2023/11/29@KEK Theory Workshop 2023

Photon sphere and quasinormal modes in AdS/CFT

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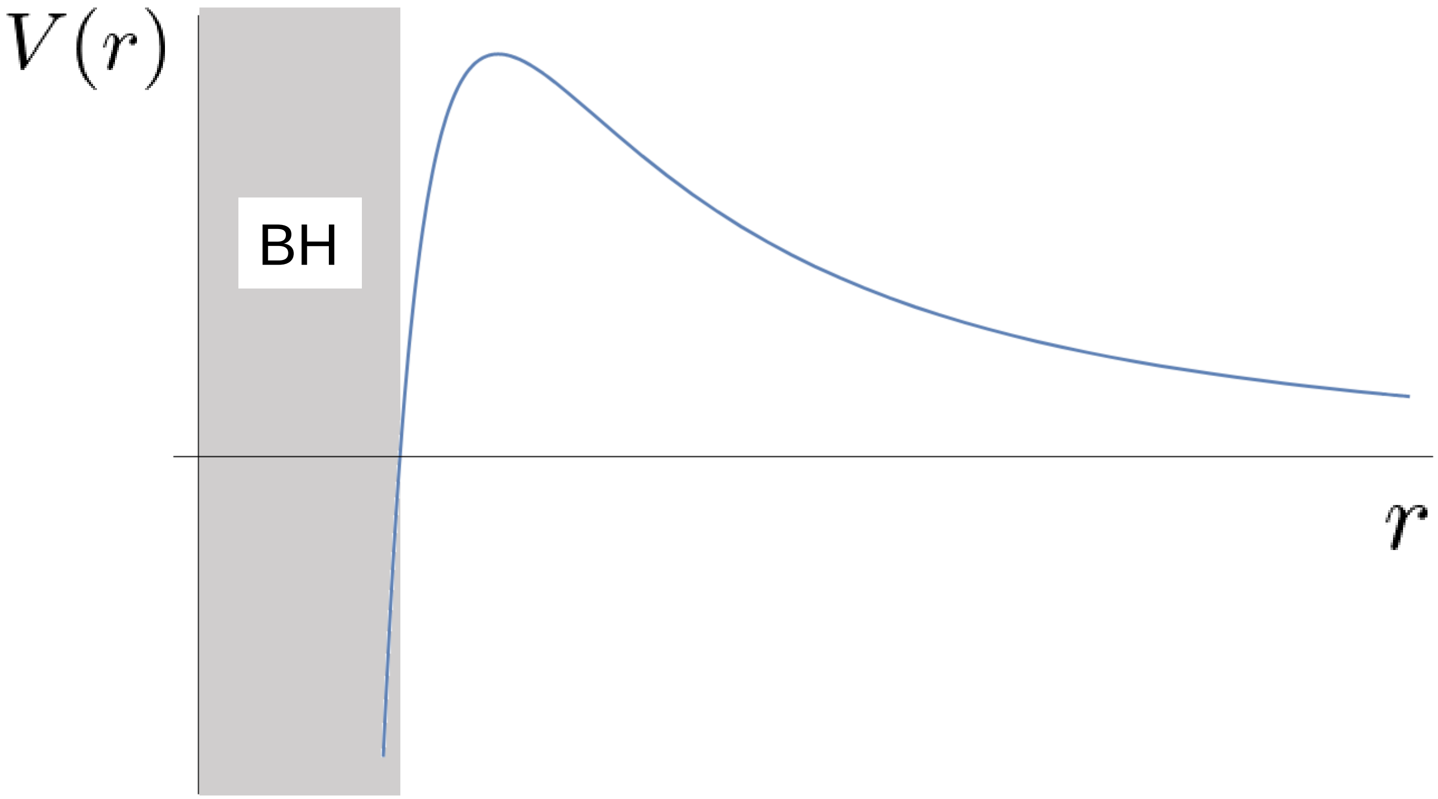
Based on arXiv:2307.00237

(with K. Hashimoto, K. Sugiyama, and T. Yoda)

Does a holographic CFT live on the photon sphere?



CFT does NOT live on the photon sphere in AdS

- In asymptotically flat BH… [Raffaelli '21] [Hadar, Kapec, Lupsasca, Strominger '22]

  Hidden conformal symmetry exists on the photon sphere

  → Photon sphere = Holographic screen?

- In asymptotically AdS BH… [Hashimoto, KS, Sugiyama, Yoda '22]
  - **The AdS boundary breaks the symmetry**
  - There is a **peculiar spectrum** near the photon sphere

    → A prediction for thermal holographic CFTs

Photon sphere in the asymptotically flat black hole

Schwarzschild BH $(d > 3)$

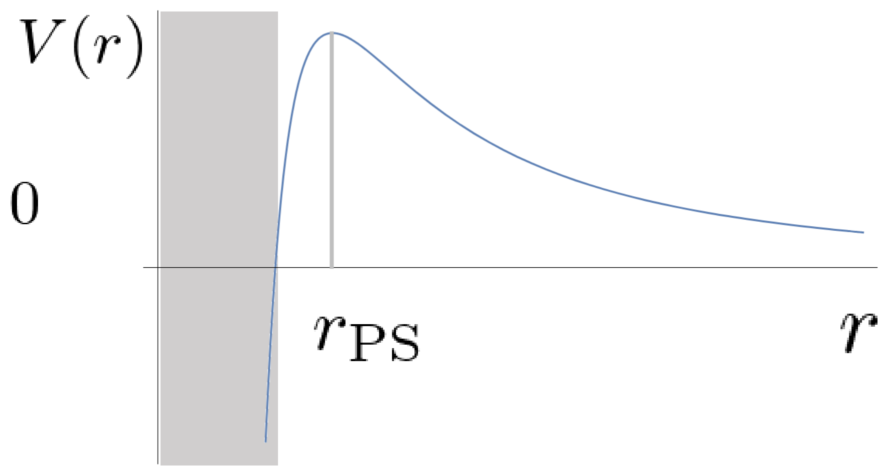
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \quad f(r) = 1 - \left(\frac{r_0}{r}\right)^{d-3}$$

