

Semiclassical analysis of the bifundamental QCD with anomaly-preserving T^2 compactification

Yui Hayashi (YITP, Kyoto U.)

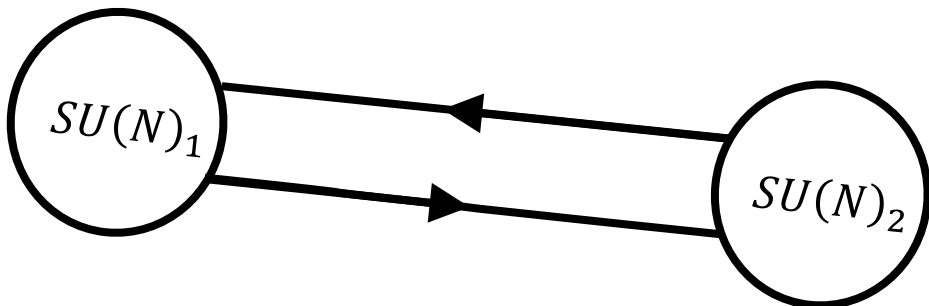
in collaboration with Yuya Tanizaki (YITP, Kyoto U.) & Hiromasa Watanabe (YITP, Kyoto U.)

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What is bifundamental QCD?

can be obtained by gauging $SU(N_f)$ of flavor-symmetric $N_f = N_c$ QCD

Bifundamental QCD =
 $SU(N)_1 \times SU(N)_2$ gauge theory + one Dirac fermion in the bi-fundamental representation $(\square, \bar{\square})$

Field contents:

a_1 : $SU(N)_1$ gauge field

a_2 : $SU(N)_2$ gauge field

Ψ : $N \times N$ matrix-valued Dirac field (fundamental in $SU(N)_1$, anti-fundamental in $SU(N)_2$)

Action:

$$S = \int \frac{1}{2g_1^2} |f_1|^2 + \frac{1}{2g_2^2} |f_2|^2 + \text{tr } \bar{\Psi} (\not{D} + m)\Psi + \frac{i\theta_1}{8\pi^2} \text{tr } f_1 \wedge f_1 + \frac{i\theta_2}{8\pi^2} \text{tr } f_2 \wedge f_2$$

Motivations: why bifundamental QCD?

1. Large N orbifold equivalence [Kachru-Silverstein '98][Bershadsky-Johansen '98] etc.etc.....

$SU(2N) \mathcal{N} = 1$ SYM

=

$SU(N)_1 \times SU(N)_2$ bifundamental QCD

at large N

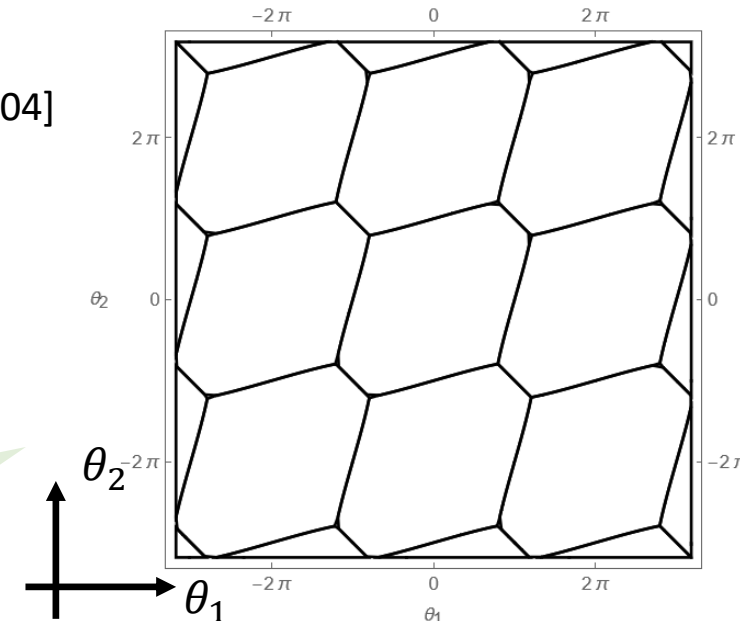
- SUSY-like structure in non-SUSY theory?
- Nonperturbative validity of equivalence?

unbroken/broken of the exchange symmetry [Kovtun-Ünsal-Yaffe '03 '04]

