The effect of fermions on the emergence of (3+1)-dimensional expanding space-time in the Lorentzian type IIB matrix model

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Anagnostopoulos-Azuma-KH-Hirasawa-Nishimura-Papadoudis-Tsuchiya, work in progress

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1. Introduction

To obtain the 4D space-time from superstring theory, which is a promising candidate of the unified theory including quantum gravity, non-perturbative effects of superstring theory are considered to be important.

◆The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)]

A non-perturbative formulation of superstring theory.

In this model, space-time does not exist a priori but emerges dynamically from the degrees of freedom of matrices.

$$S = -N\,\mathrm{Tr}\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right)$$

$$= S_{\mathrm{b}} + S_{\mathrm{f}}\ ,\quad S_{\mathrm{b}} = -\frac{N}{4}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right),\quad S_{\mathrm{f}} = -\frac{N}{2}\mathrm{Tr}\left(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right)$$

Related talks by
Piensuk, Yamamori, Tripathi, and Asano

$(\Gamma^\mu)_{\alpha\beta}$: 10D Gamma matrices
$(\mu = 0, \ldots, 9; \alpha = 1, \ldots, 16)$

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector
Ψ_α : 10D Majorana-Weyl spinor

under SO(9,1) transformation

space-time is extracted from the eigenvalue distribution of A_μ.

Does real space-time emerge?

◆The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)]

Partition function: $Z = \int dA d\Psi e^{i(S_b + S_f)} = \int dA e^{iS_b} \mathrm{Pf}\mathcal{M}(A)$

complex → sign problem!

We perform numerical simulations based on the complex Langevin method (CLM) to overcome the sign problem.

Definition of expectation values: $\langle \mathcal{O} \rangle = \frac{1}{Z} \int dA \mathcal{O} e^{iS_b} \mathrm{Pf}\mathcal{M}(A)$

Even if A_μ are Hermitian, the expectation values of eigenvalues are complex in general.

The model is not well-defined as it is.

If we define the model by the deformation of the integration contour (such as $A_0 = e^{-3\pi i u/8} \tilde{A}_0$ and $A_i = e^{\pi i u/8} \tilde{A}_i$ with $u \to +0$), complex phase of $\langle A_0 \rangle \sim e^{-3\pi i/8}$ and that of $\langle A_i \rangle \sim e^{\pi i/8}$.

→ Real space-time cannot be realized!

How does real space-time emerge in this model?

Classical solutions

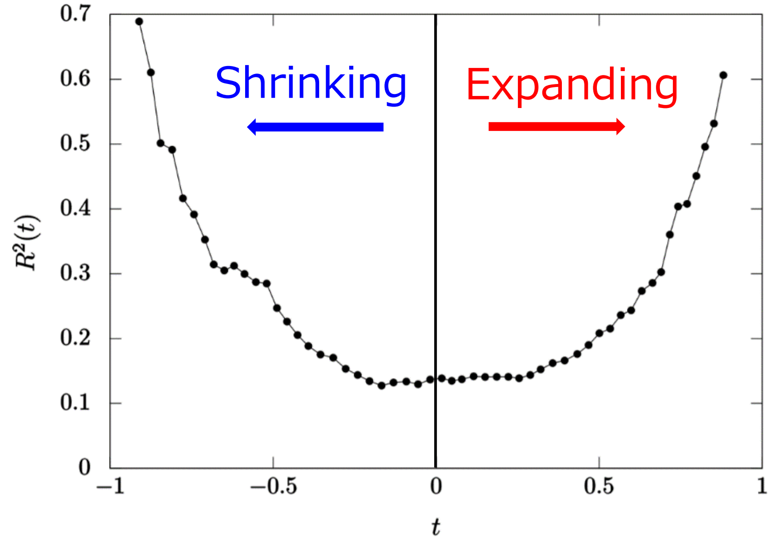
Classical EOM for the type IIB matrix model: $\frac{\delta S}{\delta A_\mu} = [A^\nu, [A_\nu, A_\mu]] = 0$

All simultaneously diagonalizable A_μ are classical solutions,
so there is no expansion in general for such solutions.

