A new perspective on thermal transition in QCD

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[arXiv:2310.01940] (& long ver. [arXiv:2310.07533])

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Introduction

[Hanada, Maltz, (2016) / Berenstein, (2018) / Hanada, Ishiki, **HW**, (2018) / Hanada, Jevicki, Peng, Wintergerst, (2019) / ⋯]

Ultimate goal :

Understanding deconfinement in gauge theories (at finite temperature)

(for phases of QCD matter, AdS/CFT correspondence, ⋯)

Large-N deconfinement exhibits a phenomenon called partial deconfinement;

- Interpreted as the coexistence of confined/deconfined sectors in the space of color dof.

  → only some modes can be excited (i.e., deconfined)

- Possible to occur in a broad class of large-N QFTs   e.g.) [Hanada, **HW**, (2023)] as a review

- Applicable to finite-N theories, such as SU(N=3) QCD?

  - An investigation on lattice QCD configurations ← this work!!

Contents

Intermediate phase at large N

[Hanada, Ishiki, HW, (2018) / ⋯]

A classification of phases at large N (and weak coupling) by

[Sundborg, (1998) / Aharony, Marsano, Minwalla, Papadodimas, V.Raamsdonk, (2003)/ Schnitzer, (2004)]

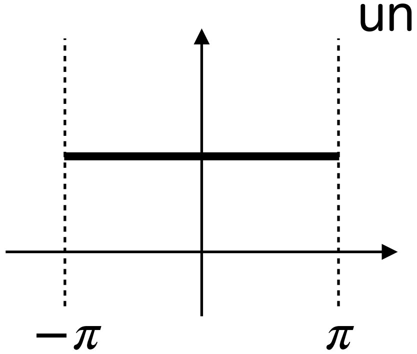
$$u_n(T) = \frac{1}{N}\mathrm{tr}\,\mathrm{P}\exp\left[\mathrm{i}n\oint A_t\right] = \int \mathrm{d}\theta\, \rho(\theta;T)\,\mathrm{e}^{\mathrm{i}n\theta}$$

: multiply-wound Polyakov loop

($u_1 = P$: Polyakov loop)

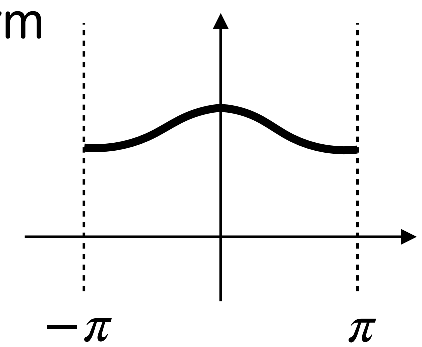
uniform | non-uniform

connected | disjointed



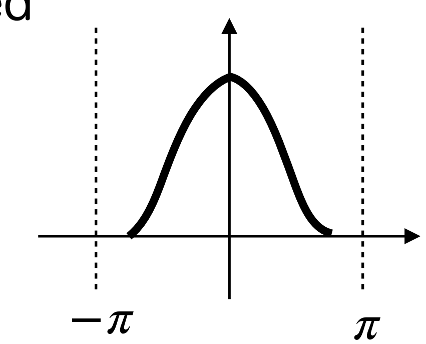
confined

Hagedorn transition



"partially-deconfined"

GWW transition



"completely-deconfined"

[Hagedorn, (1965), ⋯]

[Gross, Witten, (1980) / Wadia, (1980)]

$u_n = 0$ for all n
↕
uniform distribution

$u_n \neq 0$ for some n
&
zero or exp. small for larger n

$u_n \neq 0$ for all n
↕
(∵ no-support region)

: characterization as a phenomenon in which only some modes are excited

With finite N & infinite Volume

[Hanada, Ohata, Shimada, HW, (2023)]

How do we apply the large-N description to finite-N theory (QCD)?

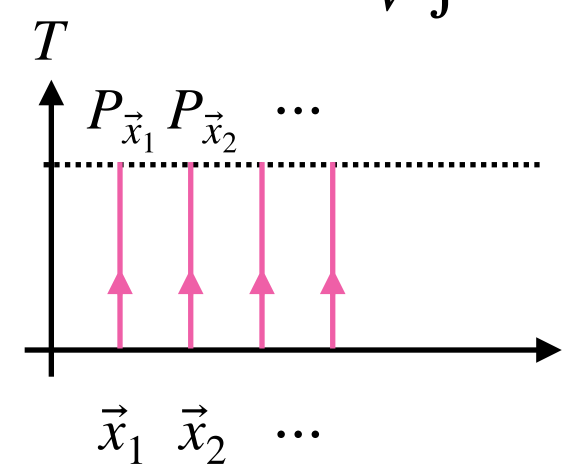
(QCD : 4d SU(N=3) gauge theory w/ N_f fundamental fermion)

Key observation : utilize the spatial extent at finite N (assuming homogeneity)

$$P = \langle P_{\vec{x}} \rangle_{\rm space} = \int {\rm d}\theta \, \rho_{\rm s}(\theta) \, {\rm e}^{{\rm i}\theta}$$

$$\langle \, \cdot \, \rangle_{\rm space} = \frac{1}{V} \int {\rm d}^3 x \, \cdot$$

$$\rho_{\rm s}(\theta) = \langle \rho_{\vec{x}}(\theta) \rangle_{\rm space} = \frac{1}{NV} \int {\rm d}^3 x \sum_{j=1}^{N} \delta(\theta - \theta_j^{(\vec{x})})$$



3 θ's at each space $\vec{x}$ → NV = 3 x (#spatial pts.) θ's

Let's apply the picture to analyses of lattice QCD!

