Operator dynamics in Lindbladian SYK

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KEK Theory workshop 2023, Tsukuba

Based on 1) arXiv: 2212.06180 (JHEP) with B. Bhattacharjee (IBS), X. Cao (ENS) and T. Pathak (IISc)
2) arXiv: 2311.00753 with B. Bhattacharjee (IBS) and T. Pathak (IISc)

Operator growth

Consider any Hamiltonian H. Start with a initial simple operator, say O_0 . Under the time evolution, the simple operator becomes a complicated operator $O(t) = e^{iHt} O_0 e^{-iHt}$.

$$O(t) = O_0 - it[H, O_0] - \frac{t^2}{2!}[H,[H,O_0]] + \frac{it^3}{3!}[H,[H,[H,O_0]]] + \cdots = O_0 - it\mathscr{L} O_0 - \frac{t^2}{2!}\mathscr{L}^2 O_0 + \frac{it^3}{3!}\mathscr{L}^3 O_0 \cdots = e^{i\mathscr{L}t} O_0 .$$

The time evolution is expanded on a basis of nested commutators.

Liouvillian $\mathscr{L} \cdot = [H, \cdot\,]$

Increasing support of many operators

$$\tilde{O}_n = \mathscr{L}^n O_0 , \quad n = 0,1,2, \cdots$$

The basis states may not be orthonormal. So we use a Gram-Schmidt (GS) orthonormalisation produces to produce **orthonormal basis (Krylov basis).**

$$\tilde{O}_n \xrightarrow{\text{GS}} \mathcal{O}_n \qquad \langle \mathcal{O}_m | \mathcal{O}_n \rangle = \delta_{mn}$$

Inner product: $\langle A | B \rangle = \frac{1}{D}\mathrm{Tr}(A^\dagger B)$

Inputs:

Hamiltonian H and the initial operator O_0

Lanczos algorithm

Outputs:

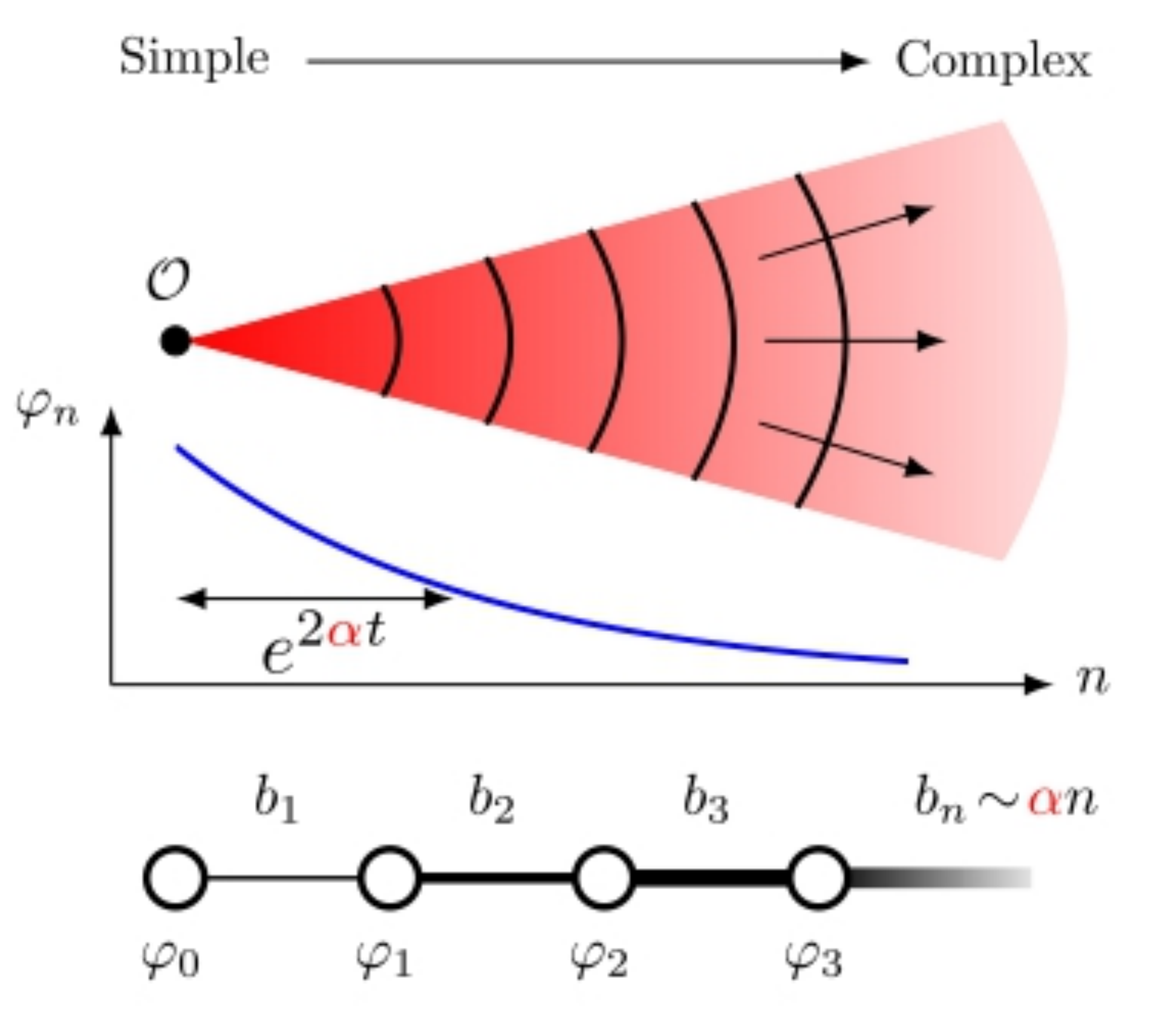
Lanczos coefficients and the Krylov basis $\{\mathcal{O}_n\}$