Decompose the scalar field as $\Phi = e^{-i\Omega t} Y_{lm}(\text{angles})\, \psi(r)\, r^{1-d/2}$

→ the EOM becomes

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} + \Omega^2 - V(r)\right)\psi(r) = 0$$

$$r_* := \int_\infty^r \frac{dr'}{f(r')}$$



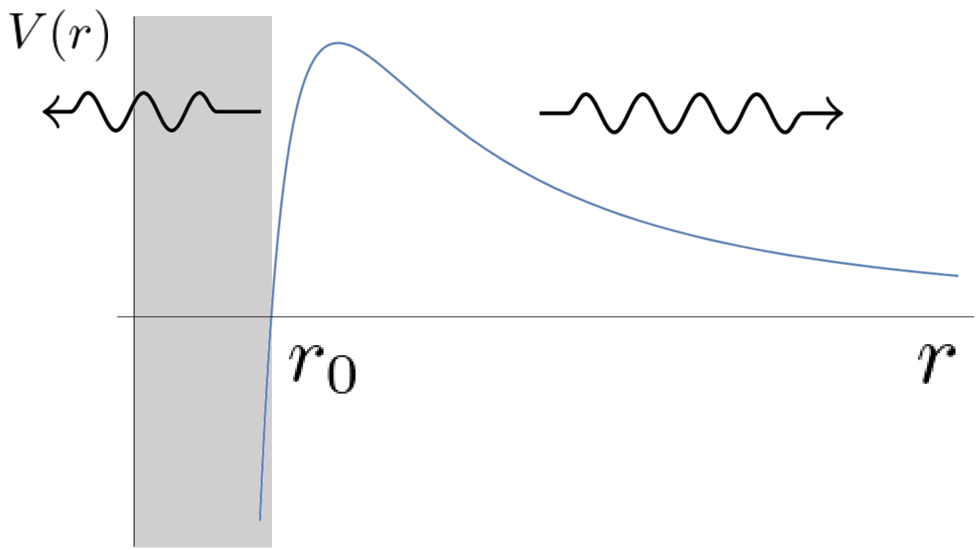
$$V(r) = f(r)\left(\frac{(l + d/2 - 1)(l + d/2 - 2)}{r^2} + \left(\frac{d}{2} - 1\right)^2 \frac{r_0^{d-3}}{r^{d-1}} + \mu^2\right)$$

QNM in the asymptotically flat black hole

<u>Quasinormal modes</u> (QNM)：damping of matter fields on BH

…solutions of the EOM $\left(\frac{\mathrm{d}^2}{\mathrm{d}r_*^2}+\Omega^2-V(r)\right)\psi(r)=0$ with the boundary condition

$$\begin{cases}\psi(r\to+\infty) \\ \qquad \sim \text{(pure outgoing)} \\ \psi(r=r_0) \\ \qquad \sim \text{(pure ingoing)}\end{cases}$$



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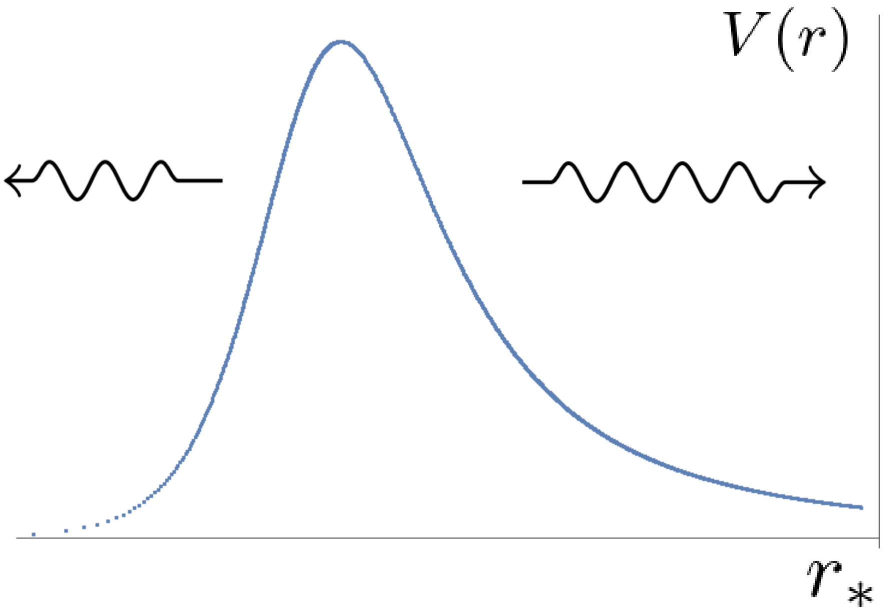
…solutions of the EOM $\left(\frac{\mathrm{d}^2}{\mathrm{d}r_*^2}+\Omega^2-V(r)\right)\psi(r)=0$ with the boundary condition

$$
\begin{cases}
\psi(r_*=0) \\
\qquad \sim \text{(pure outgoing)} \\
\psi(r_*\to-\infty) \\
\qquad \sim \text{(pure ingoing)}
\end{cases}
$$



