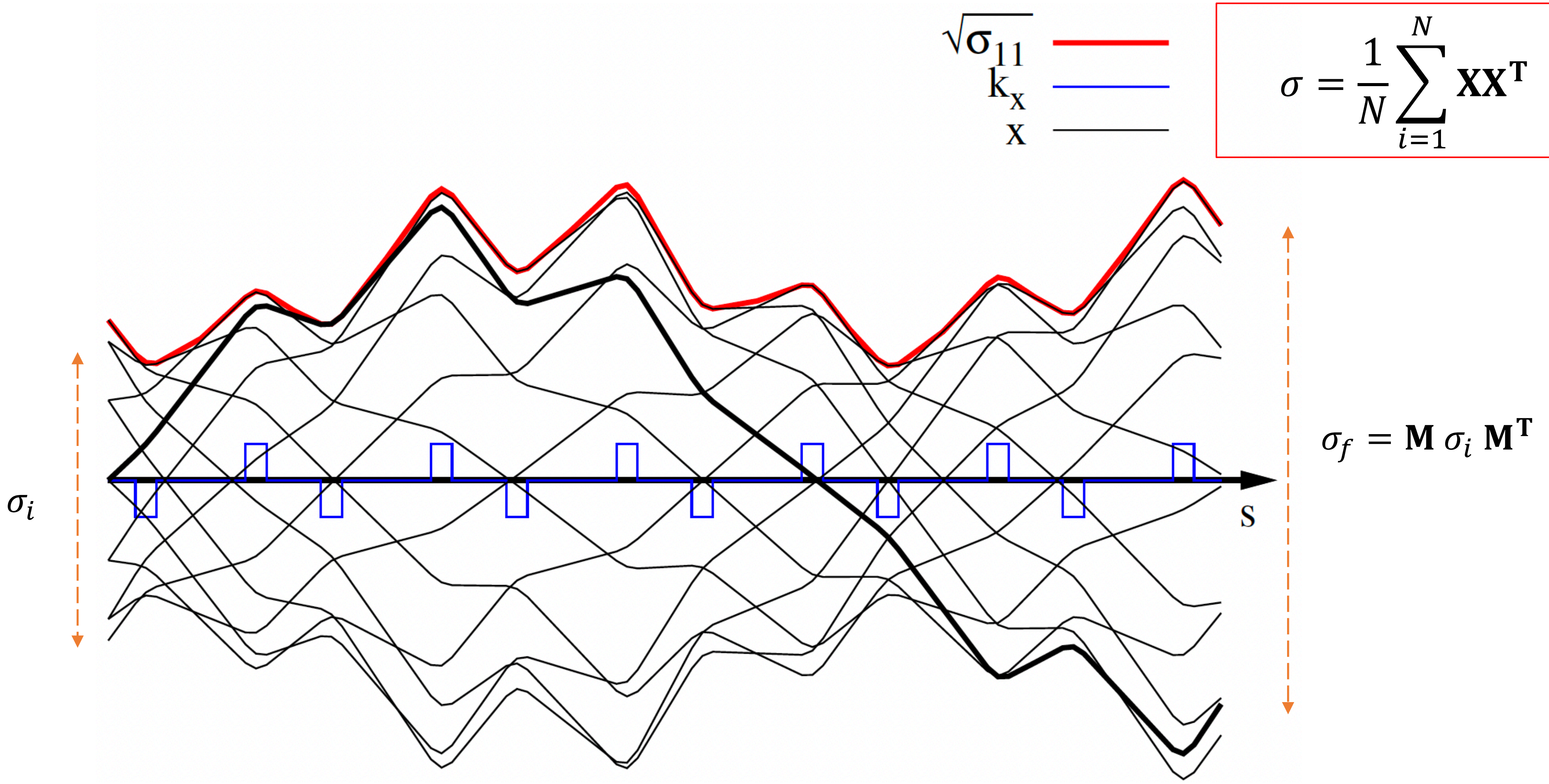
Let's say $x_f = M \cdot x_i$



Multi-particle tracking



Beam Envelope



$$\sigma = \frac{1}{N}\sum_{i=1}^{N} \mathbf{X}\mathbf{X}^\mathbf{T}$$

Statistical Approach in Beam Dynamics

$$\sigma = \frac{1}{N}\sum_{i=1}^{N} \mathbf{X}\mathbf{X}^{\mathrm{T}}$$

$$\frac{d\sigma}{ds} = \frac{1}{N}\sum_{i=1}^{N} \frac{d\mathbf{X}}{ds}\mathbf{X}^{\mathrm{T}} + \frac{1}{N}\sum_{i=1}^{N} \mathbf{X}\frac{d\mathbf{X}^{T}}{ds}$$

Given $\frac{d\mathbf{X}}{ds} = \mathbf{F}\mathbf{X}$

$$\mathbf{F}\,\mathrm{ds} = \mathbf{M} - \mathbf{I}$$

$$\therefore \sigma' = \mathbf{F}\sigma + \sigma\mathbf{F}^{\mathrm{T}}$$

Envelope equation

Infinitesimal Matrix and Hamiltonian Dynamics

Hamiltonian equation of motion:

$$x' = \frac{\partial H}{\partial P_x} \quad ; \quad P_x' = -\frac{\partial H}{\partial x}$$

Expanding x' up to second order,

$$x' = \sum_i \left.\frac{\partial x'}{\partial x_i}\right|_0 x_i = \boxed{\sum_{i=1}^{6} \left.\frac{\partial^2 H}{\partial P_x \partial x_i}\right|_0} x_i$$

$$\mathbf{X}' = \boxed{\mathbf{F}}\,\mathbf{X}$$

F can be determined from the Hamiltonian!

