The emergence of spacetime in bosonic Lorentzian IKKT matrix model with the mass term

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Definition of IKKT matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya [hep-th/9612115]

- The IKKT model: $Z = \int \mathrm{d}A \mathrm{d}\psi \, e^{iS}$ where $S = S_{\mathrm{b}} + S_{\mathrm{f}}$

Non-perturbative formulation of superstring theory

Bosonic part: $S_{\mathrm{b}} = -\frac{N}{4} \mathrm{tr}[A_\mu, A_\nu][A^\mu, A^\nu]$

Fermionic part: $S_{\mathrm{f}} = -\frac{N}{2} \mathrm{tr} \left(\bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] \right)$

$A_\mu \ (\mu = 0, 1, \ldots, 9)$ $\psi_\alpha \ (\alpha = 1, 2, \ldots, 16)$ $N \times N$ Hermitian matrices

- Shift symmetry $A_\mu \to A_\mu + \alpha_\mu \mathbf{1}$ = Translation in SUSY

⟶ Eigenvalues of A_μ = spacetime coordinates

(Spacetime emerges dynamically)

The mass term as IR regulator

Pfaffian: real polynomial in A_μ

- Lorentzian model is not absolutely convergent. $Z = \int dA\, e^{iS_b} \mathrm{Pf}\, \mathcal{M}(A)$

- To regularize: Adding the Lorentz-invariant mass term as an IR regulator.

Hatakeyama et al. [2201.13200]

$$S = S_b + S_m \qquad S_m = \frac{1}{2} N\gamma \left[e^{i\epsilon} \mathrm{Tr}(A_0)^2 - e^{-i\epsilon} \mathrm{Tr}(A_i)^2 \right]$$

ε gives convergence factor

- Euclidean model: $\tilde{S}_m = \frac{1}{2} N\gamma\, e^{3\pi i/4} \left[e^{i\epsilon} \mathrm{tr}(\tilde{A}_0)^2 + e^{-i\epsilon} \mathrm{tr}(\tilde{A}_i)^2 \right]$

$\gamma < 0$	$\epsilon \to -0$	$\mathrm{Re}(\tilde{S}_\mathrm{m}) > 0$	Euclidean model
$\gamma > 0$	$\epsilon \to +0$	$\mathrm{Re}(\tilde{S}_\mathrm{m}) < 0$	Not Euclidean model (because action is unbounded)

Classical solutions with expanding behaviour

Bosonic model with mass term

- Classical equation of motion: $A_\mu \to \hat{A}_\mu = A_\mu / \sqrt{|\gamma|}$

$$Z = \int \mathrm{d}A \, e^{i\gamma^2 S[\hat{A}]} \qquad \left(\gamma^2 \leftrightarrow \frac{1}{\hbar}\right) \text{ "classical limit" } \longrightarrow [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$$

- There are non-trivial solutions $A_\mu \neq 0$ with expanding behaviour at $\gamma > 0$.