Derive a QM model with large l approximation

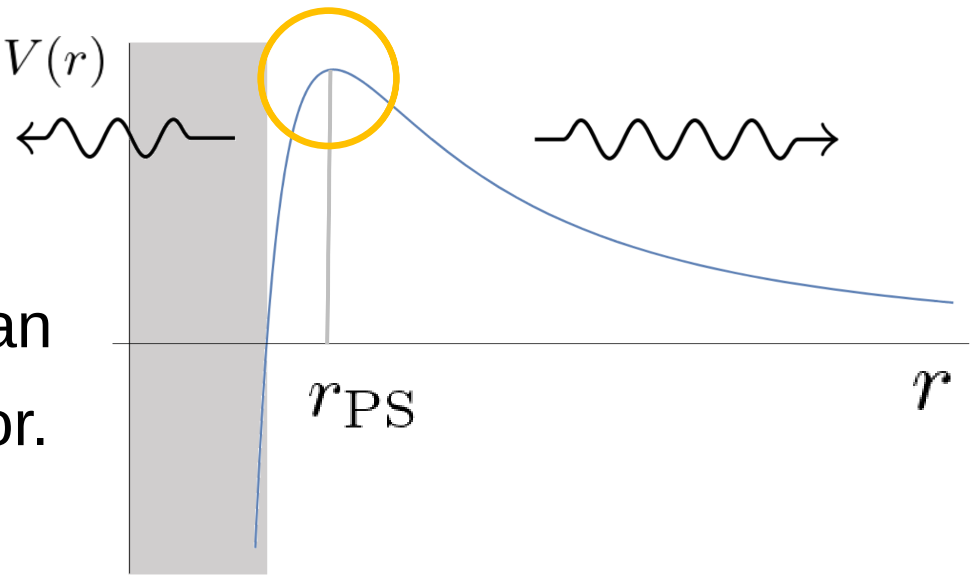
"near-ring region"

[Hadar, Kapec, Lupsasca, Strominger '22]

$$\begin{cases} |r - r_{\rm PS}| \ll r_0 & \text{(near-peak)} \\ \left| \dfrac{l}{\Omega_R} - \sqrt{\dfrac{d-1}{d-3}}\, r_{\rm PS} \right| \ll r_0 & \text{(near-critical)} \\ \Omega_R^{-1} \ll r_0 & \text{(high-frequency)} \end{cases}$$

Focus on the spectrum **near the photon sphere**

→ $V(r)$ is approximated by an inverted harmonic oscillator.



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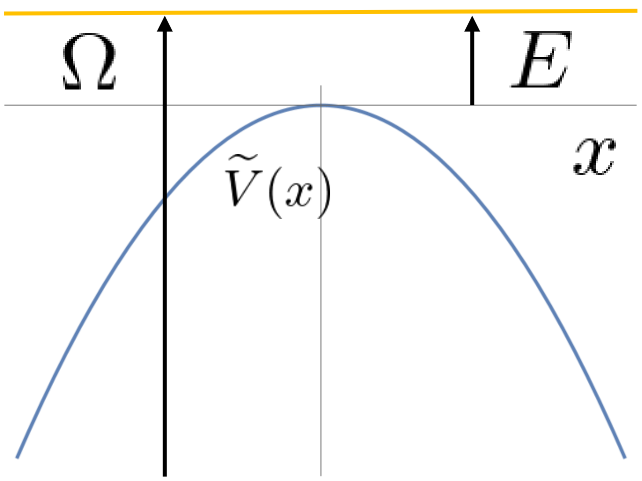
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} + \Omega^2 - V(r) \right) \psi(r) = 0$$

$$x \propto r_* - (r_*)_{\rm PS}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + E - \tilde{V}(x) \right) \psi(x) = 0$$

where $\tilde{V}(x \simeq 0) \simeq -\dfrac{x^2}{4}$, $\quad \Omega \sim \sqrt{V(r_{\rm PS}) + E + \mathcal{O}(l^{-1})}$

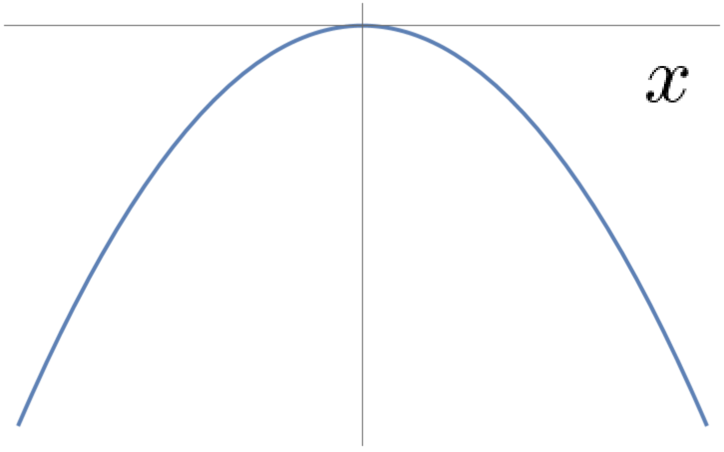


Dynamical conformal sym. near the photon sphere

$$J_1 = \frac{1}{2}\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{x^2}{4}\right) \equiv H_{\mathrm{iho}}$$: QM Hamiltonian of an inverted harmonic oscillator

$$J_2 = -\frac{i}{2}\left(x\frac{\mathrm{d}}{\mathrm{d}x} + \frac{1}{2}\right), \qquad J_3 = \frac{1}{2}\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{x^2}{4}\right)$$

$$\Longrightarrow \begin{cases} [J_1, J_2] = -iJ_3, \\ [J_2, J_3] = iJ_1, \\ [J_3, J_1] = iJ_2 \end{cases}$$



$\mathrm{SU}(1,1) \simeq \mathrm{SL}(2,\mathbb{R})$ alg.

