The analysis of the bosonic Lorentzian IKKT matrix model at large D

Naoyuki Yamamori (SOKENDAI D1)

based on collaboration with : Y. Asano, J. Nishimura, W. Piensuk

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Brief review of the bosonic Lorentzian IKKT matrix model

The bosonic Lorentzian IKKT matrix model with Lorentz invariant mass term

$$Z = \int dA e^{i(S_b + S_m)}$$ SO(9,1) symmetry

The bosonic action

$$S_b = -\frac{N}{4} \mathrm{tr}[A_\mu, A_\nu][A^\mu, A^\nu]$$

The Lorentz invariant mass term

$$S_m = \frac{1}{2} N\gamma \left\{ e^{i\varepsilon} \mathrm{tr}(A_0)^2 + e^{-i\varepsilon} \mathrm{tr}(A_i)^2 \right\}$$

convergence factor

A_μ : N×N Hermitian matrices

[talk by W. Piensuk]

For N=2, there are three classical solution $(\gamma > 0)$

- trivial solution $A_\mu = 0$
- Pauli solution $A_\mu = \sqrt{\frac{\gamma}{2}} \sigma_\mu \quad \mu = 1, 2, 3$
- squashed Pauli solution $A_\mu = \sqrt{\gamma} \sigma_\mu \quad \mu = 1, 2$

non-trivial solution breaks Lorentz symmetry spontaneously

Non-compactness of Lorentz group

⇓

$$Z_{\mathrm{Pauli}}, Z_{\mathrm{sq-Pauli}} \sim \infty$$

Introduction

Talk by W. Piensuk

- Introduced the bosonic Lorentzian IKKT matrix model with mass term
- Possibility of the divergence of partition function around the non-trivial solutions due to the non-compactness of Lorentz symmetry

This talk

- We confirmed that the partition function around the Pauli solution diverges by 1/D expansion.

1/D expansion

D: the number of bosonic matrices

[Hotta-Nishimura-Tsuchiya('98)]
(Euclidean model without mass term)

$$Z = \int dA e^{i(A^4 + \gamma A^2)}$$

SO(D-1,1)×SU(N) symmetry

$$= \int dh \int dA e^{i(h^2 + hA^2 + \gamma A^2)}$$

$$h_{ab} \sim A^a_\mu A^{b\mu}, \quad A_\mu = \sum_{a=1}^{N^2-1} A^a_\mu t^a$$

$$= \int dh e^{ih^2 - \frac{D}{2}\mathrm{logdet}K}$$

$$\tilde{h}_{ab} = \frac{N}{\sqrt{D}} h_{ab},\ K \sim \frac{1}{N}\tilde{h} + \tilde{\gamma},\ \gamma = \tilde{\gamma}\sqrt{D}$$

$$= \int d\tilde{h} e^{-D S_{\mathrm{eff}}[\tilde{h}]}$$

SU(N) symmetry **Lorentz symmetry becomes invisible**

The saddle points dominates the path integral at Large D

$$\frac{\partial S_{\mathrm{eff}}}{\partial \tilde{h}_{ab}} = \tilde{h}_{ab} + i K^{-1}_{ab} = 0$$

Saddle points for N=2

The saddle point Eq. $\tilde{h}_{ab} + iK^{-1}_{ab} = 0$

For N=2, there are three saddle points.

$$\tilde{h}_{ab} = v^{(\pm)}\mathbf{1}, \quad \tilde{h} = \tilde{\gamma}\,\mathrm{diag}\left(1, 1, \frac{i}{\tilde{\gamma}^2}\right) \qquad v^{(\pm)} = \frac{\tilde{\gamma} \pm \sqrt{\tilde{\gamma}^2 + 8i}}{4}$$

complex saddles

The relevant saddle points

$\gamma < 0$	$\tilde{h}_{ab} = v^{(+)}\mathbf{1}$
$\gamma > 0$	$\tilde{h}_{ab} = v^{(-)}\mathbf{1}, \quad \tilde{h}_{ab} = v^{(+)}\mathbf{1}, \quad \tilde{h} = \tilde{\gamma}\,\mathrm{diag}\left(1, 1, \frac{i}{\tilde{\gamma}^2}\right)$

