Phasing and Calibration of the Main Linac Cryomodule Cavities for the CBETA Energy Recover Linac

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Phasing/Calibration Goal

- In our model, each cavity has certain voltage and phase settings
- In the control system, we set voltage and phase values
- Goal: make the voltage/phase settings set in the control system correspond to what we set in the model

The CBETA MLC Linac

- Six 1.3 GHz cavities
- Beam injected at 6 MeV
- 36 MeV design energy gain, about 6 MeV/cavity
- BPMs at the entrance and exit of the linac
    - Capable of precise beam arrival measurement relative to 1.3 GHz reference

Method Overview

- Measure time of flight
- BPMs at entrance and exit of linac
- One cavity on at a time
- Voltage set to a significant fraction of beam energy
- Swing phase through 360°
- Fit measurements to model
  - Scaling factor for voltage
  - Offset for phase

Model: Cavity

- On-axis cavity field from finite-element cavity model

$$\frac{dE}{ds} = qVG(s)\cos(\omega t + \phi)$$

- Normalize $G(s)$ for $v = c$ particle:

$$\int G(s)\cos\left(\frac{\omega s}{c} + \phi\right) ds = \cos(\phi + \phi_c)$$

- Define phase ϕ_0 based on 6 MeV constant-velocity particle

$$\int G(s)\cos\left(\frac{\omega s}{\beta_0 c} + \phi + \phi_0\right) ds = A\cos\phi$$

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Model: Linac

- In our accelerator model, keep the same reference energy for timing through the entire linac for each pass
  - Design energy at the entrance to the linac for that pass
  - Reference timing doesn't change when we change machine settings
  - Calibration done here applies directly to model
- Components at known longitudinal positions
  - $s_1 \ldots s_6$ for cavities (relative to $s = 0$ in cavity model)
  - s_0 and s_7 for entrance and exit BPMs

Model: Arrival Time Difference

- Integrate ODEs within cavity from s_a to s_b, with $E(s_a) = E_0$ and $t(s_a) = s_a/\beta_0 c$:

$$\frac{dt}{ds} = \frac{E}{c\sqrt{E^2 - (mc^2)^2}}$$

$$\frac{dE}{ds} = qVG(s)\cos(\omega t + \phi + \phi_0)$$

- Result is $t(s_b) = t_m(V, \phi, E_0)$ and $E(s_b) = E_m(V, \phi, E_0)$

Model: Arrival Time Difference

- Arrival time difference between accelerated particle for cavity at s and particle with cavities off

$$T(V, \phi, E_0, s) = t_m(V, \phi, E_0) + \frac{(s_7 - s - s_b)E_m(V, \phi, E_0)}{c\sqrt{E_m(V, \phi, E_0)^2 - (mc^2)^2}} + \frac{(s_7 - s - s_a)E_0}{c\sqrt{(E_0)^2 + (mc^2)^2}}$$

The Measurement and Fit

- Measure arrival time difference between entrance and exit BPMs
  - Mathematically unnecessary, but removes noise
- Make a high precision measurement of time T_0 with cavities off
- Turn cavity i on, scan evenly spaced points in phase setting ψ_k, possibly at more than one voltage setting V_l, measure times T_{kl}
- Find phase offset ϕ_i, scaling factor λ_i, and incoming energy E_0 that minimize (fit all cavities together to get common E_0)

$$\sum_{ikl} [T_{kl} - T_0 - T(\lambda_i V_l, \phi_i + \psi_k, E_0, s_i)]^2$$

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Fast, Precise, and Robust Calculation: Newton's Method

- General least-squares problem like ours:

$$f(\vec{p}) = \sum_j \left[T_j(\vec{p}) - t_j\right]^2$$

- Minimum occurs when

$$0 = \frac{\partial f}{\partial \vec{p}} = \sum_j \frac{\partial T_j}{\partial \vec{p}} \left[T_j(\vec{p}) - t_j\right]$$

- Apply Newton's method to this: very fast and robust, will also need $\partial^2 f/\partial p_i \partial p_j$ and therefore $\partial^2 T_k/\partial p_i \partial p_j$, plus a good initial guess

