Page curves for 2d black holes with multiple injections

particle theory group, CST, Nihon university

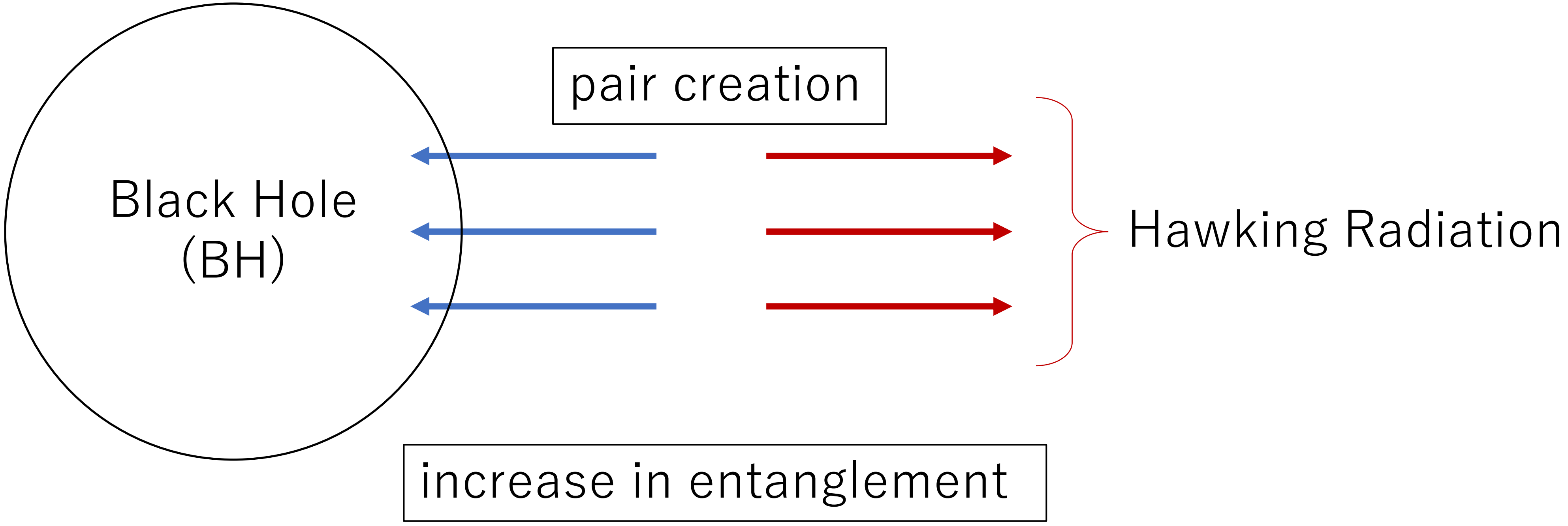
Yuuta Saito

Collaborator：Akitsugu Miwa

KEK Theory Workshop 2023
29 November 2023 to 1 December 2023 in KEK Tsukuba Campus

1. Introduction

Information Paradox



$$\log(\text{number of states in BH}) = \frac{\text{Area}}{4G_N} < S_{\mathbf{rad}}$$

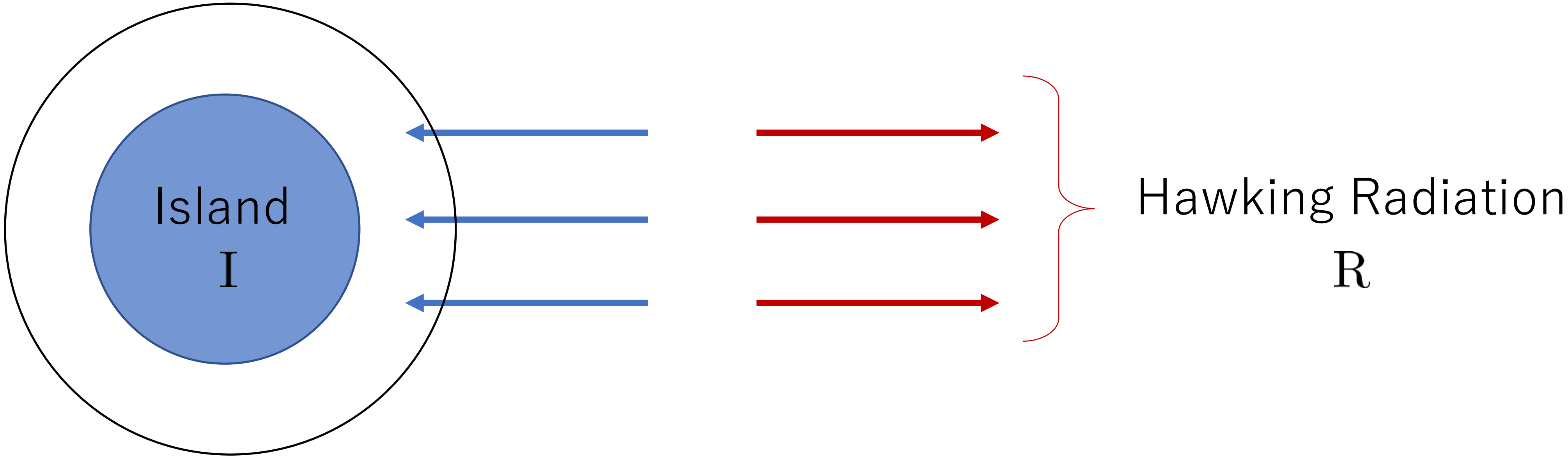
Information Paradox

$S_{\mathbf{rad}}$ should decrease after a specific time.（Page curve D. N. Page ）

Island formula

A. Almheiri, R. Mahajan, J. Maldacena and Y. Zhao, (2020).

$$S_{\mathbf{rad}} = \min_{\mathrm{I}} \operatorname*{ext}_{\mathrm{I}} \left[\frac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I}) \right]$$



island formula → **no information paradox**

… Page curve D. N. Page,

Preceding research of the island formula

K. Goto, T. Hartman and A. Tajdini, (2021).

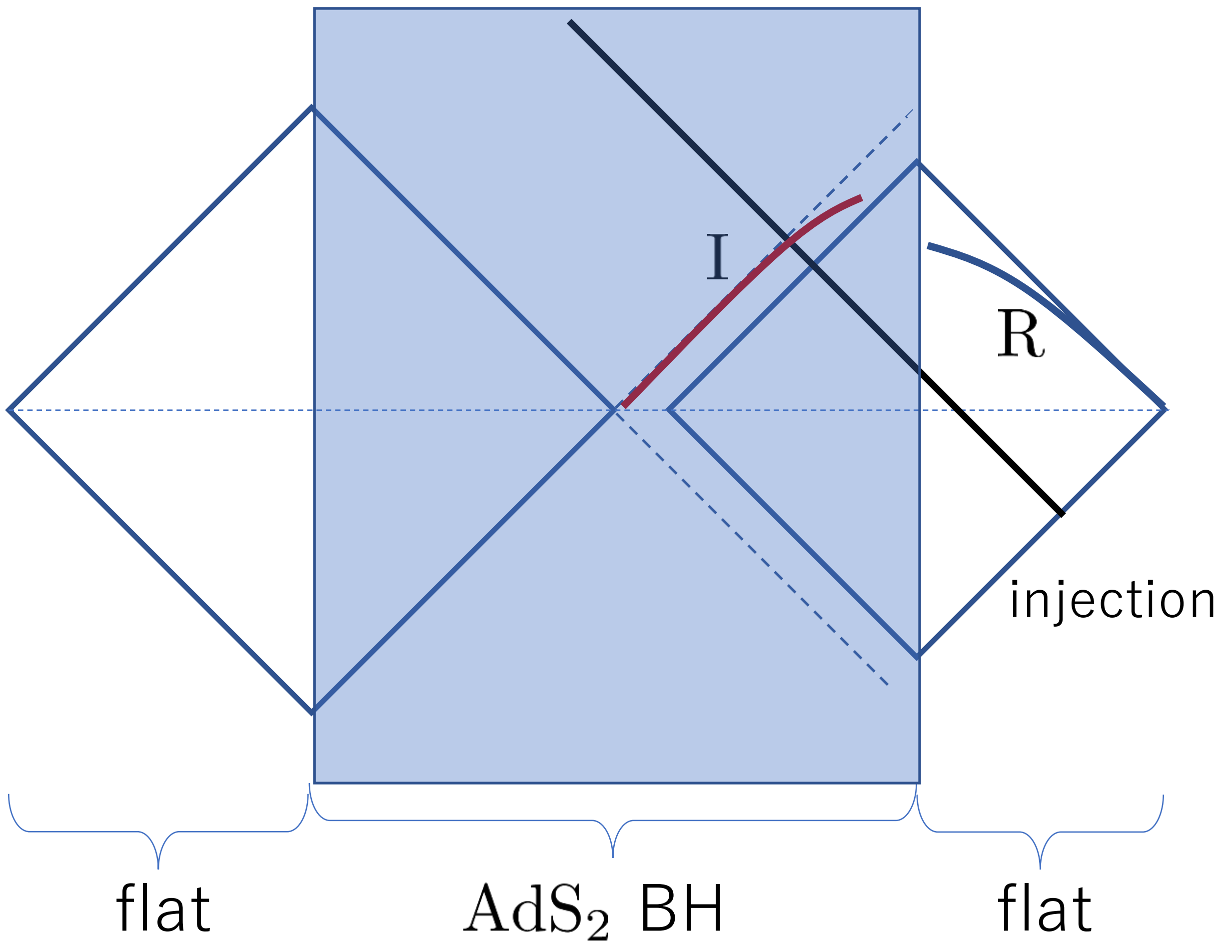
model

Jackiw-Teitelboim(JT) gravity
+
Conformal Field Theory(CFT)

spacetime

AdS_2BH with a single injection
+
2d Flat

☑ resolution of the paradox by the island formula

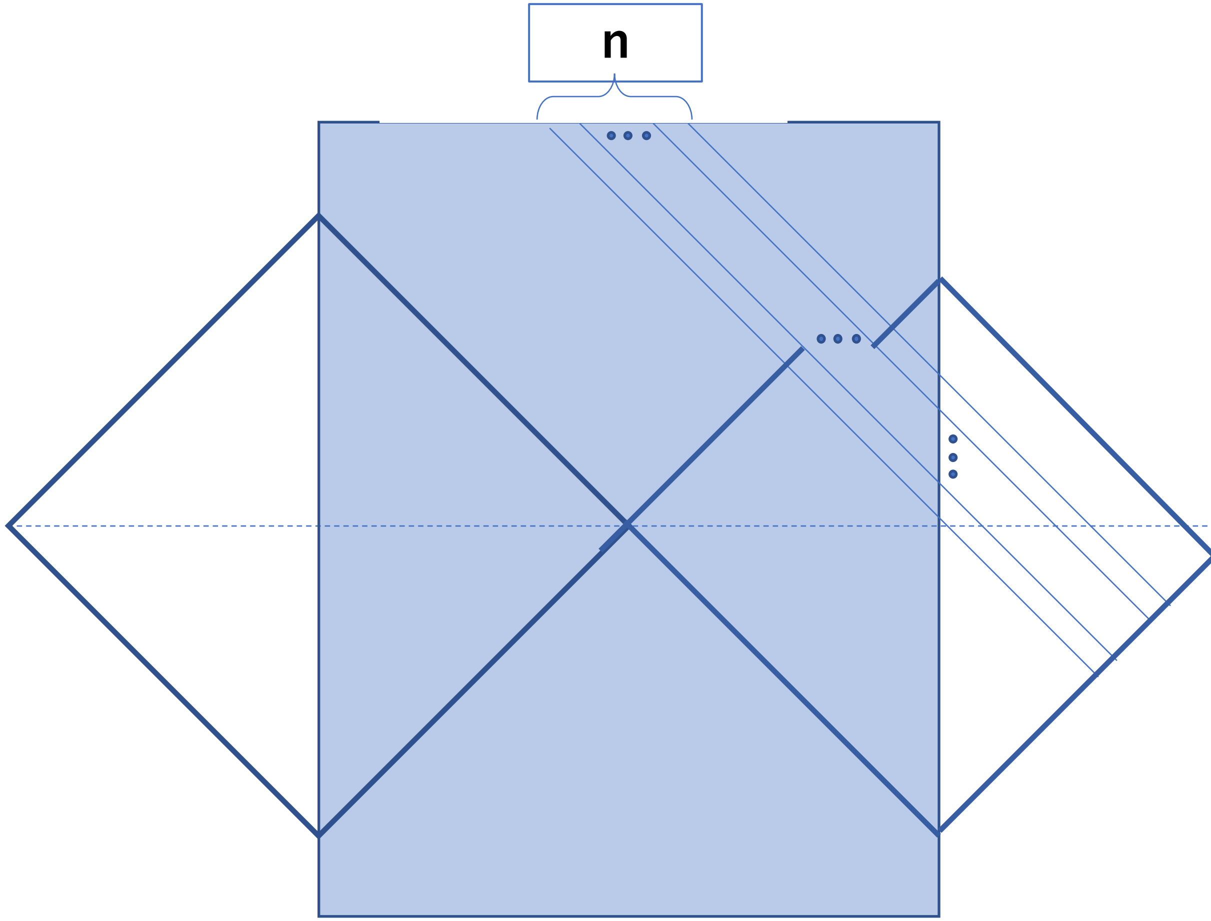


This talk：multiple injections

☑ **solution with multiple injections**

☑ **analysis of the island formula**

☑ **the Page curves of the case n=3**
(numerically)



2. spacetime with multiple injections

JT gravity and the variable of the dynamics

J. Maldacena, D. Stanford and Z. Yang,

- The action： $$I_{JT} = \frac{1}{16\pi G_N}\int \phi(R+2) + \cdots$$

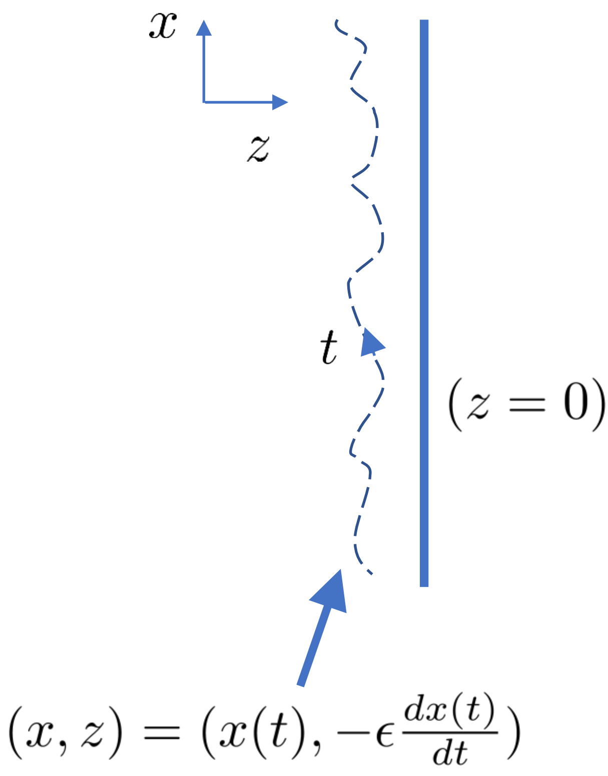
(R : scalar curvature, G_N : Newton constant, ϕ ：dilaton)

- locally AdS_2

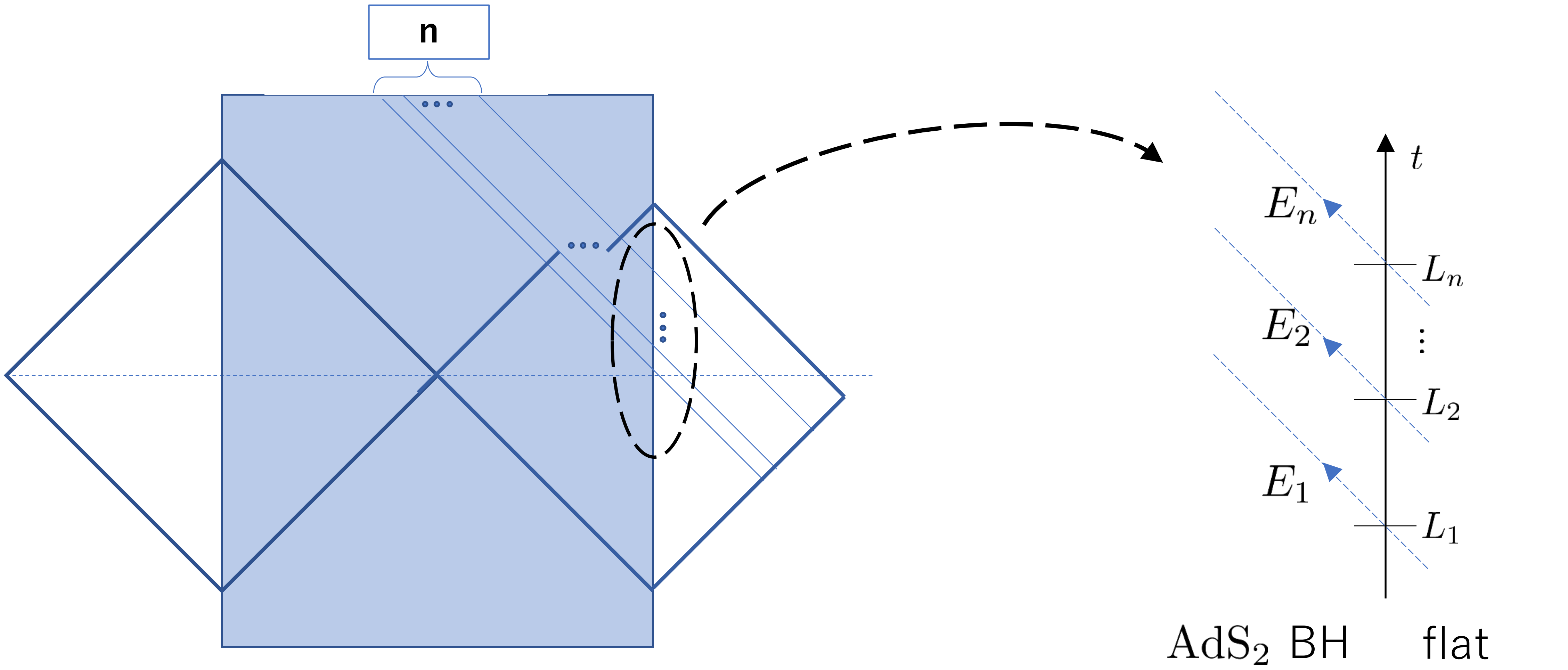
- different boundary → different solution

- $x(t)$: boundary shape (dynamical variable)