Identification of each saddle with classical solution ($\gamma > 0$)

remaining symmetries

$$\tilde{h}_{ab} = v^{(-)}\mathbf{1}, \quad \tilde{h}_{ab} = v^{(+)}\mathbf{1}, \quad \tilde{h} = \tilde{\gamma}\,\mathrm{diag}\left(1, 1, \frac{i}{\tilde{\gamma}^2}\right)$$

$\mathrm{SU}(2)$ $\mathrm{SU}(2)$ $\mathrm{U}(1)$

classical solutions [talk by W. Piensuk]

trivial solution

$$A_\mu = 0$$

(unbroken)

$\mathrm{SO}(9,1) \times \mathrm{SU}(2)$

Pauli solution

$$A_\mu = \sqrt{\frac{\gamma}{2}}\sigma_\mu \quad \mu = 1, 2, 3$$

diagonal subgroup of

$\mathrm{SO}(3) \times \mathrm{SU}(2)$

squashed Pauli solution

$$A_\mu = \sqrt{\gamma}\sigma_\mu \quad \mu = 1, 2$$

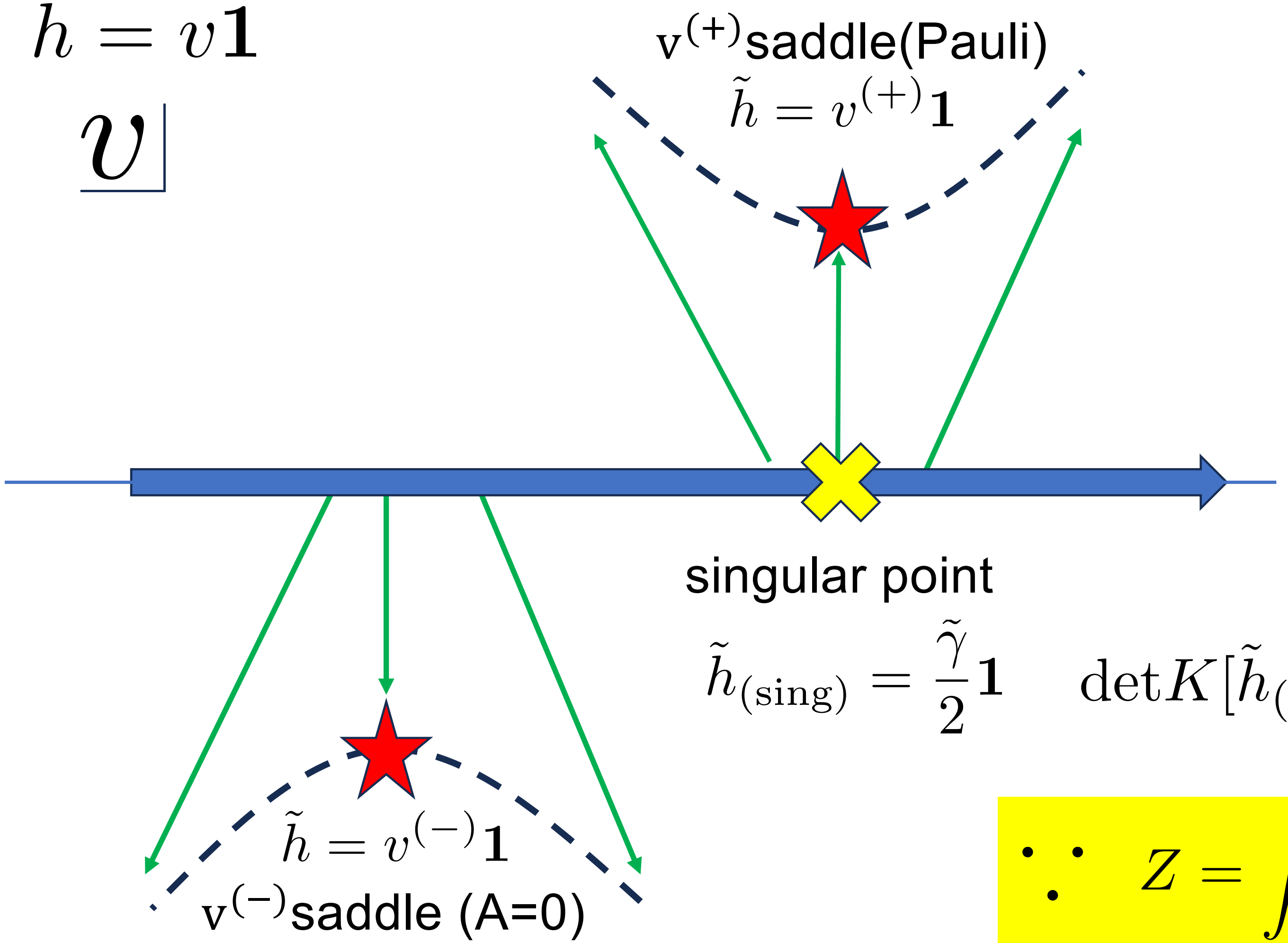
diagonal subgroup of

$\mathrm{SO}(2) \times \mathrm{U}(1)$

Singularity on the real axis

$$\tilde{h} = v\mathbf{1}$$

$$v$$



$$\tilde{h} = v^{(\pm)}\mathbf{1}$$

$$v^{(\pm)} = \frac{\tilde{\gamma} \pm \sqrt{\tilde{\gamma}^2 + 8i}}{4}$$

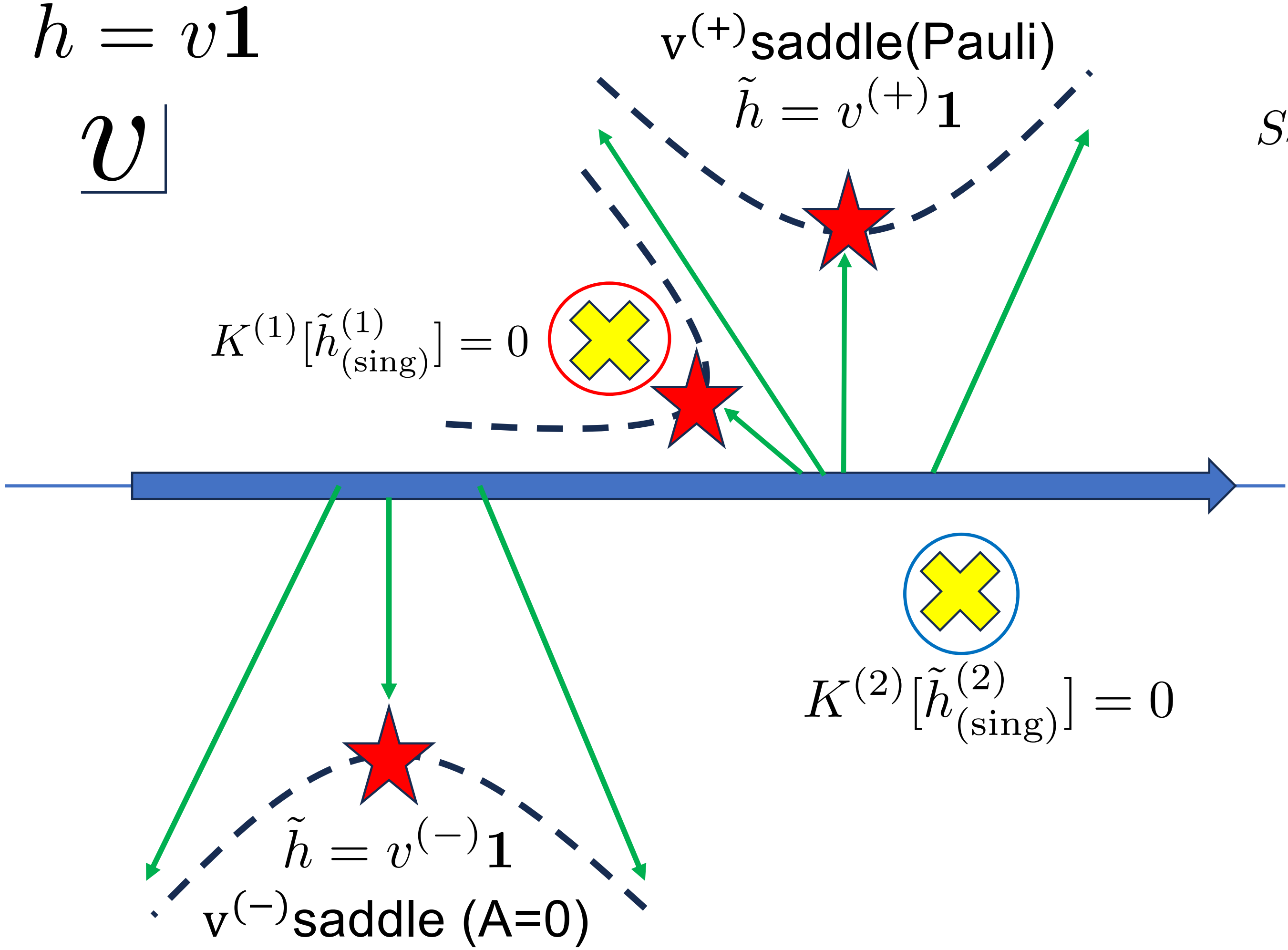
$$\tilde{h}_{(\mathrm{sing})} = \frac{\tilde{\gamma}}{2}\mathbf{1} \qquad \det K[\tilde{h}_{(\mathrm{sing})}] = 0$$