Infinitesimal Matrix and Hamiltonian Dynamics

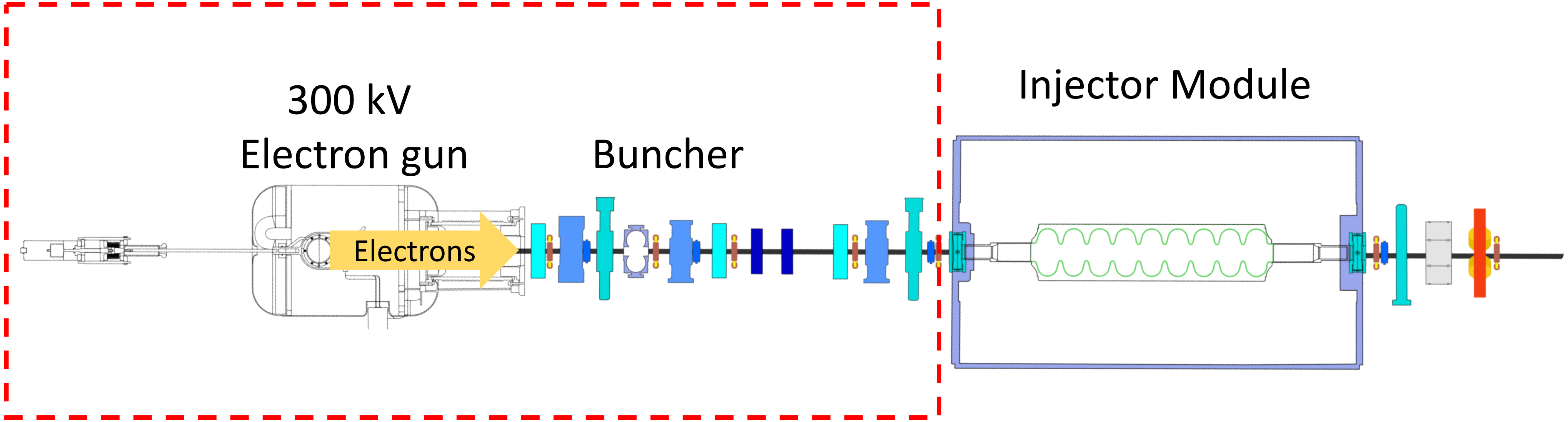
$$H(x_1, x_2, x_3, x_4, x_5, x_6; s) = \sum_i \left.\frac{\partial H}{\partial x_i}\right|_0 x_i + \frac{1}{2}\sum_{i,j} \left.\frac{\partial^2 H}{\partial x_i \partial x_j}\right|_0 x_i x_j + \cdots$$

$$\begin{pmatrix} x' \\ P_x' \\ y' \\ P_y' \\ z' \\ P_z' \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 H}{\partial P_x \partial x} & \frac{\partial^2 H}{\partial P_x^2} & \frac{\partial^2 H}{\partial P_x \partial y} & \frac{\partial^2 H}{\partial P_x \partial P_y} & \frac{\partial^2 H}{\partial P_x \partial z} & \frac{\partial^2 H}{\partial P_x \partial P_z} \\ -\frac{\partial^2 H}{\partial x^2} & -\frac{\partial^2 H}{\partial x \partial P_x} & -\frac{\partial^2 H}{\partial x \partial y} & -\frac{\partial^2 H}{\partial x \partial P_y} & -\frac{\partial^2 H}{\partial x \partial z} & -\frac{\partial^2 H}{\partial x \partial P_z} \\ \frac{\partial^2 H}{\partial P_y \partial x} & \frac{\partial^2 H}{\partial P_y \partial P_x} & \frac{\partial^2 H}{\partial P_y \partial y} & \frac{\partial^2 H}{\partial P_y^2} & \frac{\partial^2 H}{\partial P_y \partial z} & \frac{\partial^2 H}{\partial P_y \partial P_z} \\ -\frac{\partial^2 H}{\partial y \partial x} & -\frac{\partial^2 H}{\partial y \partial P_x} & -\frac{\partial^2 H}{\partial y^2} & -\frac{\partial^2 H}{\partial y \partial P_y} & -\frac{\partial^2 H}{\partial y \partial z} & -\frac{\partial^2 H}{\partial y \partial P_z} \\ \frac{\partial^2 H}{\partial P_z \partial x} & \frac{\partial^2 H}{\partial P_z \partial P_x} & \frac{\partial^2 H}{\partial P_z \partial y} & \frac{\partial^2 H}{\partial P_z \partial P_y} & \frac{\partial^2 H}{\partial P_z \partial z} & \frac{\partial^2 H}{\partial P_z^2} \\ -\frac{\partial^2 H}{\partial z \partial x} & -\frac{\partial^2 H}{\partial z \partial P_x} & -\frac{\partial^2 H}{\partial z \partial y} & -\frac{\partial^2 H}{\partial z \partial P_y} & -\frac{\partial^2 H}{\partial z^2} & -\frac{\partial^2 H}{\partial z \partial P_z} \end{pmatrix} \begin{pmatrix} x \\ P_x \\ y \\ P_y \\ z \\ P_z \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{F}\mathbf{X}$$