In the previous work [KH-Matsumoto-Nishimura-Tsuchiya-Yosprakob ('19)], to implement the effects of the IR regularization required for the Lorentzian model, we added the quadratic term of A_μ (mass term) $-\frac{1}{2}N\gamma\left[\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right]$.

Classical EOM with mass term: $\frac{\delta S}{\delta A_\mu} = [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$

- $\gamma > 0$: classical solutions with an expanding behavior



← Classical solution for only 3d space expands.
But not for $\gamma < 0$!

※Dimensionality of expanding space is arbitrary.

We introduce the same mass term
when we perform simulations of this model.

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2. Deformation of the model

Some tricks to make the CLM work

◆ Singular drift problem

$$Z = \int dAd\Psi e^{iS} = \int dAe^{iS_b}\mathrm{Pf}\mathcal{M}(A)$$

$\mathcal{M}$ that appears in $\mathrm{Pf}\mathcal{M}$ has eigenvalues accumulating near zero, which causes the singular drift problem. Due to this problem, the CLM fails. We introduce a deformation term in fermionic action to avoid the problem.

$$S_f = -\frac{N}{2}\mathrm{Tr}\left(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta] + im_f\bar{\Psi}_\alpha(\Gamma_7\Gamma_8^\dagger\Gamma_9)_{\alpha\beta}\Psi_\beta\right)$$

m_f: deformation parameter
- $m_f \to \infty$: bosonic (Due to decoupling of fermionic degrees of freedom)
- $m_f \to 0$: SUSY

◆ Stabilization

Cf.) [Attanasio-Jäger ('18)] : dynamical stabilization in CL simulation of QCD

To stabilize the CLM, $A_i \to \frac{1}{1+\eta}(A_i + \eta A_i^\dagger)$ after each Langevin step

- $\eta = 0$: do nothing
- $\eta = 1$: Hermitianize

Here, $\eta = 0.005$.

※Justifiable when dominant configurations are close to Hermitian.

Controlling the quantum fluctuation of bosonic matrices

$$S_{\rm m} = -\frac{N}{2}\gamma\,\mathrm{Tr}\left((A_0)^2 - \sum_{i=1}^{d}(A_i)^2 - \xi\sum_{i=d+1}^{9}(A_i)^2\right)$$

d, ξ : parameters that can control the quantum fluctuations of bosonic matrices

For large ξ, the bosonic degrees of freedom reduce effectively to $(d+1)$-dimensional one.

By choosing d and ξ appropriately, we can expect to realize a situation which is close to one where SUSY is respected.

The effects of fermions are reduced by $m_{\rm f}$, and that is why one needs to reduce the effects of bosons to mimic the SUSY cancellation.

If one can make $m_{\rm f}$ smaller, one does not introduce $\xi > 1$.

In the $N \to \infty, \xi \to 1, \gamma \to 0, m_{\rm f} \to 0$ limits, we can reach the original theory.

3. Results of numerical simulations

$$Z = \int dA e^{-S_{\text{eff}}}, \quad S_{\text{eff}} = i(S_{\text{b}} + S_{\text{m}}) - \log \text{Pf}\mathcal{M}$$

$$S_{\text{b}} = -\frac{N}{4} \text{Tr}\left([A^{\mu}, A^{\nu}][A_{\mu}, A_{\nu}]\right)$$

$$S_{\text{m}} = -\frac{N}{2}\gamma \,\text{Tr}\left((A_0)^2 - \sum_{i=1}^{d}(A_i)^2 - \xi \sum_{i=d+1}^{9}(A_i)^2\right)$$

