Relation between covariant and light-cone superstring field theory

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String Field Theory

- point particle(e.g. scalar field)

1st quantization

$$0 = (\hat{p}^2 + m^2)\,|\phi\rangle$$

2nd quantization

$$S = \int d^4x\,\frac{1}{2}\left(\phi(\partial^2 + m^2)\phi\right)$$

- string

conformal gauge

$$0 = Q\,|\Psi_{\mathsf{cov}}\rangle$$

covariant SFT

$$S = \int \frac{1}{2}\Psi_{\mathsf{cov}} Q \Psi_{\mathsf{cov}}$$

light-cone gauge

$$0 = (-p^+\partial_+ + L_0^\perp - 1)\,|\Psi_{\mathsf{lc}}\rangle$$

light-cone SFT

$$S = \int \frac{1}{2}\Psi_{\mathsf{lc}}(-p^+\partial_+ + L_0^\perp - 1)\Psi_{\mathsf{lc}}$$

In open bosonic SFT, the light-cone SFT action is derived from the covariant SFT action by a gauge fixing condition.

(T.Erler-H.Matsunaga '20)

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In open bosonic SFT, the light-cone SFT action is derived from the covariant SFT action by a gauge fixing condition.
⇒We extend open super SFT. (T.Erler-H.Matsunaga '20)

Contents

Goal

We impose a gauge condition to the covariant SFT action

$$\int \frac{1}{2}\Psi_{\mathsf{cov}} Q \Psi_{\mathsf{cov}} \to \int \frac{1}{2}\Psi_{\mathsf{fixed}} Q' \Psi_{\mathsf{fixed}}.$$

$$\text{Siegel gauge condition} : b_0 \Psi_{\mathsf{cov}} = 0$$
$$\text{e.o.m.} : c_0 L_0 \Psi_{\mathsf{fixed}} = 0$$

$$\text{Light-cone gauge condition} : \quad ?$$
$$\text{e.o.m.} : (-p^+ \partial_+ + L_0^\perp - 1)\Psi_{\mathsf{lc}} = 0$$

We have to make clear the relation between Ψ_{cov} and Ψ_{lc}

Covariant and Light-cone string field

A string field is a linear combination of 1st quantized state.

Covariant string field

$$\alpha^{\mu}_{-n}\dots b_{-m}\dots c_{-l}\dots e^{ik\cdot X}|0\rangle \in \mathcal{H}_{\mathsf{cov}}$$

Light-cone string field

$$\alpha^{\mu}_{-n}\dots b_{-m}\dots c_{-l}\dots e^{ik\cdot X}|0\rangle \in \mathcal{H}_{\mathsf{lc}}$$

Additionally, the following holds

$$L_0^{\|}\Psi_{\mathsf{lc}} = 0, \quad \Psi_{\mathsf{lc}} \in \mathcal{H}_{L_0^{\|}=0} \subset \mathcal{H}_{\mathsf{lc}}$$

$$L_0^{\|} := \sum_{n\neq 0}(:\alpha^{+}_{-n}\alpha^{-}_{n}: + n : b_{-n}c_{n}:)$$

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$$L_0^{\parallel} \coloneqq \sum_{n \neq 0} (: \alpha^{+}_{-n} \alpha^{-}_{n} : + n : b_{-n} c_n :)$$

The oscillator α^i_n in the light-cone gauge is embedded in the covariant state space through DDF operator

(E.Del Giudice-P.Di Vecchia-S.Fubini '72)

$$A^i_n = i\sqrt{2} \oint \frac{d\xi}{2\pi i} \partial X^i e^{\frac{in}{p_-} X^+}$$

Covariant state space and Light-cone state space

In this talk, we will understand from the point of view of a similarity transformation S.

$$\mathcal{H}_{\text{cov}} = \underbrace{\mathcal{H}_{L_{\text{long}}=0}}_{\text{DDF state}} \oplus \mathcal{H}_{L_{\text{long}}\neq 0} \iff \mathcal{H}_{\text{lc}} = \mathcal{H}_{L_0^{\parallel}=0} \oplus \mathcal{H}_{L_0^{\parallel}\neq 0}$$

$$e^{-\frac{in}{2p_-}x^+} A_n^i = S\alpha_n^i S^{-1}$$
$$L_{\text{long}} = S L_0^{\parallel} S^{-1}$$

$$Q = S Q^{\text{lc}} S^{-1}$$
$$Q-\text{closed}/Q-\text{exact} = Q^{\text{lc}}-\text{closed}/Q^{\text{lc}}-\text{exact}$$

BRS operator

The BRS operator is expanded on ghost and light-cone zero modes.

$$Q = \sum_n c_{-n} L_n^{(m)} - \frac{1}{2} \sum_{n,m} (m-n) : c_{-m} c_{-n} b_{m+n} :$$
$$= \sqrt{2} p_- \sum_{n \neq 0} c_{-n} \alpha_n^- + c_0 L_0 + \dots$$

The first term eliminates the ghost and light-cone oscillator states and the second term imposes the mass-shell condition.