F = infinitesimal transfer matrix

Examples 1 : Low energy transportation at TRIUMF e-Linac



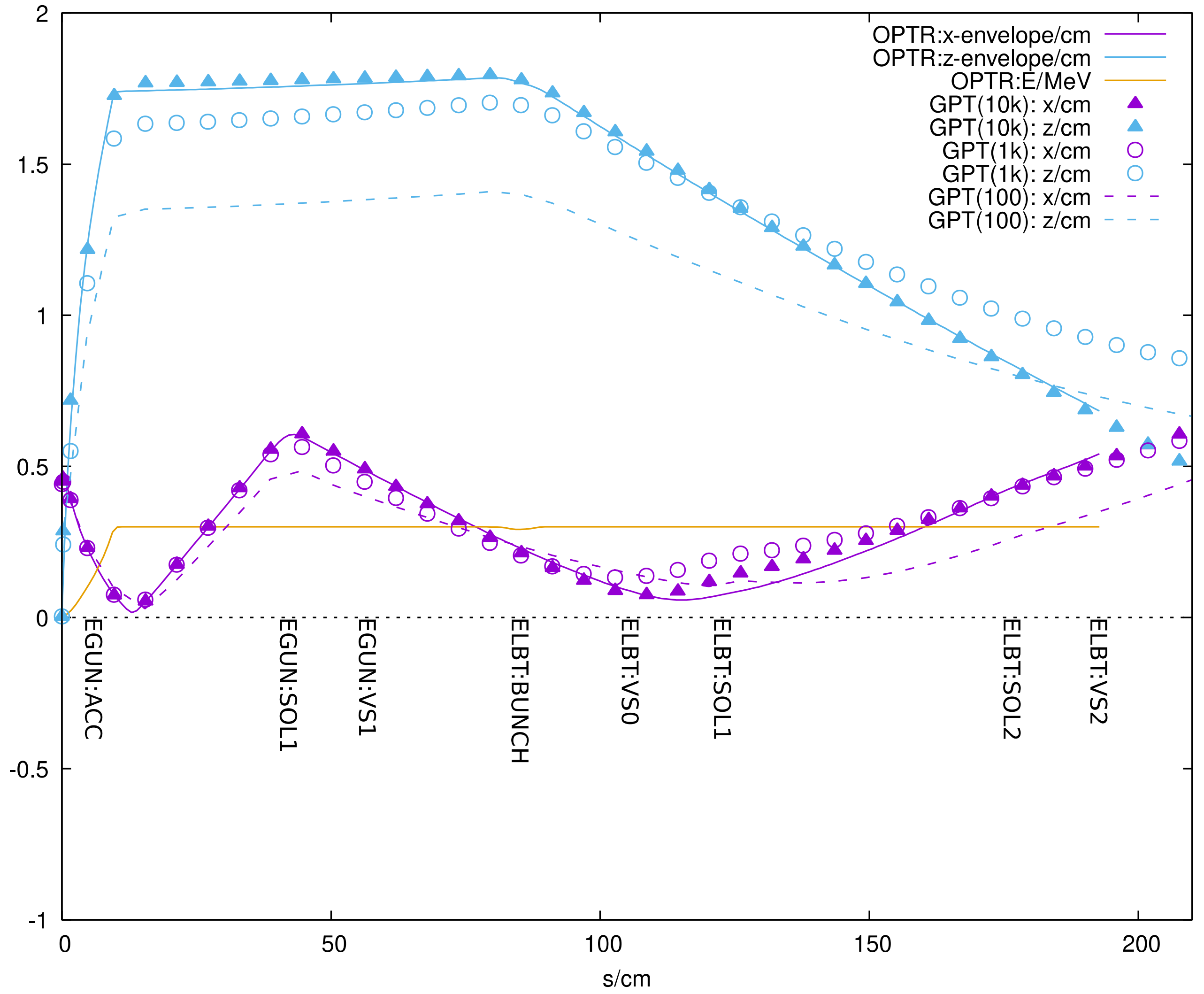
- Low energy, strong space charge
- Non-linearity always a limitation

Comparison between TRANSOPTR and GPT

Types of simulation	CPU time (s)	No. of particles
Envelope (TRANSOPTR)	0.06	-
Multi-particle (GPT)	8.4	10,000
	1.4	1,000
	0.08	100

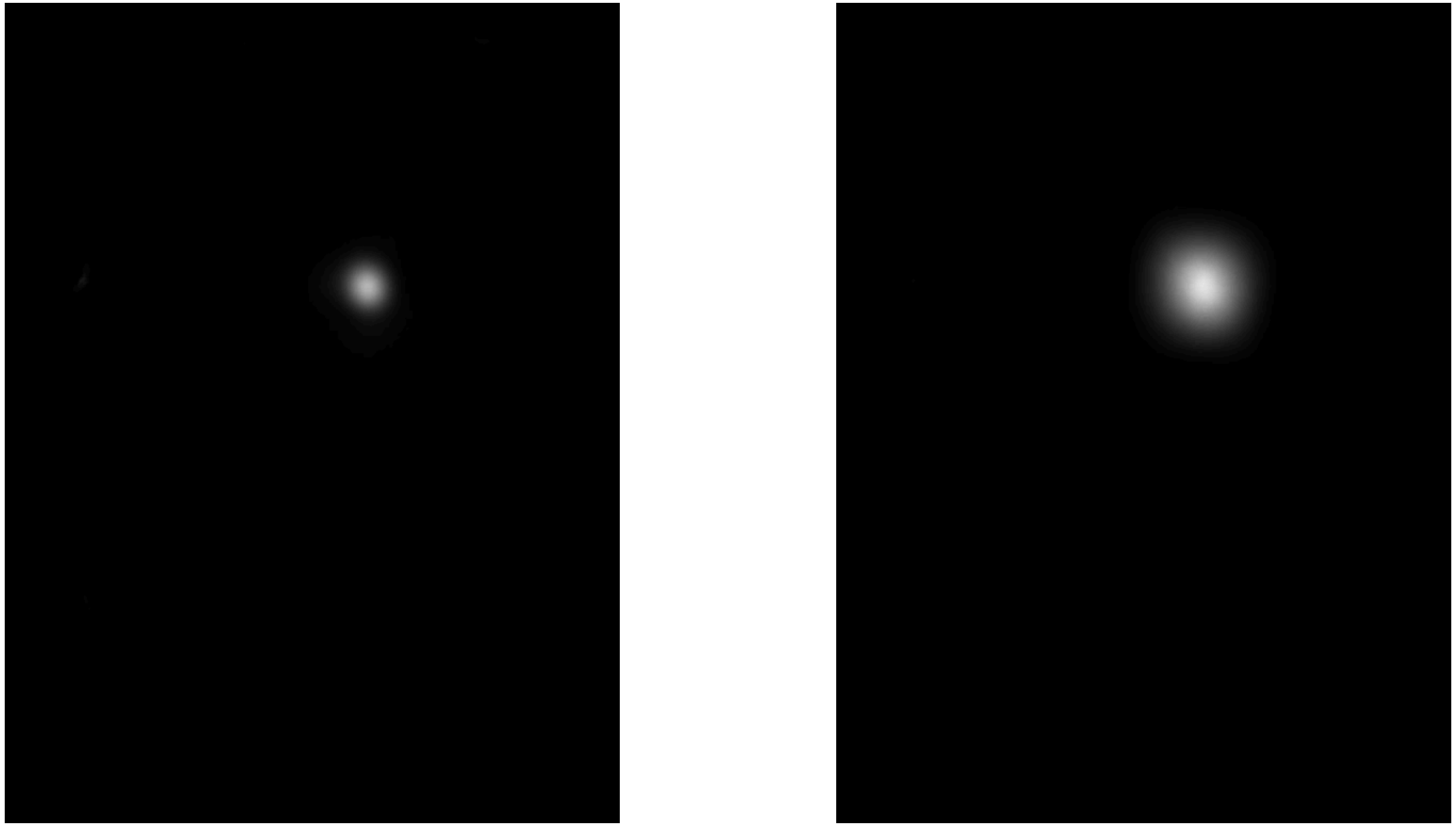
2 magnitude faster to achieve similar precision!