(t : boundary time, x : Poincare time)



multiple injections



solution with multiple injections

$$x(t)=\frac{a_kK_\nu^k(t)+b_kI_\nu^k(t)}{c_kK_\nu^k(t)+d_kI_\nu^k(t)}\quad (L_k<t<L_{k+1},\quad L_0=0,\quad \nu=\frac{6\pi\phi_r}{c\beta G_N},\quad c$$ ： central charge)

$$k=0 \qquad K_\nu^0(t)\equiv e^{\frac{\pi}{\beta}t},\qquad I_\nu^0(t)\equiv e^{-\frac{\pi}{\beta}t}$$

$$\begin{pmatrix} a_0 & c_0 \\ b_0 & d_0 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$k=1,2,\cdots,n \qquad K_\nu^k(t)\equiv K_\nu(\nu u_k(t)),\qquad I_\nu^k(t)\equiv I_\nu(\nu u_k(t))$$ (modified Bessel function)

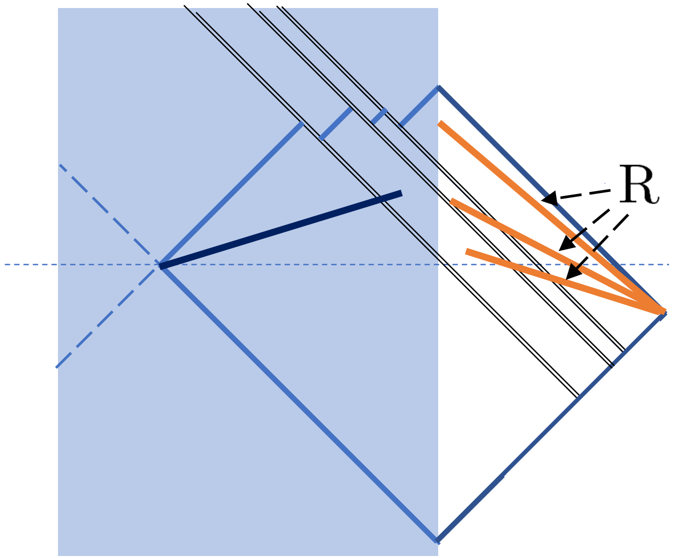
$$\left(u_k(t)=\sqrt{\frac{12\kappa\beta^2}{\pi c}\sum_{i=1}^{k}E_ie^{\kappa L_i}e^{-\frac{\kappa}{2}t}},\quad \kappa=\frac{2\pi}{\beta\nu}\right)$$

$$\begin{pmatrix} a_k & c_k \\ b_k & d_k \end{pmatrix}=\left(\frac{2}{\kappa}\right)\begin{pmatrix} \dot{K}_\nu^k(L_k) & -K_\nu^k(L_k) \\ -\dot{I}_\nu^k(L_k) & I_\nu^k(L_k) \end{pmatrix}\begin{pmatrix} I_\nu^{k-1}(L_k) & K_\nu^{k-1}(L_k) \\ \dot{I}_\nu^{k-1}(L_k) & \dot{K}_\nu^{k-1}(L_k) \end{pmatrix}\begin{pmatrix} a_{k-1} & c_{k-1} \\ b_{k-1} & d_{k-1} \end{pmatrix}$$

3. Page curves in the case n=3 (numerically)

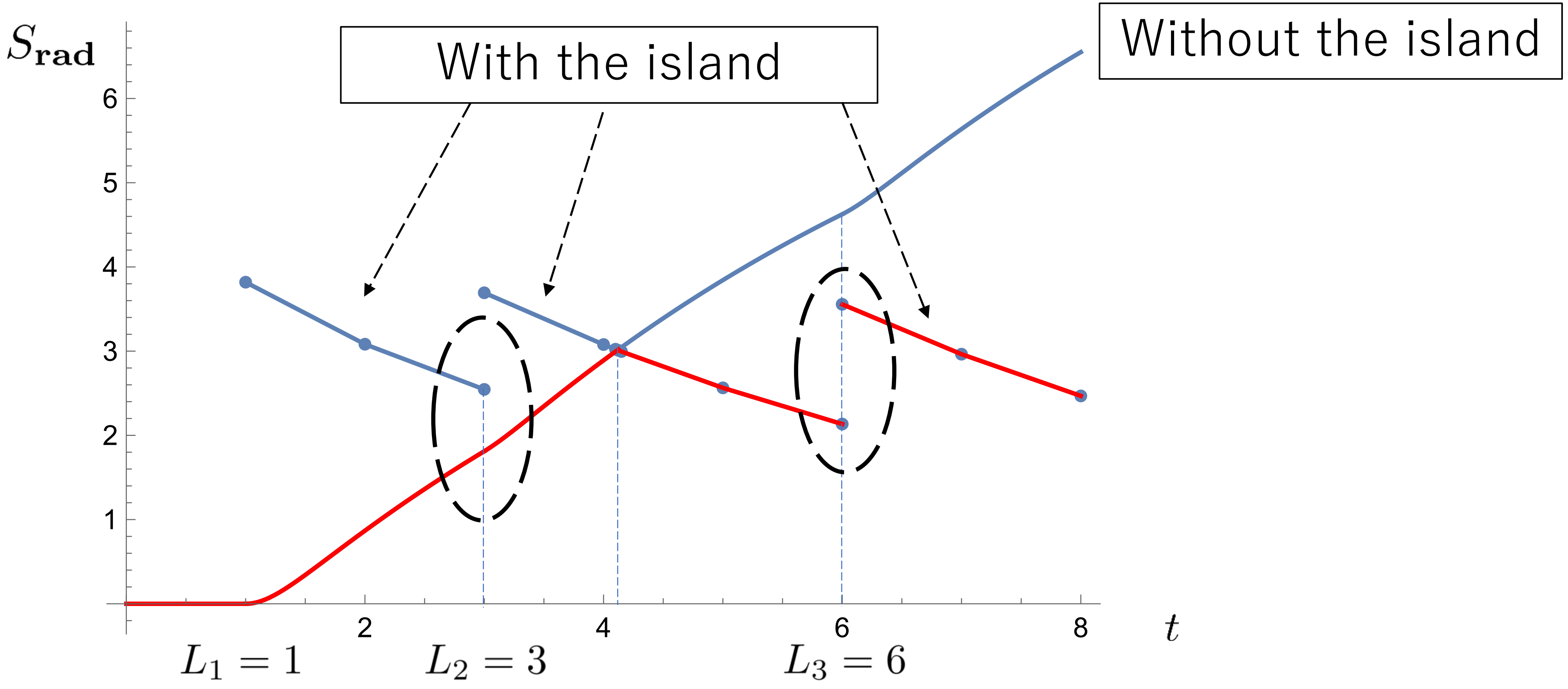
how to derive Page curve (numerically)

- software： “Mathematica” (FindMaximum, Min)
- assumption：
  - radiation region : $\mathrm{R}([0,\infty])$
  - Island’s left endpoint → bifurcation point
  - variation of $S_{\mathbf{rad}}$ → island’s right endpoint (x_A^+, x_A^-)
  - CFT → free fermion
  - following parameters

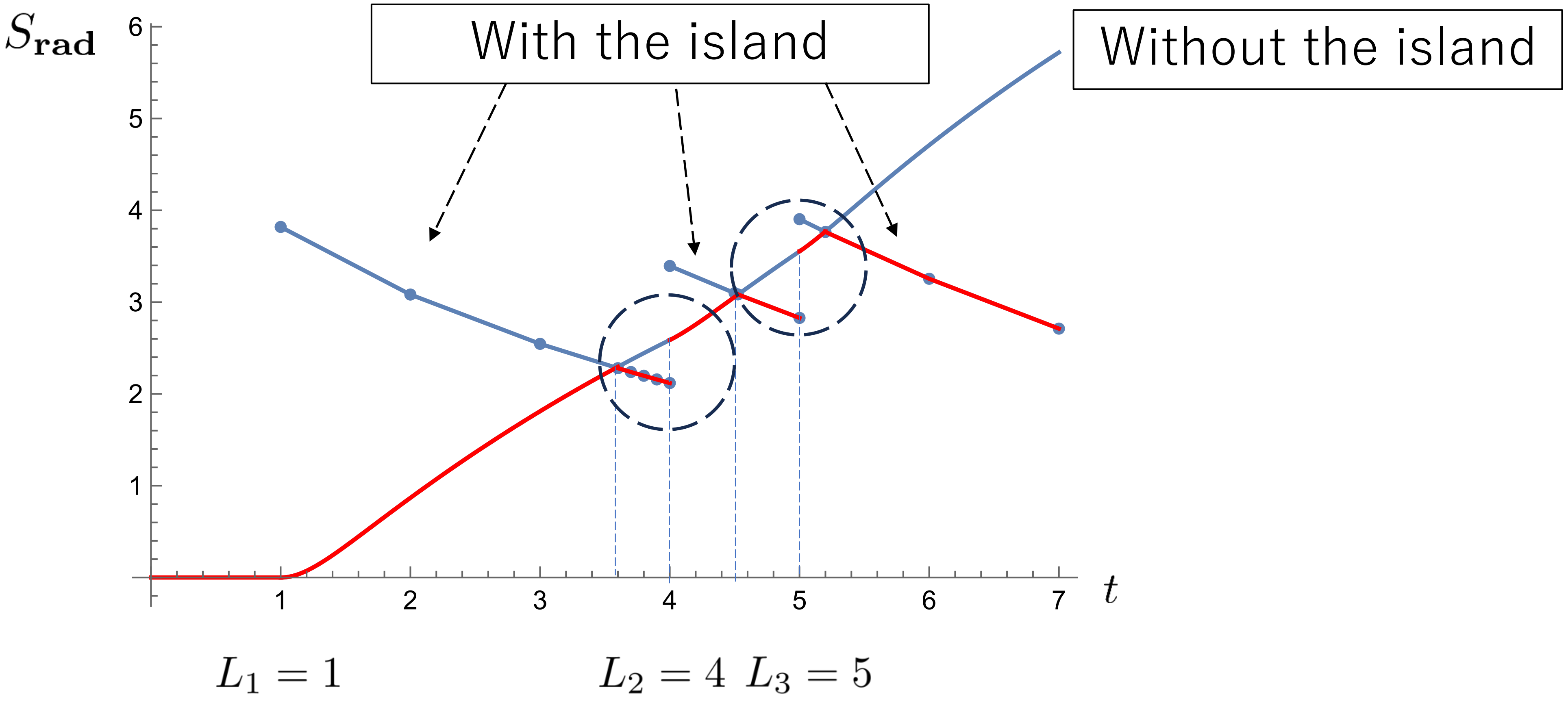


	energy	time		central charge	Newton constant	inverse temperature
first	$E_1 = 2$	$L_1 = 1$		$c = 3$	$G_N = \frac{1}{3}$	$\beta = 2\pi$
second	$E_2 = 1$	$L_2 = 3$	$L_2 = 4$			
third	$E_3 = 1$	$L_3 = 6$	$L_3 = 5$			

Page curve $(L_2 = 3,\ L_3 = 6)$



Page curve $(L_2 = 4,\ L_3 = 5)$



Summery

☑ We generalized the single-injection solution of K. Goto et. al. (2021) to the solution with multiple injections.

☑ We discussed numerical computation of Page curves in the case n=3.

- The injections cause the increase in entanglement entropy, which means the expansion of island.

- Island could either disappear or survive, up to parameters.

Thank you for listening !

Summery

●We generalized K. Goto, T. Hartman and A. Tajdini, (2021).

☑ solution with multiple injections

☑ Page curves in the case n=3 (numerically)

- increase in entanglement entropy by the injection
  (expansion of island)

- change from down line to up line at a special injection time
  (Injections destroy island??)

Summery

☑ We generalized K. Goto, T. Hartman and A. Tajdini, (2021).

☑ In JT gravity, we made the solution in the black hole spacetime with multiple injections.

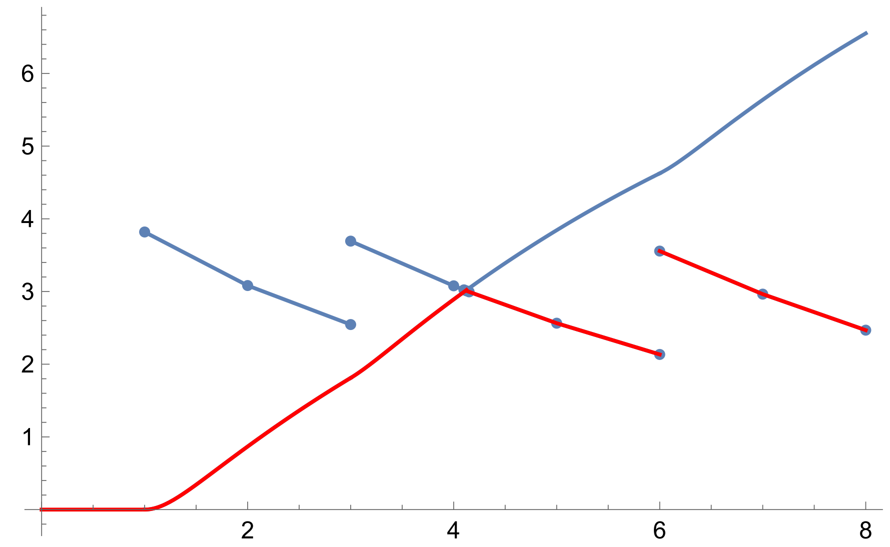
☑ We derived the Page curve in the case n=3 by using the Mathematica as the numerical computation.

☑

☑ In the special parameter, when we injected the energies, the spacetime is no longer one after Page time, so the island disappears.

(Destruction of the island by the injections)

interpretation $(L_2 = 3,\ L_3 = 6)$



how to derive Page curve (numerically)

- software： “Mathematica” (FindMaximum, Min)
- assumption：
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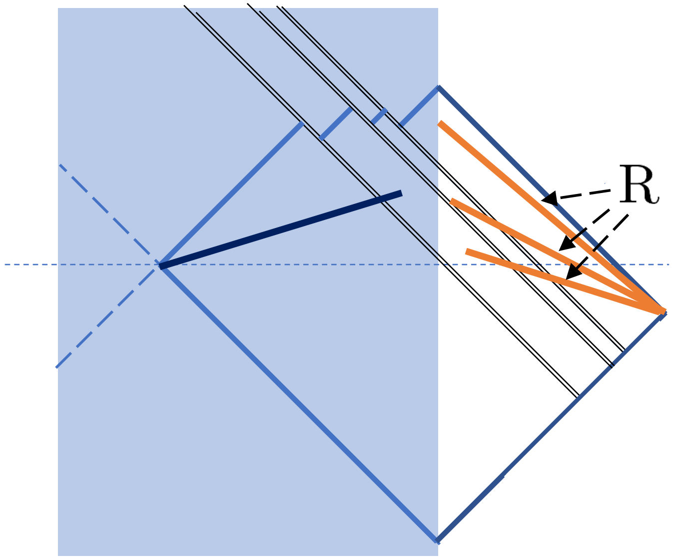

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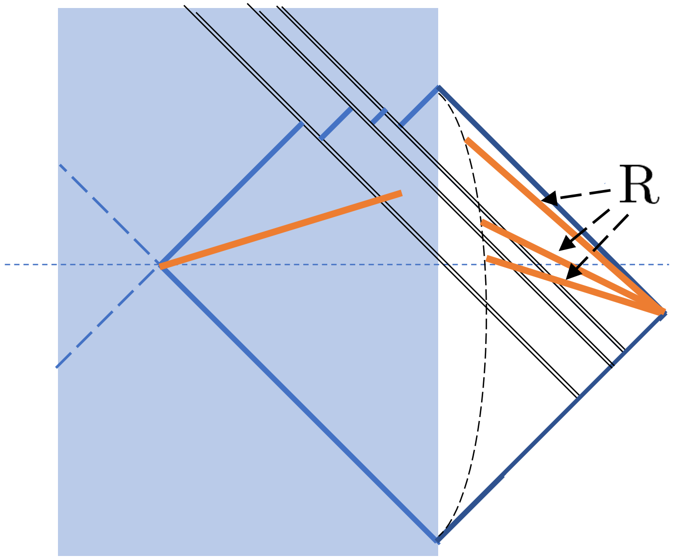

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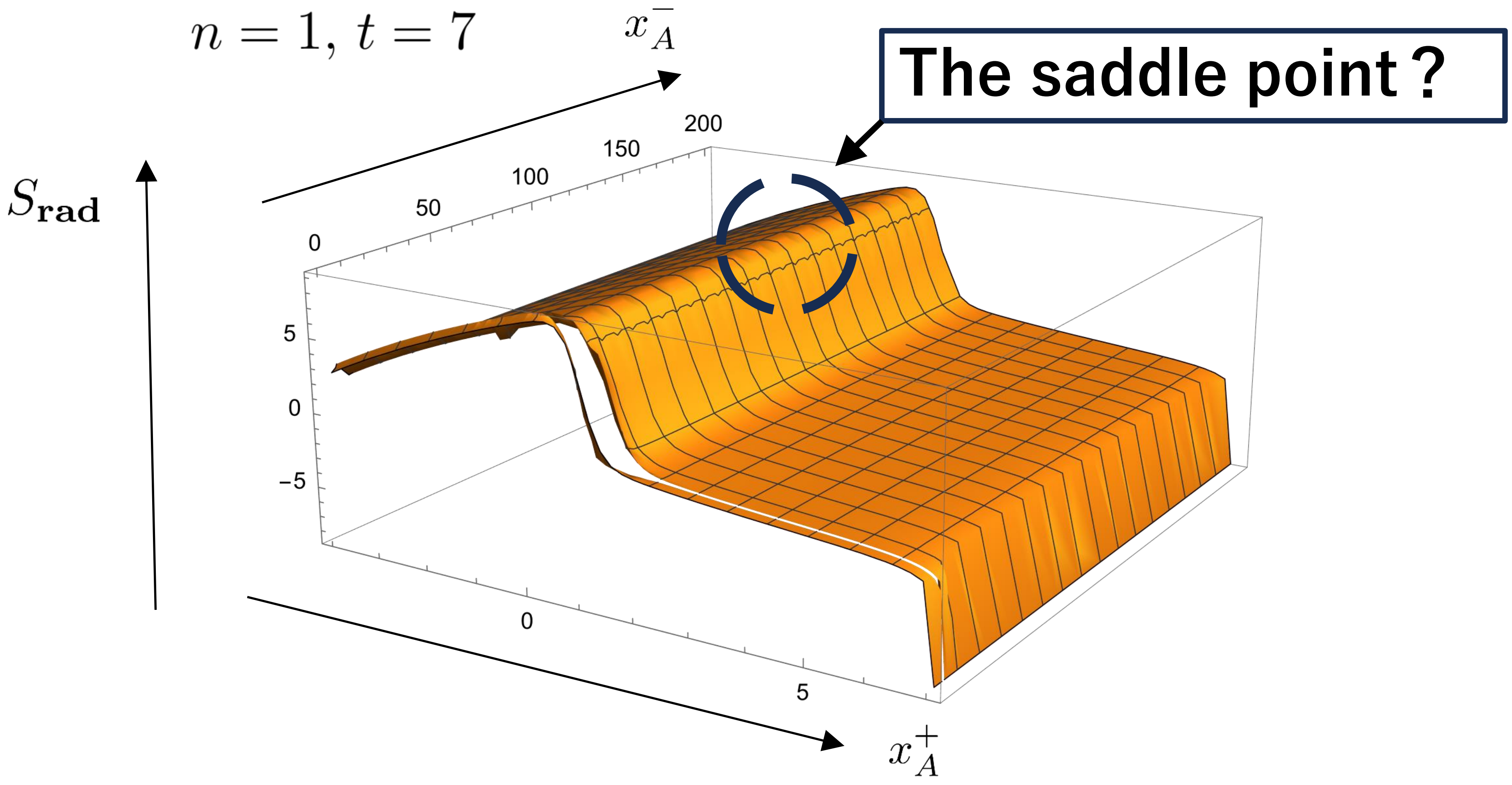
interpretation $(L_2 = 4,\ L_3 = 5)$

The computation of the extremization for the island formula

$$S_{\mathbf{rad}} = \min_{\mathrm{I}} \operatorname*{ext}_{\mathrm{I}} [S_{\mathrm{gen}}(\mathrm{I})] \quad \left(S_{\mathrm{gen}}(\mathrm{I}) = \frac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I}) \right)$$

This !