Conformal symmetry breaks in AdS black hole

AdS-Schwarzschild BH $(d > 3)$

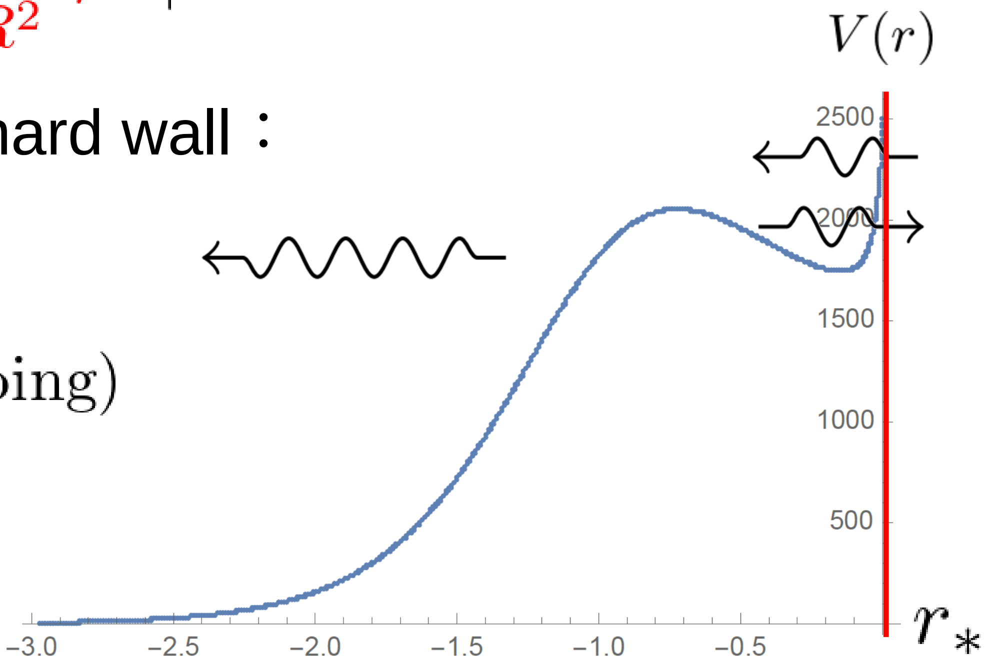
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \quad f(r) = 1 - \left(\frac{r_0}{r}\right)^{d-3} + \frac{r^2}{R^2}$$

$$\longrightarrow \quad V(r) = V_{\text{flat}}(r) + \frac{\mu_{\text{eff}}^2}{R^2} r^2 + \cdots$$

AdS boundary works as a hard wall：

$$\begin{cases} \psi(r_* = 0) = 0 \\ \psi(r_* \to -\infty) \sim (\text{pure ingoing}) \end{cases}$$

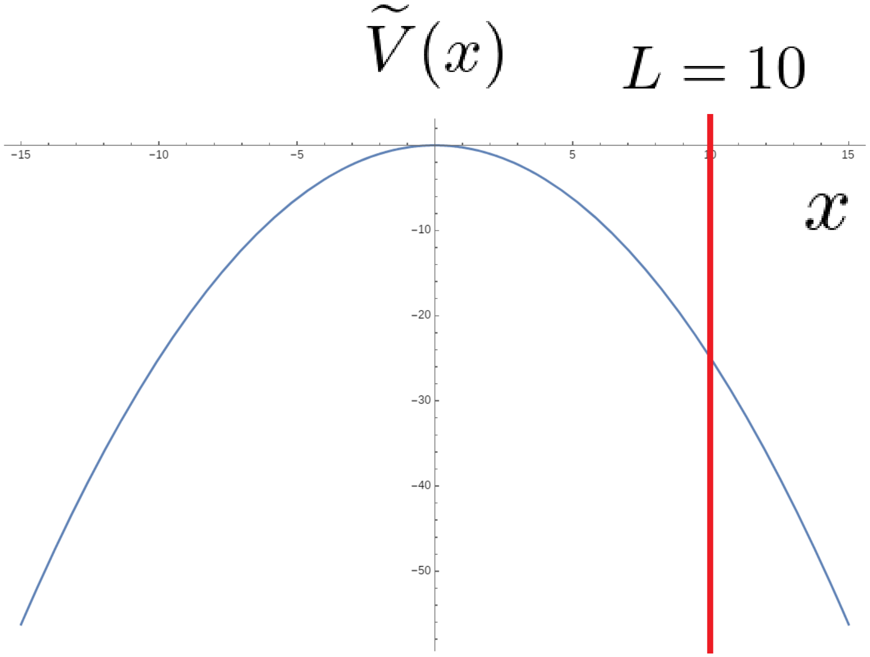
$\to \mathrm{SL}(2, \mathbb{R})$ is broken!