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- Introduction
- Brief look at partial deconfinement
- Numerical analysis on lattice QCD configurations
- Summary & Prospects

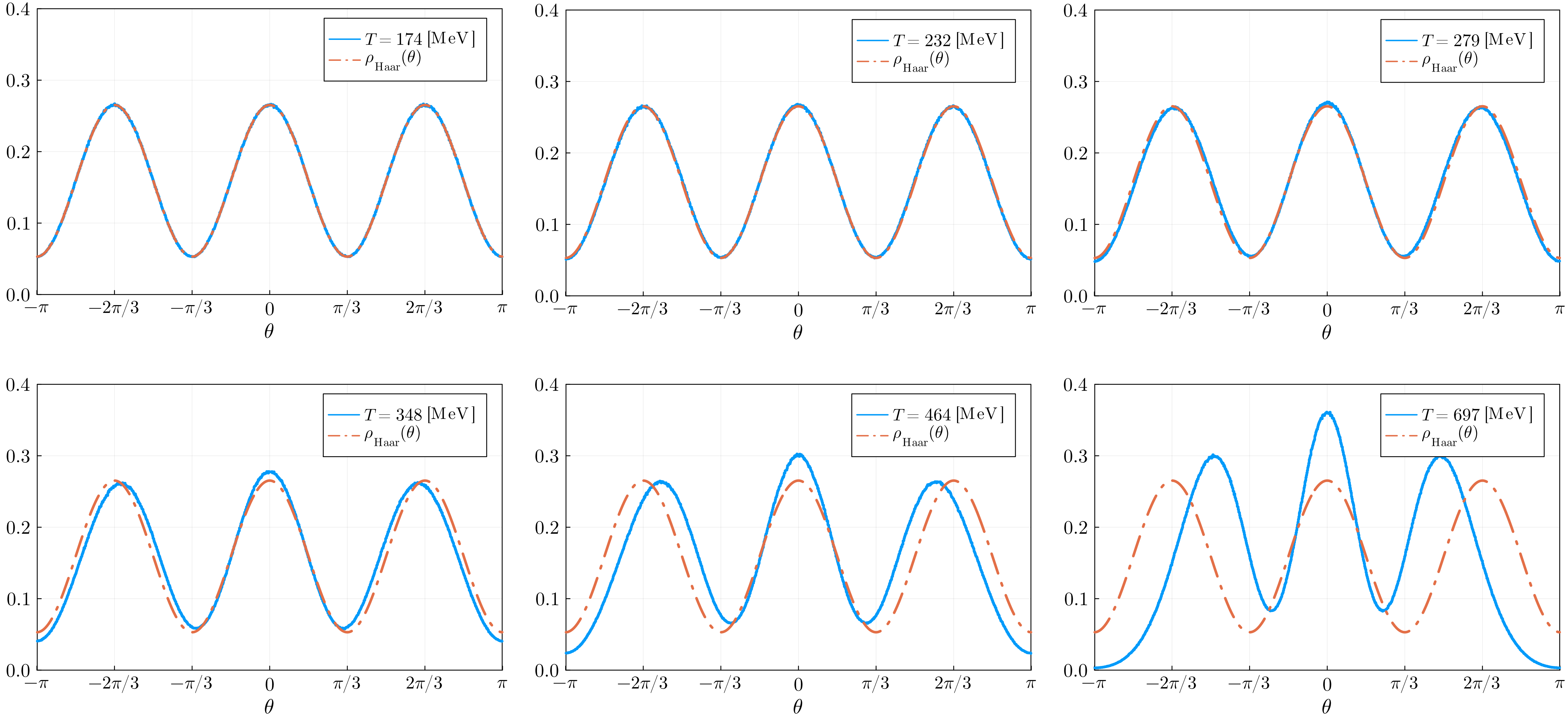
Profile of lattice QCD configuration

Gauge configuration by WHOT-QCD collaboration [Phys. Rev. D. 85, 094508 (2012)]

- $N_f = 2 + 1$, RG-improved Iwasaki gauge + NP $O(a)$-improved Wilson quarks
  - $T = 0$ config. of CP-PACS & JLQCD collaboration [Phys. Rev. D. 78, 011502 (2008)]

- $\beta = 2.05, \quad a^{-1} = 2.79$ GeV, $\quad (a \simeq 0.07$ fm$)$
- $32^3 \times n_t$ lattice with $n_t = 4,\ 6,\ \cdots,\ 16$ ($T \simeq 174,\ 199,\ \cdots,\ 697$ MeV)
- $m_\pi / m_\rho \simeq 0.63$: heavy quark
- $T_{\rm pc} \approx 190$ MeV

