Giant graviton expansions for $\mathcal{N} = 2$ SCFTs

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Abstract

Giant graviton expansions of superconformal indices gives the finite rank corrections via AdS/CFT correspondence. We apply and check the relation for $\mathcal{N} = 2$ superconformal field theories constructed on 7-brane background.

Introduction

Superconformal Index

We calculate the superconformal index (SCI).

Superconformal Index(SCI)

- Partition function counts only BPS states.
- Independent of coupling constant ($\frac{dI}{dg}=0$)

[Römelsberger('05)][Kinney,Maldacena,Minwalla,Raju('05)]

We consider 4d $\mathcal{N}=4$ superconformal field theory (SCFT) w/ $SO(4)_{j_1,j_2}\times SO(6)_{R_x,R_y,R_z}$ sym.

Definition of SCI

$$I(q,p,x,y,z):=\mathrm{tr}[(-1)^F q^{j_1}p^{j_2}x^{R_x}y^{R_y}z^{R_z}]\quad (qp=xyz)$$

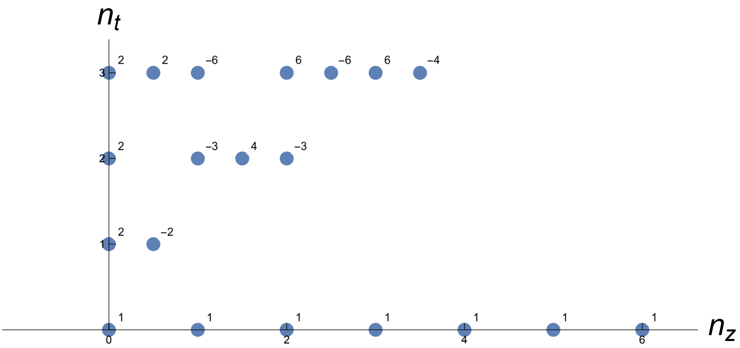
we take $(q,p,x,y,z)\to(tz^{\frac{1}{2}},tz^{\frac{1}{2}},t,t,z)$ for simplicity.

2d Plot

We denote the numerical results of SCI as the 2d plot,

n_z (order of z) - n_t (order of t).

$$I_{U(1)} = (1+z+z^2+\cdots)+t(2-2z^{\frac{1}{2}})+t^2(2-3z+\cdots)+\cdots$$



Index and AdS/CFT correspondence

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The correspondence of SCIs of the AdS side and of the CFT side in the large N limit is known for a long time. [Kinney et. al. ('05)]

The Simplest AdS/CFT Correspondence

CFT

4d $\mathcal{N}=4$
$U(N)$ SYM

$\Longleftrightarrow$

AdS

Type IIB on $AdS_5 \times S^5$
w/ AdS radius $L \propto N^{\frac{1}{4}}$.

In the large N limit, we can calculate the SCI of the AdS side as classical SUGRA.

Relation for SCIs between AdS/CFT Correspondence

$$I_{U(N=\infty)}(t,z) = I_{\text{SUGRA}}(t,z)$$

Giant Graviton

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In finite N region, we must consider the contribution of giant graviton (GG) as finite N correction. [Arai,Imamura('19)]

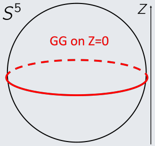
$$I_{U(N)} = I_{\mathrm{SUGRA}}(1 + \cdots)$$

Giant Graviton(GG)

The D3-branes wrapping 3-cycle on the compact space

[McGreevy et. al.('00)]

	0	1	2	3	4	X	Y	Z
	AdS_5					S^5		
GG($Z=0$)	✓					✓✓	✓✓	



CFT side: determinant operators [Gaiotto,Lee('21)]

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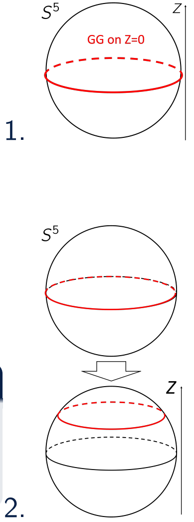
Index of Giant Gravitons

We can separate the SCI into two factors:

1. GG wrapped on great circle $Z=0$
2. Fluctuation = CFT on GG
   Equivalent to the boundary CFT.
   - $z \to z^{-1}$
     The volume becomes smaller.
   - $t \to tz^{\frac{1}{2}}$ $(q, p \leftrightarrow x, y)$
     $so(4)_{j_1,j_2}$ and $so(4)_{R_x,R_y}$ charges are exchanged