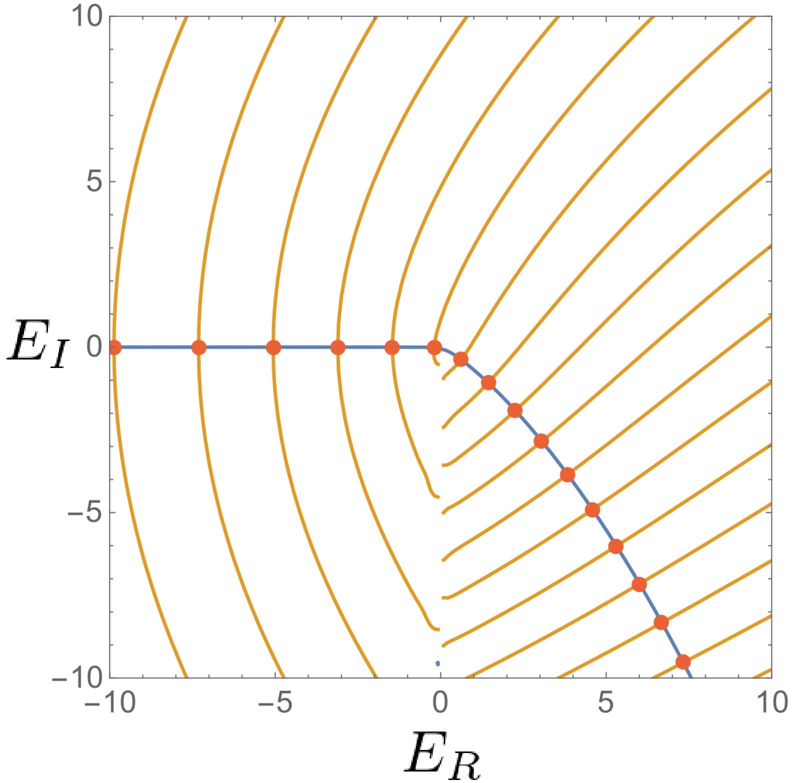


QNM spectrum of the AdS BH from a QM model

Model 1：Inverted harmonic oscillator $\quad \widetilde{V}(x) = -\frac{x^2}{4}$

＋ a Dirichlet boundary



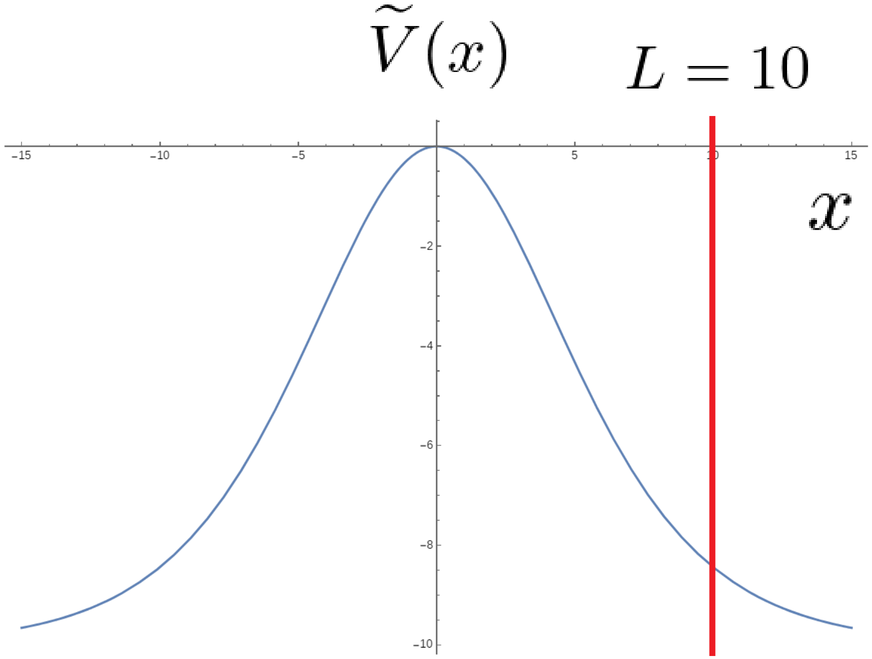


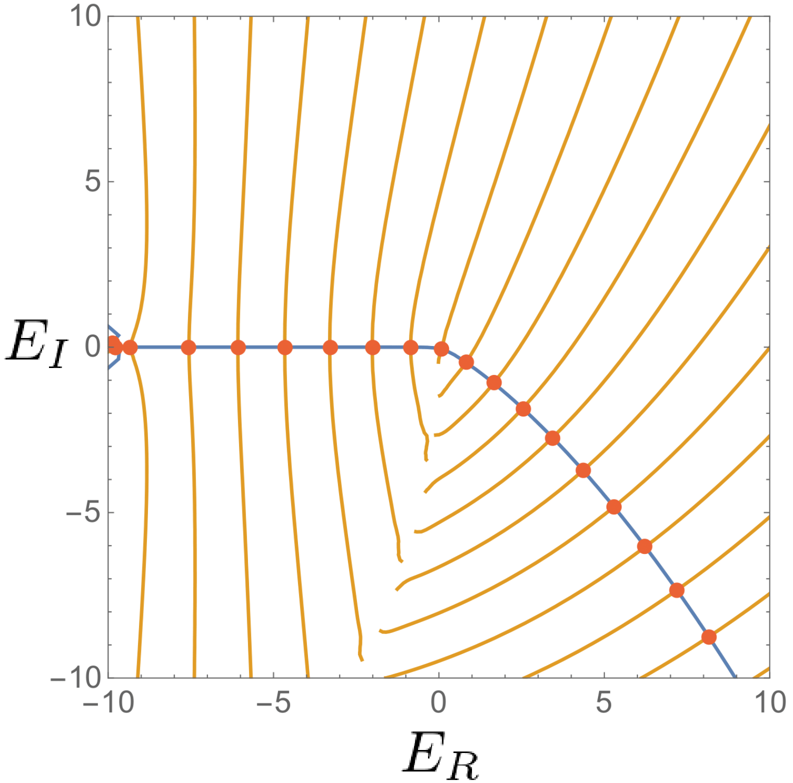
QNM spectrum is insensitive to the potential tail