$\because$ $Z = \int dA e^{iS}$ The integral is not absolutely convergent

The case with convergence factor ε

$$\tilde{h} = v\mathbf{1}$$

v



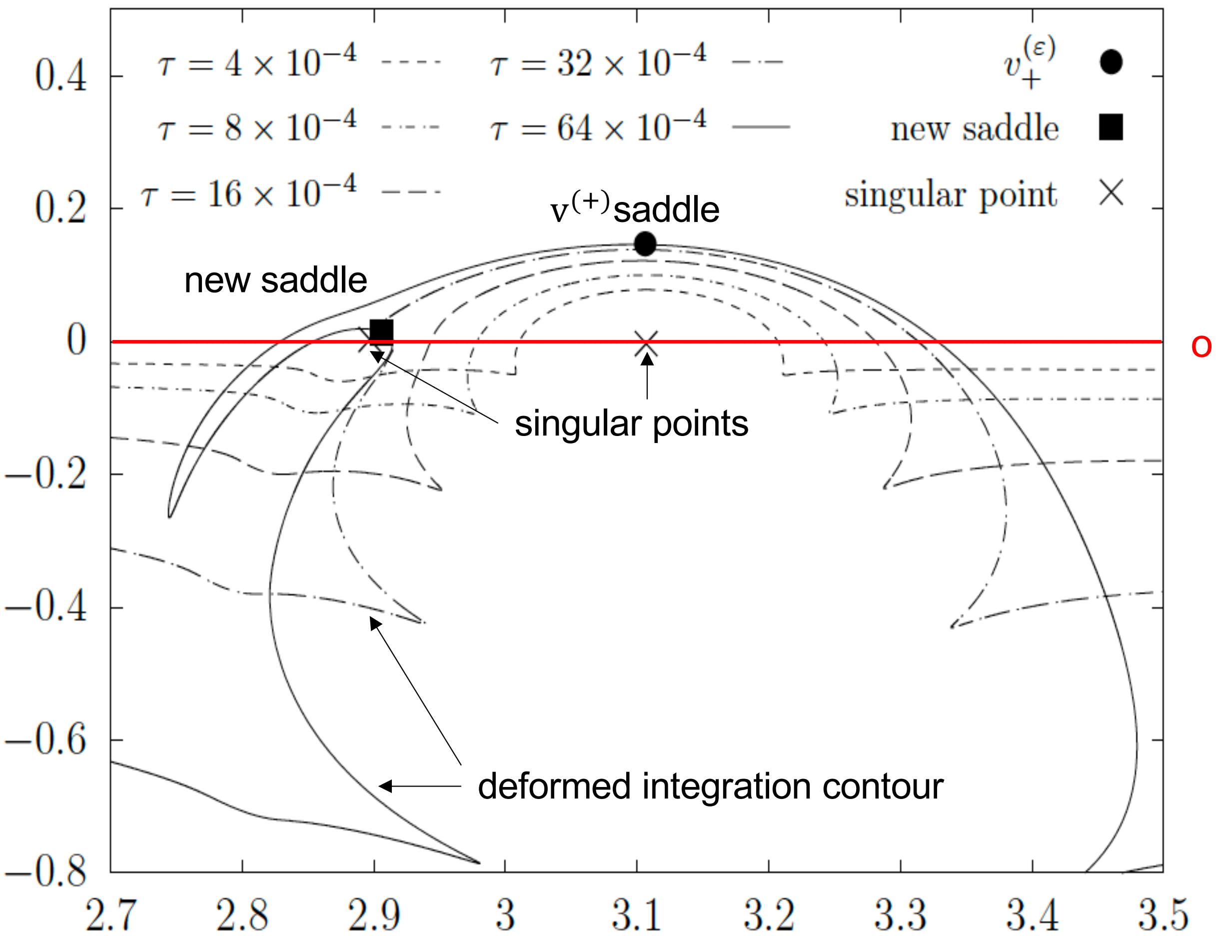
$$S_m = \frac{1}{2} N\gamma \left\{ e^{i\varepsilon} \mathrm{tr}(A_0)^2 - e^{-i\varepsilon} \mathrm{tr}(A_i)^2 \right\}$$