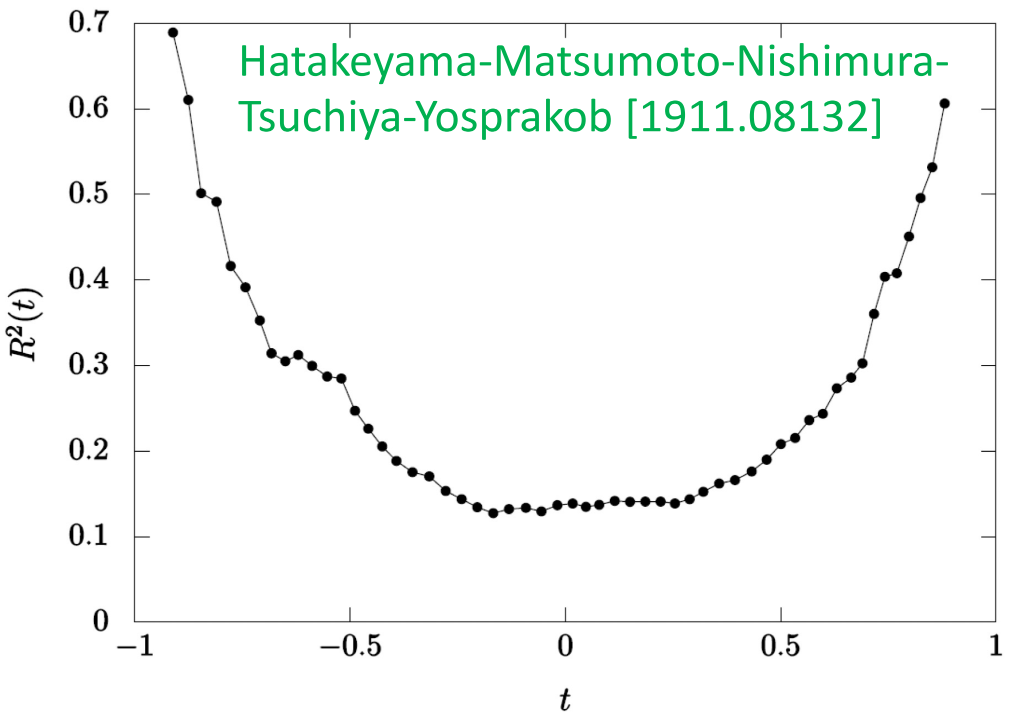
  No such solutions at $\gamma < 0$ Steinecker [1709.10480]

- 3D space are expected with fermions.

  Configurations A_μ with $A_1 = A_2 \neq 0$ and $A_{3-10} = 0$ gives Pf(M) = 0. ⇒ suppressed

  Krauth-Nicolai-Nishimura [hep-th/9803117]

  Nishimura-Vernizzi [hep-th/0003223]



Complete set of classical solutions

D-dim bosonic model at N=2

("D" is not restricted by SUSY)

	$\gamma > 0$	$\gamma < 0$
	Trivial: $A_\mu = 0$	Trivial: $A_\mu = 0$
Non-trivial solutions $A_\mu \neq 0$	Pauli: $A_\mu = \sqrt{\frac{\gamma}{2}}\sigma_\mu \quad \mu = 1,2,3$ $\mathrm{SO}(3) \times \mathrm{SO}(D-4,1)$ — 3 extended spatial directions	
	squashed Pauli: $A_i = \sqrt{\gamma}\sigma_i \quad i = 1,2$ $\mathrm{SO}(2) \times \mathrm{SO}(D-3,1)$ — 2 extended spatial directions	

(Other solutions are irrelevant.)

(Non-trivial solutions represent lower-dimensional "space".)

Non-compact Lorentz group

- Non-trivial solutions have flat directions $\Rightarrow$ Infinitely many equivalent configurations.
  symmetry breaking

	SO(D) model	SO(D-1,1) model
Definition	$Z_{\mathrm{E}} = \int \frac{dk}{2\pi} \int d^D x \, e^{ik(x_1^2 + x_2^2 + \cdots + x_D^2 - 1)}$	$Z_{\mathrm{L}} = \int \frac{dk}{2\pi} \int d^D x \, e^{ik(x_1^2 + x_2^2 + \cdots - x_0^2 - 1)}$
Saddle points	$x_1^2 + x_2^2 + \cdots + x_0^2 - 1 = 0$ Sphere	$x_1^2 + x_2^2 + \cdots - x_0^2 - 1 = 0$ Hyperboloid
Divergence/ Convergence	$Z_{\mathrm{E}} = \frac{1}{2} \mathrm{Vol}\left(S^{D-1}\right) < \infty$	$Z_{\mathrm{L}} = \frac{1}{2} \mathrm{Vol}\left(H_{D-1}^{(1)}\right) = \infty$

The emergence of 3D space

- "The most diverging partition function = The most dominant configuration"

Solutions	Trivial	Pauli-matrix	squashed Pauli-matrix
Z	finite	$\varepsilon^{-3D/2}\ (\varepsilon^{-14.7})$	$\varepsilon^{-4.6}$

Results at large D (non-perturbative in γ) (N. Yamamori's talk)

Numerical results at D=10 (A. Tripathi's talk)