Phase distribution $\rho(\theta; T)$

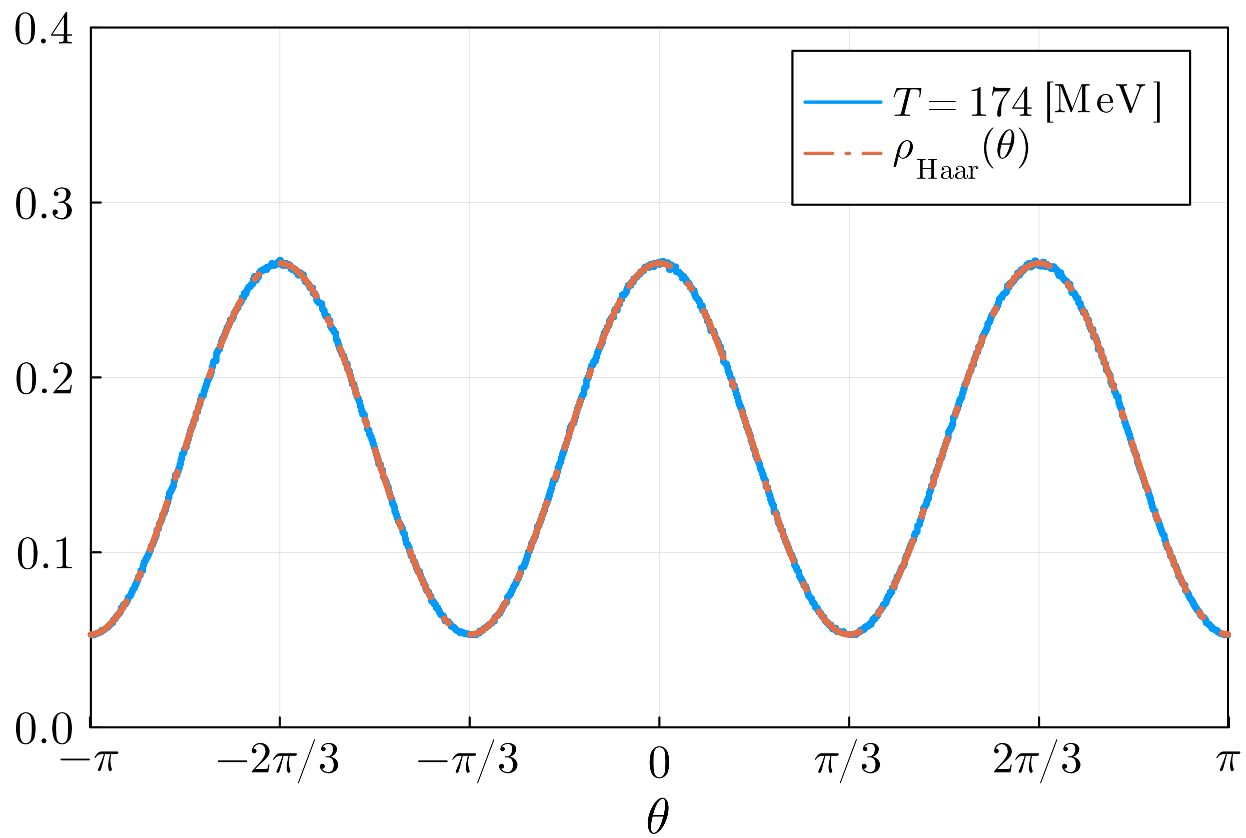


Deviation from a distribution becomes visible as raising the temperature.

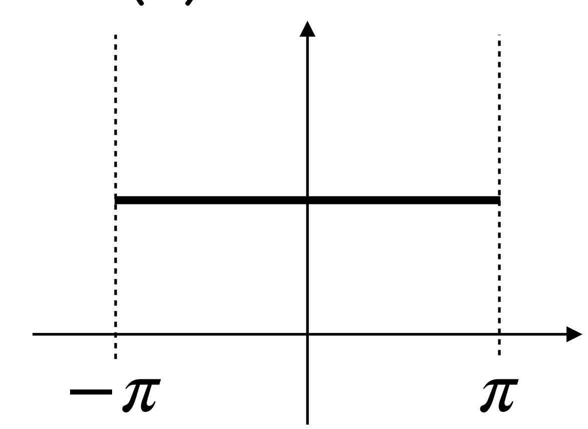
Haar randomness

The uniform distribution at large N → SU(N) Haar-random distribution

$$\rho_{\text{Haar}}(\theta; N) = \frac{1}{2\pi}\left(1 - (-1)^N \frac{2}{N}\cos(N\theta)\right)$$



“SU(∞)” Haar random



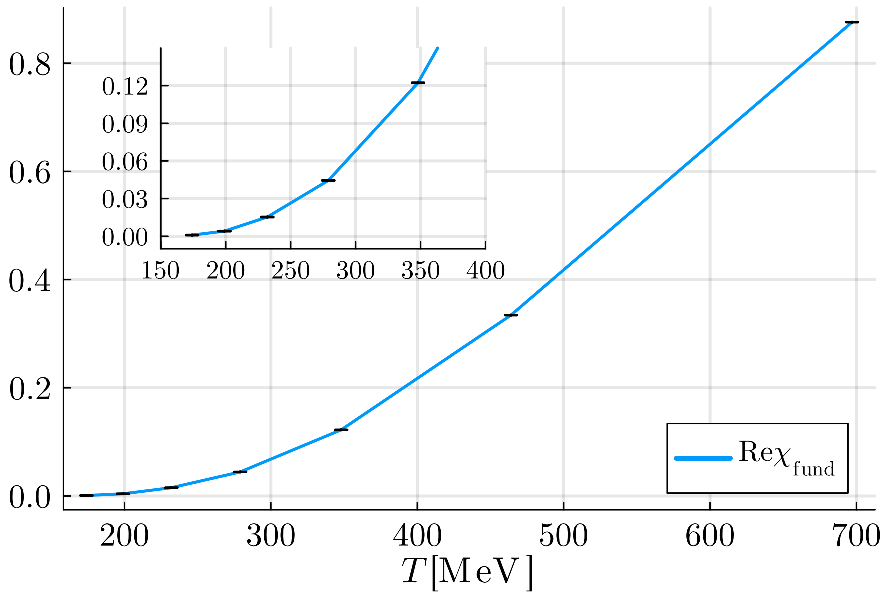
- Almost-random SU(3) matrix lives on lattice sites c.f. [Polyakov, (1975)/(1978)]
- Deviation from $\rho_{\text{Haar}}(\theta)$ is quantified by characters $\chi_r(\{\theta\})$ (discussed later)