Universal operator growth hypothesis

Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

"For chaotic systems, the Lanczos coefficients grow linearly, and this is the maximum growth possible" $b_n \sim \alpha\, n\,.$

The time evolution is in Krylov basis: $|O(t)\rangle = \sum_n i^n \varphi_n(t) |\mathcal{O}_n\rangle$



The φ_n's satisfy the following equation: $\dot{\varphi}_n(t) = b_n \varphi_{n-1}(t) - b_{n+1}\varphi_{n+1}(t)$

K-complexity is the average location of the particle in Krylov chain: $K(t) := \sum_n n\, |\varphi_n(t)|^2$

For chaotic systems, K-complexity grows exponentially: $K(t) \sim e^{2\alpha t}$

What happens in open quantum systems?

We are interested in the Markovian dynamics, where he evolution of any operator is governed by the adjoint of Lindbladian

$$\mathcal{O}(t) = e^{i\mathcal{L}_o^\dagger t}\,\mathcal{O}_0\,, \qquad \mathcal{L}_o^\dagger \mathcal{O} = [H, \mathcal{O}] - i\sum_k \left[\pm L_k^\dagger \mathcal{O} L_k - \frac{1}{2}\left\{L_k^\dagger L_k, \mathcal{O}\right\}\right].$$

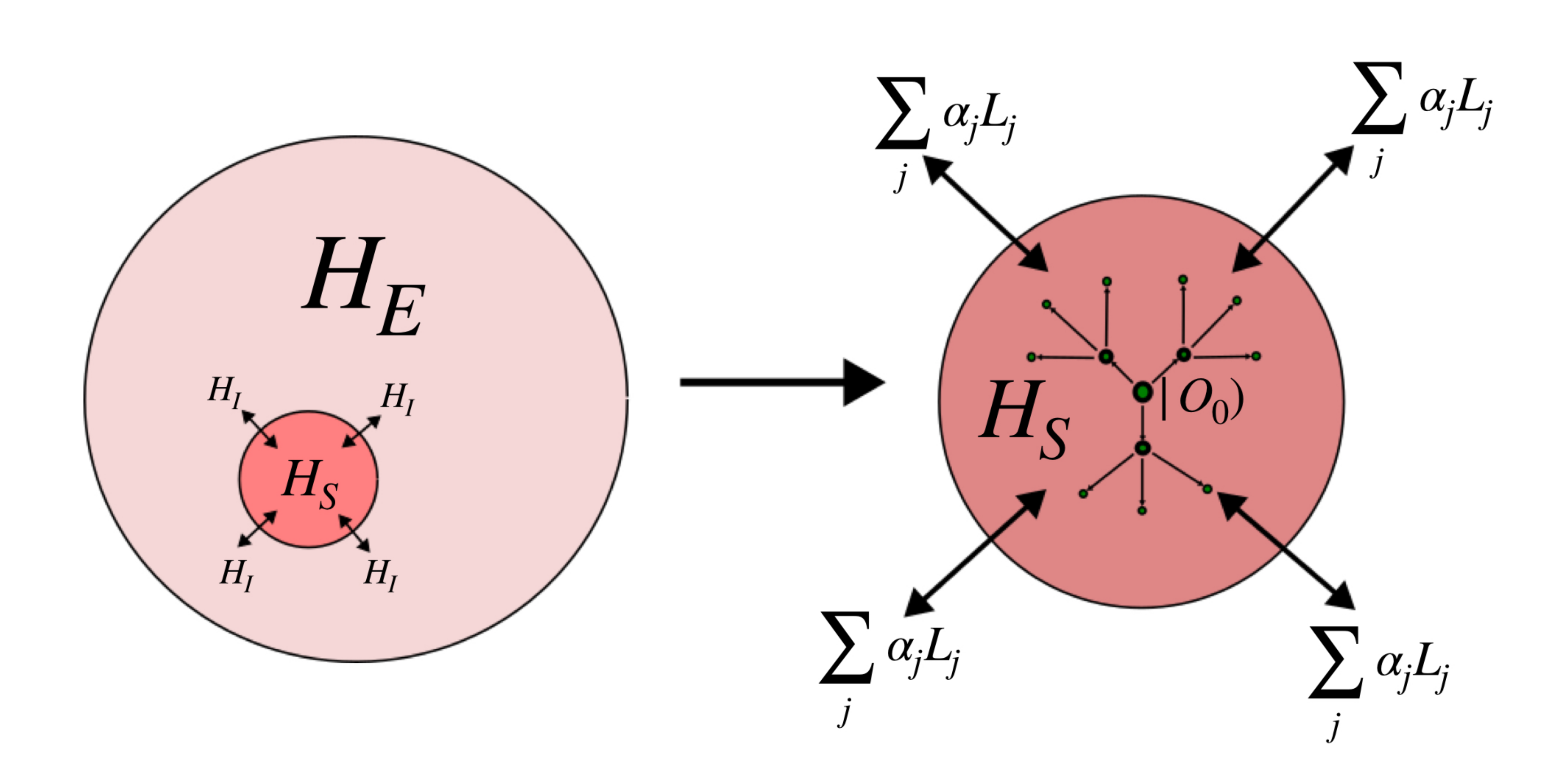
Lindblad (1976)

Gorini-Kossakowski-Sudarshan (1976)

The operators L_k are known as jump (Lindblad) operators and they encode the information between the system and the interaction.

We are mostly ignorant about the specific details of the environment.

Due to the non-unitary evolution, the Lanczos algorithm fails.



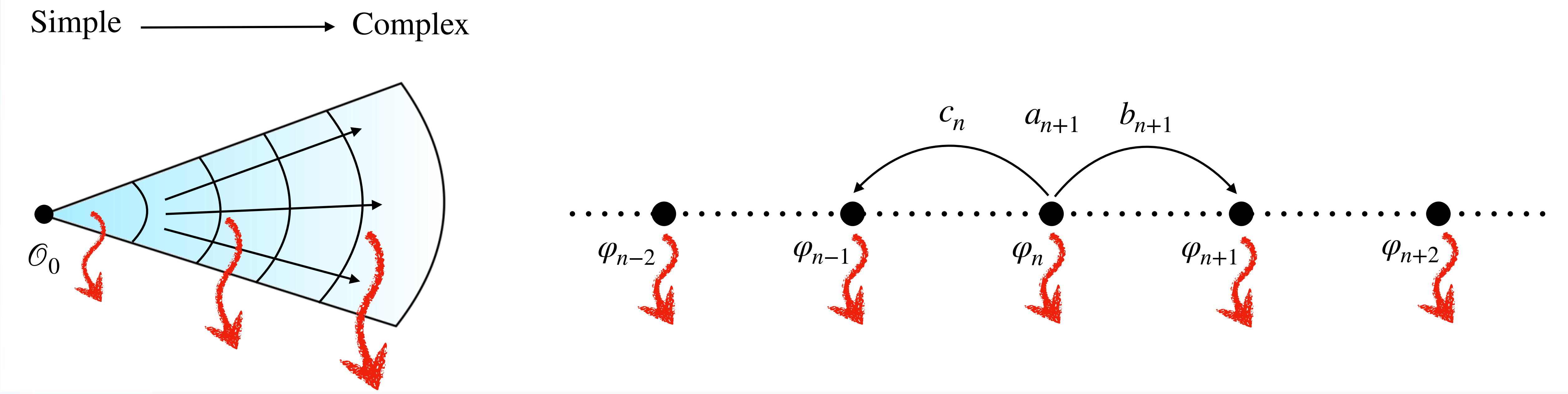
System is the SYK:

Sachdev-Ye (1993), Kitaev (2015)

$$H = i^{q/2} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} j_{i_1 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q} \quad \text{with} \quad \langle j_{i_1 \cdots i_q} \rangle = 0 \quad \text{and} \quad \langle j^2_{i_1 \cdots i_q} \rangle = 2^{q-1} \frac{(q-1)!\,\mathcal{J}^2}{qN^{q-1}}$$

Class 1: Linear jump operators:

$$L_k = \sqrt{\lambda}\, \psi_k\,, \qquad k = 1,2,\cdots,N\,.$$



Every fermions dissipate at equal rate. Mimics to a two-sided Keldysh wormhole.

Class 2: Random quadratic jump operators

$$L^a = \sum_{1 \le i \le j \le N} V^a_{ij} \psi_i \psi_j\,, \qquad a = 1,2,\cdots,M\,.$$

$$\langle V^a_{ij} \rangle = 0 \qquad \langle |V^a_{ij}|^2 \rangle = \frac{2V^2}{N^2} \quad \forall i,j,a$$

Kulkarni-Numasawa-Ryu (2021)
Sa-Ribeiro-Prosen (2021)

The action of the dissipative part of the Lindbladian results

$$\mathscr{L}_D^\dagger \mathcal{O}_n = i\zeta q R V^2 n\, \mathcal{O}_n\,,$$

$$a_n \sim iRV^2\, n\,.$$

where $\zeta \sim o(1)$. Similar expressions also hold for generic p-body dissipation.

We verify the result using bi-Lanczos algorithm, which generalises Lanczos algorithm for non-unitary evolution.

Construct to separate bi-orthonormal Krylov spaces

$$\mathbb{K}^j(\mathscr{L}_o^\dagger, |p_1\rangle) = \{\,|p_1\rangle, \mathscr{L}_o^\dagger\,|p_1\rangle, (\mathscr{L}_o^\dagger)^2\,|p_1\rangle, \ldots\}\,,$$
$$\mathbb{K}^j(\mathscr{L}_o, |q_1\rangle) = \{\,|q_1\rangle, \mathscr{L}_o\,|q_1\rangle, \mathscr{L}_o^2\,|q_1\rangle, \ldots\}\,.$$

$$\langle q_m | p_n\rangle = \delta_{mn}$$

Bhattacharya-**PN**-Nath, Sahu (2023)

Bhattacharjee-**PN**-Pathak (2023)

In this bi-orthonormal basis, the Lindbladian takes an “ideal” tridiagonal form

$$\mathscr{L}_o^\dagger = \begin{pmatrix} i\,|a_1| & b_1 & 0 & \cdots & 0 \\ b_1 & i\,|a_2| & b_2 & \cdots & 0 \\ 0 & b_2 & i\,|a_3| & b_3 & \cdots \\ \cdots & \cdots & b_3 & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \cdots \end{pmatrix}.$$

We propose an operator growth hypothesis in open systems based on the results of Sachdev-Ye-Kitaev (SYK) model.

Growth is completely determined by the asymptotic behavior of (two sets of) Lanczos coefficients.

$$a_n \sim i\chi\mu n\,, \quad b_n \sim \alpha n\,.$$

Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

Bhattacharjee-Cao-**PN-**T. Pathak (2022)

Bhattacharjee-**PN-**Pathak (2023)