E = 300 keV ; Q = 14 pC



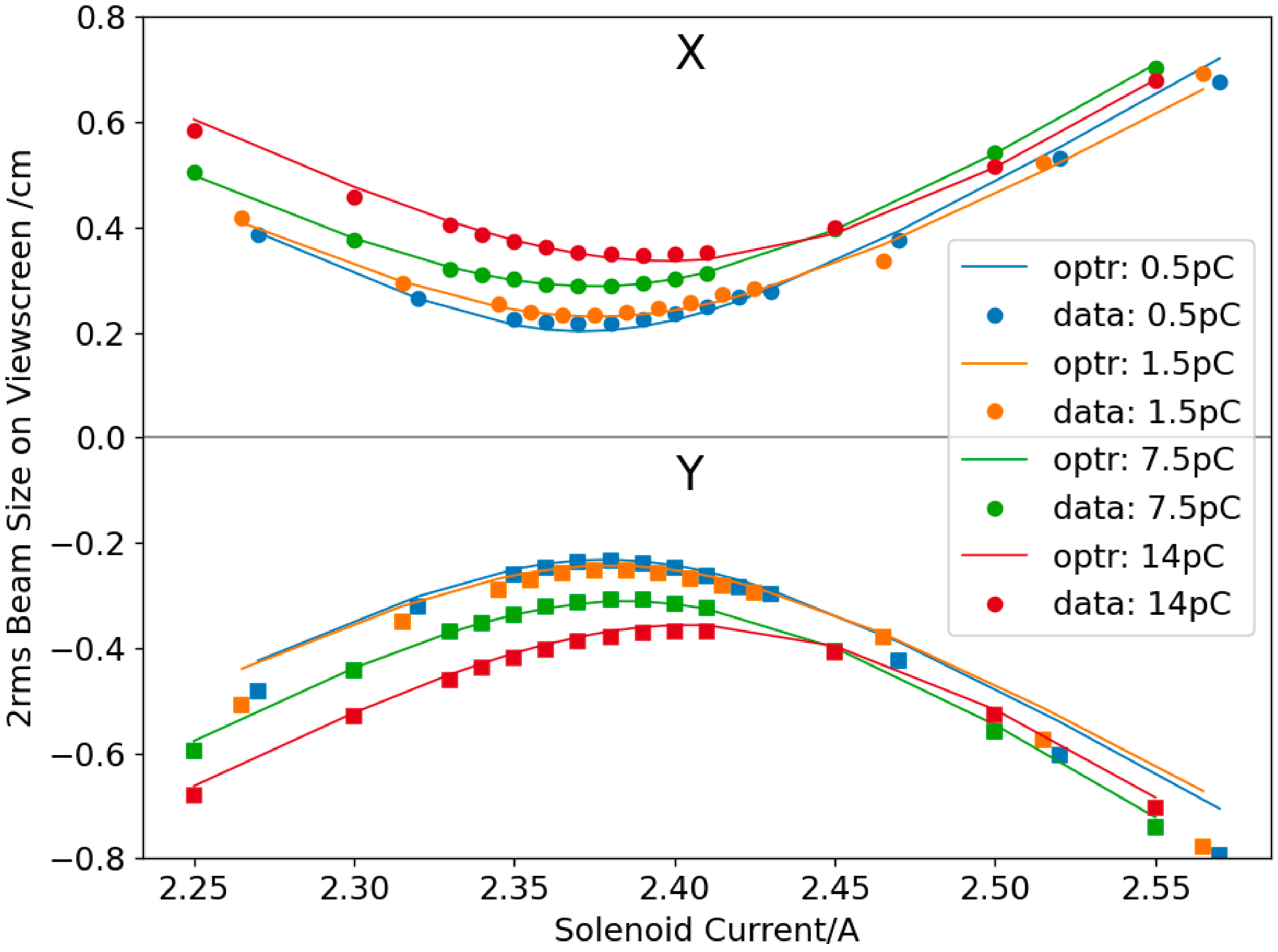
Comparison with Experimental Data

Electron Beam on a Viewscreen