$-\log \text{Pf}\mathcal{M}$ is obtained from $S_{\text{f}} = -\frac{N}{2} \text{Tr}\left(\bar{\Psi}_{\alpha}(\Gamma^{\mu})_{\alpha\beta}[A_{\mu}, \Psi_{\beta}] + i m_{\text{f}} \bar{\Psi}_{\alpha}(\Gamma_7 \Gamma_8^{\dagger} \Gamma_9)_{\alpha\beta} \Psi_{\beta}\right)$

We will show results at $N = 64, 96, 128, \gamma = 4, d = 5, \xi = 10, m_{\text{f}} = 3.5.$

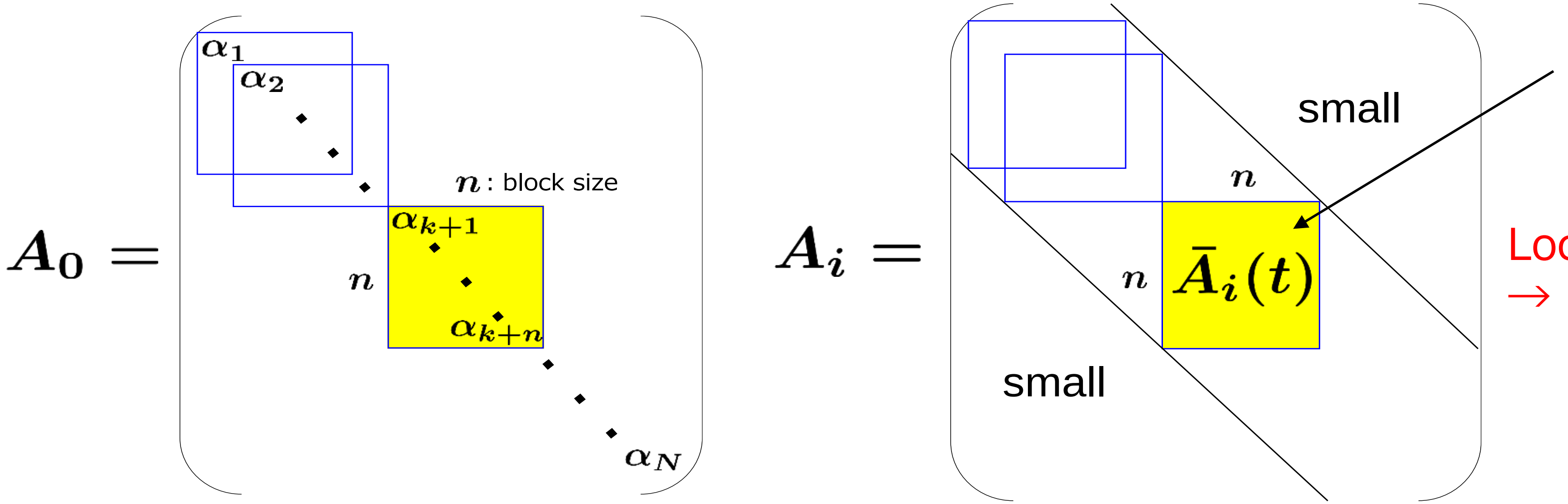
How to determine whether time and space are real

- Distribution of α_i (eigenvalues of A_0) $\Delta\langle\alpha_i\rangle \equiv \langle\alpha_{i+1}\rangle - \langle\alpha_i\rangle \propto e^{i\theta_t}$

$\theta_t = 0$: real time

Definition of time: $\bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n}\alpha_{k+i} \in \mathbb{C},\ t_\rho = \sum_{k=1}^{\rho}|\bar{\alpha}_{k+1} - \bar{\alpha}_k|$