Krylov complexity

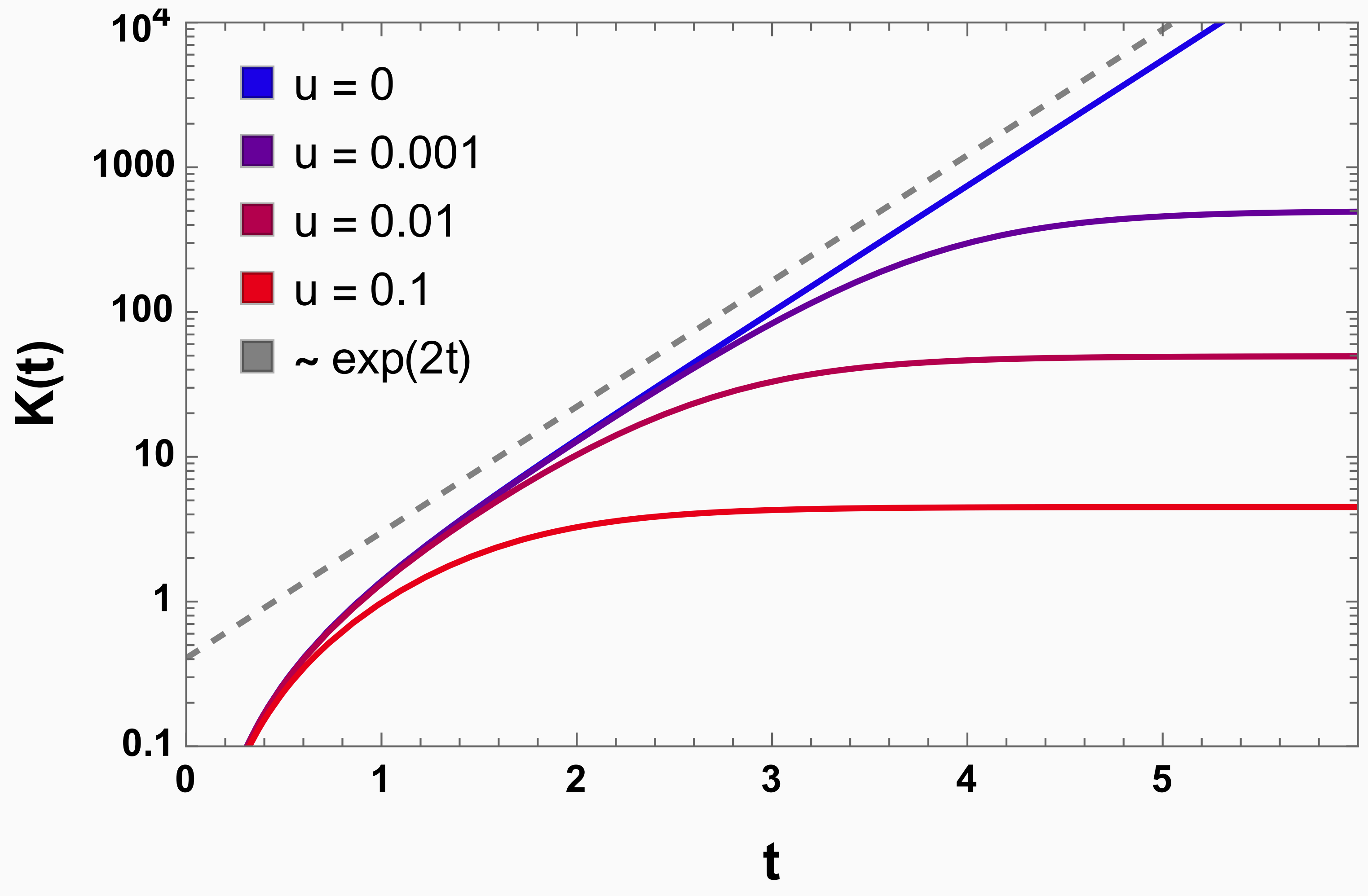
$$K(t) = \frac{\eta\left(1-u^2\right)\tanh^2(t)}{1+2u\tanh(t)-\left(1-2u^2\right)\tanh^2(t)}\,.$$

Weak dissipation limit

$$K(t) = \eta\left[\sinh^2(t) - 2u\sinh^3(t)\cosh(t) + O(u^2)\right]\,,$$

A systematic asymptotic analysis gives

$$K(t) \sim 1/u \qquad t_* \sim \ln(1/u)$$



Growth of the K-complexity in presence of dissipation.

Environment acts like an indirect probe which perform continuous measurement.

Outlook:

1. We motivate to understand “dissipative quantum chaos”.

2. We believe that the dissipative timescale and and the saturation is generic and robust for any dissipative chaotic systems. A valid question is to understand how this dissipative time scale is related to the scrambling lime.

3. Generalizing chaos bound for open quantum systems. What happens for non-Markovian evolution?

4. Is there any connection with the level statistics of Ginibre ensemble and the Krylov complexity?

5. Other interesting questions…

Thank you…

どうもありがとうございます