Computing Derivatives

- Only nontrivial derivatives to compute are for t_m and E_m
- General differential equation for $\vec{z}(\vec{z}_0, \vec{p}, s)$, $\vec{z}(\vec{z}_0, \vec{p}, 0) = \vec{z}_0$:

$$\frac{d\vec{z}}{ds} = \vec{g}(\vec{z}(\vec{z}_0, \vec{p}, s), \vec{p}, s)$$

- Can write differential equations for derivative:

$$\frac{d}{ds}\frac{\partial z_i}{\partial p_j} = \sum_k \frac{\partial g_i}{\partial z_k}\frac{\partial z_k}{\partial p_j} + \frac{\partial g_i}{\partial p_j} \quad \frac{d}{ds}\frac{\partial z_i}{\partial z_{0j}} = \sum_k \frac{\partial g_i}{\partial z_k}\frac{\partial z_k}{\partial z_{0j}} \quad \left.\frac{\partial z_i}{\partial p_j}\right|_0 = 0 \quad \left.\frac{\partial z_i}{\partial z_{0j}}\right|_0 = \delta_{ij}$$

- Similar equations for second derivative
- Derivatives of $\vec{g}$ with respect to parameters are easy for this system
- Thus find all derivatives with coupled ODEs over a single cavity

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Initial Guess

- Use data points with maximum and minimum arrival times $T_{\pm}$
- Guess phase offset ϕ_i assuming these are trough/crest
- For λ_i and E_0, assume all energy gained at cavity center, solve

$$T_{\pm} - T_0 \approx \frac{(s_7 - s_i)(E_0 \pm \lambda_i V_i)}{c\sqrt{(E_0 \pm \lambda_i V_i)^2 - (mc^2)^2}} - \frac{(s_7 - s_i)E_0}{c\sqrt{(E_0)^2 - (mc^2)^2}}$$

- Guess: expand to second order in λ_i, solving for E_0 and λ_i

$$\frac{c(T_{\pm} - T_0)}{s_7 - s_i} \approx \pm \frac{\lambda_i V_i (mc^2)^2}{[(E_0)^2 - (mc^2)^2]^{3/2}} + \frac{3E_0 (mc^2)^2 \lambda_i^2 V_i^2}{[(E_0)^2 - (mc^2)^2]^{5/2}}$$

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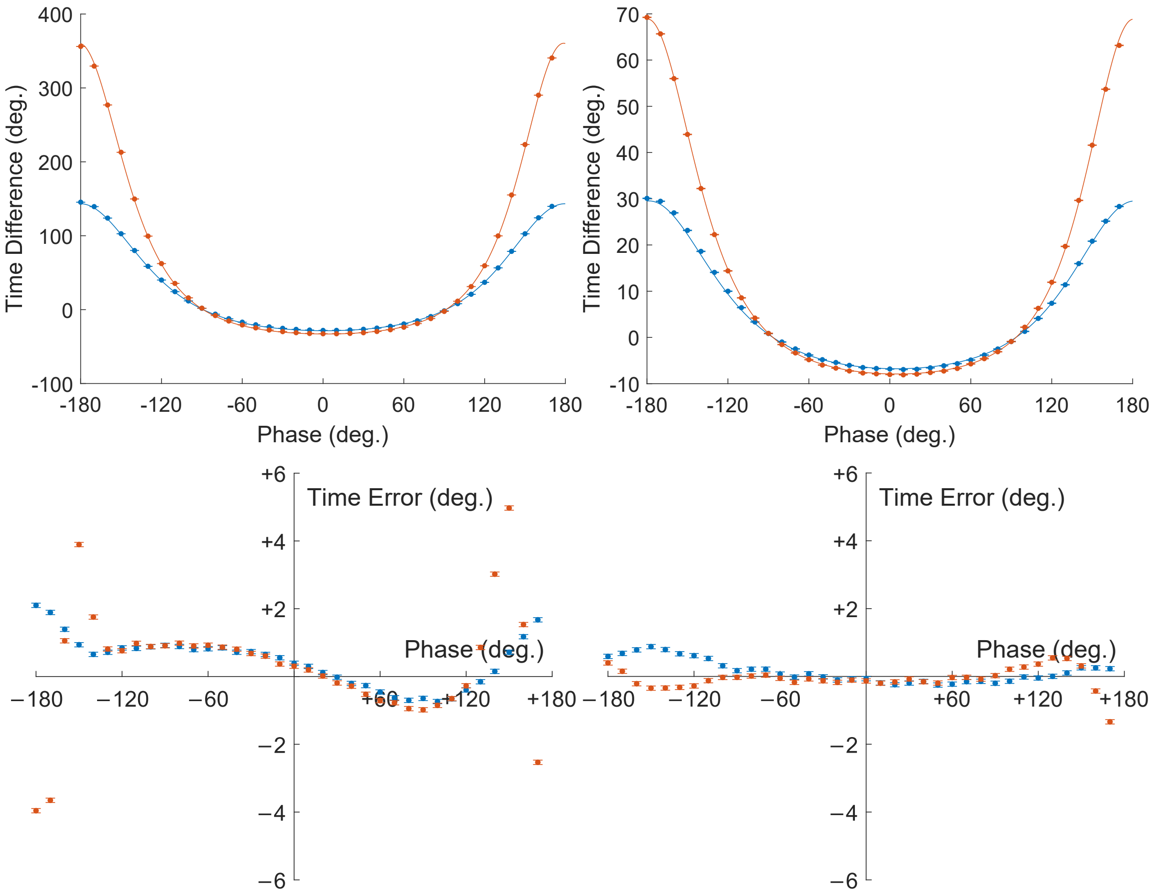
Sample Run