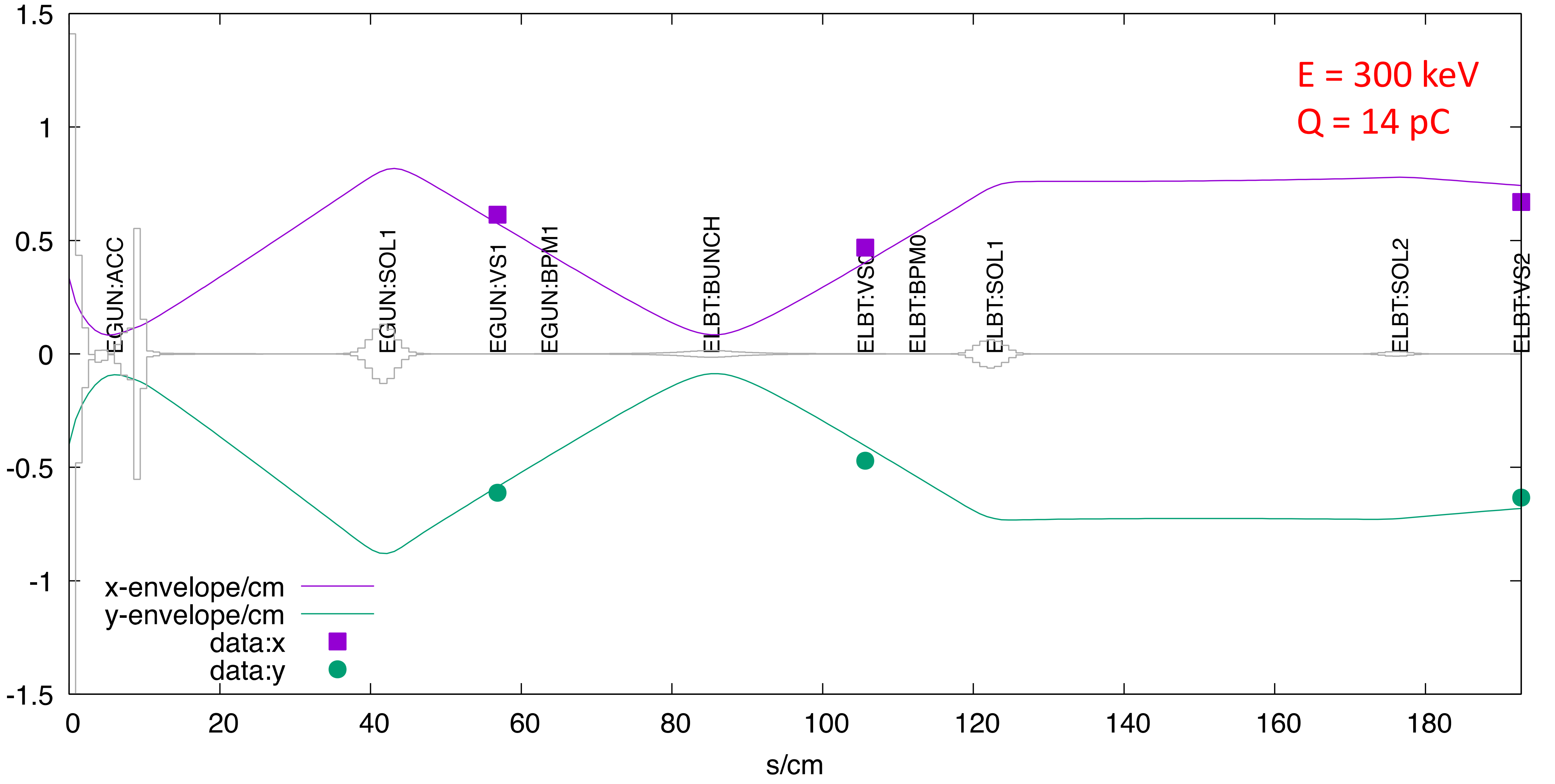


0.5 pC   14 pC

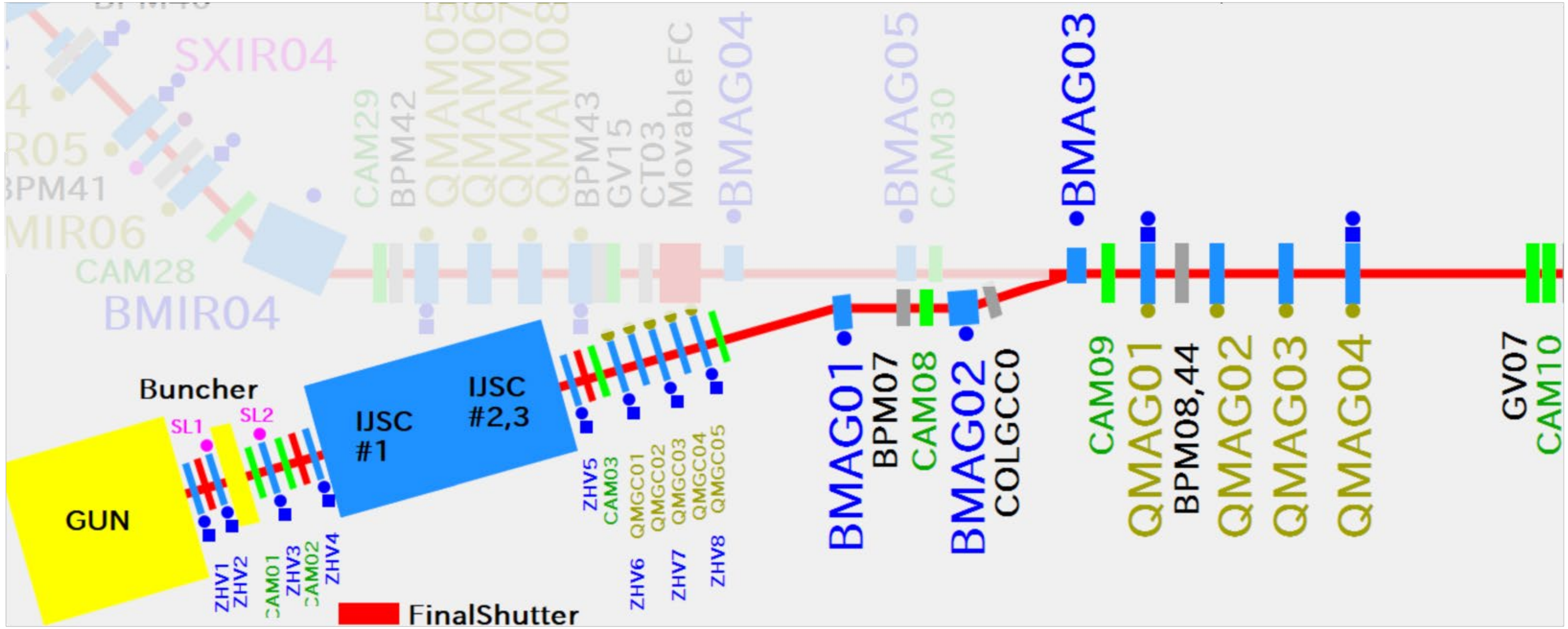
Fitting for the initial beam condition