2. Rich phase diagram [Tanizaki-Kikuchi '17][Karasik-Komargodski '19]

- Good playground for understanding confining vacuum

(expected) typical phase diagram



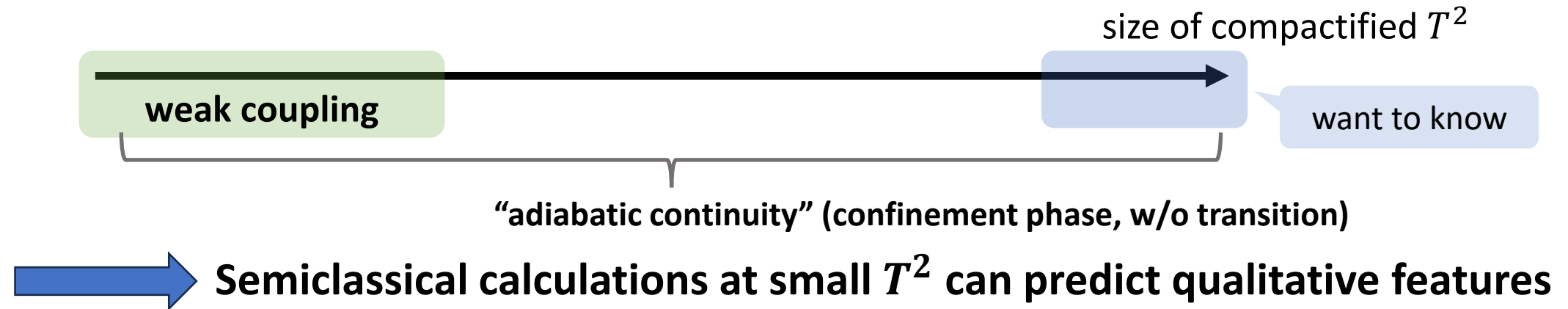
Methodology: Semiclassics on $\mathbb{R}^2 \times T^2$ with 't Hooft flux

Main ansatz: adiabatic continuity conjecture

theory on $\mathbb{R}^2 \times T^2$ with 't Hooft flux (small T^2)

\approx

theory on \mathbb{R}^4



☑ this method successfully gives a reasonable picture for confining vacuum in SU(N) YM, SU(N) N=1 SYM, QCD(F), QCD(Sym), QCD(AS) [Tanizaki-Ünsal '22 '23]. (cf. [Yamazaki-Yonekura '17])

Example: Semiclassics on $\mathbb{R}^2 \times T^2$ in $SU(N)$ YM [Tanizaki-Ünsal '22]

- 't Hooft flux on T^2 ($\leftrightarrow \mathbb{Z}_N^{[1]}$ background)

In gauge theory on T^2 , the field is periodic up to a gauge transformation $g_3(x_4), g_4(x_3)$ (transition function). Without fundamental matter, we can choose

$$g_3(0)g_4(L)g_3^\dagger(L)g_4^\dagger(0) = e^{\frac{2\pi i}{N}}$$

We can take $g_3 = S, g_4 = C$ (shift and clock matrices of $SU(N)$)

- **Center vortex**

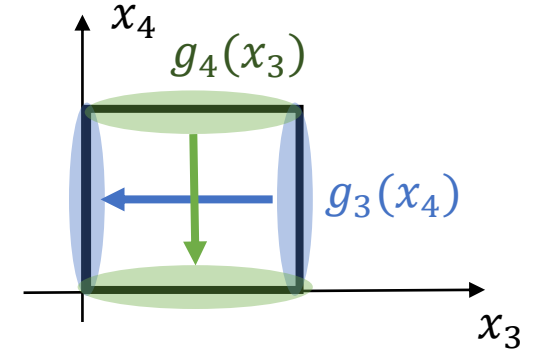
If we further compactify \mathbb{R}^2 with 't Hooft flux, the minimal topological charge is $Q_{\text{top}} = \frac{1}{N}$. $\rightarrow Q_{\text{top}} = \frac{1}{N}, S_{YM} = \frac{8\pi^2}{Ng^2}$ (self-dual solution).

It is numerically confirmed that such a self-dual solution (**fractional instanton/center vortex**) exists and will survive in $\mathbb{R}^2 \times T^2$ [Gonzalez-Arroyo-Montero '98, Montero '99].

Semiclassics on small T^2

→ confining vacuum!

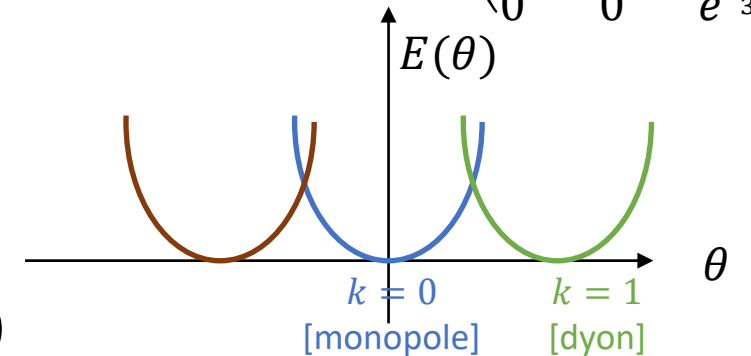
(gapped gluon, confinement by center vortices, multi-branch structure)