Characters & Polyakov loops

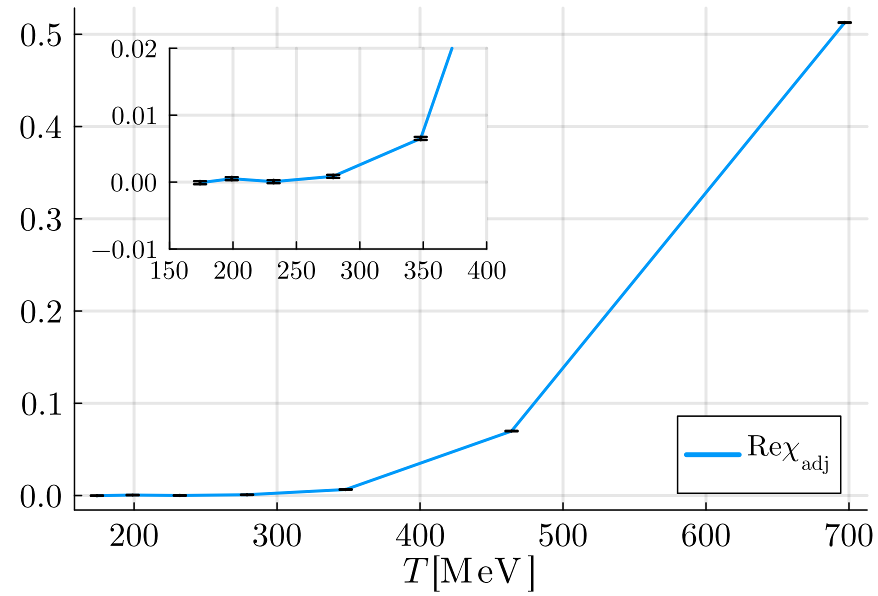
Characters have a clearer physical meaning;

- Discriminable which degrees of freedom (: representation) are excited.

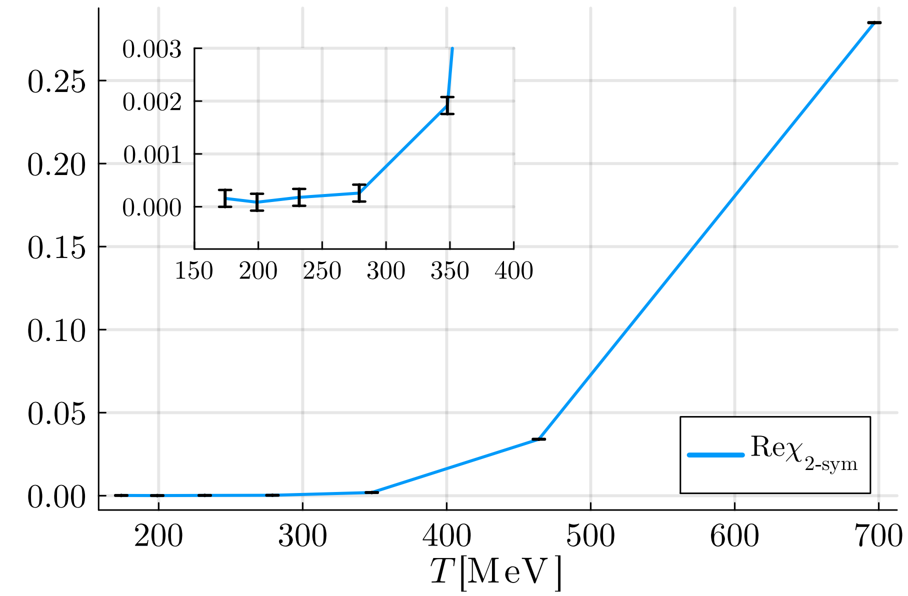
$\chi_{\text{fund}} = \sum_j \mathrm{e}^{\mathrm{i}\theta_j}$



$\chi_{\text{adj}} = 2 + \sum_{j\neq k} \mathrm{e}^{i(\theta_j-\theta_k)}$



$\chi_{2\text{-sym}} = \sum_j \mathrm{e}^{2i\theta_j} + \sum_{j\neq k} \mathrm{e}^{i(\theta_j+\theta_k)}$



χ_r's other than χ_{fund} start to depart at different T above 348 MeV $(\simeq 2T_{\text{pc}})$

$\rightarrow$ a supporting evidence of the presence of an intermediate region

Characters & Polyakov loops

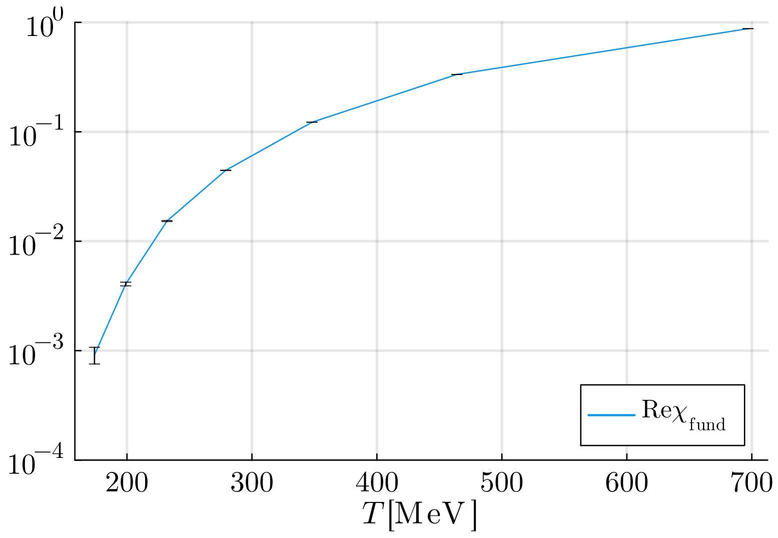
Characters have a clearer physical meaning;

- Functions on group mfd. can be expanded by them (c.f. Peter-Weyl theorem)

$$\longrightarrow \quad \rho(\{\theta\}) = \sum_r \rho_r \chi_r(\{\theta\}) = \rho_{\mathrm{Haar}}(\{\theta\}) + \sum_{r \neq \mathrm{trivial}} \rho_r \chi_r(\{\theta\}),$$

- $\langle \chi_{\mathrm{fund}} \rangle_{\mathrm{space}} \sim \mathrm{e}^{-m/T}$ w/ some mass gap

  (It may correspond to hadron gas excitation.)