Model 2：Pöschl-Teller potential

＋ a Dirichlet boundary

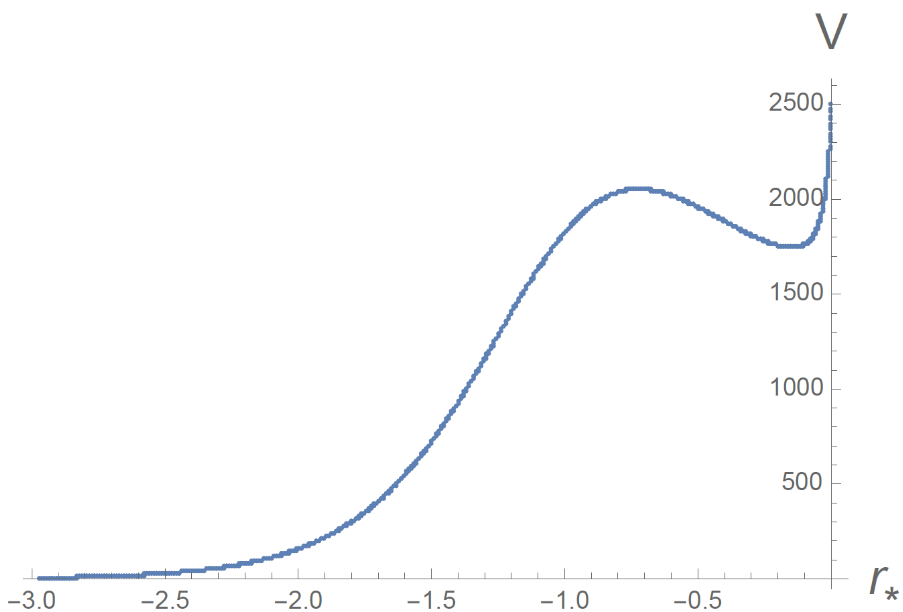
$$\widetilde{V}(x) = 10\left(\frac{1}{\cosh^2\left(x/2\sqrt{10}\right)} - 1\right)$$

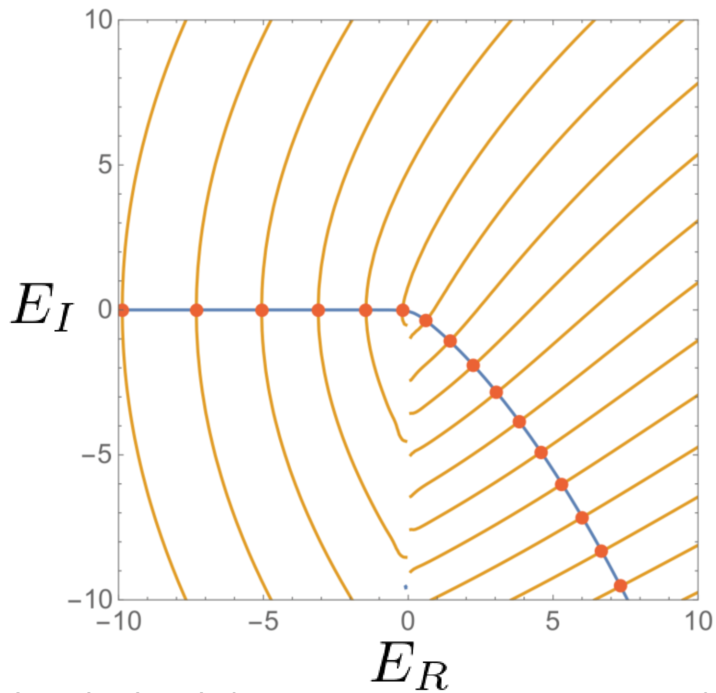




"Photon-sphere subsector" in holographic CFTs

<u>Prediction</u>：In **thermal holographic CFTs** on a sphere, there exists the **large angular momentum** spectrum with the **universal** pattern below：





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Backups

"Photon-sphere subsector" in holographic CFTs

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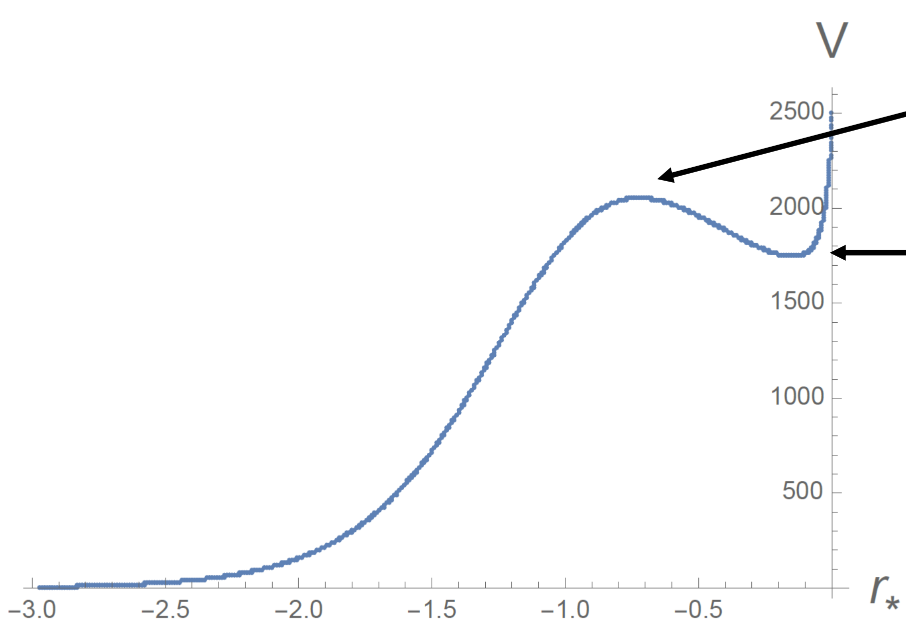