$A_\mu \to U A_\mu U^\dagger$, U: unitary matrix diagonalizing A_0



represent space at t

Locality of time is guaranteed.
→ Emergence of time evolution

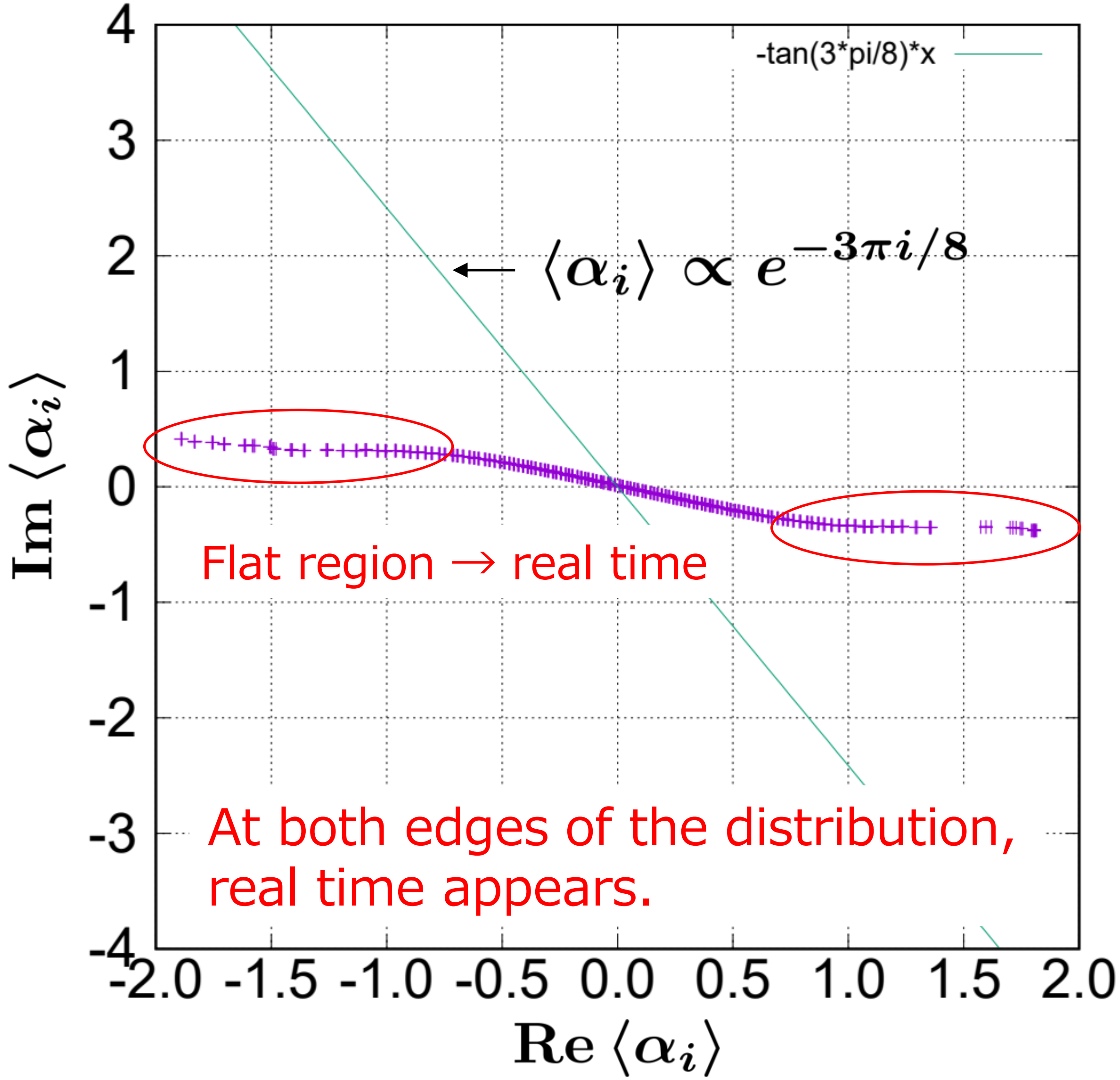
($|R^2(t)|$: extent of space)