Index of n GGs

$$\underline{z^{nN}}\underline{I_{U(n)}(tz^{\frac{1}{2}}, z^{-1})}$$



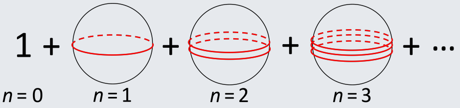
Giant Graviton Expansion

With the GGs, the SCI with finite N region is given in the AdS side:

[Gaiotto,Lee('21)][Imamura('22)]

Giant Graviton Expansion

$$I_{U(N)}(t,z) = I_{U(\infty)}(t,z) \sum_{n=0}^{\infty} z^{nN} I_{U(n)}(tz^{\frac{1}{2}}, \tfrac{1}{z})$$

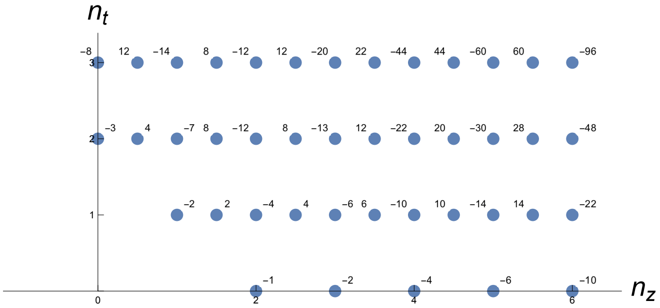


The same function $I_{U(N)}$ emerges on both sides.

Numerical Test for $U(1)$

If the GG expansion holds, the plot will vanish as we gradually add the contributions of GGs.

$$I_{U(N)}(t,z) - I_{U(\infty)}(t,z)$$



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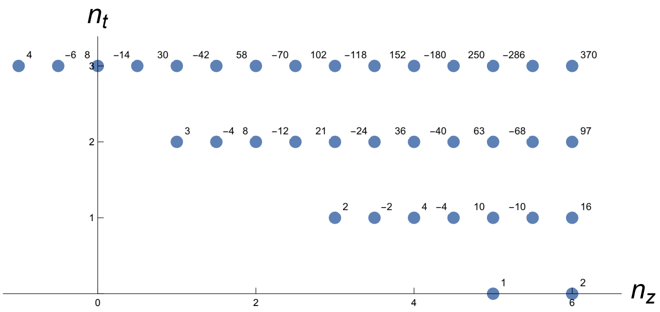
Numerical Test for $U(1)$

If the GG expansion holds, the plot will vanish as we gradually add the contributions of GGs.

$$I_{U(N)}(t,z) - I_{U(\infty)}(t,z)\sum_{n=0}^{1} z^{nN} I_{U(n)}(tz^{\frac{1}{2}}, \tfrac{1}{z})$$



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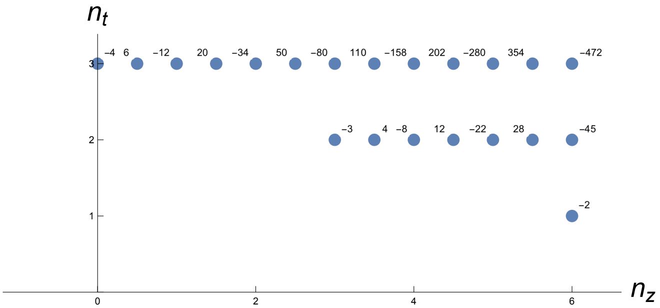
Numerical Test for $U(1)$

If the GG expansion holds, the plot will vanish as we gradually add the contributions of GGs.

$$I_{U(N)}(t,z) - I_{U(\infty)}(t,z)\sum_{n=0}^{2} z^{nN} I_{U(n)}(tz^{\frac{1}{2}},\tfrac{1}{z})$$



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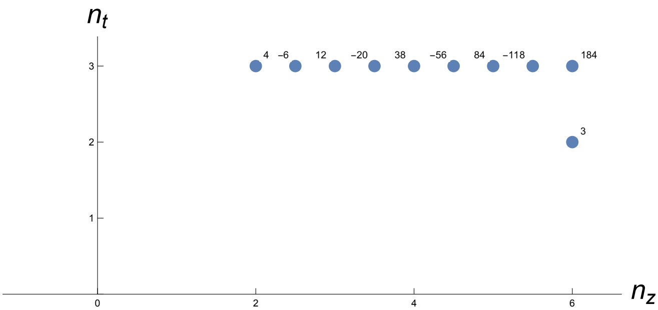
Numerical Test for $U(1)$

If the GG expansion holds, the plot will vanish as we gradually add the contributions of GGs.

$$I_{U(N)}(t,z) - I_{U(\infty)}(t,z) \sum_{n=0}^{3} z^{nN} I_{U(n)}(tz^{\frac{1}{2}}, \tfrac{1}{z})$$



The expansion correctly reproduces the SCI.