Polyakov loop $P \propto \chi_{\mathrm{fund}}$ is responsible to deviation from $\rho_{\mathrm{Haar}}(\theta)$ at low-T

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Summary & Prospects

- Deconfinement transition is intriguing from several viewpoints.
  - In large N theories at finite T, a nontrivial intermediate phase appears.
- Valuable concept for QCD w/ quarks (unveiled by WHOT-QCD lattice configs.)
  - Haar-random distribution $\rho_{\mathrm{Haar}}(g)$ is characteristic at low temperatures.
  - Intermediate regions appear, determined by the deviation from $\rho_{\mathrm{Haar}}(g)$.
  - Haar randomness yields stronger condition than center symmetry.
  - As a relation to other QCD scale, the instanton condensation fits nicely.
- Further investigation on QCD (and pure YM).
  - Checking other observables, such as correlators c.f.) [Bergner, Gautam, Hanada, (2023)]
  - Simulations w/ finer lattice along T, at physical point.
- Revisiting the quantum gravity with the technique of characters

c.f.) [Berenstein, Yan, (2023)]

Backup

Viewpoint from characters

Good basis : characters (i.e., Polyakov loops in representation r, Schur polynomial)

$$\chi_r(g) = \mathrm{tr}\, R_r(g)$$
$$= \chi_r(\{\theta\})$$

$R_r(g)$: Representation matrix of rep. r

$g \in G = \mathrm{SU}(3)$: holonomy (~Polyakov line $\mathrm{e}^{\mathrm{i}\Theta}$)

- Orthonormality

$$\frac{1}{\mathrm{Vol}(G)} \int_G \mathrm{d}g\, \bar{\chi}_r(g) \chi_{r'}(g) = \delta_{rr'}$$

(r, r' : irreducible rep.)

- Functions on G can be expanded by the characters (: Peter-Weyl theorem)

$P(g)$: distribution function

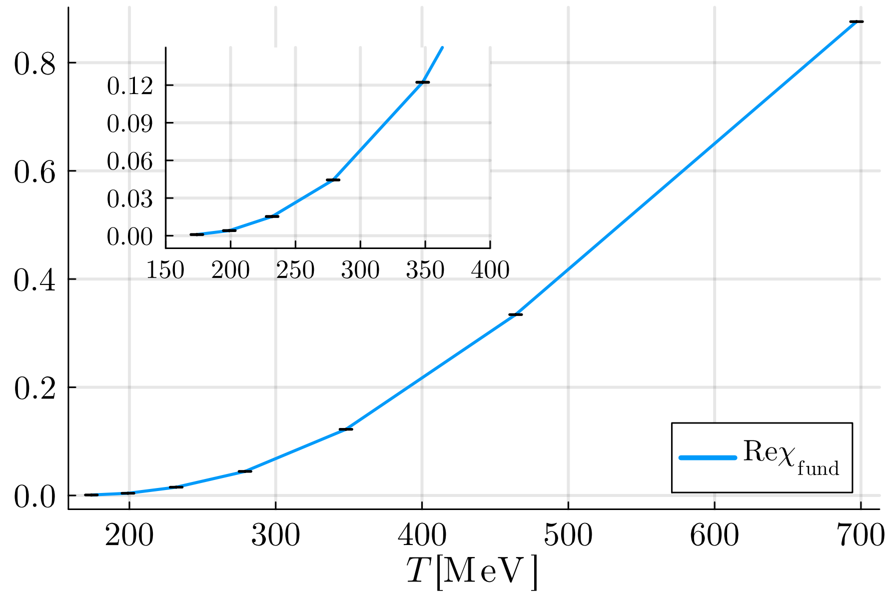
$$1 = \frac{1}{\mathrm{Vol}(G)} \int \mathrm{d}g\, \rho(g) \qquad \rho(g) = \sum_r \rho_r \chi_r(g), \qquad \chi_{\mathrm{trivial}}(g) = 1$$

Haar random;

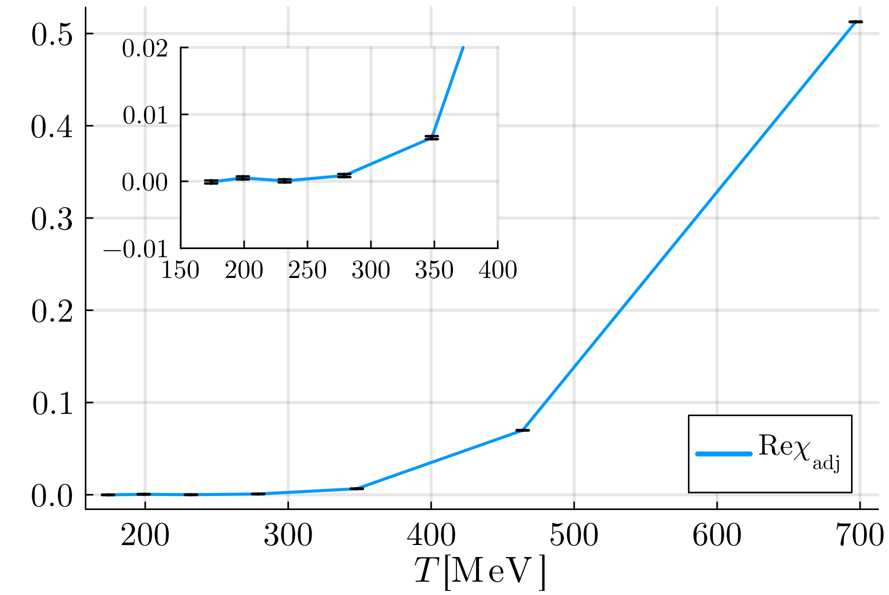
$\rho(g)$: constant (uniform distrib.) <=> $\rho_r = \begin{cases} 1 & r : \text{trivial} \\ 0 & r : \text{otherwise} \end{cases}$