$$Z(\tilde{h}) \sim \det K^{(1)}[\tilde{h}]^{-\frac{1}{2}} \cdot \det K^{(2)}[\tilde{h}]^{-\frac{(D-1)}{2}}$$

ε splits the singular point

The v$^{(+)}$ saddle becomes relevant

The new saddle points



τ: the parameter deforming the integration contour based on Cauchy's theorem

new saddle point appears between the singular points

the new saddle and $v^{(+)}$saddle describe the contribution of the Pauli solution

Divergence of the partition function as $\varepsilon \to 0$

Pauli solution

$$\begin{cases} Z(v^{(+)}) \sim e^{-\frac{3}{8}iD\tilde{\gamma}^2} \left(\dfrac{\tilde{\gamma}}{2}\right)^{\frac{3}{2}D} \\ Z(v^{\text{new}}) \sim e^{-\frac{3}{8}iD\tilde{\gamma}^2} (\tilde{\gamma}\varepsilon)^{-\frac{3}{2}D} \end{cases}$$

diverges as $\varepsilon \to 0$

trivial solution

$$Z(v^{(-)}) \sim e^{-\frac{3}{4}\frac{D}{\tilde{\gamma}}} (2\tilde{\gamma})^{-\frac{3}{2}D}$$

$$\therefore \; Z_{\text{Pauli}} \sim \varepsilon^{-\frac{3}{2}D}, \quad Z_{\text{trivial}} \sim \text{finite}$$

due to the Lorentz symmetry

transition at finite ε : $|Z(v^{\text{new}})| > |Z(v^{(+)})|$ for $\tilde{\gamma} < \tilde{\gamma}_c = \sqrt{\dfrac{2}{\varepsilon}}$ (large D)

confirmed by numerical simulation [talk by A. Tripathi]