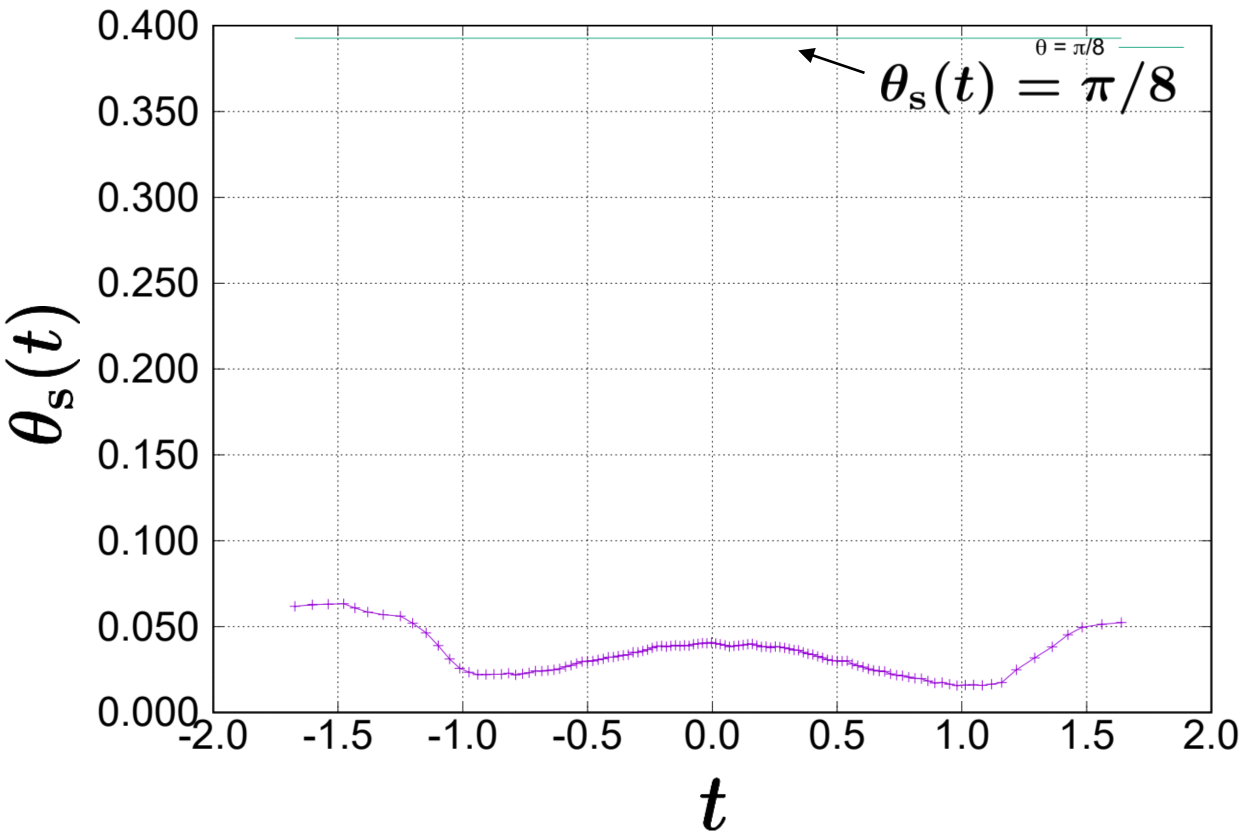
- $\theta_s(t)$ $\theta_s(t) = 0$: real space $R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta_s(t)}|R^2(t)|$
- $\lambda_i(t)$: eigenvalues of $T_{ij}(t) = \left\langle \frac{1}{n}\mathrm{tr}\left(X_i(t)X_j(t)\right)\right\rangle$ (order parameter of SSB) $X_i(t) = \frac{\bar{A}_i(t) + \bar{A}_i^\dagger(t)}{2}$