- 10 degree steps, 3000 kV and 4000 kV settings
- First and last cavity shown
- Errors dominated by systematics: model, measurement
- Uncertainties computable: phase uncertainties between 0.04° and 0.21°, calibration from 0.17% to 0.50%, E_0 6.6 keV

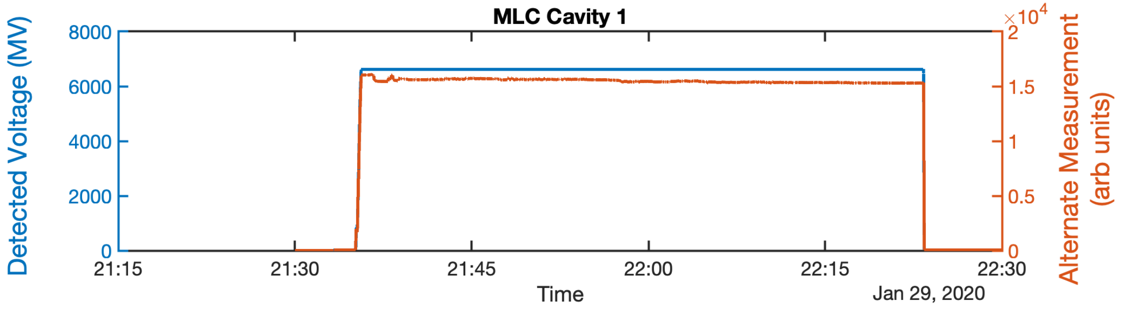


Practical Details

- Beam going off-axis messes up time of flight measurement, but difficult to quantify exactly how
  - We measured the MLC misalignment and steered the beam to the center
- The method did not work well for our injector cryomodule
  - Steering by cavities was severe
  - Low energy makes beam transmission for all phases difficult
- In practice used 30° steps and one voltage, only calibrating phases and checking injection energy
  - RF phases relative to gun reset on each machine start/trip

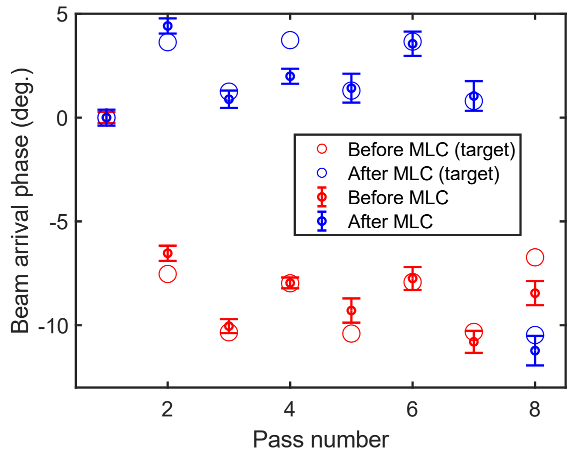
Practical Details

- Actual voltage assumed to be linear in control system voltage
- Voltage calibration *should* not be needed very often
- But we did have a slow drift in actual voltage, likely because our control readback did not accurately reflect the voltage
- Algorithm fast & robust: never failed unless data taking failed



Summary

- Created a method to quickly calibrate phases and voltages of our main linac cavities by scanning phases of cavities one at a time with injected beam
- Using a good cavity model, Newton's method, and a good initial guess, the computation runs very quickly and never failed
- Careful cavity phase calibration and RF modeling enabled 4-pass energy recovery in CBETA



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