Summary

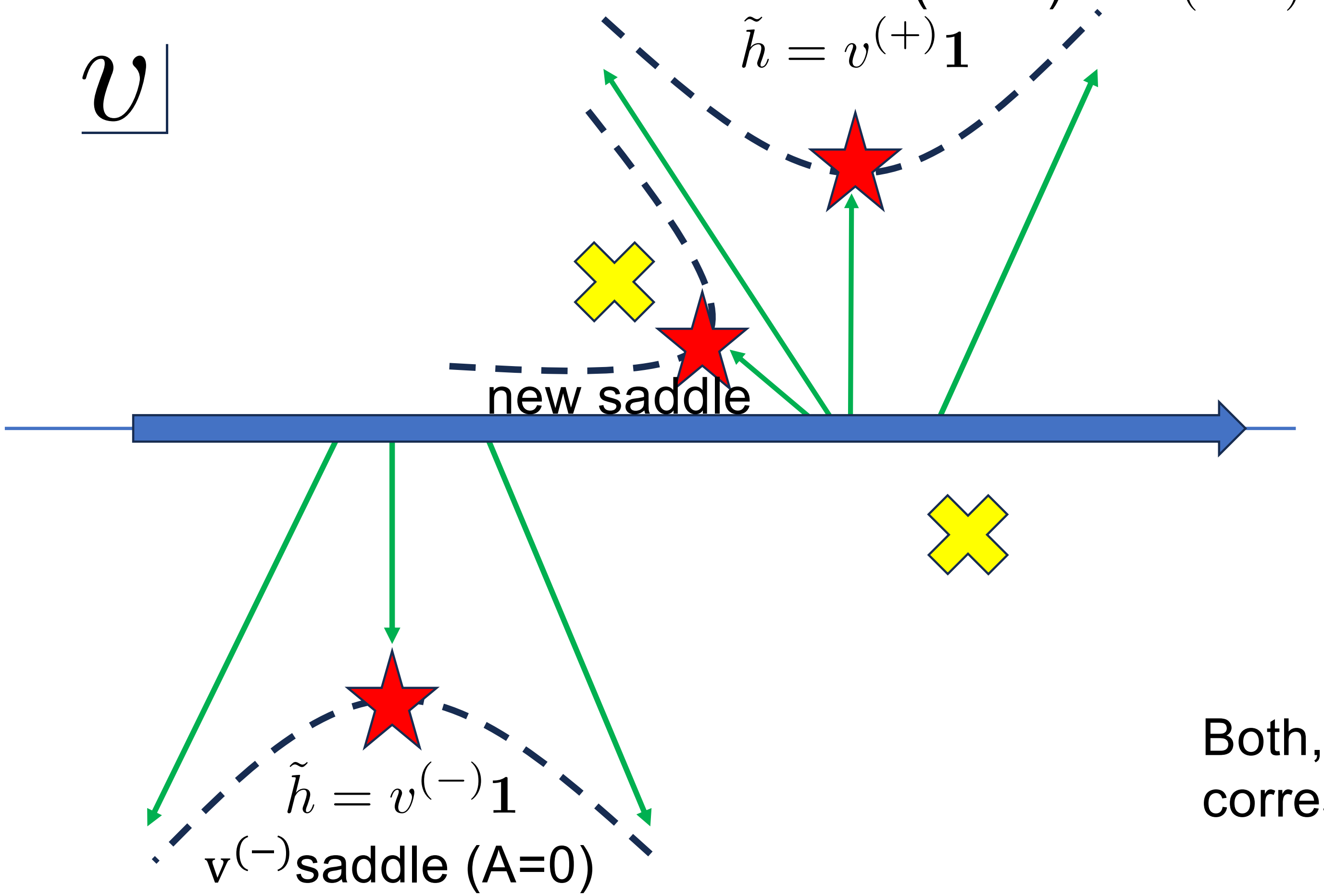
- 1/D expansion is useful in probing the nonperturbative properties of the N=2 bosonic IKKT matrix model with mass term

- We have confirmed the divergence of the partition function around Pauli solution due to the non-compactness of Lorentz symmetry group
- New saddle appears after introducing convergence factors
- Partition function associated with new saddle diverges as $\varepsilon \to 0$

Backup Slide

The new saddle point

$$\tilde{h} = v\mathbf{1}$$



$$Z(v^{(+)}) \sim e^{-\frac{3}{8}iD\tilde{\gamma}^2}\left(\frac{\tilde{\gamma}}{2}\right)^{\frac{3}{2}D}$$

new saddle point appears between the singular points

Both, the new saddle and $v^{(+)}$saddle correspond to the Pauli solution

Numerical simulation around the Pauli solution

$$Z = \int dA\, e^{i(S_b + S_m)}$$

$$S_m = \frac{1}{2} N\gamma \left\{ e^{i\varepsilon} \mathrm{tr}(A_0)^2 + \mathrm{tr}(A_i)^2 \right\}$$

convergence factor due to the Lorentz symmetry

initial configuration: $A_\mu = \sqrt{\frac{\gamma}{2}}\sigma_\mu \quad \mu = 1, 2, 3$

results obtained by the generalized thimble method:

$$Z \sim \varepsilon^{-c}$$

$$\langle \mathrm{tr}(A_0)^2 \rangle \sim -\frac{\partial}{\partial \varepsilon} \log Z = \frac{c}{\varepsilon}$$

$$\langle \mathrm{tr}(A_0)^2 \rangle \sim \frac{c}{\varepsilon}$$

$c \sim 14.7$ (thimble calculation)

$c = \frac{3}{2} D = 15$ (1/D expansion)

$$\langle -\mathrm{tr}(A_0)^2 + \mathrm{tr}(A_i)^2 \rangle = \text{finite} \sim \frac{3}{4}\gamma$$ (large γ)