The computation of the extremization for the island formula



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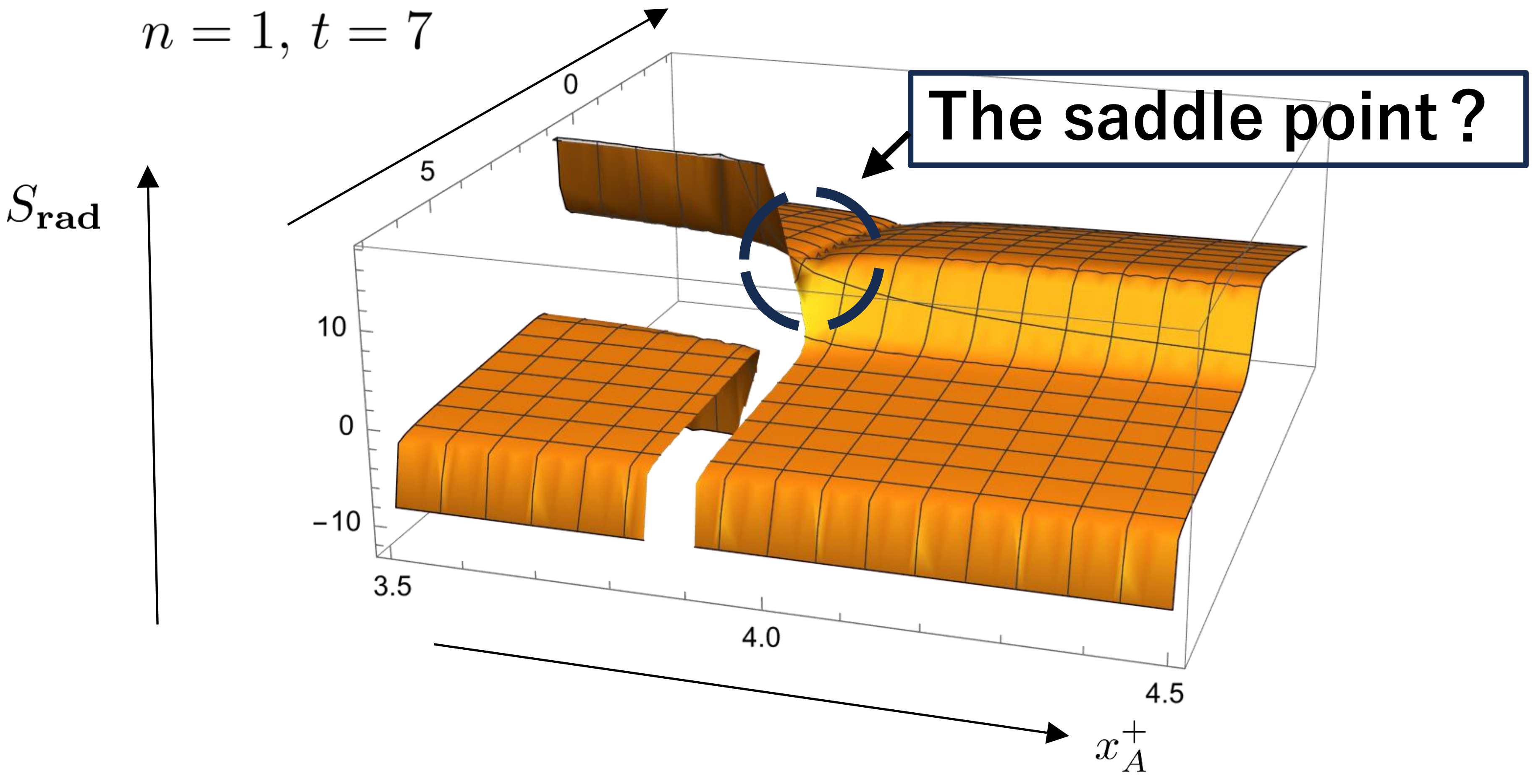
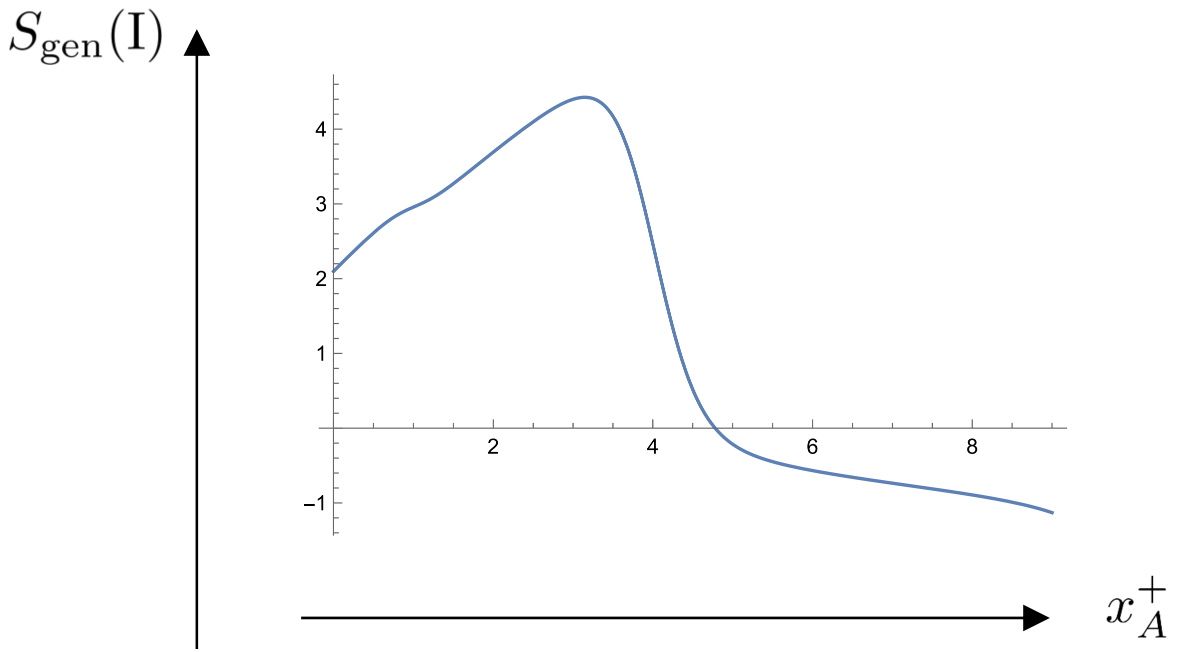


The computation of the extremization for the island formula

1. We move x_A^+ with keeping x_A^- fixed in the arbitrary time t.

   - We can find the maximal value of $S_{\text{gen}}(\text{I})$.

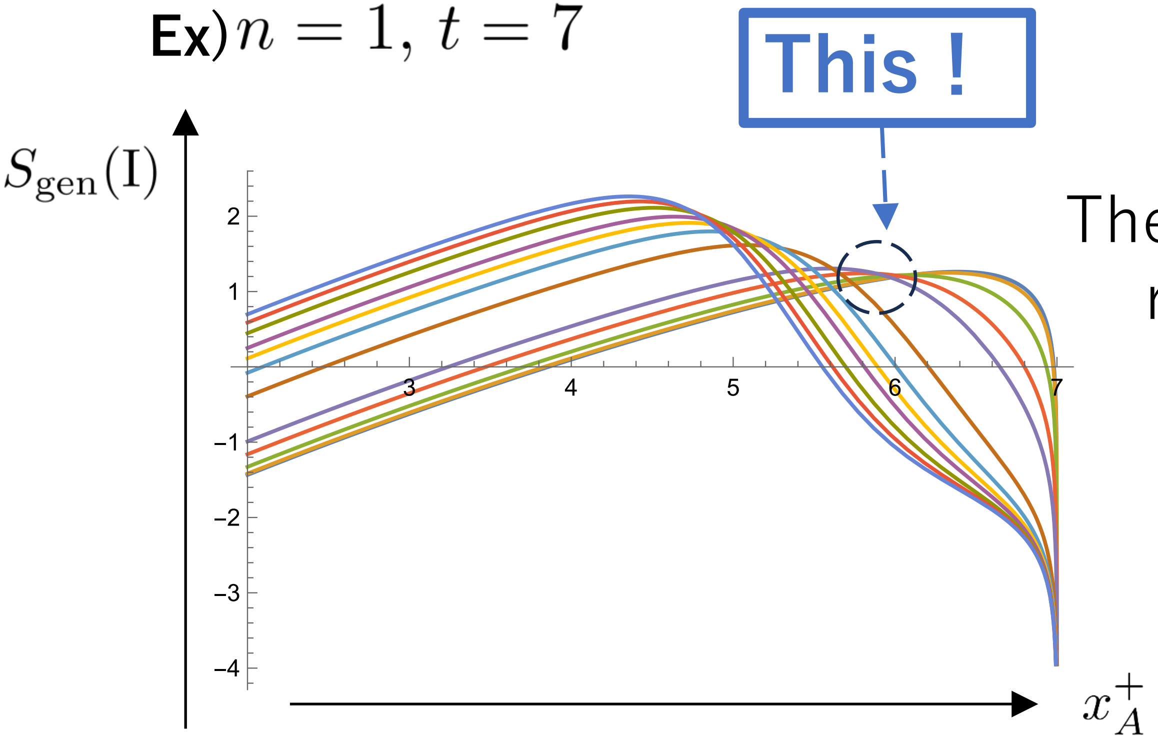
Ex) $n = 1,\, t = 7$



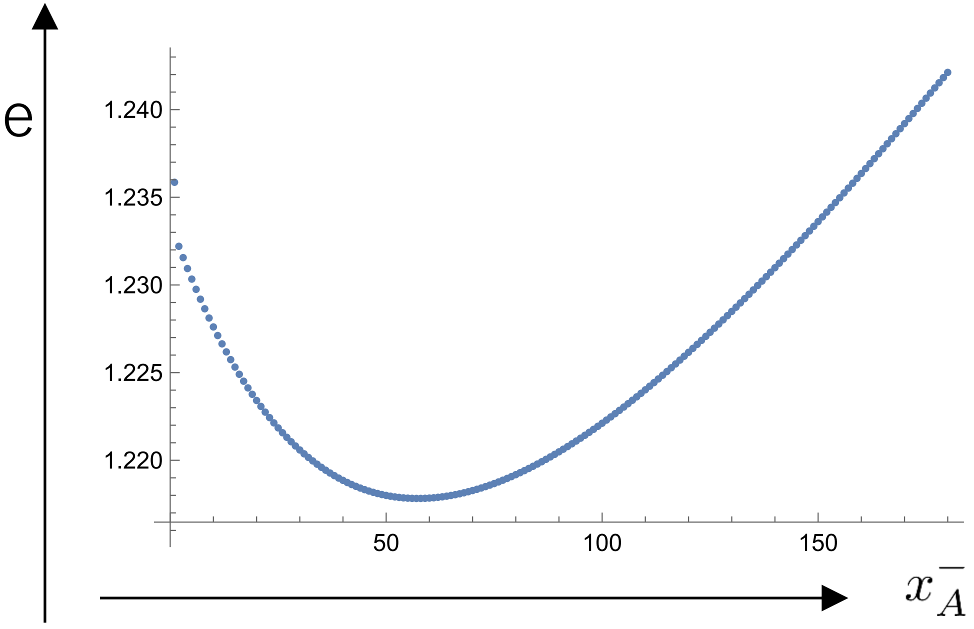
The computation of the extremization for the island formula

2. We move x_A^- , so research the variation of the maximal value.
   - We can find the minimal value of $S_{\text{gen}}(\text{I})|_{x_A^+}$.

Ex) $n = 1,\ t = 7$



The maximal value related to x_A^+ , $S_{\text{gen}}(\text{I})|_{x_A^+}$



The computation of the minimization for the island formula

$$S_{\mathbf{rad}} = \min_{\mathrm{I}} \operatorname{ext}_{\mathrm{I}} \left[S_{\mathrm{gen}}(\mathrm{I}) \right] \quad \left(S_{\mathrm{gen}}(\mathrm{I}) = \frac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I}) \right)$$

This !

The computation of the minimization for the island formula

3. We compare the value of the saddle point with the no-island value , so consider the smaller value as $S_{\mathbf{rad}}$.

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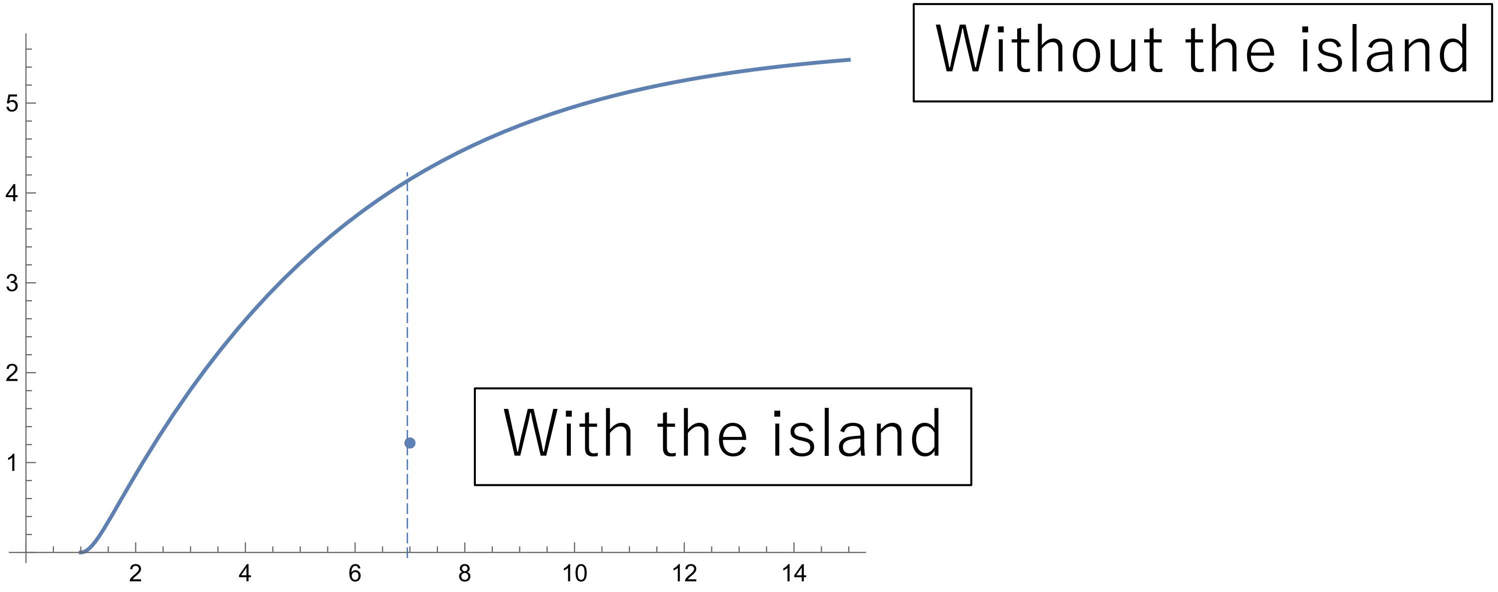

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4. We repeat the manipulation in the other time at region R.

