Derivation of 11 dimensional spacetime without assuming supersymmetry

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Based on

TM 2305.15161

Today: Spacetime dimensions can be strongly constrained by

Scale invariance & Electric-Magnetic duality

(Simple dimensional analysis is enough to show it.)

→ 10/11 dimensions are obtained as the solutions.

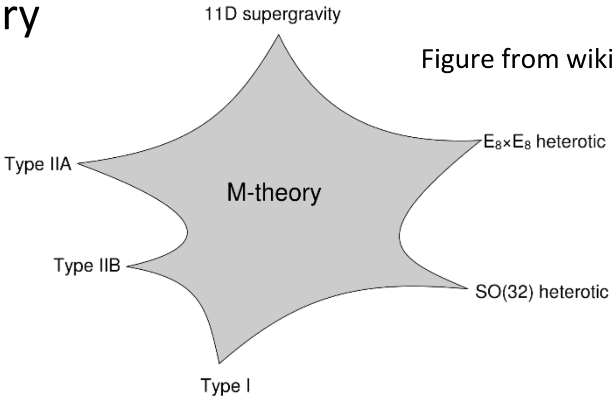
Scale invariance

Electric-Magnetic duality

(Do not assume SUSY/SUGRA.)

⇒ Essential ingredients of superstring theory can be derived.

Figure from wikipedia



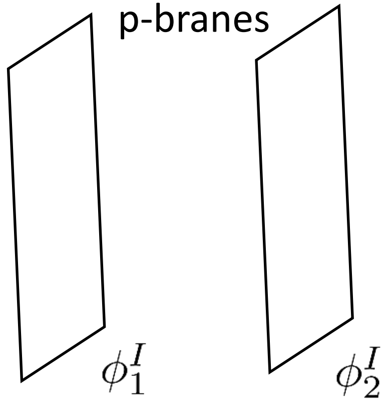
Derivation

★ Set up:

Our underlying theory
- p-brane in D-dimensional spacetime.
- Dirac's quantization → EM-dual (D-p-4)-brane

Q. What is the low energy effective theory of almost parallel two p-branes?

D-dim



canonical normalization
(For Dimensional analysis)

$$S = \int d^{p+1}x \sum_{a=1}^{2} \frac{1}{2}(\partial\phi_a^I)^2 + \cdots$$

ϕ_a^I: coordinates in the D-dimensions

$$\begin{cases} I = 1, \cdots, D-p-1 \\ a = 1, 2 \end{cases}$$

Assumption: The p-branes are not tensionless.

★ Set up:

Our underlying theory
- p-brane in D-dimensional spacetime.
- Dirac's quantization → EM-dual (D-p-4)-brane

Q. What is the possible interaction between the p-branes at long distance?

(low energy)

→ Derivative expansion

Assumption: Scale invariance & translation invariance.

(focus on relative motion)

n: a non-negative integer

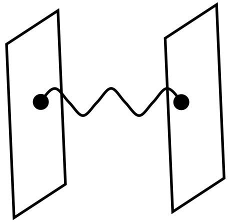
$n = 0, 1, 2, \cdots$

$$S = \int d^{p+1}x \sum_{a=1}^{2} \frac{1}{2}(\partial\phi_a^I)^2 + \frac{(\partial\phi_1 - \partial\phi_2)^{2n}}{|\phi_1 - \phi_2|^X} \cdots$$

no dimensionful coupling

X is determined through a dimensional analysis.

$$[\phi] = \frac{p-1}{2} \quad \rightarrow \quad X = 2(n-1) + \frac{4(n-1)}{p-1}$$



If this interaction is induced via bulk massless modes, X should be an integer and related to D and p via:

D = X+p+3 (e.g. D=4, p=0, X=1)

→ (n,p,D) are constrained.

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→ EM-dual (D-p-4)-brane

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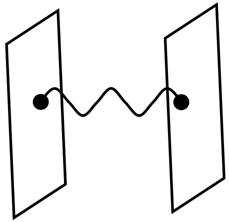

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→ (n,p,D) are constrained.