(Kato-Ogawa '83)

$$\left(\sqrt{2} p_- \sum_{n \neq 0} c_{-n} \alpha_n^- \right)^2 = (c_0 L_0)^2 = 0 \quad \text{and} \quad \left\{ \sqrt{2} p_- \sum_{n \neq 0} c_{-n} \alpha_n^-, c_0 L_0 \right\}$$

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Similarity transformation

In general, we assign an appropriate quantum number N

$$Q = Q_0 + Q_1 + \dots, \quad N = \{K, Q_0\}$$

$$S = P\exp\left(-\int_0^1 dt\,\left(t\{K, Q_1\} + t^2\{K, Q_2\} + \dots\right)\right)$$

$$Q = SQ_0S^{-1}$$

We use this result. Our task is

- What is an appropriate number?
  $Q_0 = Q^{\mathrm{lc}}$
- Does K exist?

N-number

We assign an following quantum number called "N-number".

$$\begin{aligned} +1 &: \alpha_n^+, c_n \quad \text{for} \quad n \neq 0 \\ -1 &: \alpha_n^-, b_n \quad \text{for} \quad n \neq 0 \\ 0 &: \text{otherwise} \end{aligned}$$

N-number counting operator and K are given by

$$N := \sum_{n \neq 0} \left(\frac{1}{n} \alpha_{-n}^+ \alpha_n^- + c_{-n} b_n \right), \quad K = \frac{1}{\sqrt{2} p_-} \sum_{n \neq 0} \frac{1}{n} \alpha_{-n}^+ b_n.$$

We can check

$$N = \left\{ K, Q^{\mathrm{lc}} \right\}.$$

Similarity transformation

$$Q = Q_0 + Q_1 + Q_2$$

$$Q^{\mathsf{lc}} = \sqrt{2}\,\underbrace{p_-}_{0}\sum_{n\neq 0}\underbrace{c_{-n}}_{+1}\underbrace{\alpha_n^-}_{-1} + \underbrace{c_0 L_0}_{0} = Q_0$$

$$S = \exp\left(-\frac{1}{\sqrt{2}p_-}\sum_{n\neq 0}\frac{1}{n}\alpha_{-n}^+\left(L_n^{(m)}\Big|_{p^\pm=0} - \sum_m (n+m) b_{-m} c_{n+m}\right)\right)$$

We can confirm

$$e^{-\frac{in}{2p_-}x^+} A_n^i = S\alpha_n^i S^{-1}$$

It is correct transformation.

Light-cone gauge condition

L_{long} is BRS-exact

$$L_{\text{long}} = S L_0^{\|} S^{-1} = \left\{ Q, b_0 + i p_- \oint \frac{d\xi}{2\pi i} \frac{b}{\partial X^+} \right\}.$$

We impose

$$\left(b_0 + i p_- \oint \frac{d\xi}{2\pi i} \frac{b}{\partial X^+} \right) \Psi_{\text{cov}} = 0$$

$$\int \frac{1}{2} \Psi_{\text{cov}} Q \Psi_{\text{cov}} \to \int \frac{1}{2} \Psi_{\text{lc}} (-p^+ \partial_+ + L_0^{\perp} - 1) \Psi_{\text{lc}}$$
$$+ \int \frac{1}{2} \tilde{\Psi}_{L_0^{|} \neq 0} \underbrace{\left(\sqrt{2} p_- \sum_{n \neq 0} c_{-n} \alpha_n^- \right)}_{\text{no derivative}} \tilde{\Psi}_{L_0^{|} \neq 0}$$