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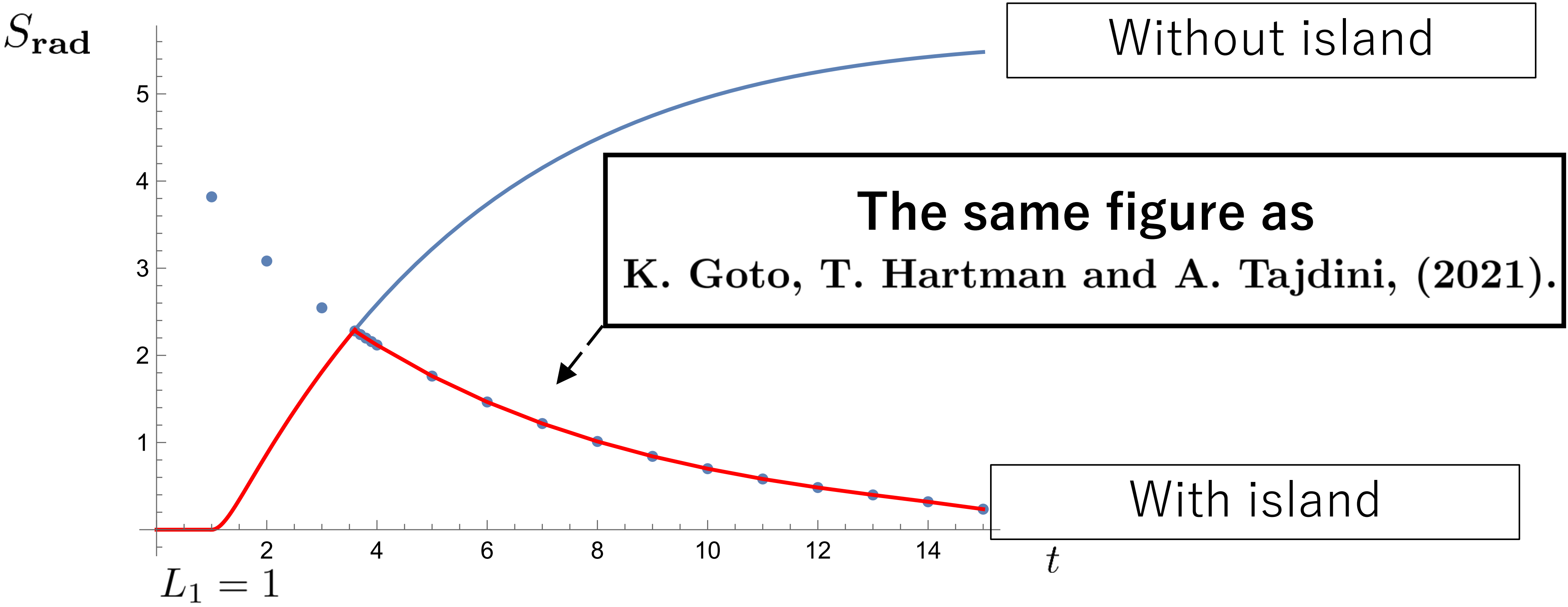
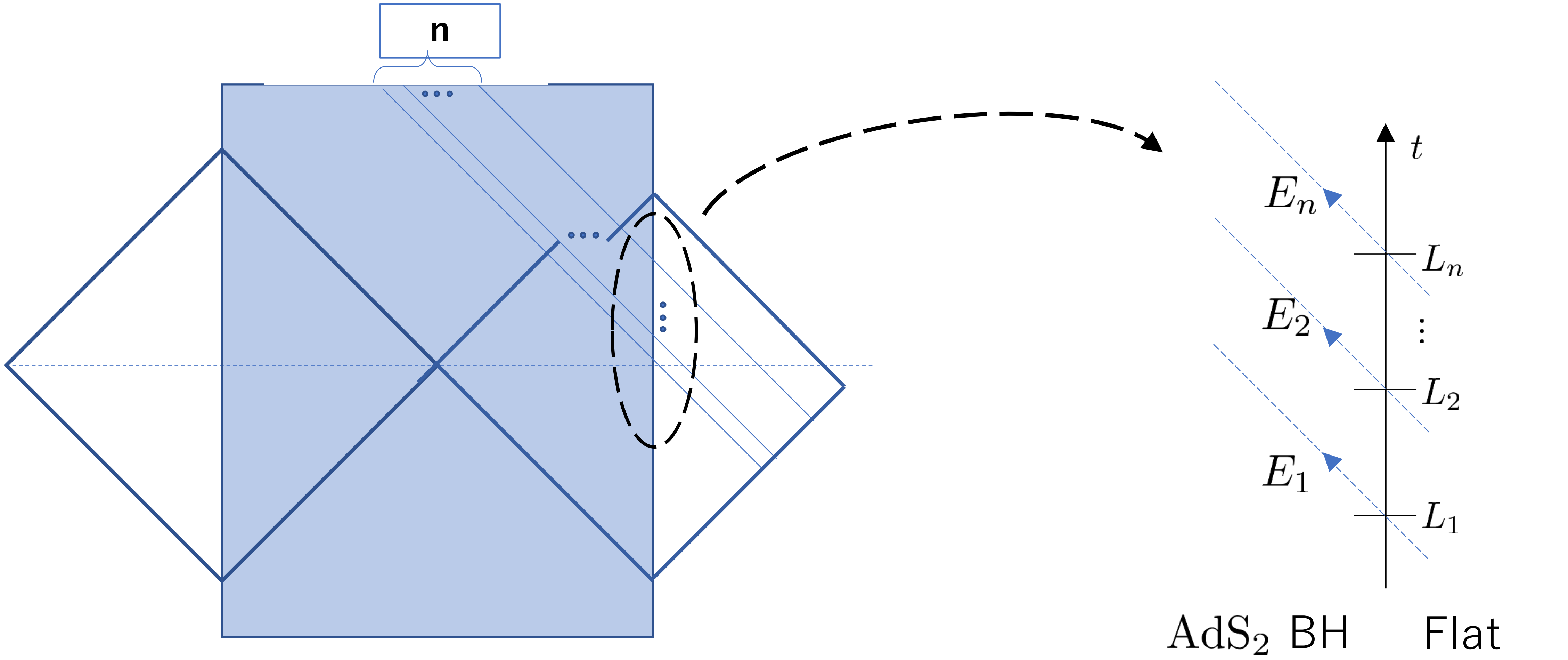


The main in this talk

In the same manipulation,
We draw the Page curve in the case $\mathbf{n = 3}$.

The black hole spacetime with “n” injections



The solution of the spacetime with n injections

$$k=0 \qquad K_\nu^0(t)\equiv e^{\frac{\pi}{\beta}t}, \qquad I_\nu^0(t)\equiv e^{-\frac{\pi}{\beta}t}$$

$$\begin{pmatrix} a_0 & c_0 \\ b_0 & d_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

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$$x(t) = \frac{a_k K_\nu^k(t) + b_k I_\nu^k(t)}{c_k K_\nu^k(t) + d_k I_\nu^k(t)} \qquad \left(L_k < t < L_{k+1}, \quad L_0 = 0, \quad \nu = \frac{6\pi\phi_r}{c\beta G_N}, \quad c\right.$$ ： central charge)

3. The derivation of the Page curve by the numerical computation in the case n=3

（**The main talk**）

The derivation of Page curve by using the numerical computation

- The software：Formula manipulation system “Mathematica” (FindMaximum, Min)
- The assumption：
  - We observe the Hawking radiation at $\mathrm{R}([0,\infty])$.
  - There is a left endpoint of the island in the bifurcation point.
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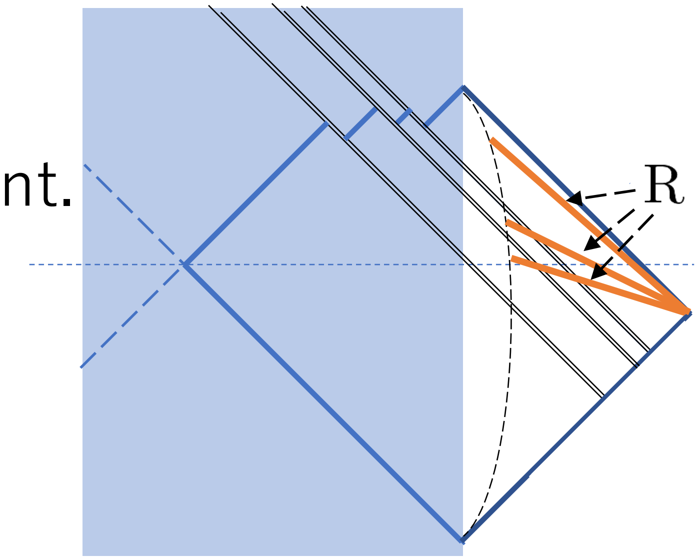

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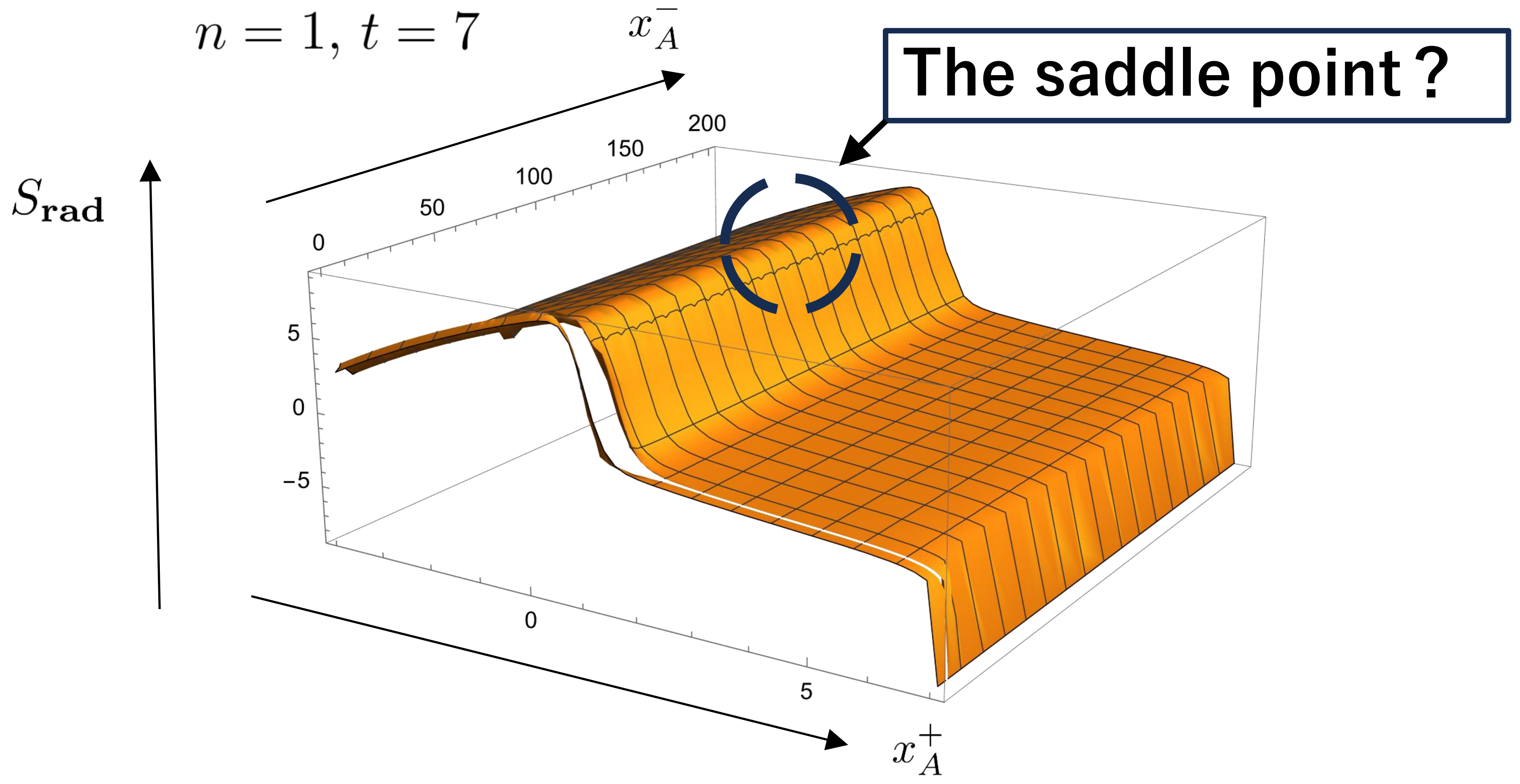

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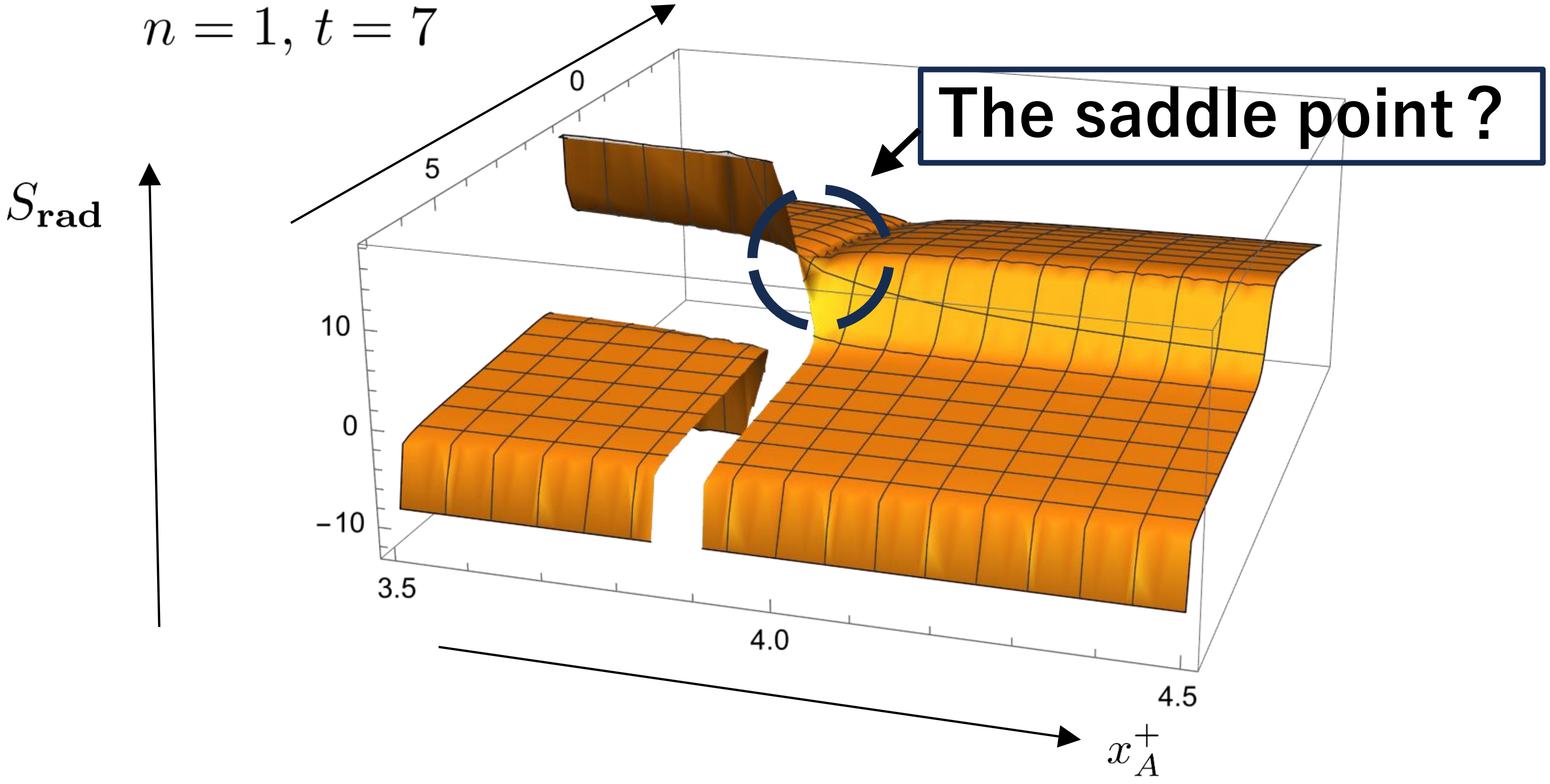

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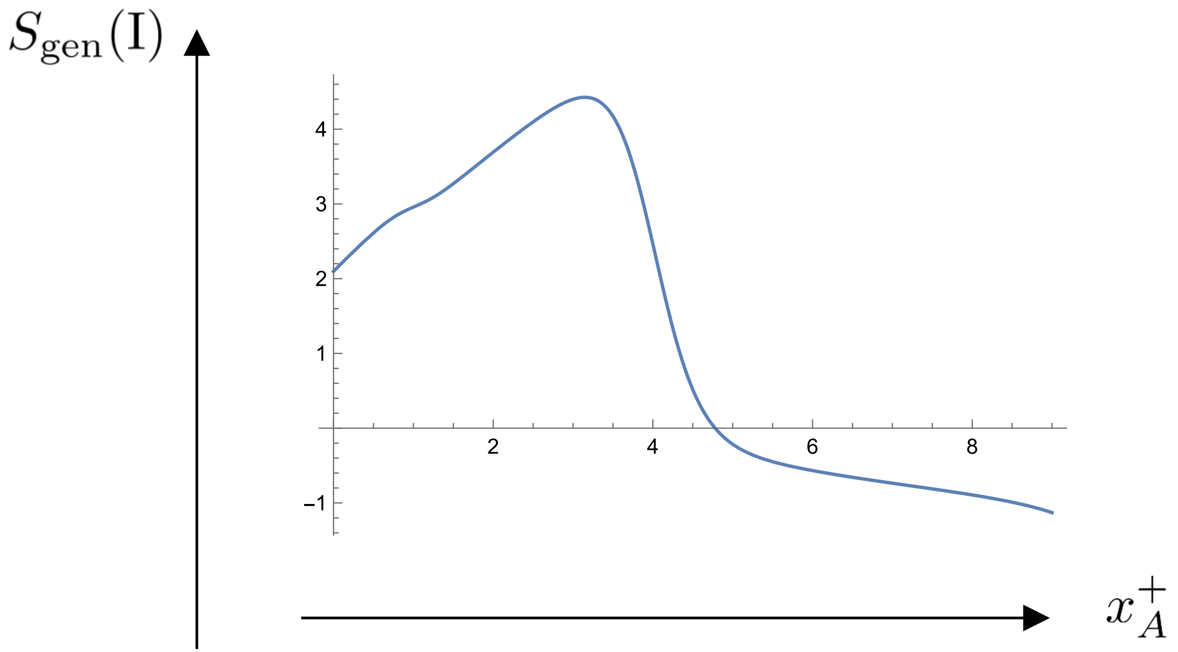