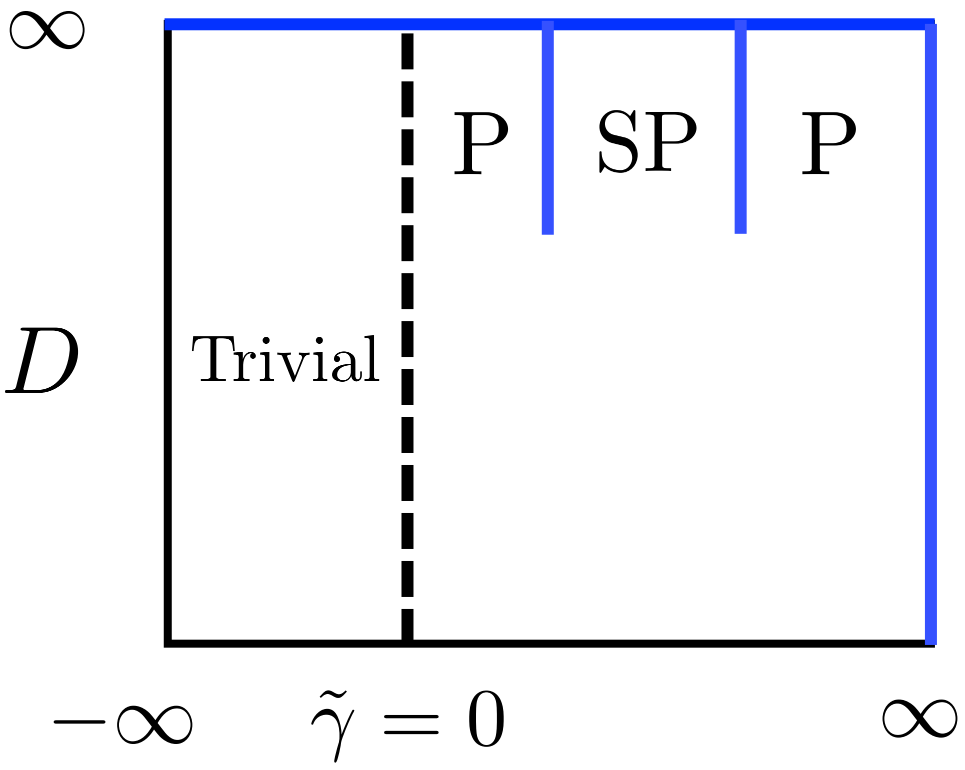
- As $\varepsilon \to 0$, only Pauli solution contributes = emerging 3D "space" at N=2.
- This is true for any $\gamma > 0$.