A. n=0 p=0, X=2 → D=5

n=2 p=2, X=6 → D=11

p=3, X=4 → D=10

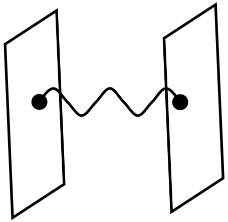
p=5, X=3 → D=11

n=3 p=2, X=12 → D=17

p=3, X=8 → D=14

p=5, X=6 → D=14

p=9, X=5 → D=17



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p=5, X=6 → D=14
~~p=9, X=5 → D=17~~

p-brane → EM-dual (D-p-4)-brane

Common “n” is assumed.

= The bulk dynamics is similar.

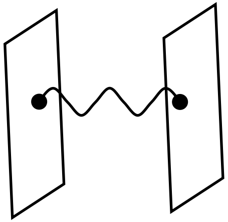
rm

$$\rightarrow \quad X = 2(n-1) + \frac{4(n-1)}{p-1}$$

★ General Solutions: Only two solutions exist

①: (n,p,D)=(2,2,11) and (2,5,11)
→ M-theory (M2 & M5)

n=2 interaction is also consistent with SUGRA.



$$S=\int d^{p+1}x\sum_{a=1}^{2}\frac{1}{2}(\partial\phi_a^I)^2+\frac{(\partial\phi_1-\partial\phi_2)^{2n}}{|\phi_1-\phi_2|^X}\cdots$$

②: (n,p,D)=(n,2n-1,4n+2) $n=1,2,3,\cdots$
→ EM dual of p-brane is p-brane
(p=D-p-4 = 2n-1)
→ self-dual branes?

n=1 → (p,D)=(1,6) → self-dual string in little string theory (non-gravitational theory)
Gustavsson 2003

n=2 → (p,D)=(3,10) → (self-dual) D3-brane in IIB superstring

n=3 → (p,D)=(5,14) → ?
n=4 → (p,D)=(7,18) → ?
n=5 → (p,D)=(9,22) → ?
n=6 → (p,D)=(11,26) → ?
....

} self-dual brane + gravity
→ gravitational anomaly
Álvarez-Gaumé Witten 1984
→ Higher-spin theory?

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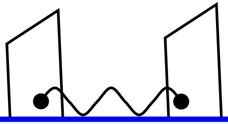

Only these two cases may correspond to conventional gravitational theory.

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Summary

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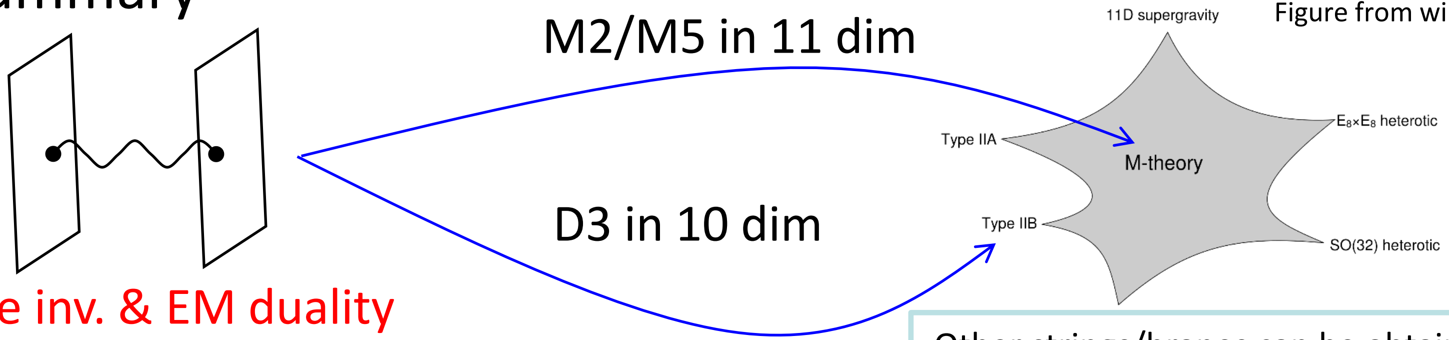

Figure from wikipedia

Scale inv. & EM duality

(& conventional gravity)

Other strings/branes can be obtained via string dualities.

★ Future Direction: Guiding principle of superstring theory?

U(1) gauge symmetry → Maxwell theory

general covariance → General relativity

?? → superstring theory/M-theory

Scale inv. & EM duality can be clues?