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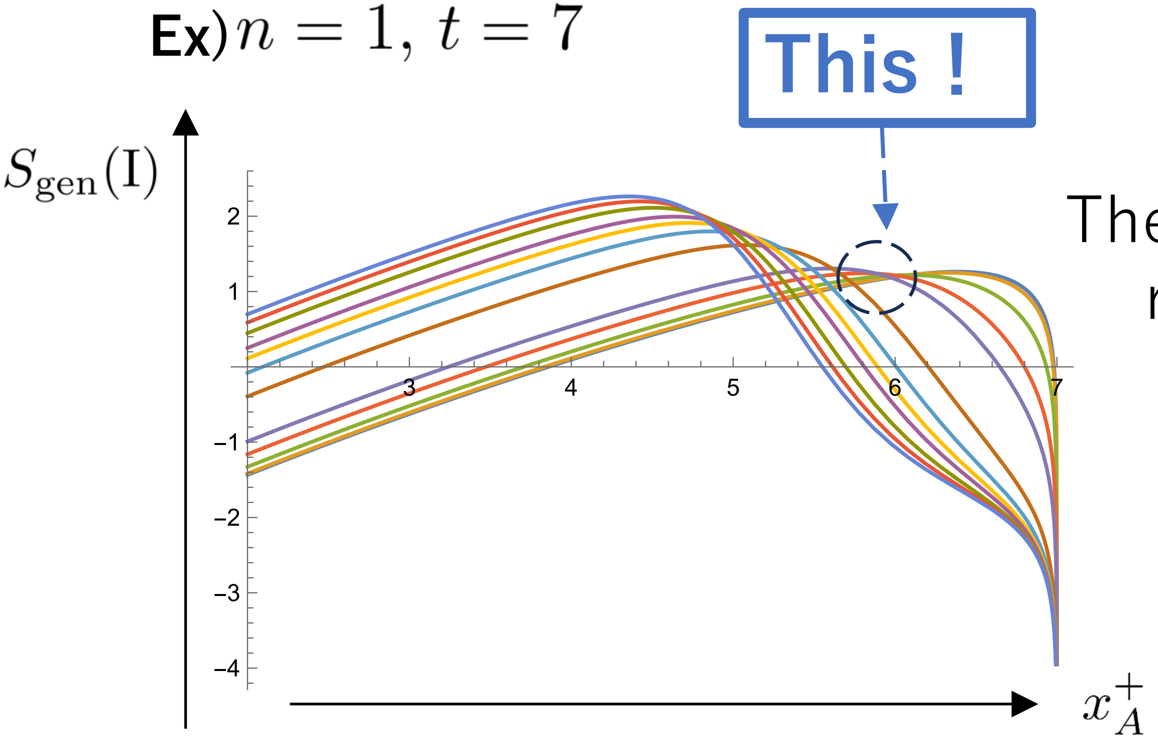
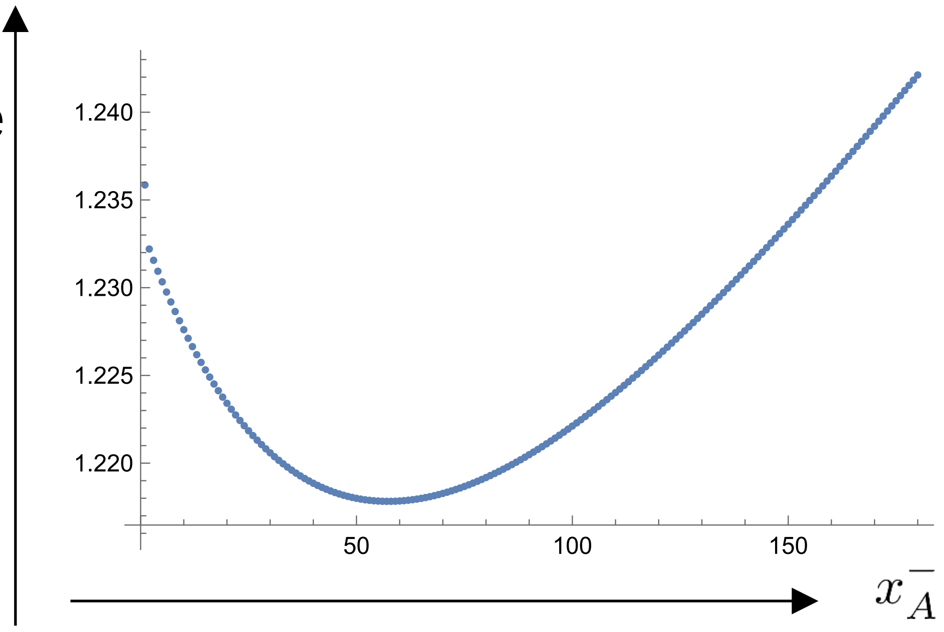


The maximal value related to x_A^+ , $S_{\text{gen}}(\text{I})|_{x_A^+}$



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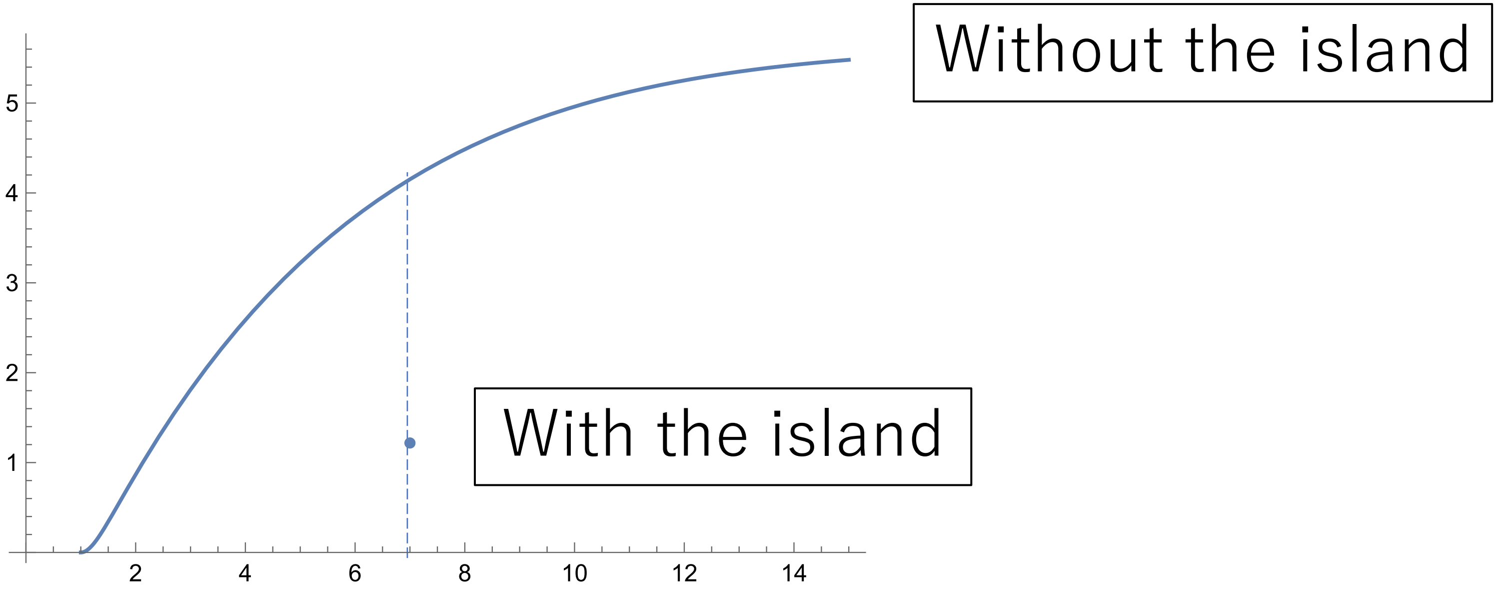

The computation of the minimization for the island formula

4. We repeat the manipulation in the other time at region R.

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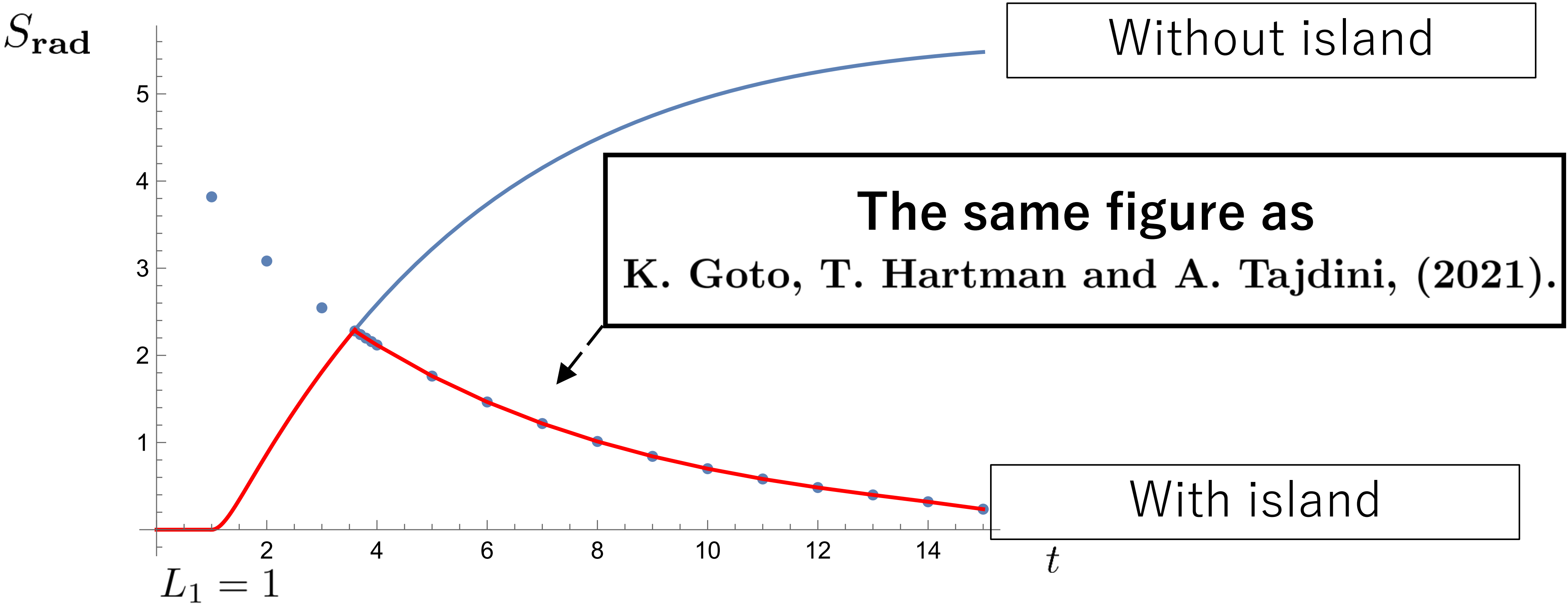

The main in this talk

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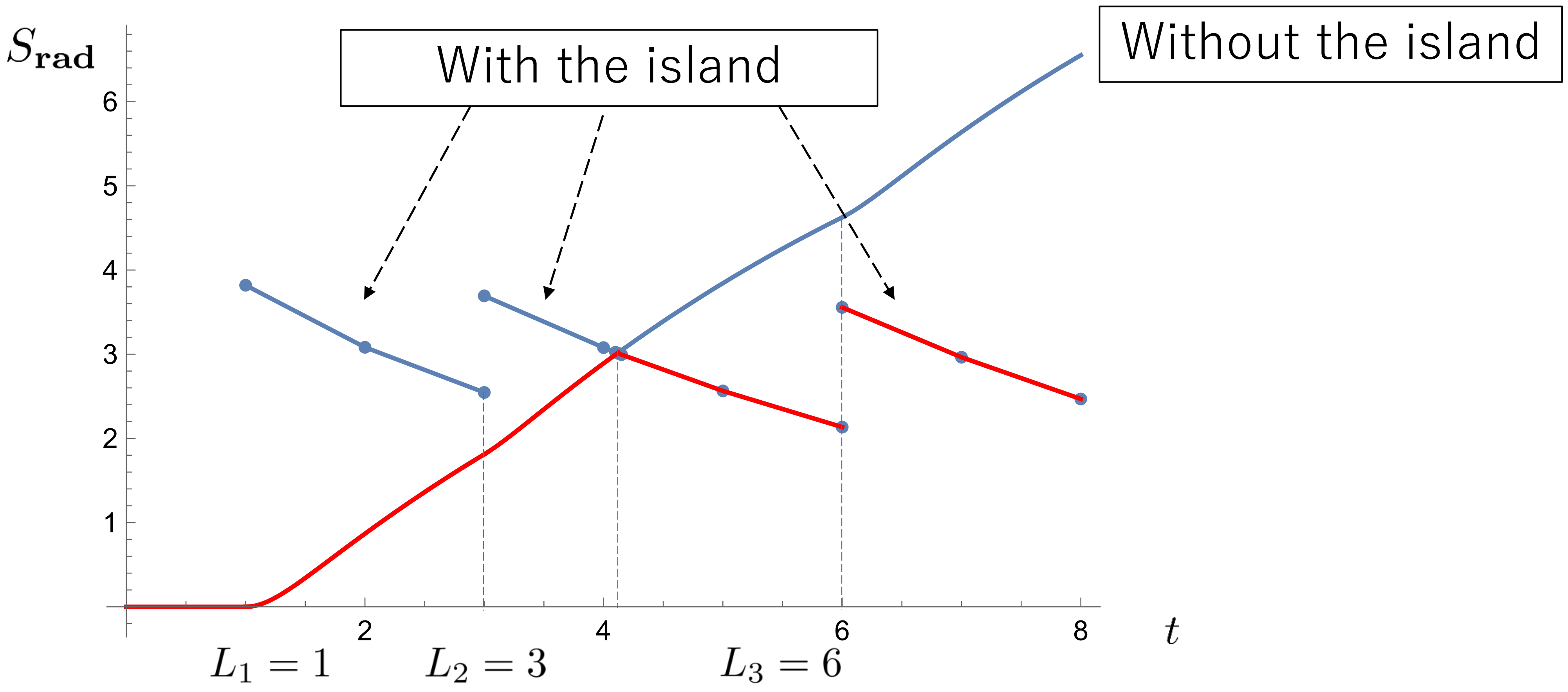
Page curve $(n = 3)$



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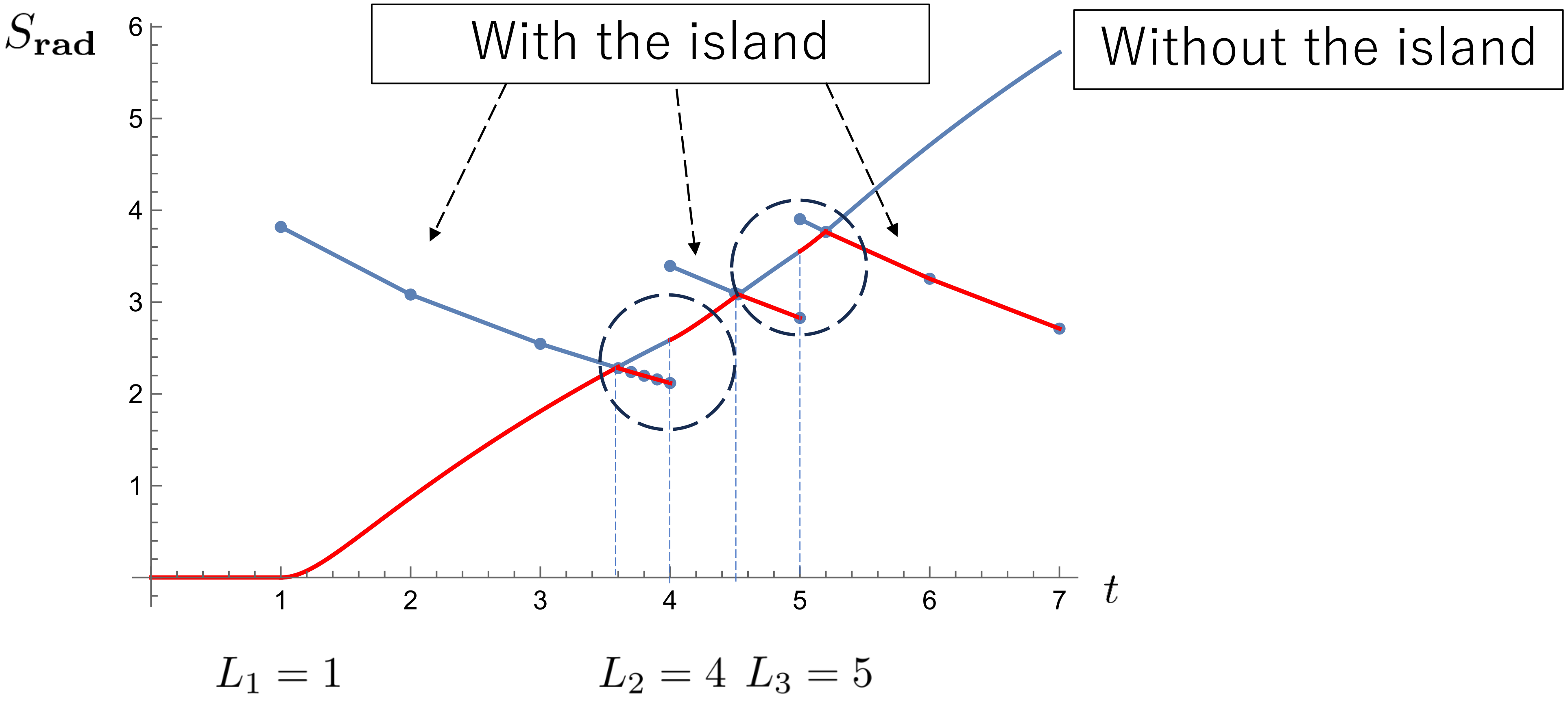

Summery

- We generalized K. Goto, T. Hartman and A. Tajdini, (2021).
- In JT gravity, we made the solution in the black hole spacetime with "n" injections.
- We derived the Page curve in the case n=3 by using the Mathematica as the numerical computation.
- In the special parameter, when we injected the energies, the spacetime is no longer one after Page time, so the island disappears.