$$\Omega_R \sim l\sqrt{\frac{1}{R^2}+\frac{1}{r_0^2}}$$

$$\Omega_R \sim \frac{l}{R}$$

$$\mathrm{tr}[F\overbrace{D_\theta D_\theta \cdots D_\theta D_\theta}^{l} F]$$

c.f.) [Festuccia, Liu '08]

Quasinormal modes as descendants

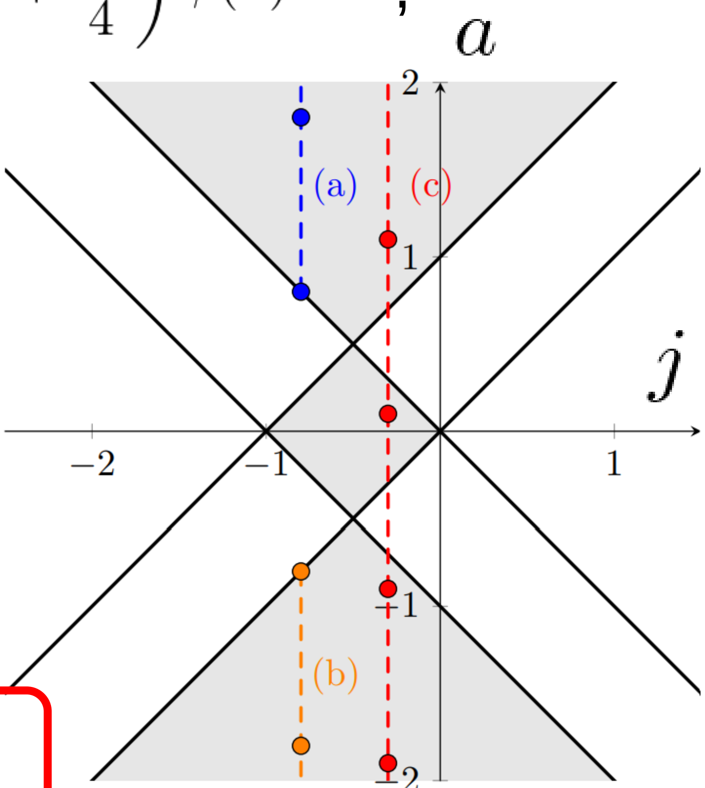
From the Schrödinger eq. $\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + E + \frac{x^2}{4}\right)\psi(x) = 0$,

$$H_{\mathrm{iho}}\psi \simeq -i\frac{E_I}{2}\psi \qquad (\because \ E_R \simeq 0)$$

$$\longrightarrow \boxed{\hat{J}_3\psi \simeq -\frac{\Omega_I}{2b}\psi}$$

QNMs should have $\Omega_I < 0$ and decay. c.f.) $\Phi \propto e^{-i\Omega t}\psi$

→ They should be in the discrete rep. (a) of $\mathrm{SU}(1,1)$.



$$\hat{J}_3\psi = a\psi$$
$$\hat{\boldsymbol{J}}^2\psi = -j(j+1)\psi$$

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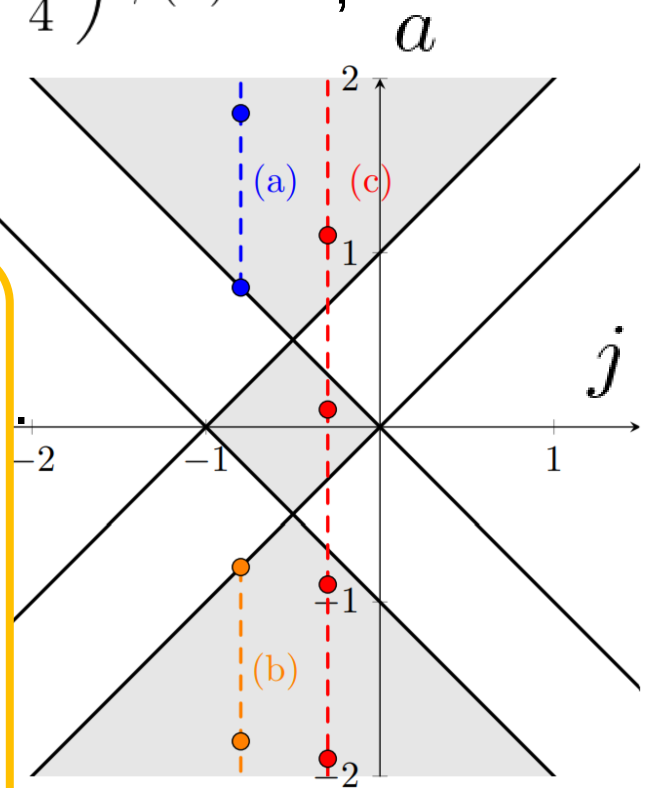
$$\hat{J}_3\psi \simeq -\frac{\Omega_I}{2b}\psi$$

- The ground state is the highest-weight mode

$$\hat{J}_-\Phi_j = 0 \qquad \left(j=\frac{1}{4},\frac{3}{4}\right)$$

- Higher overtones are obtained as $\mathrm{SU}(1,1)$ descendants $\hat{J}_+^N\Phi_j$.