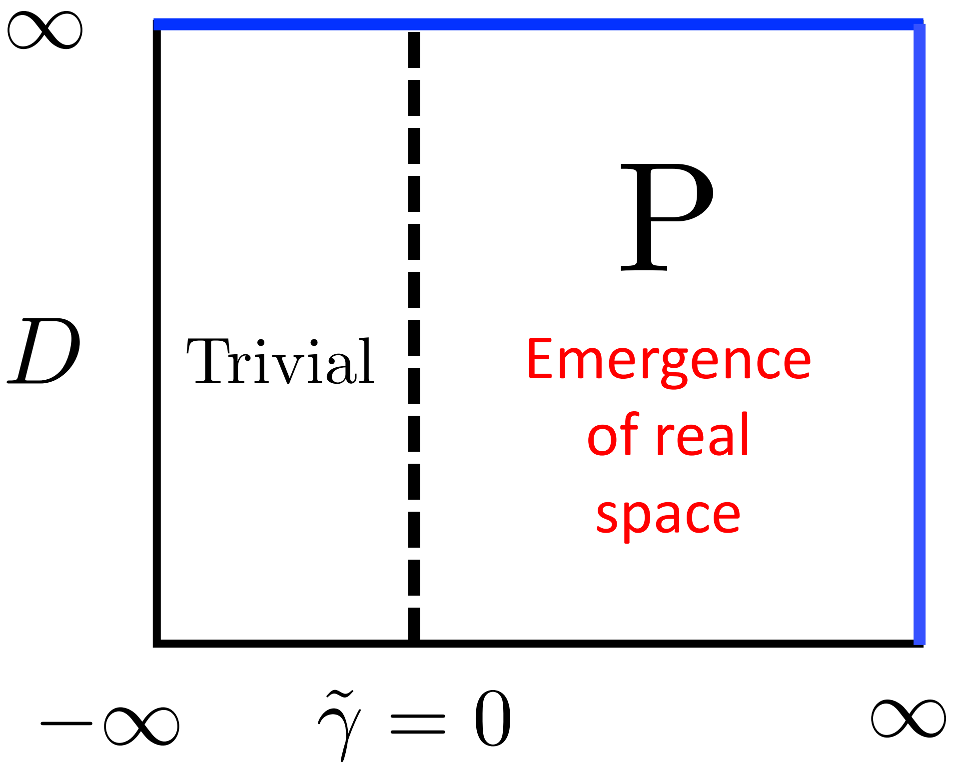
Phase diagram at N=2

$A_0 \to iA_D$ $\quad Z \sim e^{iS_E}$

compact symmetry
$\Rightarrow$ finite

SO(D) model ← They are different → SO(D-1,1) model





Summary

- We exhaust all classical solutions at N=2.

$\gamma < 0$	$\gamma > 0$
Trivial solution	Trivial solution
	Pauli solution
	squashed Pauli solution

- Non-trivial solutions representing lower-dimensional “space” appear only at $\gamma > 0$.
- Pauli solution dominating at $\gamma > 0$ at N=2 may be regarded as the emergence of 3D real “space”.
- At larger N, there are more non-trivial solutions $\Rightarrow$ (3+1)D spacetime?

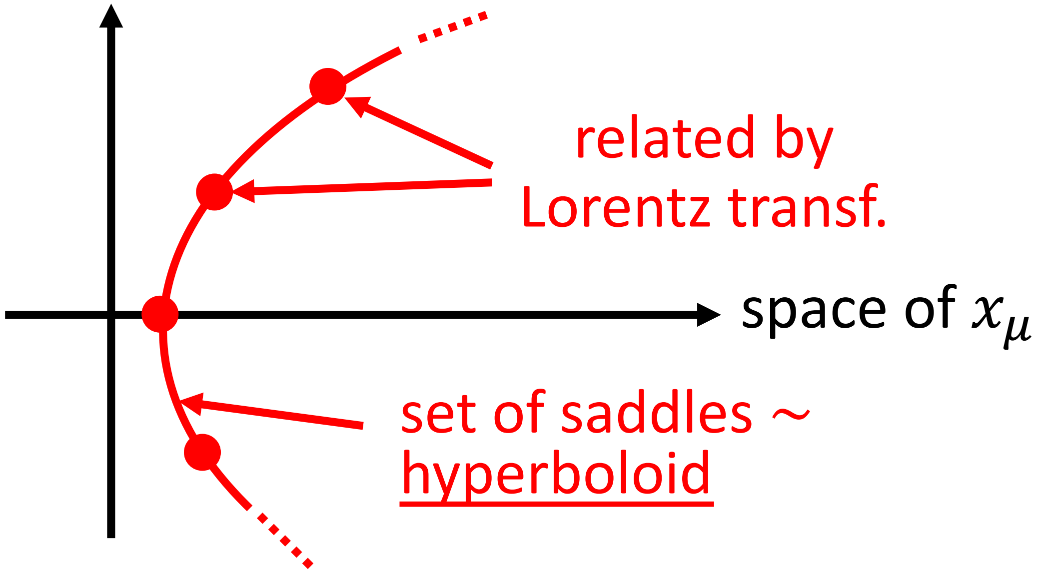
Backup

Non-compactness of Lorentz symmetry

- Consider SO(D-1,1)-symmetric model

$$Z = \int \frac{\mathrm{d}k}{2\pi} \int \mathrm{d}^D x \, \underbrace{e^{ik(-x_0^2 + x_1^2 + \cdots + x_{D-1}^2 - 1)}} = \frac{1}{2} \mathrm{vol}\, H_{D-1} = \infty$$

saddles satisfy $-x_0^2 + x_1^2 + \cdots + x_{D-1}^2 - 1 = 0$ [Hyperboloid]



- This is how Lorentzian model can be associated with divergent partition function for non-trivial solutions.