(Destruction of the island by the injections)

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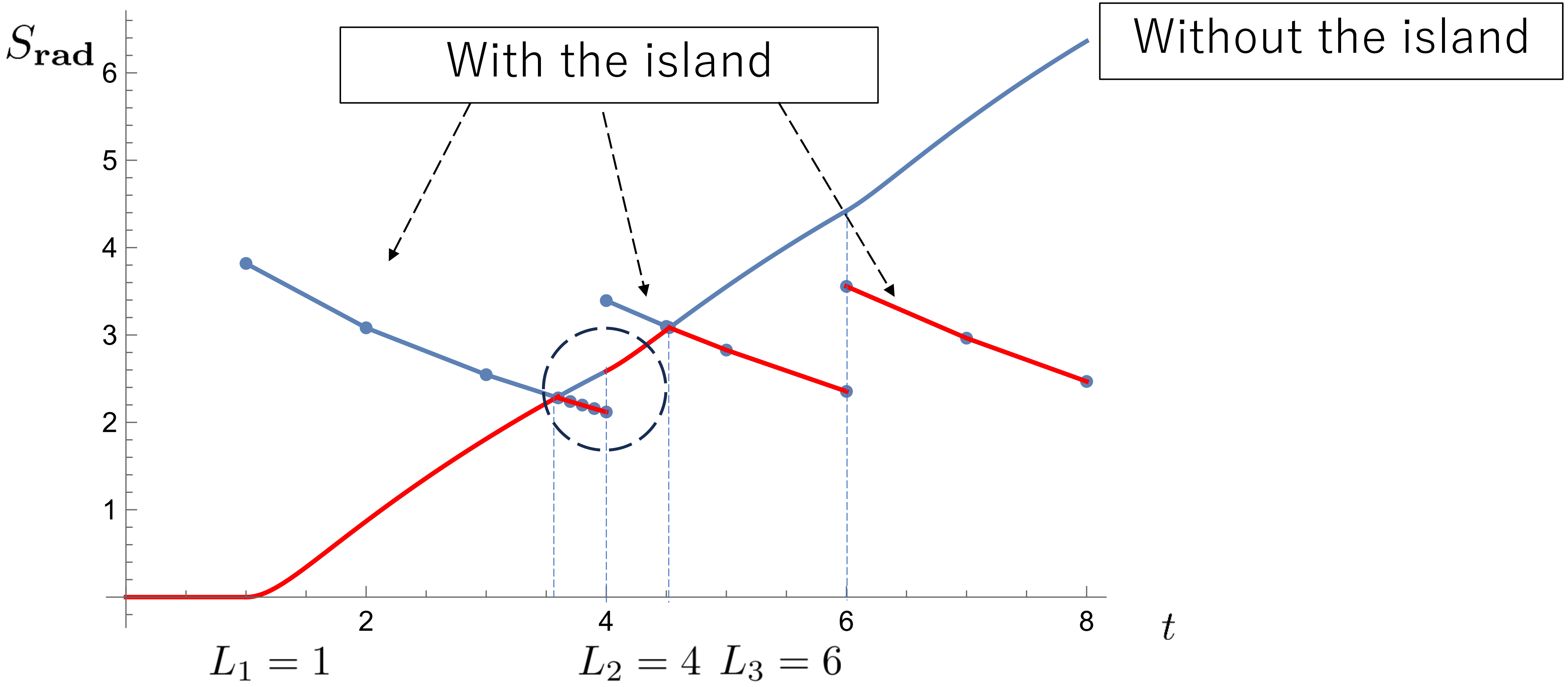

Summery

- We generalized K. Goto, T. Hartman and A. Tajdini, (2021).
- JT重力理論において, 光的エネルギーをn回入射したブラックホール時空での解を構成した.
- $n=3$ の時空でMathematicaを用いてPage曲線を数値計算によって導出した.
- エネルギーを打ち込むごとに Page 時間後ではなくなり, アイランドが消滅する. (エネルギー入射によりアイランドを“破壊”)

The derivation of Page curve by using the numerical computation

- The software：Formula manipulation system “Mathematica” (FindMaximum, Min)
- The assumption：
  - We observe the Hawking radiation at $\mathrm{R}([0,\infty])$.
  - There is a left endpoint of the island in the bifurcation point.
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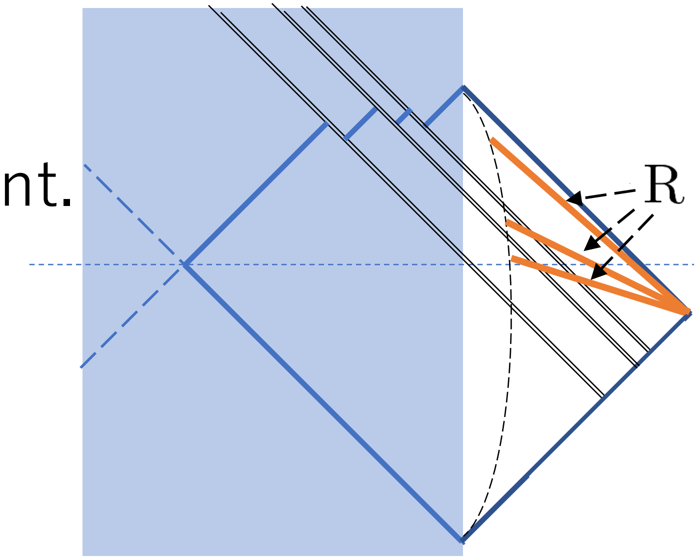

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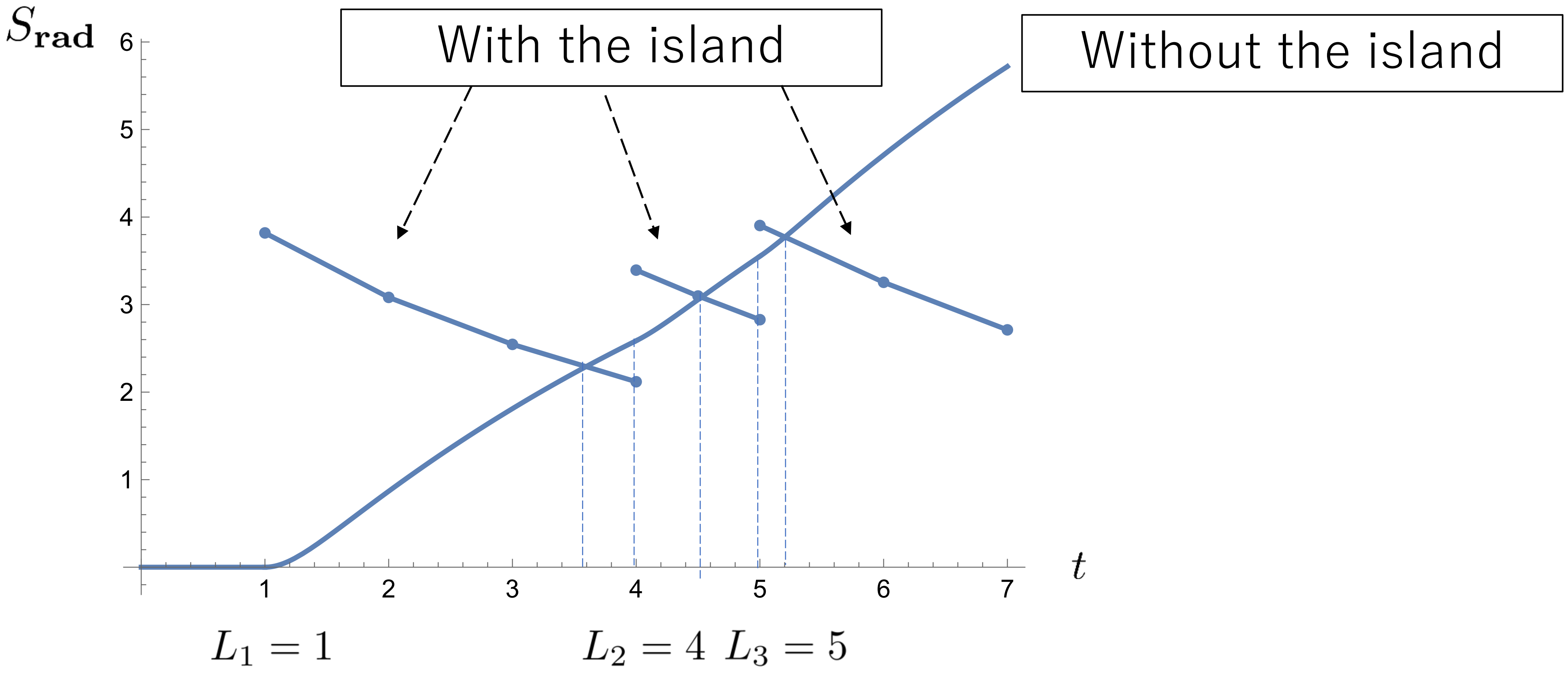

アイランド公式における極値の計算

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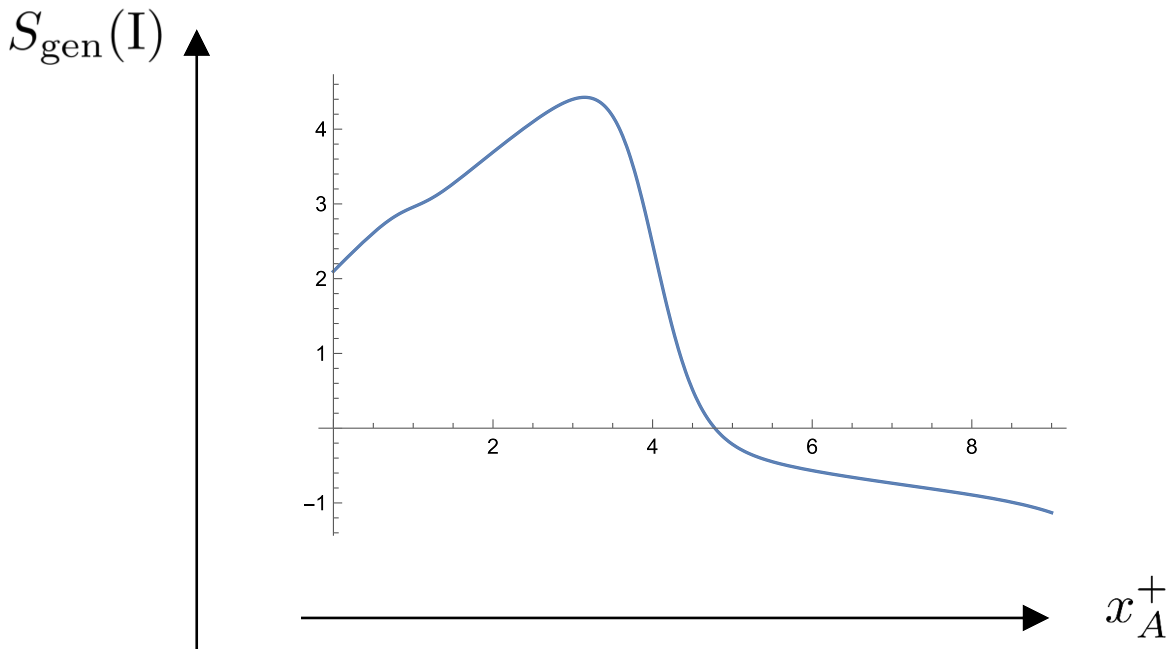
ここ！

The computation of the extremization for the island formula

1. 領域Rの時刻 t で, x_A^- を固定し x_A^+を動かす.

- $S_{\rm gen}(\mathrm{I})$ において極大点が見られる.

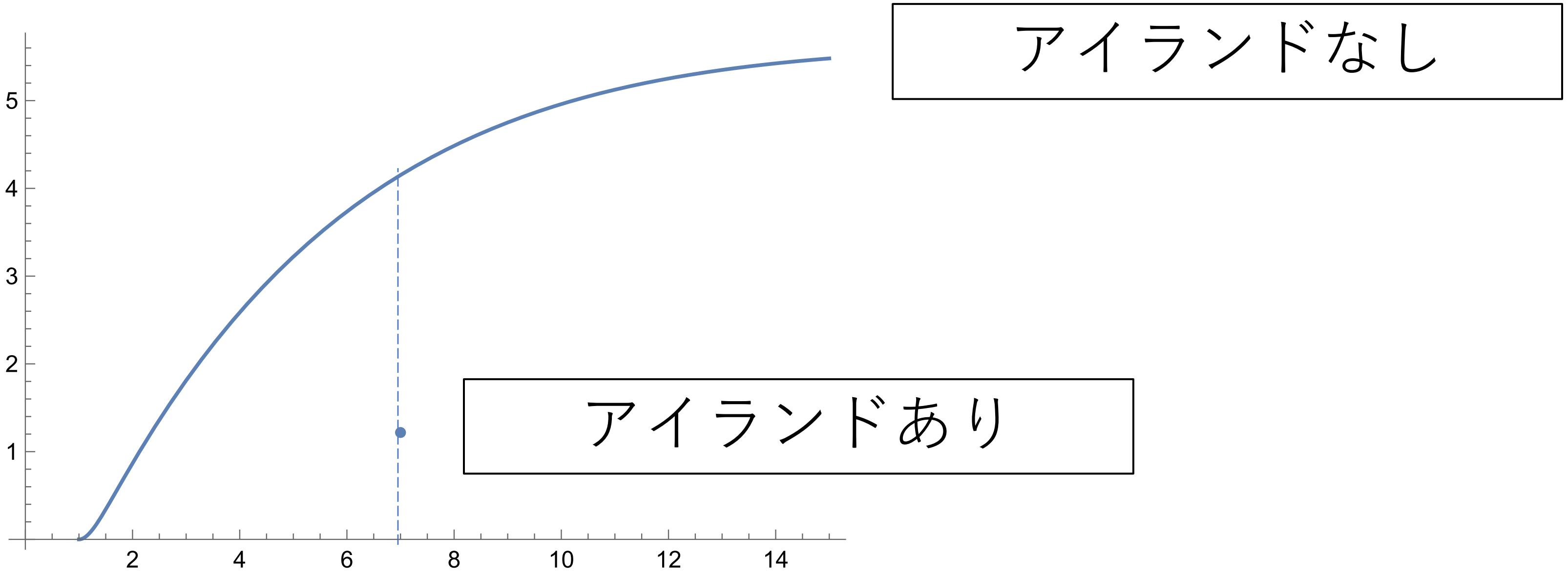
例 $n=1,\, t=7$



The computation of the minimization for the island formula

3. アイランドのない場合の $S_{\mathrm{gen}}(\mathrm{I})$ の値と鞍点での値を比較し，
小さいほうの値を選び$S_{\mathbf{rad}}$ とする．

例 $n=1, t=7$

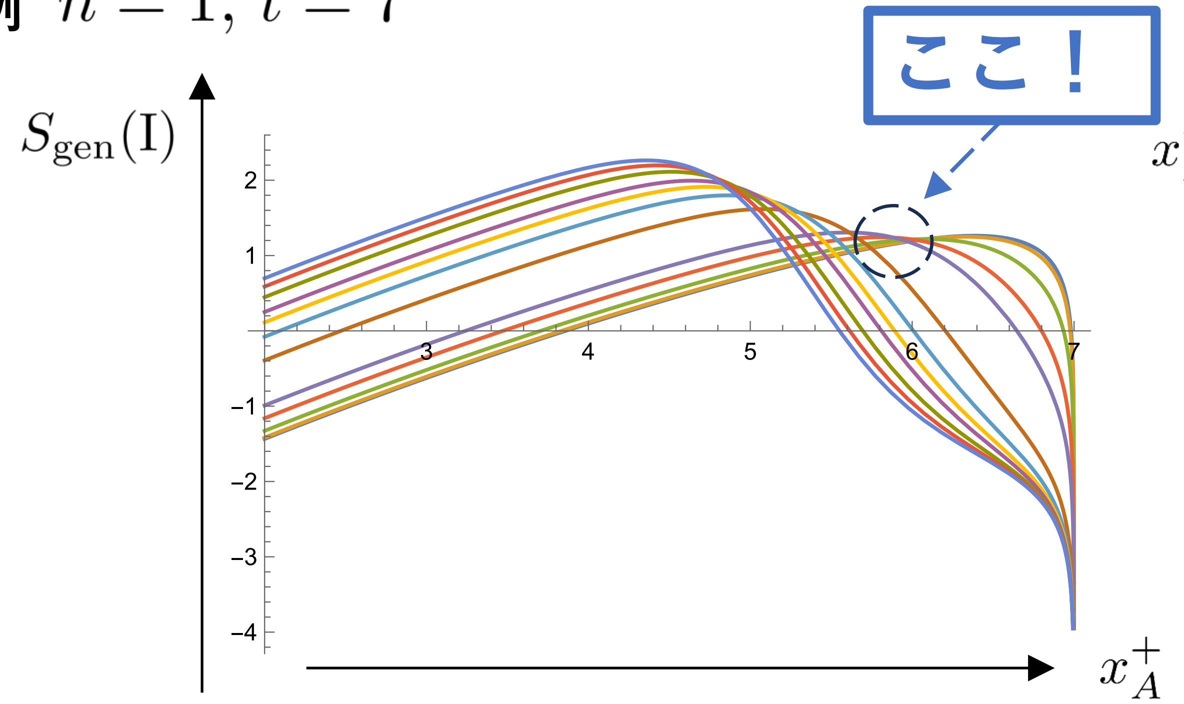


The computation of the extremization for the island formula

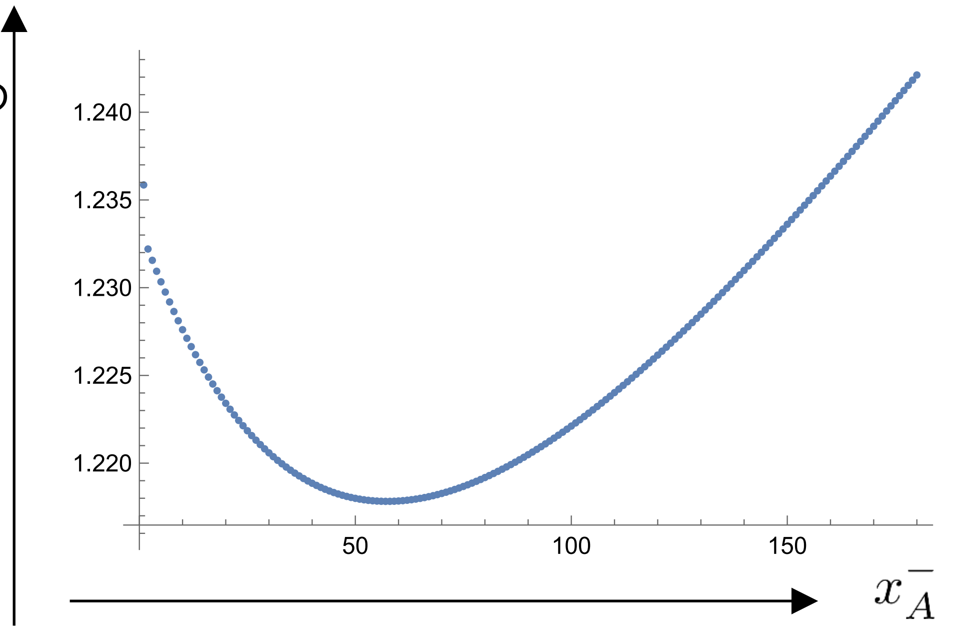
2. x_A^-の値を変化させて極大値の変化を調べる.

- $S_{\rm gen}(\mathrm{I})$において極小点が見られる.

例 $n=1,\, t=7$



x_A^+に関する極大値 $S_{\rm gen}(\mathrm{I})|_{x_A^+}$



The main in this talk

$n=1$の場合と同様な方法で
$n=3$でのPage曲線を描く.