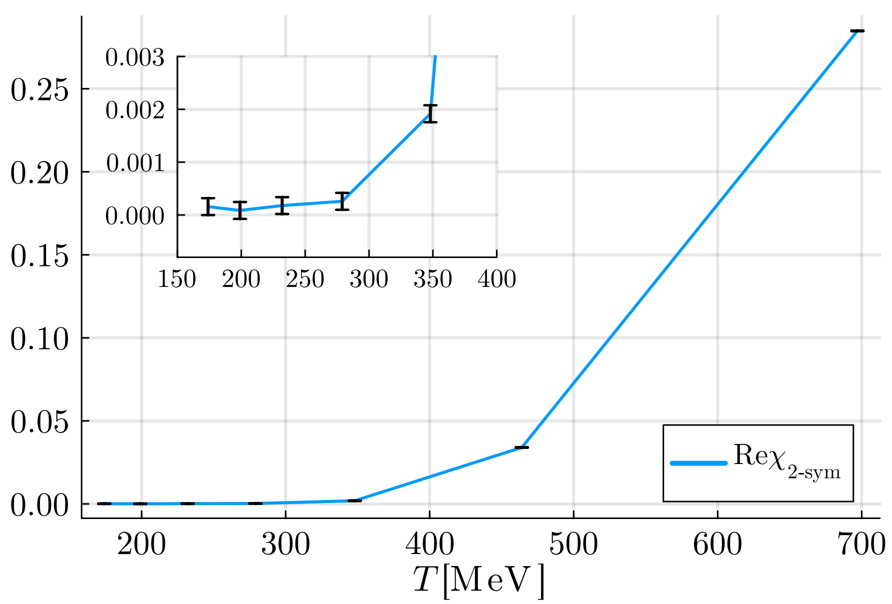
Numerical results of characters

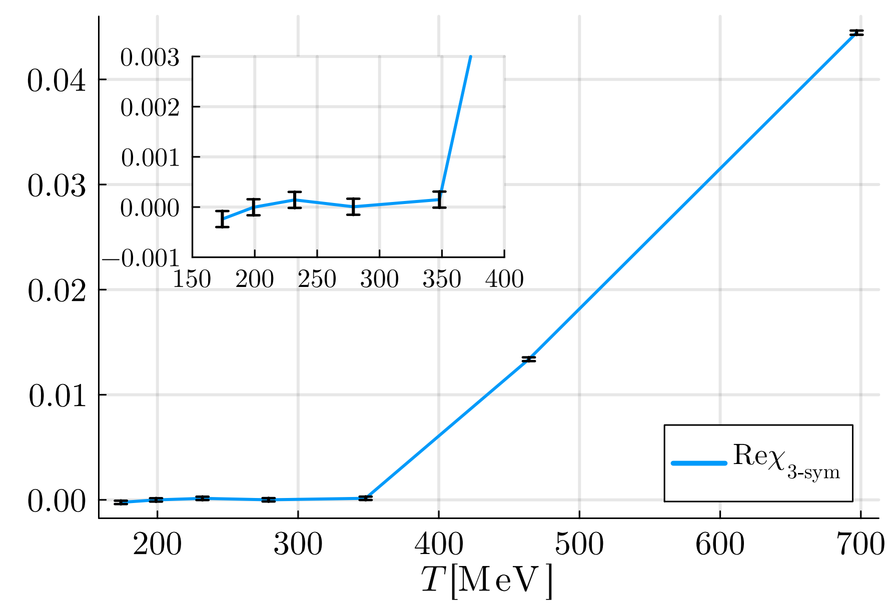
$\chi_{\text{fund}} = \sum_j e^{i\theta_j}$



$\chi_{\text{adj}} = 2 + \sum_{j \neq k} e^{i(\theta_j - \theta_k)}$







$\chi_{2\text{-sym}} = \sum_j e^{2i\theta_j} + \sum_{j \neq k} e^{i(\theta_j + \theta_k)}$

$\chi_{3\text{-sym}} = 1 + \sum_j e^{3i\theta_j} + \sum_{j \neq k} e^{i(2\theta_j + \theta_k)}$

Latter 3 χ_r's start to depart at different T above 348 MeV ($\simeq 2T_{\text{pc}}$)