Emergence of real space-time

$N = 128, \gamma = 4, d = 5, \xi = 10, m_f = 3.5$

$$R^2(t) = e^{2i\theta_s(t)} |R^2(t)|$$





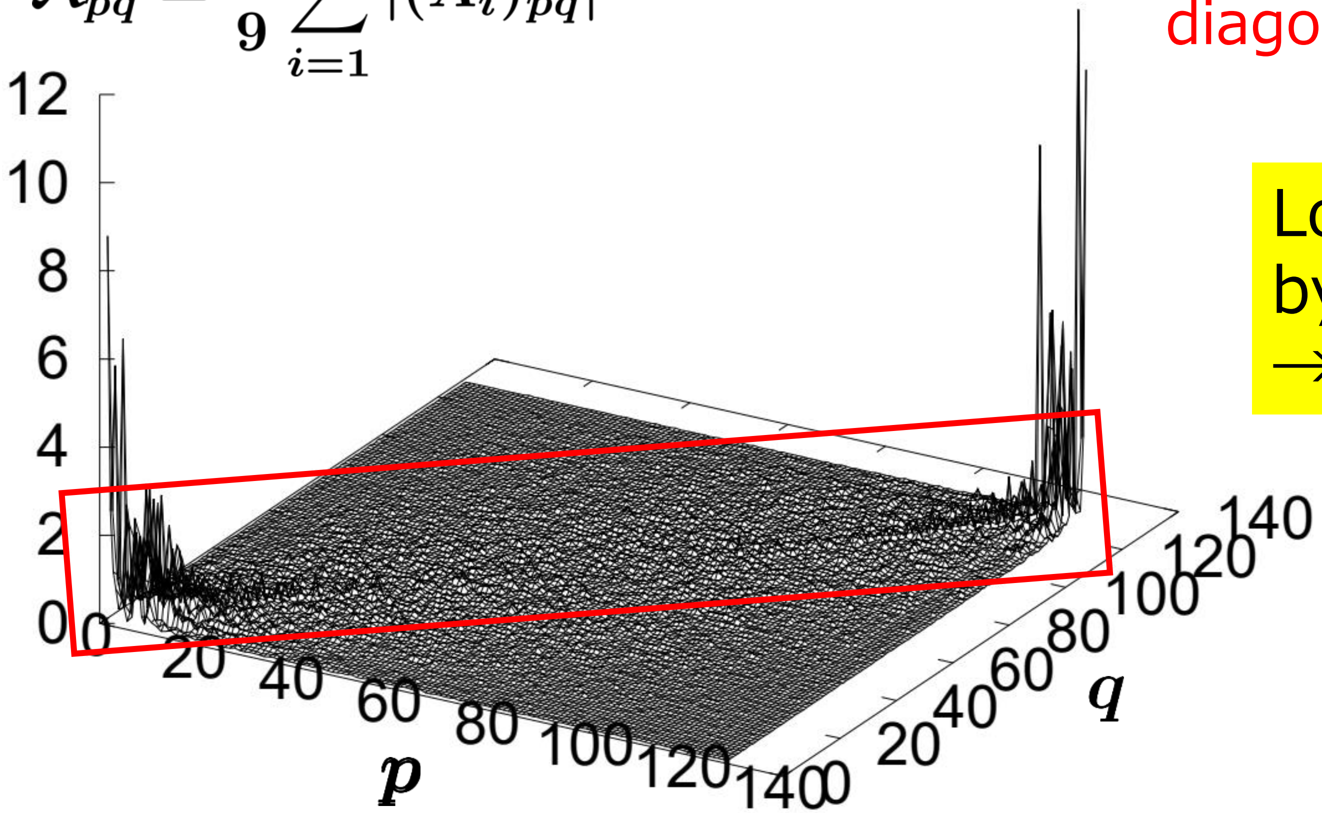
$\theta_s(t)$ are almost zero at any t, namely, real space appears.

At N=64 and 96, real space-time also appears for the same values of other parameters.

Band-diagonal structure

$$N = 128, \gamma = 4, d = 5, \xi = 10, m_{\mathrm{f}} = 3.5$$

$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} |(A_i)_{pq}|^2$$



diagonal elements > off-diagonal elements

Locality of time is ensured
by the band-diagonal structure.
→One can read off the time evolution.