Contents

BRS operator

The most important point is how to treat ψ and $\beta\gamma$-ghost zero mode

$$Q = \sum_{n} c_{-n} L_{n}^{(m)} - \sum_{n,m} \frac{1}{2}(n-m) b_{-m-n} c_m c_n$$
$$+ \sum_{r \in \mathbb{Z}+\nu} \gamma_{-r} G_{r}^{(m)} + \sum_{m} \sum_{r \in \mathbb{Z}+\nu} \left(\frac{1}{2}(2r-m) \beta_{-m-r} c_m \gamma_r - b_{-m} \gamma_{m-r} \gamma_r \right)$$

BRS operator

The most important point is how to treat ψ and $\beta\gamma$-ghost zero mode

$$Q = \sum_n c_{-n} L_n^{(m)} - \sum_{n,m} \frac{1}{2}(n-m) b_{-m-n} c_m c_n$$
$$+ \sum_{r\in\mathbb{Z}+\nu} \gamma_{-r} G_r^{(m)} + \sum_m \sum_{r\in\mathbb{Z}+\nu} \left(\frac{1}{2}(2r-m)\beta_{-m-r} c_m \gamma_r - b_{-m}\gamma_{m-r}\gamma_r \right)$$

$$= \underbrace{\sqrt{2}p_- \left(\sum_{n\neq 0} c_{-n}\alpha_n^- + \sum_{r\in\mathbb{Z}+\nu} \gamma_{-r}\psi_r^- \right) + c_0 L_0}_{Q^{\mathrm{lc}}} + \dots$$

The first and second terms eliminate the ghost and light-cone oscillator states and impose

$$\psi_0^- \Psi_{\mathrm{lc}} = 0 \quad \text{for} \quad \text{R sector}$$

N-number

N-number is also

$$\begin{aligned} +1 &: \alpha_n^+, c_n, \psi_r^+, \gamma_r \\ -1 &: \alpha_n^-, b_n, \psi_r^-, \beta_r \\ 0 &: \text{otherwise} \end{aligned}$$

where $n \neq 0$ and all r. N-number counting operator is

$$N := \sum_{n\neq 0}\left(\frac{1}{n}\alpha_{-n}^+\alpha_n^- + c_{-n}b_n\right) + \sum_{r\in\mathbb{Z}+\nu}\left(\psi_{-r}^+\psi_r^- - \gamma_{-r}\beta_r\right)$$

$$K = \frac{1}{\sqrt{2}p_-}\left(\sum_{n\neq 0}\frac{1}{n}\alpha_{-n}^+ b_n - \sum_{r\in\mathbb{Z}+\nu}\psi_{-r}^+\beta_r\right)$$

Similarity transformation

$$Q = Q_0 + Q_1 + Q_2$$

$$Q^{\mathsf{lc}} = \sqrt{2}p_- \left(\sum_{n\neq 0} c_{-n}\alpha_n^- + \sum_{r\in\mathbb{Z}+\nu} \underbrace{\gamma_{-r}\psi_r^-}_{0} \right) + c_0 L_0 = Q_0$$

$$S = e^{-R}$$

$$R = \frac{1}{\sqrt{2}p_-} \sum_{n\neq 0} \frac{1}{n}\alpha_{-n}^+ \left(L_n^{(m)}\Big|_{p^\pm=0} - \sum_m (n+m) b_{-m} c_{n+m} + \sum_{r\in\mathbb{Z}+\nu} \left(r - \frac{n}{2}\beta_{n-r}\gamma_r\right)\right)$$

$$+ \sum_{r\in\mathbb{Z}+\nu} \psi_{-r}^+ \left(G_r^{(m)}\Big|_{p^\pm=0} - \sum_{n\neq 0} b_{-n}\gamma_{n+r} - \sum_{n\neq 0} n c_{-n}\beta_{n+r} \right)$$

This transformation is consistent with the previous work for NS sector

(M.Kohriki-H.Kunitomo-M.Murata '11)

Summary and Future work

Summary

- We constructed the similarly transformation between the covariant and light-cone superstring state space.
- Our transformation relates the oscillators α_n^i, ψ_r^i to the DDF operators.
- We can derive the light-cone SFT free action from the covariant SFT free action for NS sector.

$$\int \frac{1}{2}\Psi_{\mathsf{cov}} Q \Psi_{\mathsf{cov}} \to \int \frac{1}{2}\Psi_{\mathsf{lc}} Q^{\mathsf{lc}} \Psi_{\mathsf{lc}}$$

Future work

- Can we derive the light-cone SFT free action from the covariant SFT free action for R sector?
  It is necessary to understand the transformation for PCO.