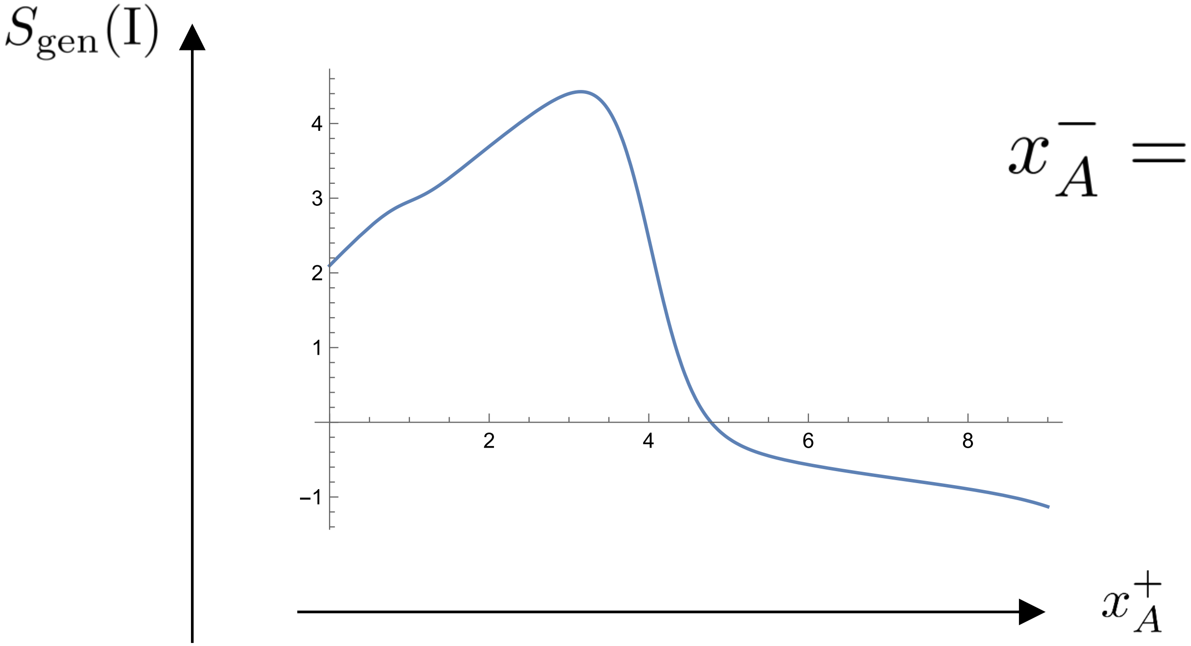
Page曲線の数値計算による導出の方法

アイランド公式：$S_{\mathbf{rad}} = \min_{\mathrm{I}} \mathrm{ext}_{\mathrm{I}} \left[S_{\mathrm{gen}}(\mathrm{I})\right]$ $\left(S_{\mathrm{gen}}(\mathrm{I}) = \dfrac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I})\right)$

1. 領域Rの時刻 t で, x_A^- を固定し x_A^+ を動かす.

    - $S_{\mathrm{gen}}(\mathrm{I})$ において極大点が見られる.

例 $n = 1, t = 7$



$$x_A^- = 3.8723827737768537$$

Page曲線の数値計算による導出の方法

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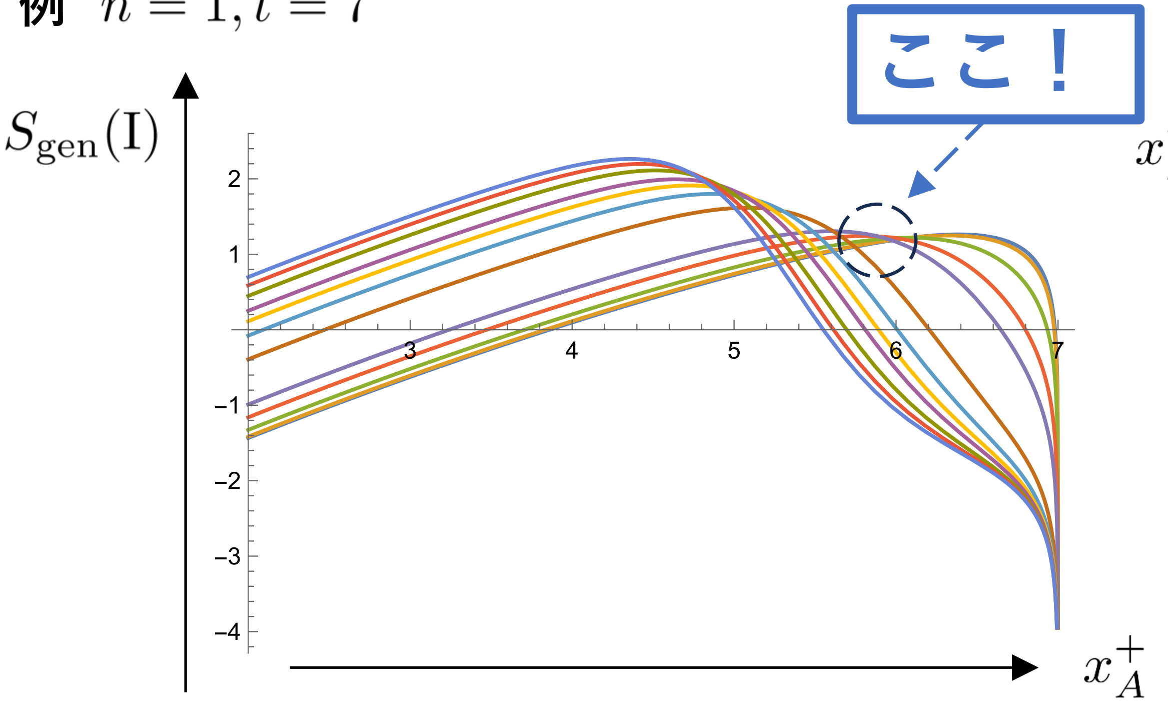
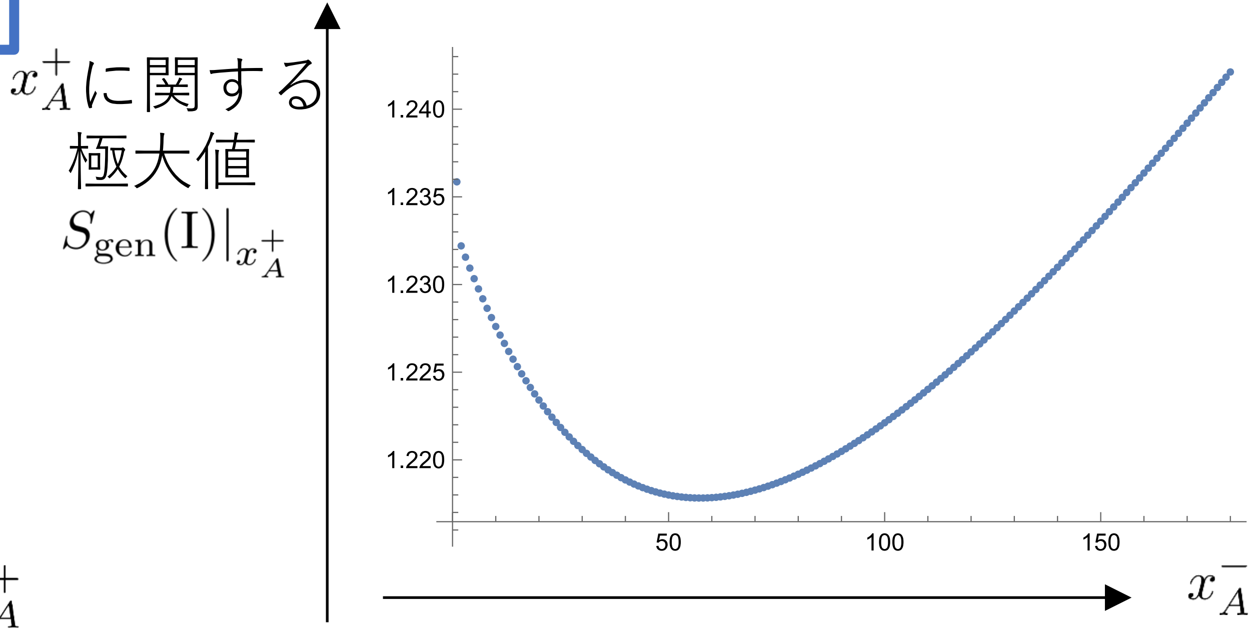
2. x_A^-の値を変化させて極大値の変化を調べる.

- $S_{\mathrm{gen}}(\mathrm{I})$において極小点が見られる.

例　$n = 1, t = 7$

$x_A^- = 3.8723807738831035$





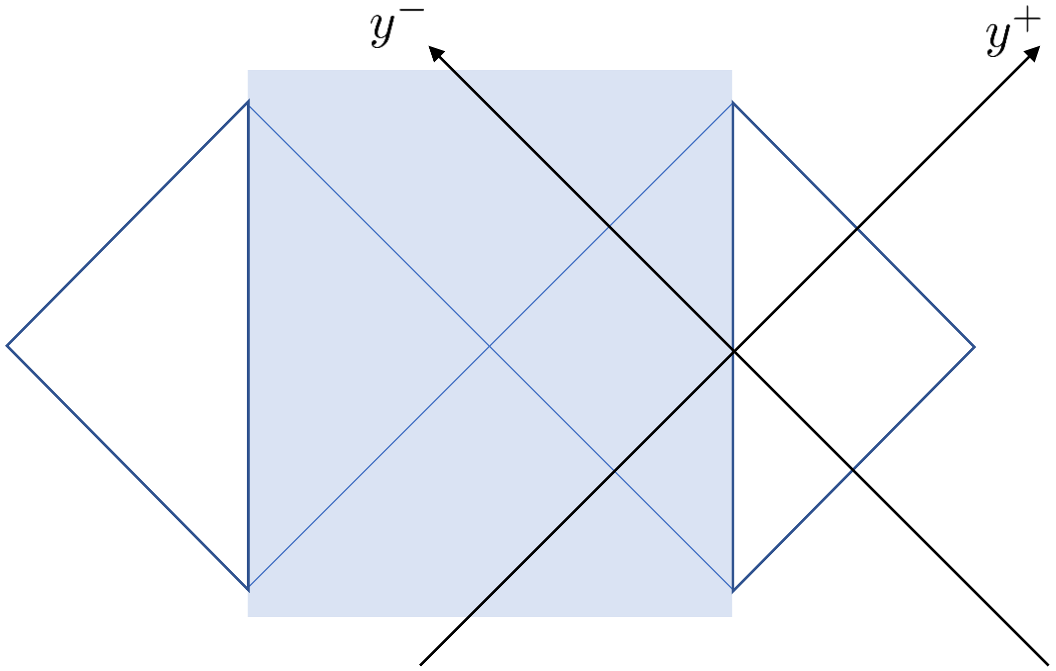
平坦時空の座標

- 座標：$(y^+, y^-) = (t+\sigma, t-\sigma)$

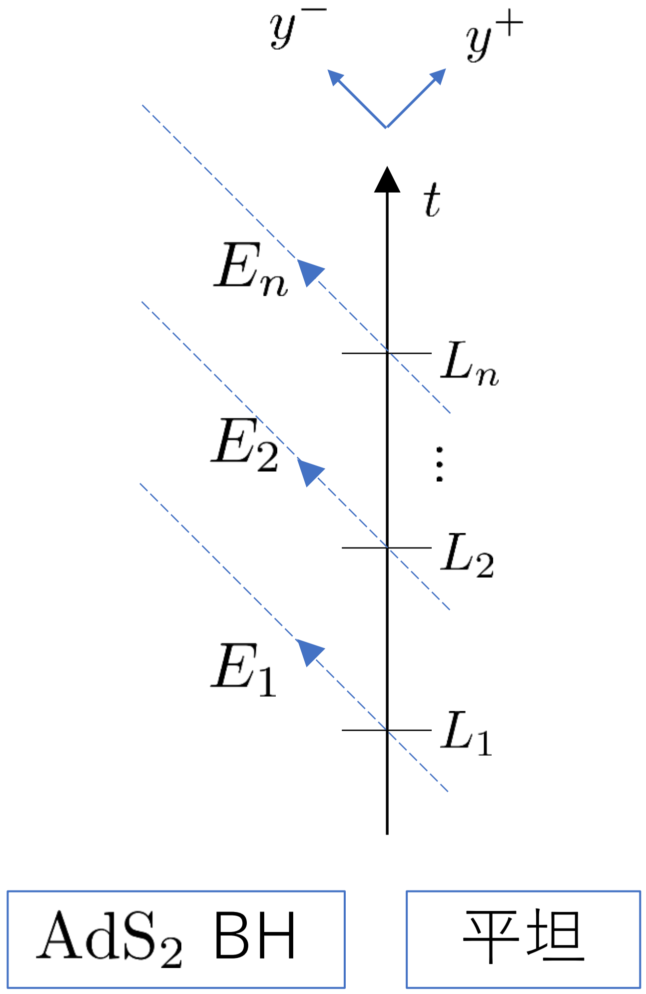
- 計量 $ds^2 = -\frac{4dy^+dy^-}{\epsilon^2}$

$\sigma = -\epsilon$ で貼り合わせ, $y^\pm$を内部へ拡張
（境界上の t と同一視する）

$$x^+ = x(y^+),\ x^- = x(y^-)$$



n 回の光的エネルギー入射を含むブラックホール時空



- 逆温度 β のブラックホール　　c ：中心電荷

$$T_{y^+y^+} = \frac{\pi c}{12\beta^2}, \qquad T_{y^-y^-} = \frac{\pi c}{12\beta^2}$$

- $t = L_1, L_2, \cdots, L_n$ でエネルギー $E_1, E_2, \cdots, E_n$ を入射

$$T_{y^+y^+} = \frac{\pi c}{12\beta^2} + \sum_{i=1}^{n} E_i \delta(y^+ - L_i), \qquad T_{y^-y^-} = -\frac{c}{24\pi}\{x(y^-), y^-\}$$

放出量が増える

n 回の光的エネルギー入射を含むブラックホール時空

- エネルギー保存：

$$-\frac{\phi_r}{8\pi G_N}\frac{d}{dt}\{x(t),t\} = T_{y^+y^+}(t) - T_{y^-y^-}(t) \ = \frac{\pi c}{12\beta^2} + \sum_{i=1}^{n} E_i\delta(t-L_i) + \frac{c}{24\pi}\{x(t),t\}$$

- 一般解：

$$x(t) = \frac{a_k K_\nu^k(t) + b_k I_\nu^k(t)}{c_k K_\nu^k(t) + d_k I_\nu^k(t)} \qquad (L_k < t < L_{k+1},\quad L_0 = 0,\quad \nu = \frac{6\pi\phi_r}{c\beta G_N})$$

$$k = 0 \qquad K_\nu^0(t) \equiv e^{\frac{\pi}{\beta}t}, \qquad I_\nu^0(t) \equiv e^{-\frac{\pi}{\beta}t}$$

$$k = 1, 2, \cdots, n \qquad K_\nu^k(t) \equiv K_\nu(\nu u_k(t)), \quad I_\nu^k(t) \equiv I_\nu(\nu u_k(t))$$ （変形Bessel関数）

$$\left(u_k(t) = \sqrt{\frac{12\kappa\beta^2}{\pi c}\sum_{i=1}^{k} E_i e^{\kappa L_i}} e^{-\frac{\kappa}{2}t}, \quad \kappa = \frac{2\pi}{\beta\nu} \right)$$

JT重力理論と力学変数

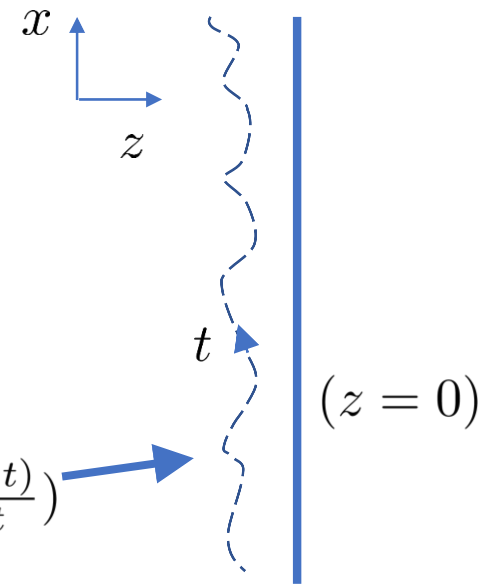
J. Maldacena, D. Stanford and Z. Yang,

- 作用：$I_{JT} = \frac{1}{16\pi G_N} \int dx^2 \sqrt{-g}\phi(R+2) + \cdots$ （R：スカラー曲率,G_N：Newton定数,ϕ：ディラトン）

局所的には AdS_2 , 時空の違いは境界の違いで記述される.