$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - i g_3^\dagger d g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - i g_4^\dagger d g_4 \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$



Semiclassics in bifundamental QCD (1)

Apply the previous method to bifundamental QCD:

- 't Hooft flux for both $SU(N)_1$ and $SU(N)_2$
- **Two types of center vortices:** $(Q_{\text{top}}^{(1)}, Q_{\text{top}}^{(2)}) = (\frac{1}{N}, 0)$ and $(0, \frac{1}{N})$
- The boundary condition for the bifundamental fermion,

$$\Psi(\vec{x}, x_3 + L, x_4) = S^\dagger \Psi(\vec{x}, x_3, x_4) S, \quad \Psi(\vec{x}, x_3, x_4 + L) = C^\dagger \Psi(\vec{x}, x_3, x_4) C$$
 allows one constant mode $\Psi \propto 1_{N \times N}$: \exists **dynamical fermion ψ in 2d effective theory.**

- Effect of center vortex on the fermion: **a center vortex changes the chirality**

$$2 \text{Index}(D) = 2N \left(Q_{\text{top}}^{(1)} + Q_{\text{top}}^{(2)} \right) = 2$$

Thus, the vertex operator of a center vortex carries $\bar{\psi}_L \psi_R$.

e.g.) the $(1/N, 0)$ center vortex gives $-K^{(1)} e^{-\frac{8\pi^2}{Ng_1^2} + i\theta_1/N} \bar{\psi}_L \psi_R$.

Semiclassics in bifundamental QCD (2)

- Dilute gas approximation

$$Z_{YM} = \sum_{k_1, k_2 \in \mathbb{Z}_N} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[-S_{2d}[\psi, \bar{\psi}] - \int d^2x (M_{k_1, k_2} \bar{\psi}_L \psi_R + c.c.) \right]$$

$$S_{2d}[\psi, \bar{\psi}] = \int d^2x \bar{\psi} (\not{\partial}_{2d} + m) \psi, \quad M_{k_1, k_2} = K^{(1)} e^{-\frac{8\pi^2}{Ng_1^2} + i \frac{\theta_1 - 2\pi k_1}{N}} + K^{(2)} e^{-\frac{8\pi^2}{Ng_2^2} + i \frac{\theta_2 - 2\pi k_2}{N}}$$

fermion “constant” mode

Summing up center vortices

- 2d low-energy effective description:

N^2 semiclassical vacua $(k_1, k_2) \in \mathbb{Z}_N \times \mathbb{Z}_N$ + a dynamical 2d fermion on each vacuum