Characters & Polyakov loops

- For $u_{n\neq 3}$

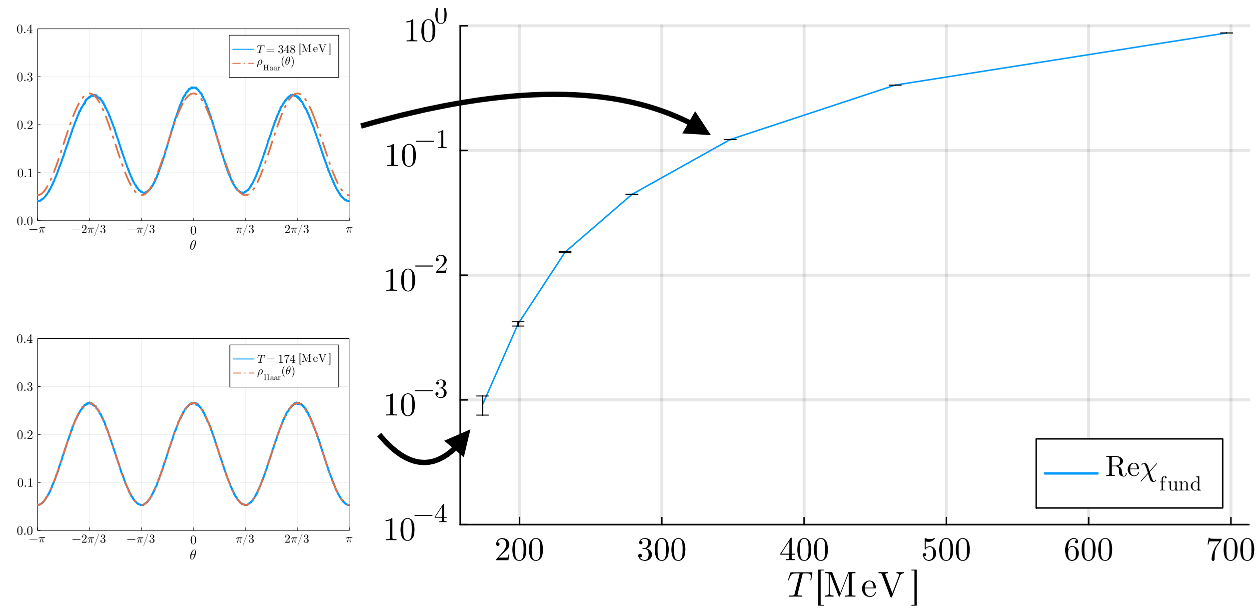
$$P = u_1 \propto \chi_{\text{fund}} \qquad u_2 \propto \chi_{2-\text{sym}} - \chi^*_{\text{fund}} \qquad u_4 \propto \chi_{4-\text{sym}} - \chi_{\yng(3,1)} + \chi_{\text{fund}} \qquad \cdots$$

P is sensitive at low-T to the deviation from Haar random

$$\rho(\{\theta\}) = \rho_{\text{Haar}}(\{\theta\}) + \sum_{r\neq\text{trivial}} \rho_r \chi_r(\{\theta\})$$

$$\langle \chi_{\text{fund}} \rangle_{\text{space}} \sim \mathrm{e}^{-m/T}$$

w/ some mass gap



- From orthonormality,

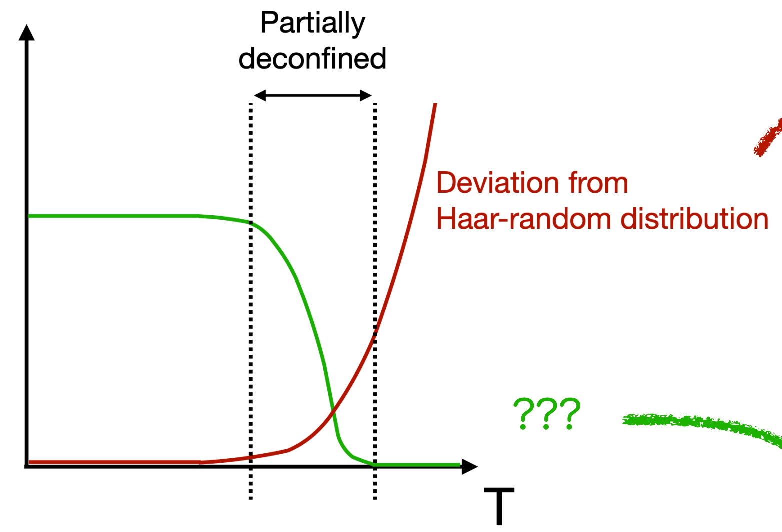
$$\frac{1}{\mathrm{Vol}(G)}\int_G \mathrm{d}g\, u_n(g) = \frac{1}{\mathrm{Vol}(G)}\int_G \mathrm{d}g\, \chi_{\text{trivial}}(g) u_n(g) = \frac{\delta_{n,3}}{3}$$