Our Work

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The relation of giant graviton expansion is satisfied for other theories.

In this talk, we propose the giant graviton expansion for $\mathcal{N}=2$ SCFTs:

Giant Graviton Expansion for $\mathcal{N}=2$ SCFTs

$$I_{G[N]}(t,z) \stackrel{!}{=} I_{G[\infty]}(t,z)\sum_{n=0}^{\infty} z^{\frac{2}{2-\alpha_G}nN} I_{G[n]}(tz^{\frac{1}{2}},\tfrac{1}{z})$$

and we checked it works by summing a few GGs ($n \leq 3$).

Expansion for $\mathcal{N} = 2$ SCFT

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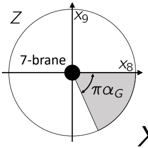
D3 & 7-brane System

Some specific $\mathcal{N}=2$ superconformal field theory $G[N]$ exist on N D3-branes with the backgroud: [Aharony et al.(1998)]

- Deficit angle $\pi\alpha_G$ on Z
- 7-brane with global symmetry G

	0	1	2	3	X	Y	Z
D3	✓	✓	✓	✓			
7	✓	✓	✓	✓	✓✓	✓✓	



$X, Y, Z \in \mathbb{C}$

7 series of CFT

Type	α_G	G	Theory
H_0	$\frac{1}{3}$	None	Argyres-Douglas
H_1	$\frac{1}{2}$	A_1	
H_2	$\frac{2}{3}$	A_2	
D_4	1	D_4	$Sp(N)$
E_6	$\frac{4}{3}$	E_6	Minahan-Nemeschnsky
E_7	$\frac{3}{2}$	E_7	
E_8	$\frac{5}{6}$	E_8	

AdS/CFT Correspondence for $\mathcal{N}=2$ SCFTs

AdS/CFT Correspondence about $\mathcal{N}=2$ SCFTs

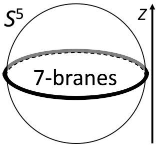
CFT

4d $\mathcal{N}=2$ SCFT
$(H_0,\ H_1,\ H_2,\ D_4,$
$E_6,\ E_7,\ E_8)$

$\Longleftrightarrow$

AdS

Superstring
theory
w/ 7-brane
on $AdS_5 \times S^3$



[Fayyazuddin,Spalinski('98)][Aharony et al.('98)]

We mainly consider $D_4[N]$ since we can calculate SCIs easily:

CFT: $Sp(N)$ gauge theory

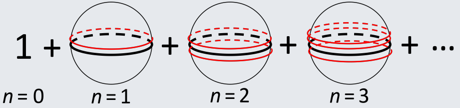
AdS: Orientifold theory w/ O7& D7s

Giant Graviton Expansion for $\mathcal{N}=2$ SCFT

Similar to $\mathcal{N}=4$ $U(N)$ SYM, the SCI in finite N region should be given through the giant graviton expansion:

Giant Graviton Expansion for $\mathcal{N}=2$ SCFTs

$$\underline{I_{G[N]}(t,z)} \stackrel{!}{=} I_{G[\infty]}(t,z)\sum_{n=0}^{\infty} z^{\frac{2}{2-\alpha_G}nN}\underline{I_{G[n]}(tz^{\frac{1}{2}},\tfrac{1}{z})}$$



(D_4: $\alpha_G = 1$)

The corrections from single-wrapping GG are already calculated. [SM,Imamura('21)]