- 計量 ：$ds^2_{\text{int}} = -\frac{4dx^+dx^-}{(x^+ - x^-)^2}$　　$(x^\pm = x \pm z)$
- 境界を指定する関数 $x(t)$ （力学自由度）

（ t ：境界上での時間座標）



時空の境界

$(x,z) = (x(t), -\epsilon\frac{dx(t)}{dt})$

n 回の光的エネルギー入射を含むブラックホール時空

$$x(t) = \frac{a_k K_\nu^k(t) + b_k I_\nu^k(t)}{c_k K_\nu^k(t) + d_k I_\nu^k(t)} \qquad (L_k < t < L_{k+1}, \quad L_0 = 0)$$

境界条件： $x(t),\ \dot{x}(t),\ \ddot{x}(t)$ が連続になるようにつなぐ

➡ 係数の漸化式が得られる

$k = 0$

$$\begin{pmatrix} a_0 & c_0 \\ b_0 & d_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$k = 1, 2, \cdots, n$

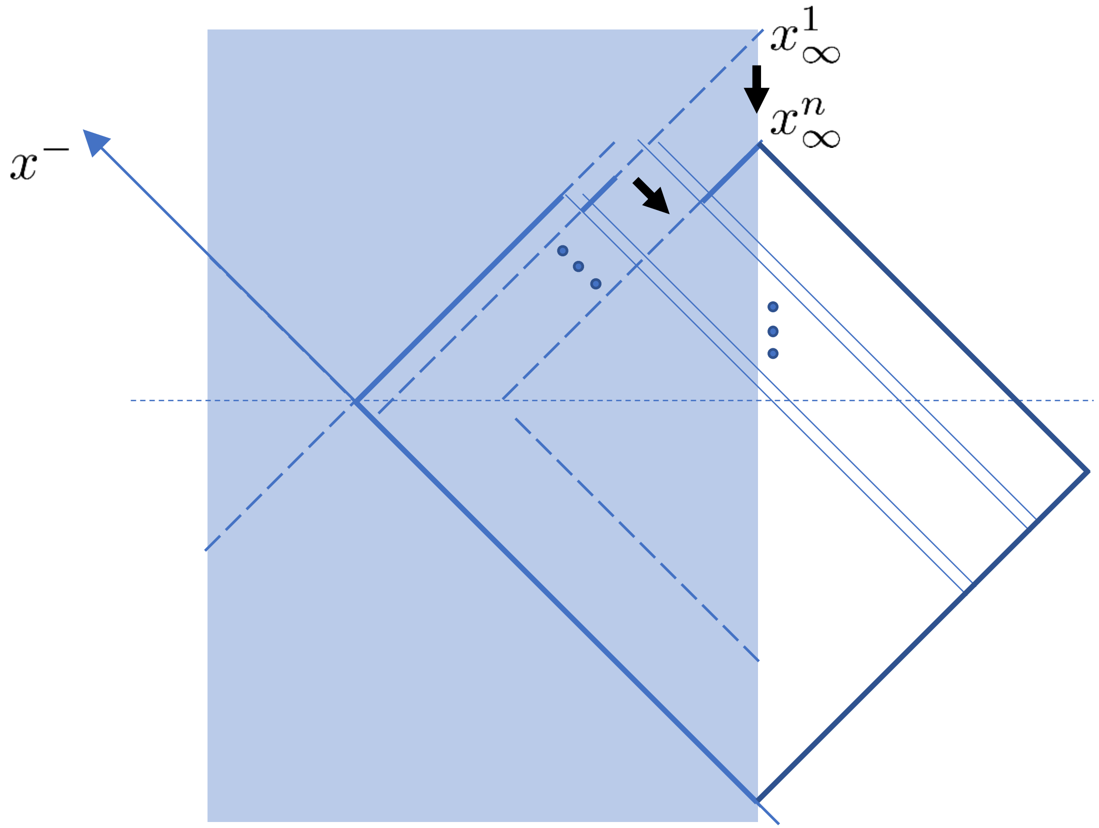
$$\begin{pmatrix} a_k & c_k \\ b_k & d_k \end{pmatrix} = -\left(\frac{2}{\kappa}\right) \begin{pmatrix} \dot{I}_\nu^k(L_k) & -I_\nu^k(L_k) \\ -\dot{K}_\nu^k(L_k) & K_\nu^k(L_k) \end{pmatrix} \begin{pmatrix} K_\nu^{k-1}(L_k) & I_\nu^{k-1}(L_k) \\ \dot{K}_\nu^{k-1}(L_k) & \dot{I}_\nu^{k-1}(L_k) \end{pmatrix} \begin{pmatrix} a_{k-1} & c_{k-1} \\ b_{k-1} & d_{k-1} \end{pmatrix}$$

$$(\dot{K}_\nu^k = \tfrac{d}{dt} K_\nu^k, \quad \dot{I}_\nu^k = \tfrac{d}{dt} I_\nu^k)$$

ブラックホールの膨張

- 光的エネルギー入射前後の変化　$x(t=\infty) \equiv x_\infty^k \quad (L_k < t < L_{k+1}, \quad L_0 = 0)$

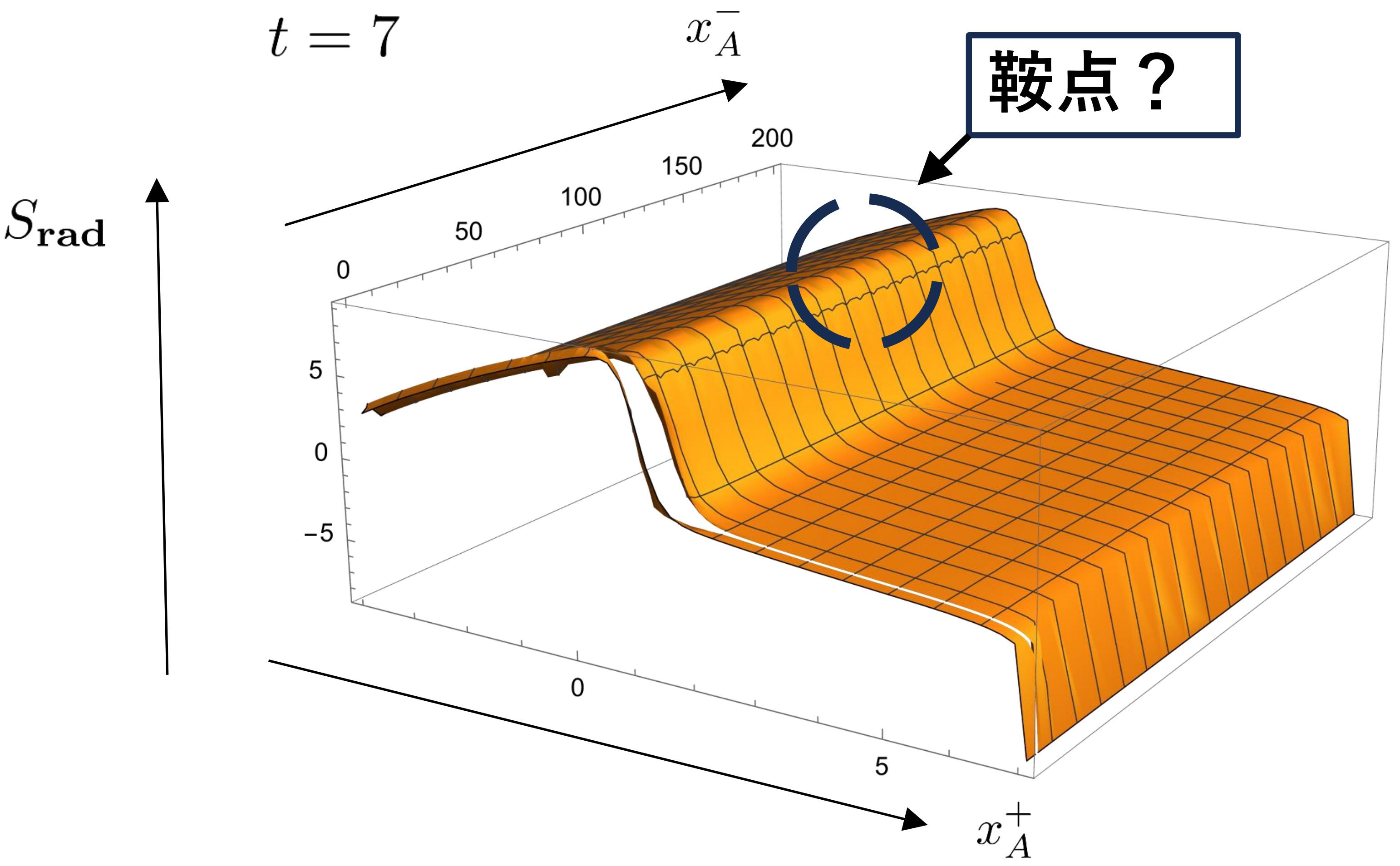
$$
\begin{aligned}
x_\infty^{k-1} - x_\infty^k &= \frac{a_{k-1}}{c_{k-1}} - \frac{a_k}{c_k} \\
&= \frac{a_{k-1}c_k - a_k c_{k-1}}{c_{k-1}c_k} > 0
\end{aligned}
$$



→ 光的エネルギー入射によりブラックホールは膨張する

Page曲線の数値計算による導出

アイランド公式：$S_{\mathbf{rad}} = \min_{\mathrm{I}} \mathrm{ext}_{\mathrm{I}} \left[S_{\mathrm{gen}}(\mathrm{I}) \right]$ $\left(S_{\mathrm{gen}}(\mathrm{I}) = \dfrac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I}) \right)$



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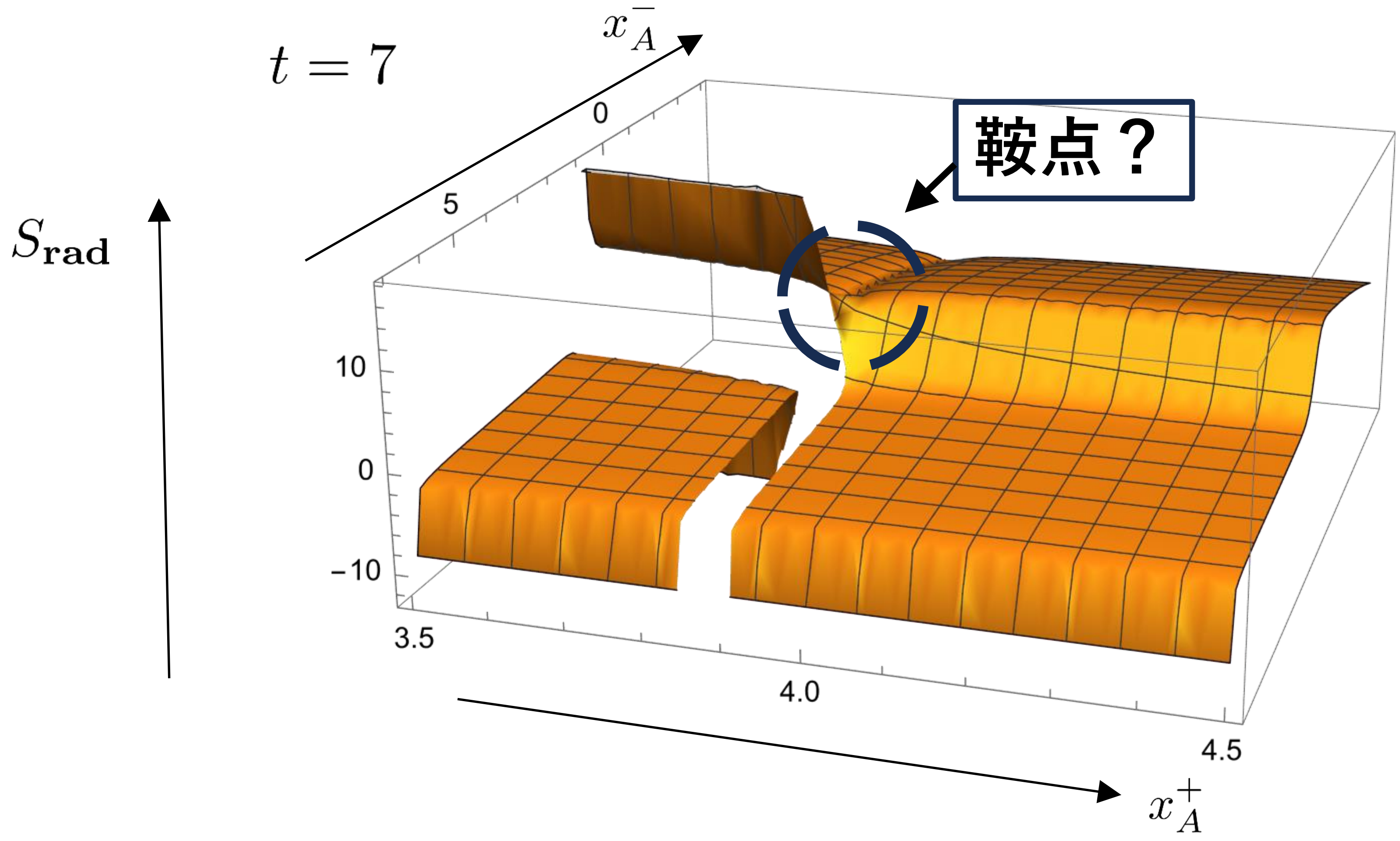

Page曲線の数値計算による導出の方法

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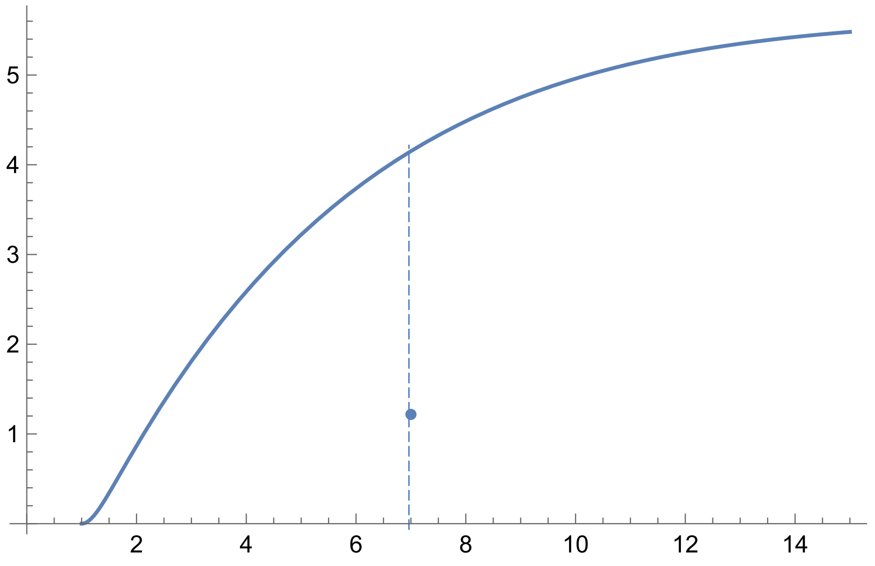
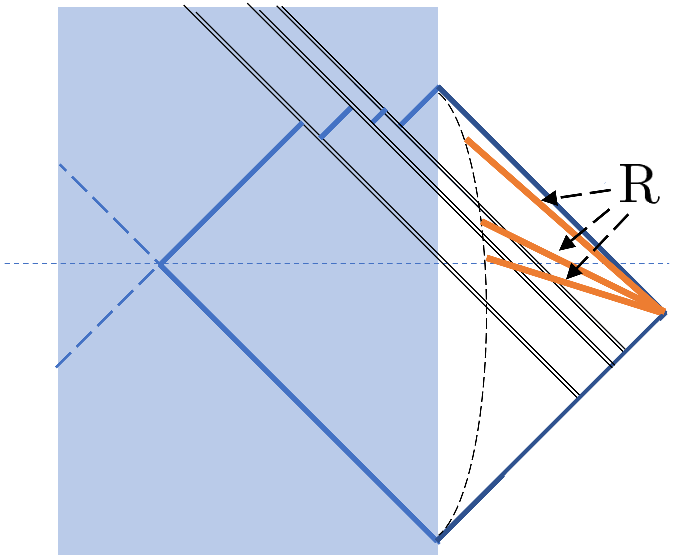
3. アイランドのない場合の$S_{\mathrm{gen}}(\mathrm{I})$ の値と鞍点での値を比較し, 小さいほうの値を選び $S_{\mathbf{rad}}$ とする．

例 $n = 1, t = 7$



Page曲線の数値計算による導出

- ソフトウェア：数式処理ソフト "Mathematica" (Findmaximum, Minを使用)
- 仮定：
  - 領域 $\mathrm{R}([0,\infty])$でHawking輻射を観測する.
  - アイランドの左端点はbifurcation point に存在する.
  - アイランドの右端点 (x_A^+, x_A^-)を変数として$S_{\mathbf{rad}}$を扱う.
  - 自由フェルミオンを用いる.
  - 以下のようなパラメータを用いる.



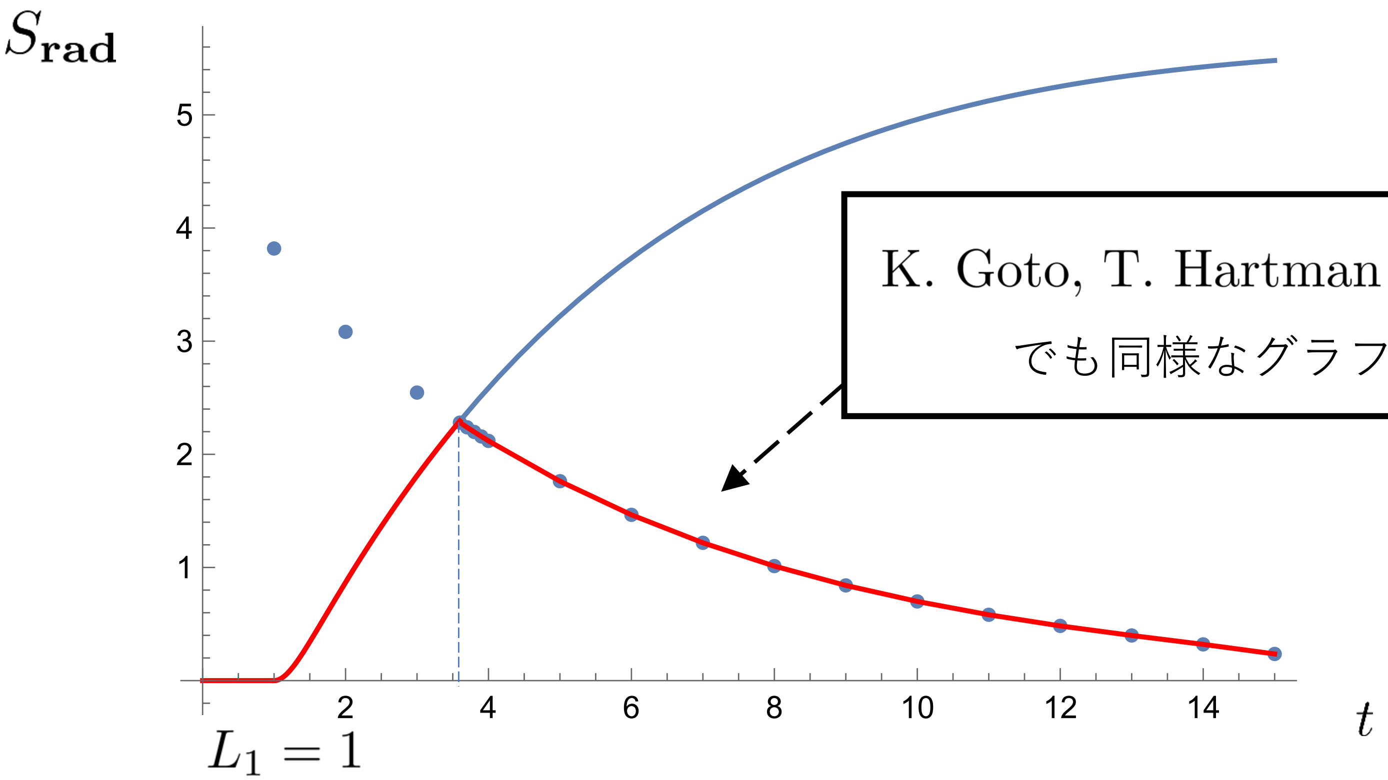
	入射エネルギー	入射時刻	中心電荷	Newton定数	定常状態での逆温度
1回目	$E_1 = 2$	$L_1 = 1$	$c = 3$	$G_N = \frac{1}{3}$	$\beta = 2\pi$
2回目	$E_2 = 1$	$L_2 = 4$			
3回目	$E_3 = 1$	$L_3 = 5$			

Page曲線の数値計算による導出

アイランド公式：$S_{\mathbf{rad}} = \min_{\mathrm{I}} \operatorname{ext}_{\mathrm{I}} [S_{\mathrm{gen}}(\mathrm{I})]$ $\left(S_{\mathrm{gen}}(\mathrm{I}) = \dfrac{\mathrm{Area}(\partial \mathrm{I})}{4G_N} + S_{\mathrm{ent}}(\mathrm{R} \cup \mathrm{I}) \right)$

4. 領域 R の時刻を変えて同様な操作を行う

例 $n = 1$



K. Goto, T. Hartman and A. Tajdini, (2021).
でも同様なグラフが描かれていた.