: Only u_3 contains χ_{trivial}

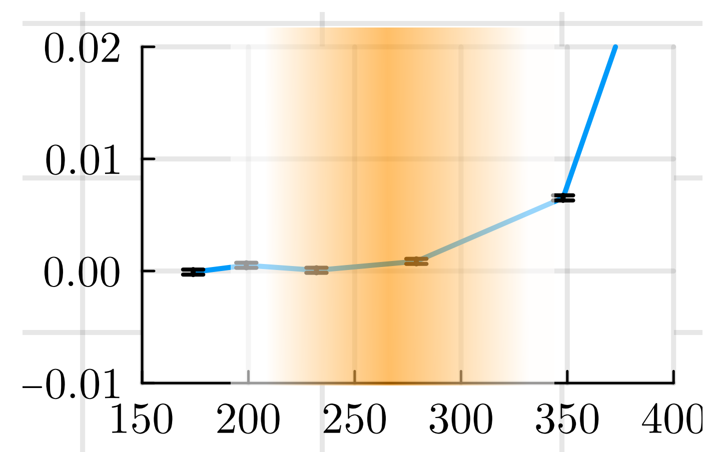
→ stronger than the center symmetry

Relation to typical scale in QCD

Conceptually,

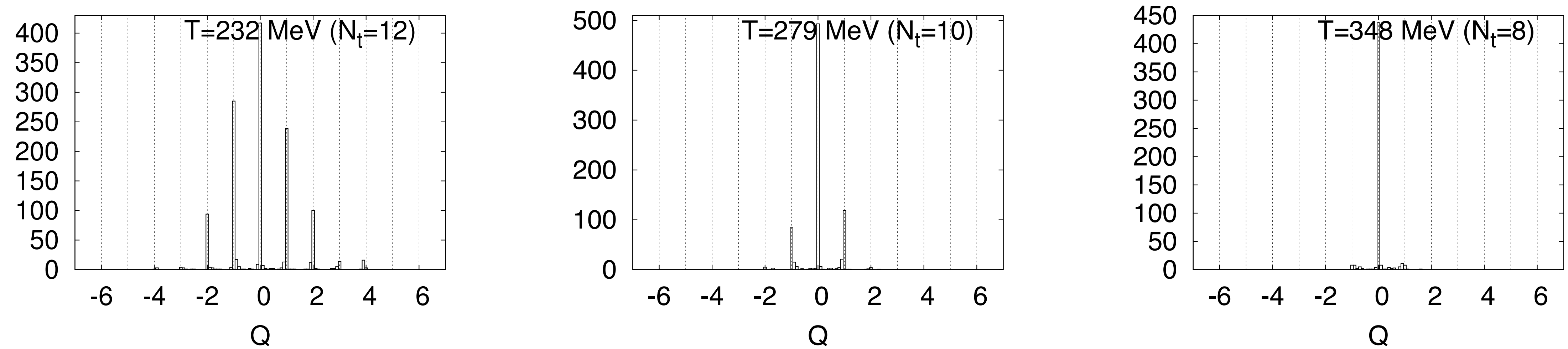


We saw e.g. $\chi_{\rm adj} = 2 + \sum_{j\neq k} e^{i(\theta_j - \theta_k)}$



One answer: instanton condensation c.f.) [Taniguchi, Kanaya, Suzuki, Umeda, (2017)]

topological charge $Q(t)$ obtained by the gradient flow method



- Consistent picture with large-N model analyses [Buividovich, Dunne, Valgushev, (2017)] [Hanada, Holden, Knaggs, O'Bannon, (2021)]
- Nontrivial properties have been recognized in $T_c \leq T \lesssim 3T_c$ [Asakawa, Hatsuda, (1997)] [Glozman, (2023), ⋯]

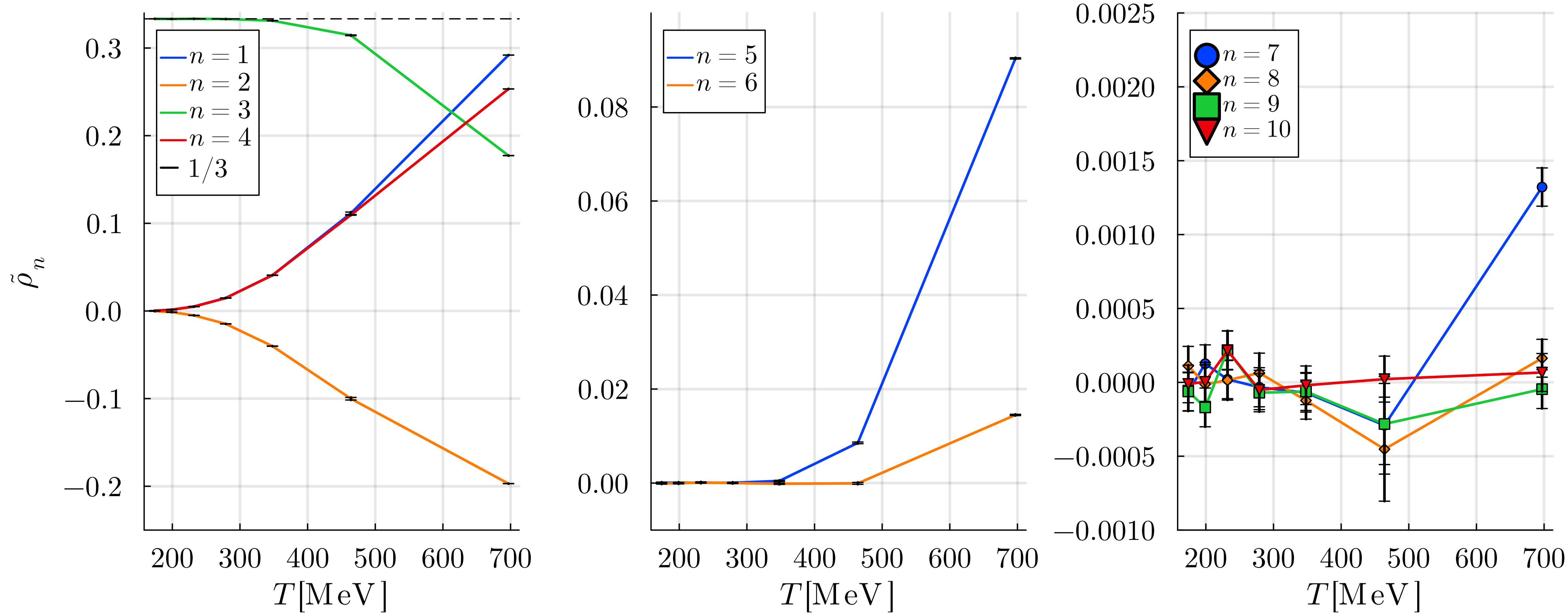
n-wound Polyakov loops

$$\tilde{\rho}_n := \langle u_n \rangle_{\text{space}} = \frac{1}{N} \left\langle \mathrm{tr}\,\mathrm{P}\exp\left[in \oint A_t(x) \right] \right\rangle_{\text{space}} = \int \mathrm{d}\theta\, \rho_{\mathrm{s}}(\theta)\, \mathrm{e}^{in\theta}$$

: n-wound Polyakov loops

If SU(3) Haar random,

$$\tilde{\rho}_3 = \frac{1}{3}, \qquad \tilde{\rho}_{n \neq 3} = 0$$



$\tilde{\rho}_n$ starts to depart at different temperatures → nontrivial intermediate region