Numerical Test

Numerical Test for $D_4[1]$

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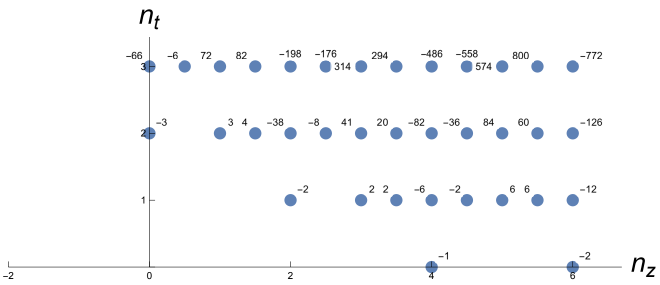
$$I_{D_4[1]}(t,z) - I_{D_4[\infty]}(t,z)$$



Numerical Test for $D_4[1]$

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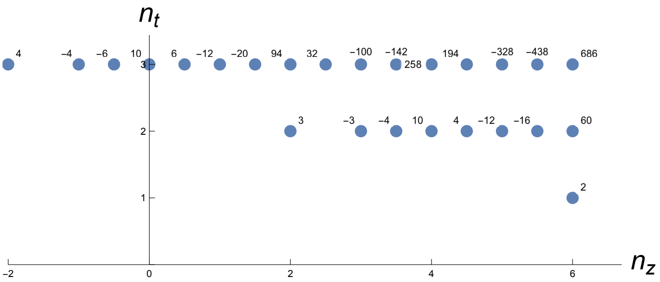
$$I_{D_4[1]}(t,z) - I_{D_4[\infty]}(t,z)\sum_{n}^{1} z^{2n} I_{D_4[n]}(tz^{\frac{1}{2}},\frac{1}{z})$$



Numerical Test for $D_4[1]$

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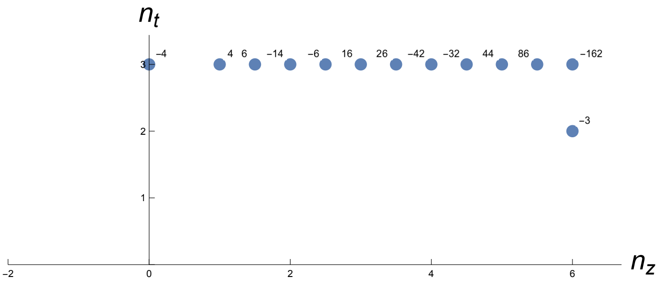
$$I_{D_4[1]}(t,z) - I_{D_4[\infty]}(t,z)\sum_n^{2} z^{2n} I_{D_4[n]}(tz^{\frac{1}{2}}, \tfrac{1}{z})$$



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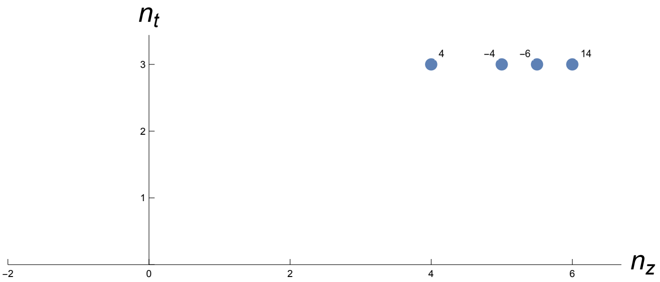
Numerical Test for $D_4[1]$

$$I_{D_4[1]}(t,z) - I_{D_4[\infty]}(t,z)\sum_{n}^{\color{red}{3}} z^{2n} I_{D_4[n]}(tz^{\frac{1}{2}}, \tfrac{1}{z})$$

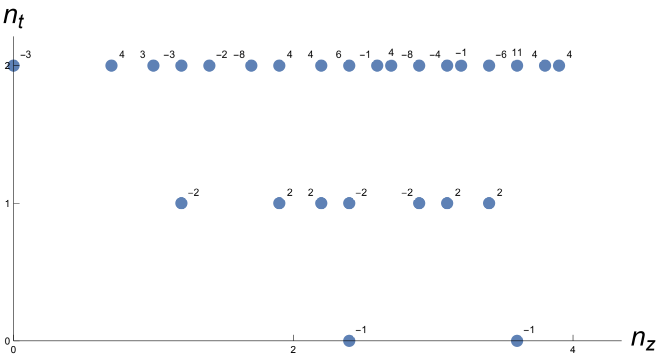


The relation of GG expansion also holds.