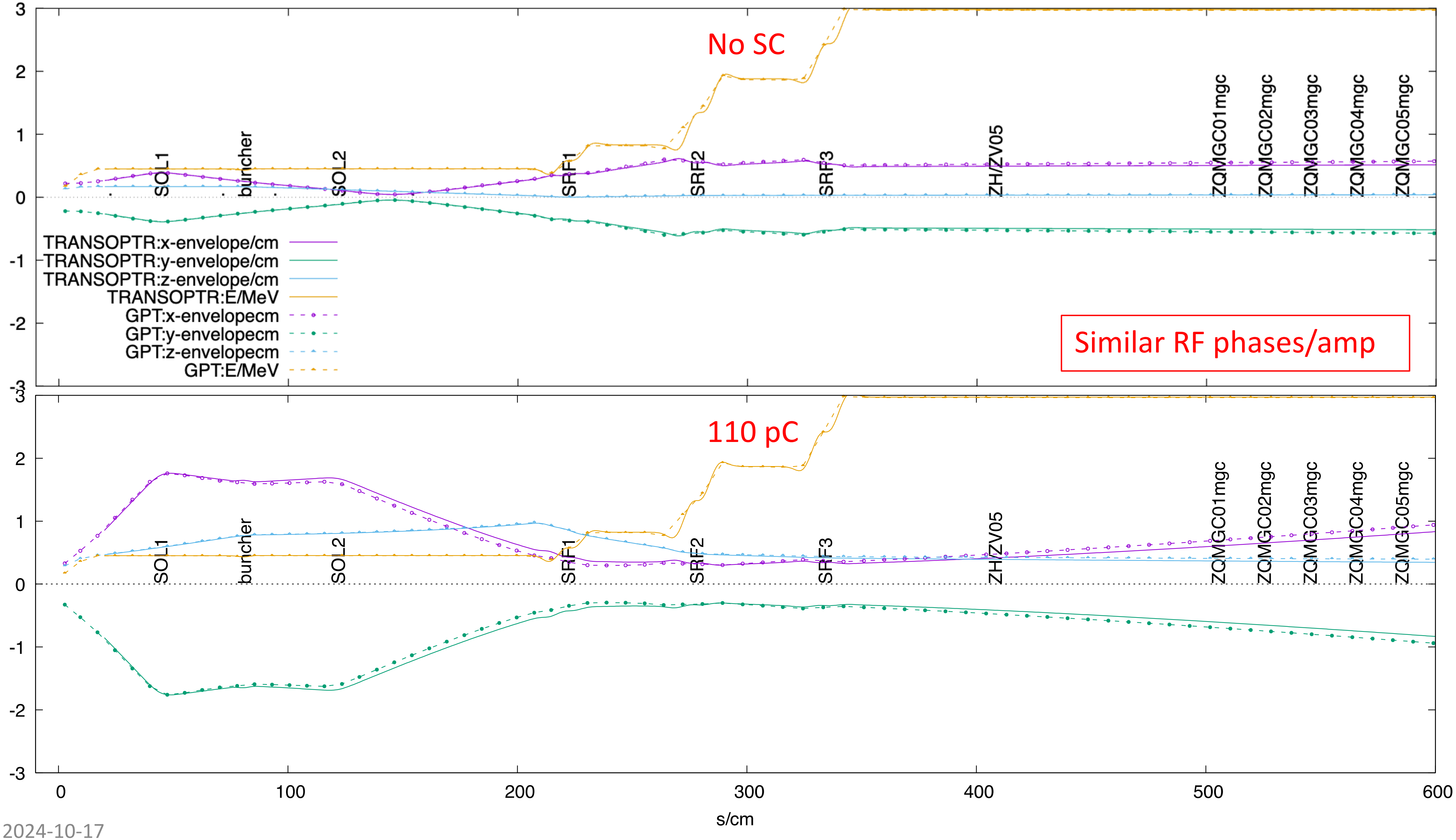
Comparison with Experimental Data



Example 2: Injection Beamline at cERL

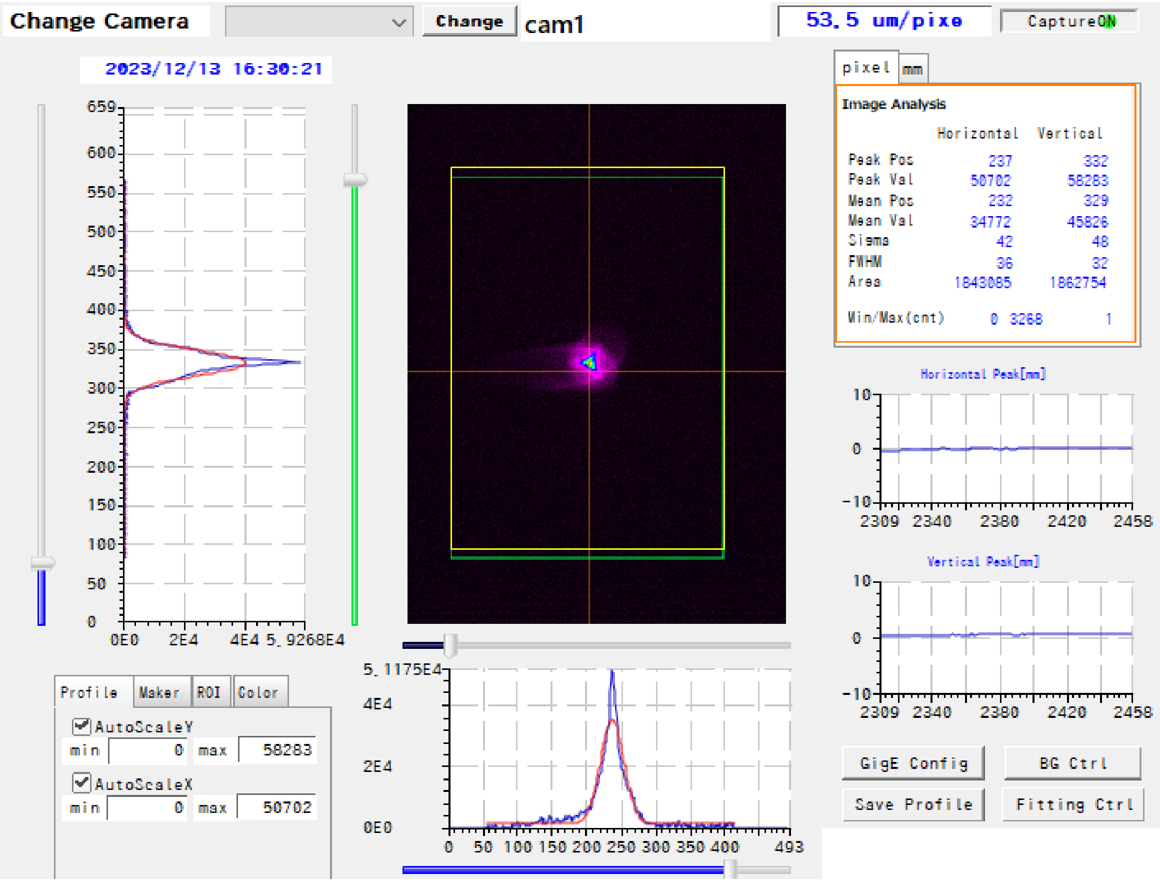