[Hadar, Kapec, Lupsasca, Strominger '22]



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Solving a QM model respecting AdS boundary

Model 1：Inverted harmonic oscillator $\widetilde{V}(x) = -\dfrac{x^2}{4}$

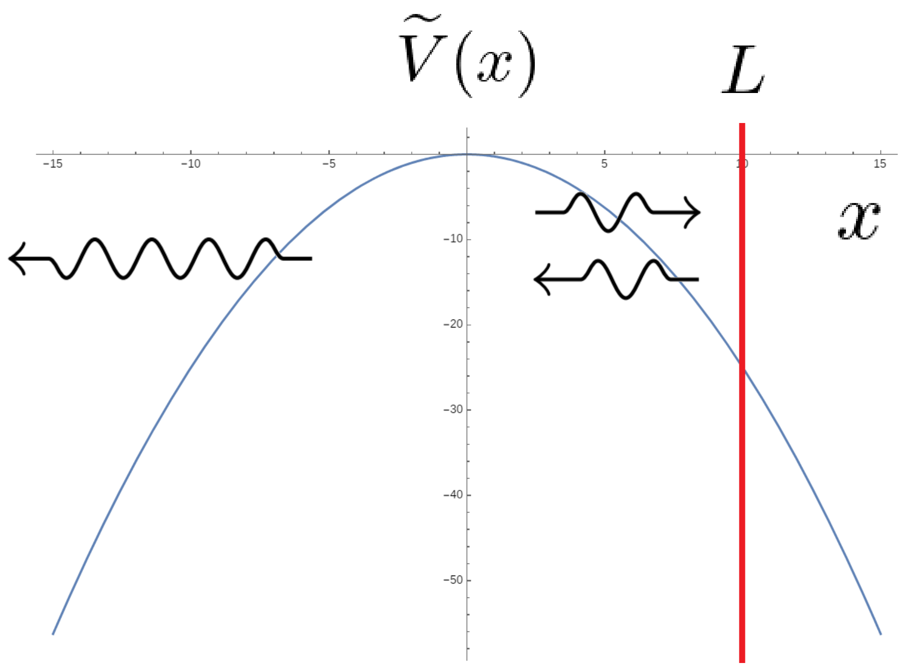
＋ a Dirichlet boundary

EOM

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + E + \frac{x^2}{4}\right)\psi(x) = 0$$

Boundary condition

$$\begin{cases} \psi(x = L \gg 1) = 0 \\ \psi(x \to -\infty) \\ \qquad \sim \text{(pure ingoing)} \end{cases}$$



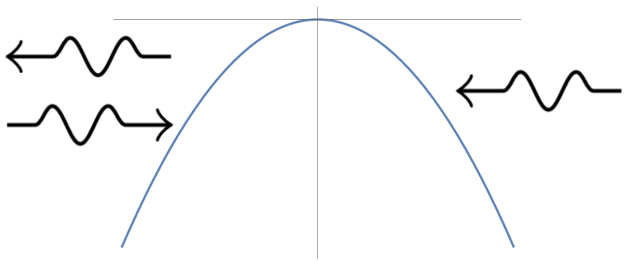
Boundary condition at the BH horizon

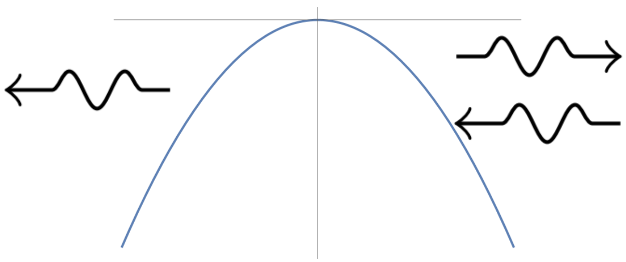
The general form of the solution is

$$z := e^{i\pi/4}x$$

$$\psi = A\,D_\nu(z) + B\,D_{-\nu-1}(iz) \qquad E =: i\left(\nu + \frac{1}{2}\right)$$

where $D_\nu(z)$ is the parabolic cylinder function.

$D_\nu(z) =$ 

$D_{-\nu-1}(iz) =$ 

→ In this case, $\psi = B\,D_{-\nu-1}(iz)$

Boundary condition at the AdS boundary

Near the AdS boundary $x \sim L \gg 1$,

$$\psi \sim e^{-(iz)^2/4}(iz)^{-\nu-1}\left(1 - e^{i\vartheta_+(-\nu-1;iz)}\right)$$

where $\vartheta_+(\nu;z) := -\frac{iz^2}{2} + i(2\nu+1)\ln z + \pi\nu - i\ln\frac{\sqrt{2\pi}}{\Gamma(-\nu)}$.

→ The Dirichlet boundary condition $\psi(x=L)=0$ yields a quantization condition

$z \equiv e^{i\pi/4}x$

$$\vartheta_+(-\nu-1; ie^{i\pi/4}L) \in 2\pi\mathbb{Z}$$

Future works

- To explain the "photon-sphere subsector" on the CFT side

  …Why and when does such robust stability appear?

- A characteristic of spacetime-emergent materials

  …An experimental verification of holography?

- The QNMs define the Lyapunov exponent of chaos

  …Could the chaos bound work as a bound for the QNM?