Future Work: Numerical Trial for $H_0[1]$

We considered the GG expansion for $H_0[1]$ (Argyres-Douglas theories), guessing the SCI $I_{H_0[n\geq 2]}$.

$$I_{H_0[1]}(t,z) - I_{H_0[\infty]}(t,z)$$



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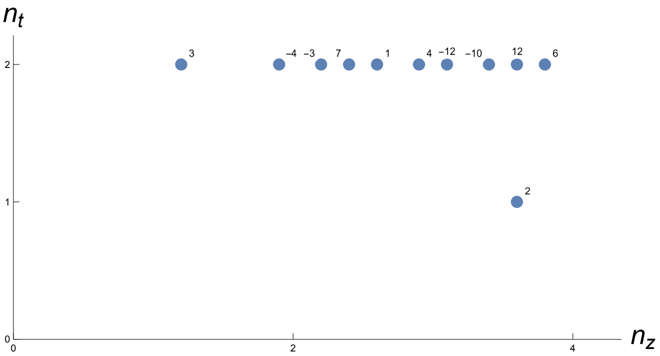
$$I_{H_0[1]}(t,z) - I_{H_0[\infty]}(t,z)\sum_n^{\color{red}1} I_{H_0[n]}(tz^{\frac{1}{2}},\frac{1}{z})$$



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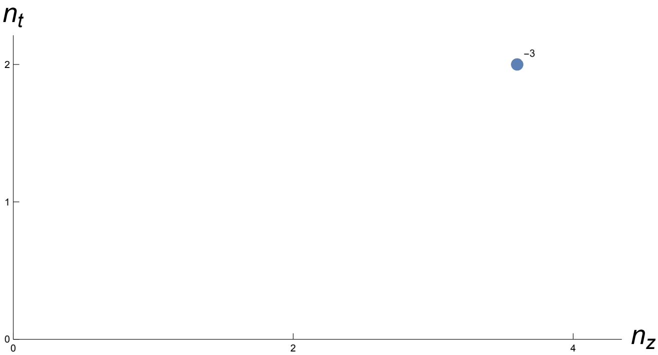

Summary

- The superconformal index is a physical quantity similar to the partition function, counting up only BPS states.
- The relation between the SCIs for the CFT and the AdS sides is described as the giant graviton expansion.
- The giant graviton expansion is also satisfied other than $\mathcal{N} = 4$ $U(N)$ SYM, such as $\mathcal{N} = 2$ SCFT.
- Calculating the SCI of H_0 up higher-order with giant graviton expansion remains a future work.

In addition, we can use GG expansion to

- $AdS_4 \times S^7$, $AdS_7 \times S^4$ (M-theory)
- $AdS_5 \times S^5/\mathbb{Z}_k$
- O3-D3 system ($AdS_5 \times S^5/\mathbb{Z}_2+$ orientifold)

and so on. . . [SM,Imamura,Mori,Fujiwara,Yokoyama('23)]

Thank you for listening

AD& MN

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The other theories belong to . . .

H_0, H_1, H_2 : Argyres-Douglas(AD) theories

E_6, E_7, E_8 : Minahan-Nemeschansky(MN) theories.

Argyres-Douglas and Minahan-Nemeschansky Theories

Specific 4d $\mathcal{N} = 2$ strong-coupled SCFT

- They sometimes emerge in IR limits of supersymmetric gauge or superstring theories.
  - 4d supersymmetric gauge theories [Argyres,Douglas('95)]
  - M-theory [Xie('12),Gaiotto('12)]
- Except for a few theories, their Lagrangians are unknown.

GG expansion for AD and MN theories

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Except for D_4 theories, we do not know the SCIs completely.

N	H_0	H_1	H_2	D_4	E_6	E_7	E_8
1	✓	✓	✓	✓	✓	✓	
2				✓			
3				✓			

✓ (red) : a-maximization

[Maruyoshi,Song('17)]

[Agarwal et. al.('18)]

✓ (blue) : Argyres-Seiberg duality

[Gadde et al.('10)]

It is difficult to calculate the fluctuations in GG expansion...

$$\frac{I_{H_0[N]}}{I_{H_0[\infty]}}(t,z) = 1 + z^{\frac{6}{5}N} I_{H_0[1]}(tz^{\frac{1}{2}}, \tfrac{1}{z}) + \underline{z^{\frac{12}{5}N} I_{H_0[1]}(tz^{\frac{1}{2}}, \tfrac{1}{z}) + \cdots}$$

We guessed the SCI of $H_0[n]$ from $D_4[N]$ and $H_0[1]$ theories up to t^2.