Comparison between TRANSOPTR and GPT

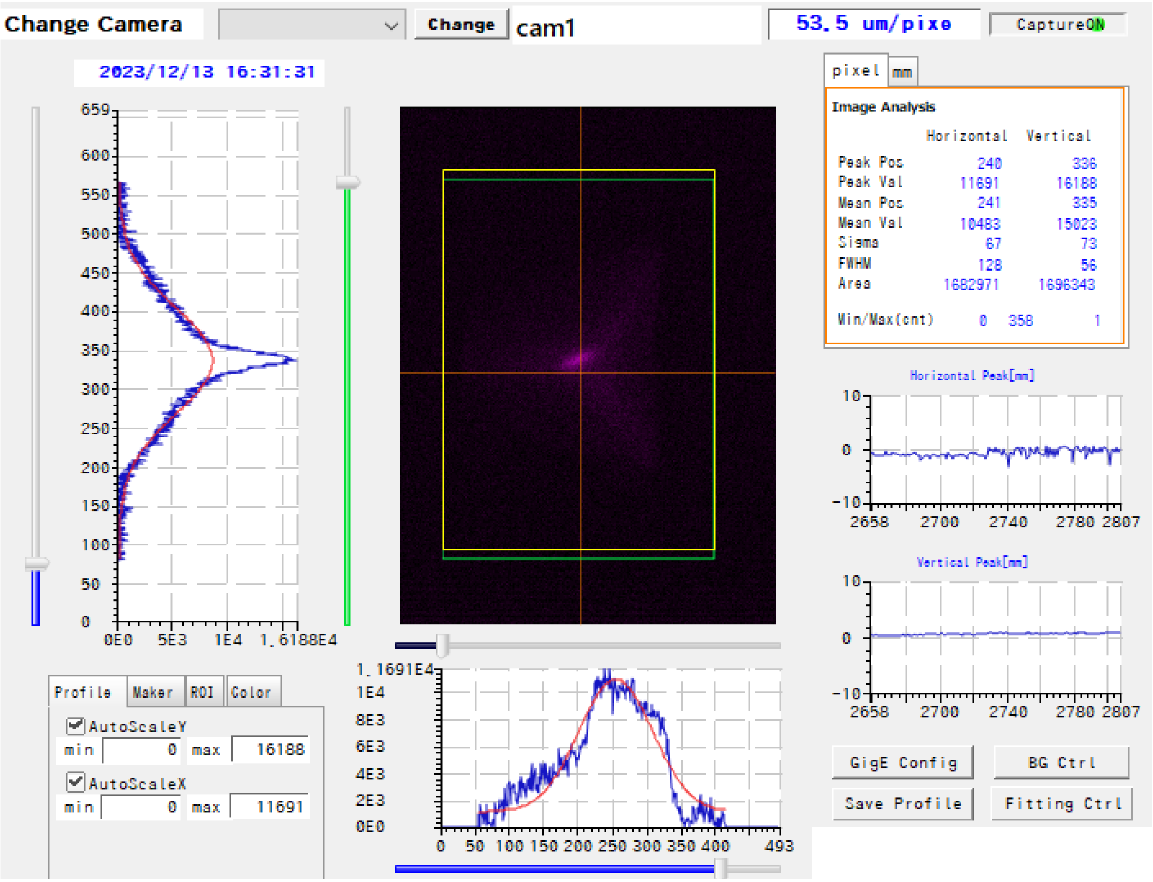


Comparison with Experimental Data (cERL)