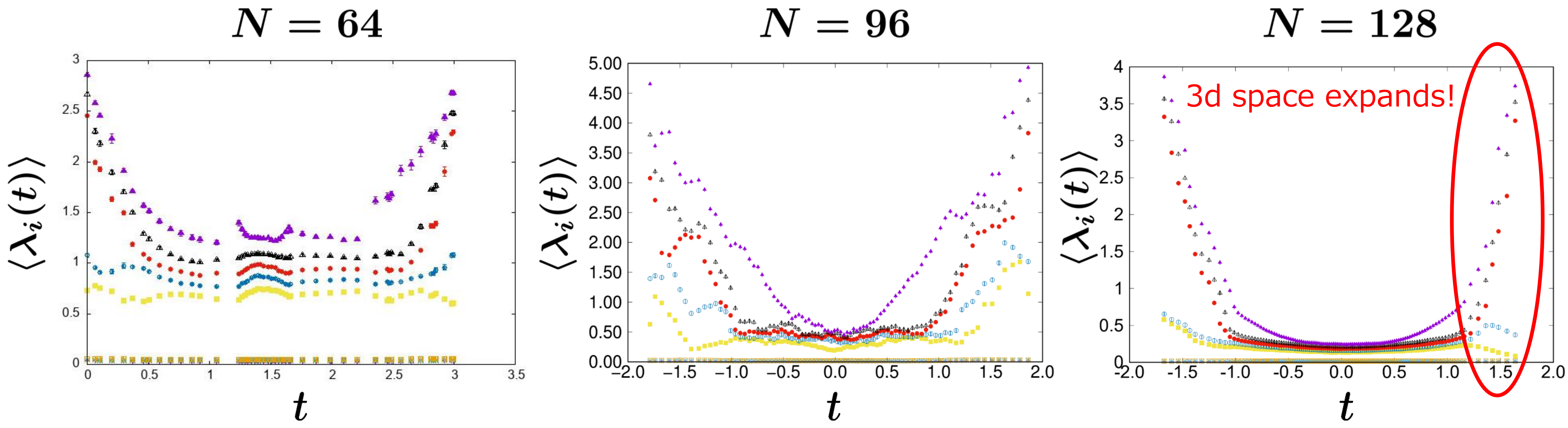
※This structure appears when the space expands.

At N=64 and 96,
this structure also appears for the same values of other parameters.

N-dependence of the dimensionality of space-time

$\gamma = 4, d = 5, \xi = 10, m_{\mathrm{f}} = 3.5$

$\lambda_i(t)$: eigenvalues of $T_{ij}(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(X_i(t) X_j(t)\right) \right\rangle$ $\quad X_i(t) = \frac{\bar{A}_i(t) + \bar{A}_i^{\dagger}(t)}{2}$



At N=64 and 96, (5+1)-dimensional space expands at late times, while at N=128, (3+1)-dimensional space expands at late times!
→The large-N limit should be taken.

4. Summary and outlook

- We perform numerical simulations of the type IIB matrix model, which is a candidate for the non-perturbative formulation of superstring theory by using the complex Langevin method (CLM) to overcome the sign problem.
- If we define the model by the deformation of the integration contour, $\langle A_\mu \rangle$ are complex. → Space-time cannot be real!
- We introduce the mass term (coefficient: γ) to realize the real space-time.
- We add the deformation term (coefficient: m_{f}) in the fermionic action to avoid the failure of the CLM and d, ξ to control the quantum fluctuation of bosonic matrices.
- We observe the (3+1)-dimensional expanding real space-time at $N = 128, \gamma = 4, d = 5, \xi = 10, m_{\mathrm{f}} = 3.5.$
- Do we observe the (3+1)-dimensional expanding space-time when we take the $N \to \infty, \xi \to 1, \gamma \to 0, m_{\mathrm{f}} \to 0$ limits to obtain the original theory?