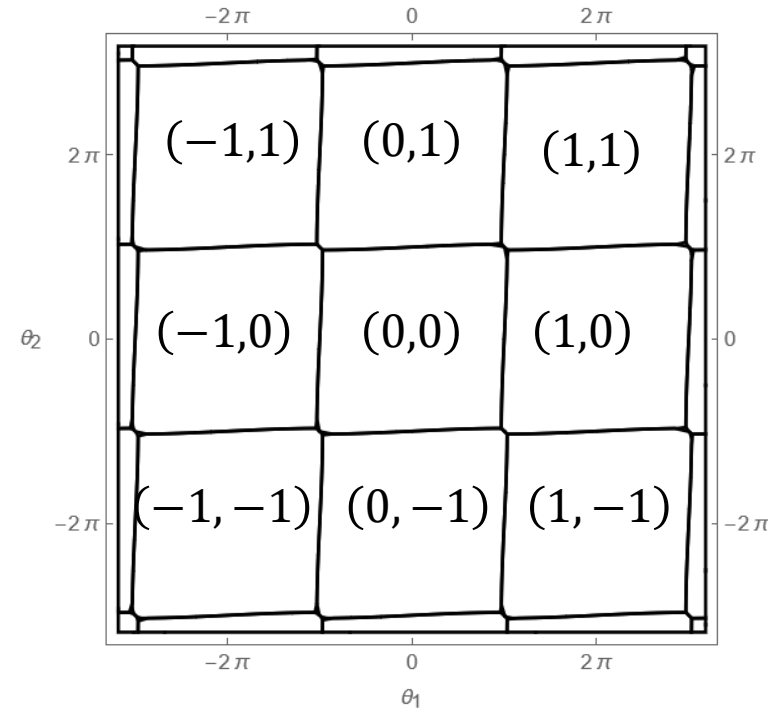
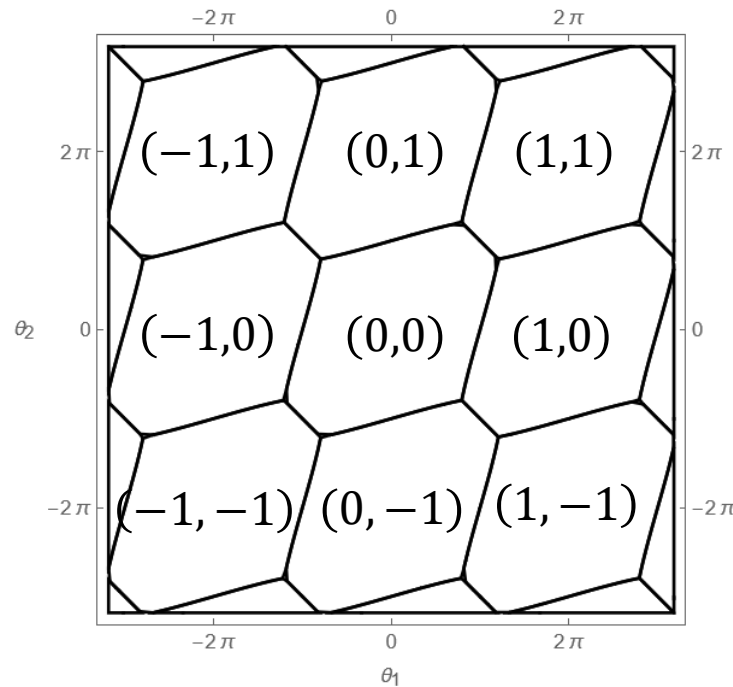
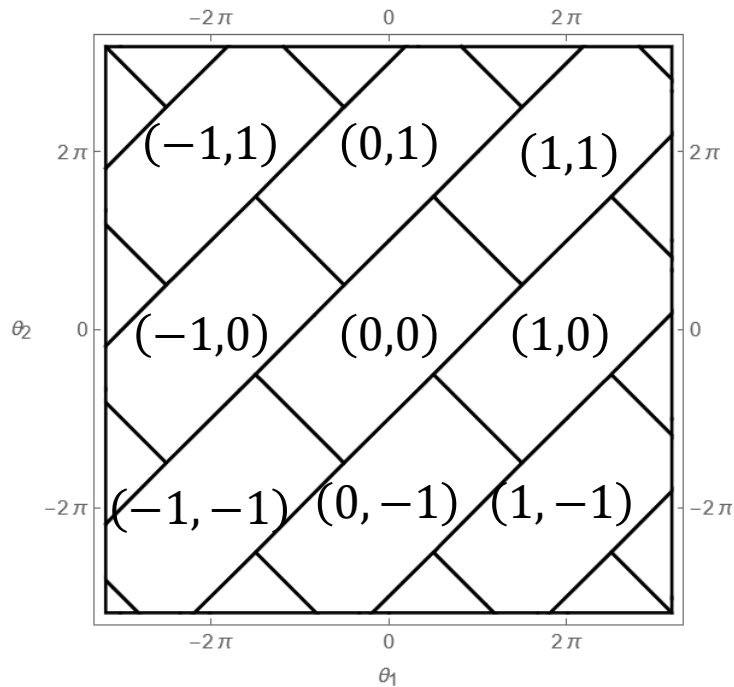
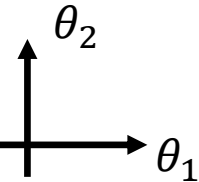
Fermion vacuum energy splits the N^2 degenerate vacua; **the semiclassical vacuum (k_1, k_2) with the largest $|m + M_{k_1, k_2}|$ is the ground state.**

Semiclassics in bifundamental QCD (3)

For simplicity, let us draw phase diagrams at $g_1 = g_2$ ($\rightarrow K^{(1)} = K^{(2)}$)

massless

fermion mass m



Summary

describing a confining vacuum by semiclassics of **center vortices** [Tanizaki-Ünsal '22]

We study the bifundamental QCD through semiclassics on $\mathbb{R}^2 \times T^2$ with 't Hooft flux.

2d effective theory = N^2 semiclassical vacua $(k_1, k_2) \in \mathbb{Z}_N \times \mathbb{Z}_N$
+ a dynamical 2d fermion on each vacuum

→ phase diagram on (θ_1, θ_2) -plane

✓ phase diagrams from explicit calculations

(new for small/intermediate fermion mass)

✓ consistent with the conjectured phase diagrams

[Tanizaki-Kikuchi '17][Karasik-Komargodski '19]

✓ on exchange-symmetric line $(\theta_1 = \theta_2)$, the exchange symmetry is unbroken

→ evidence for nonperturbative orbifold equivalence.

Adiabatic continuity works well!