Electron Beam on a Viewscreen

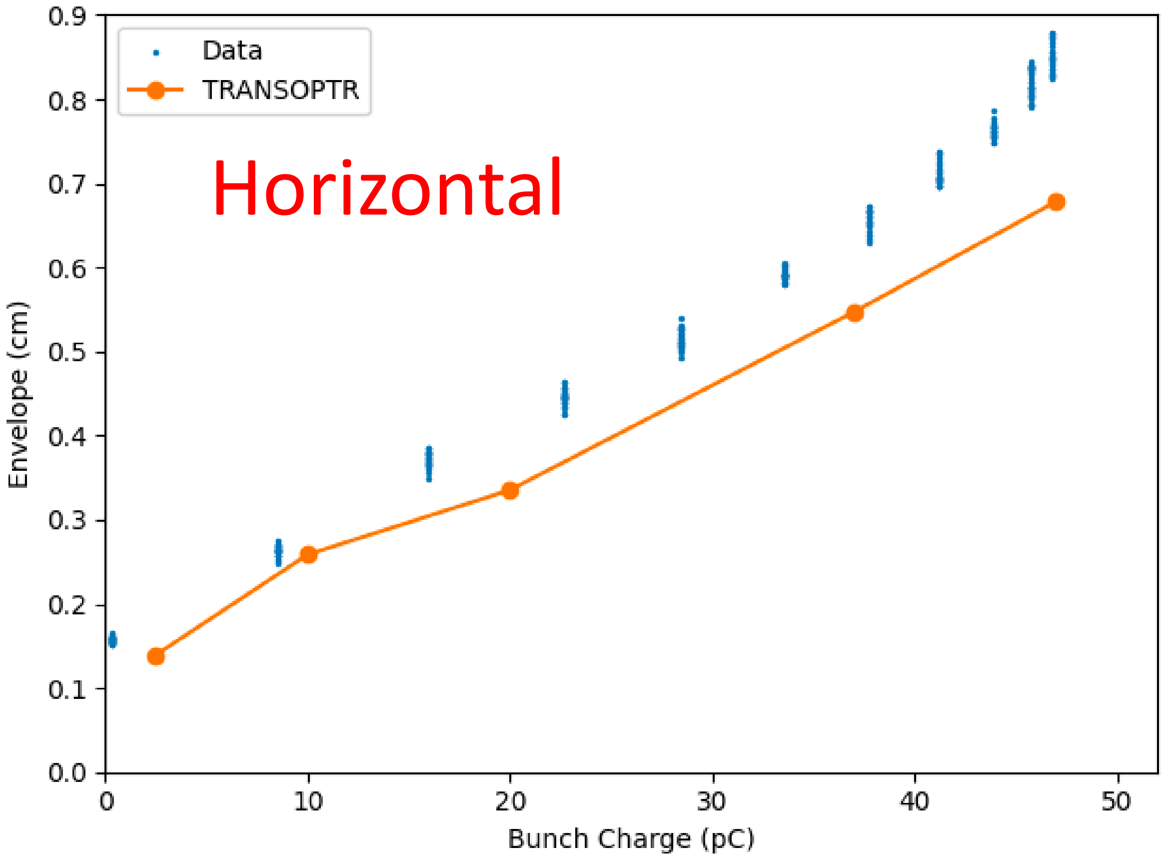


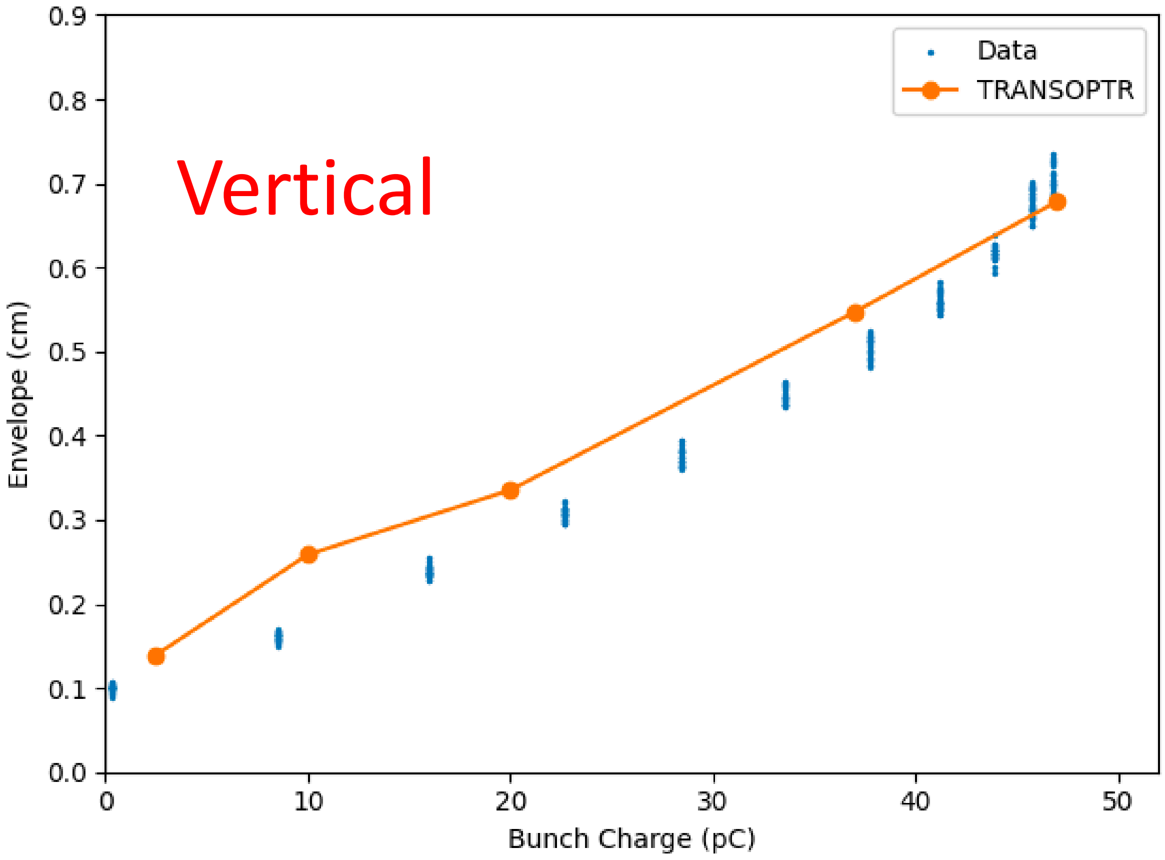
Low Space Charge Beam
(<5 pC)



High Space Charge Beam
(>50 pC)

Comparison with Experimental Data





Conclusions

- Envelope tracking produces results that are compatible to multi-particle tracking code at a speed of more than 2 order of magnitudes faster
- The experimental data agrees with the envelope tracking within $\pm 20\%$.
- The rapid turnaround using an envelope code can assist swift beam tuning and beamline design in early phase.

Acknowledgement

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Thank you!