Perturbative superstring theory and the IKKT Matrix Model

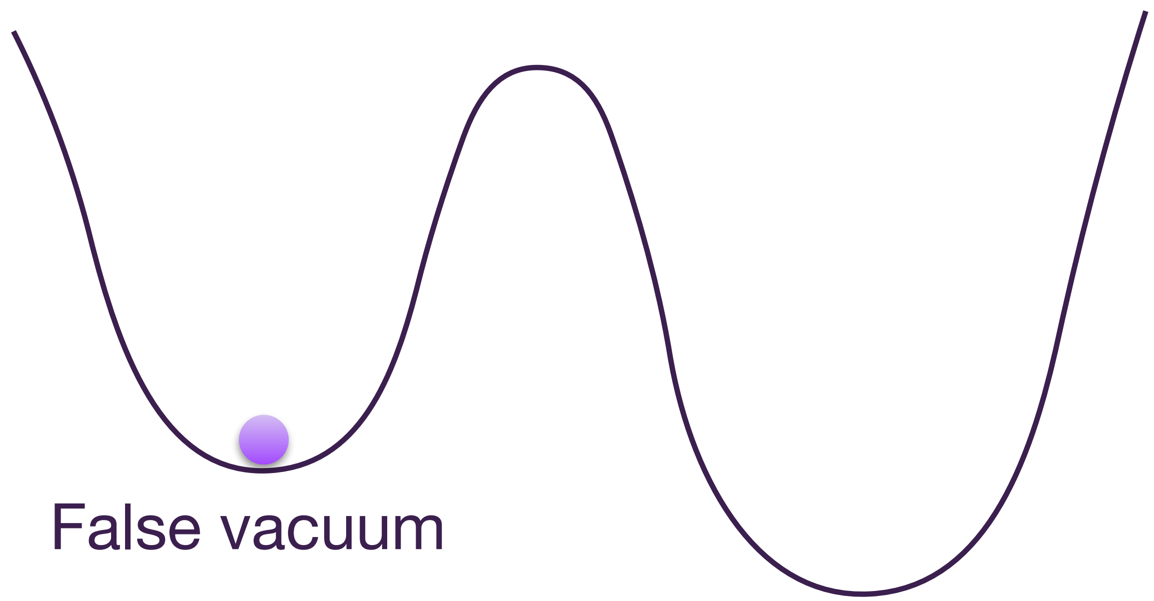
Yuhma Asano (University of Tsukuba)
Nov. 29, 2023 @KEK-TH

Introduction

What's wrong with superstring theory?

The established string theory is merely based on perturbation theory.

- There're infinitely many candidates of the vacuum, and it's unpredictable.



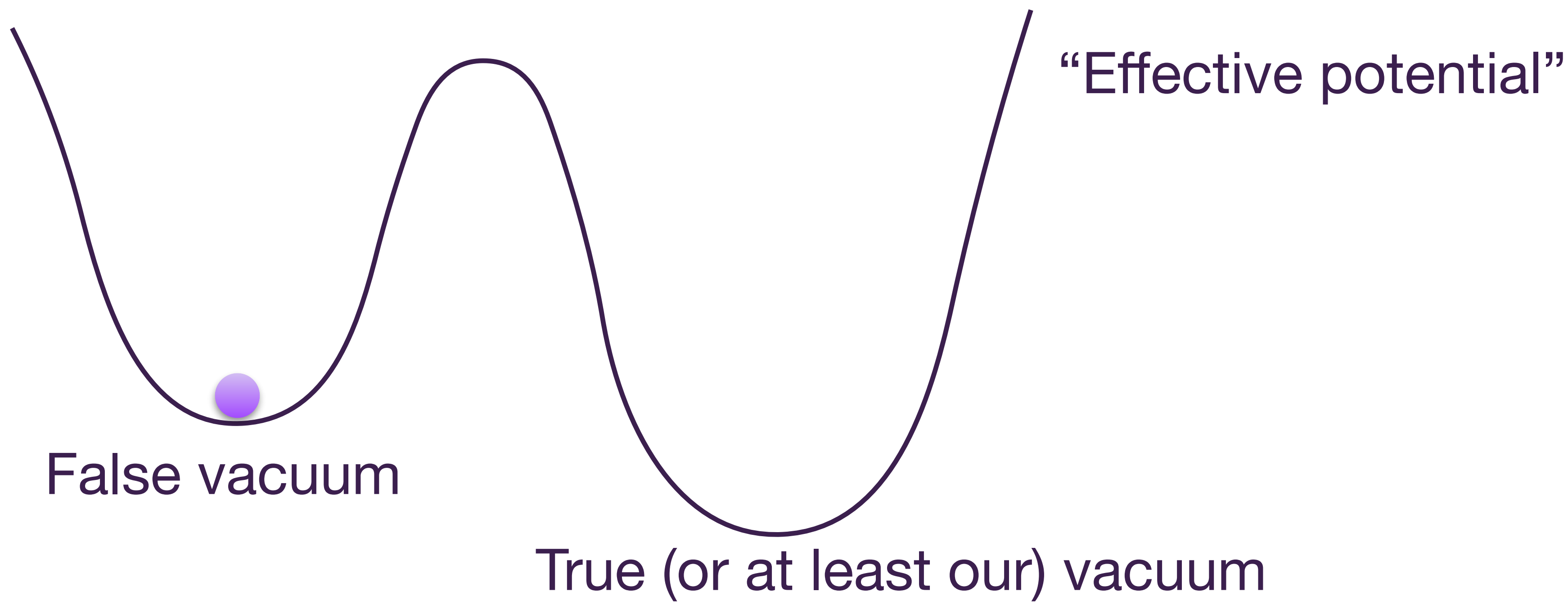
How is our 4D spacetime realised?

Introduction

What's wrong with superstring theory?

The established string theory is merely based on perturbation theory.

- There're infinitely many candidates of the vacuum, and it's unpredictable.

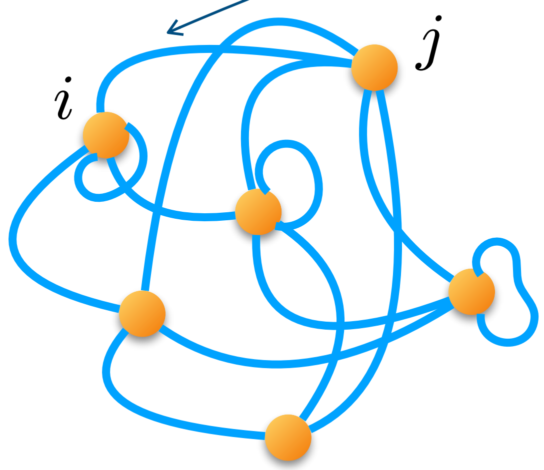


How is our 4D spacetime realised?

We need non-perturbative formulation
to make it genuinely predictable quantum gravity theory!

Introduction

Matrix Model



branes connected by strings

$$X^\mu = \begin{pmatrix} X^\mu_{11} & X^\mu_{12} & \cdots & \cdots & X^\mu_{1N} \\ X^\mu_{21} & \ddots & & & \vdots \\ \vdots & & \ddots & X^\mu_{ij} & \vdots \\ \vdots & & & \ddots & \vdots \\ X^\mu_{N1} & \cdots & \cdots & \cdots & X^\mu_{NN} \end{pmatrix}$$

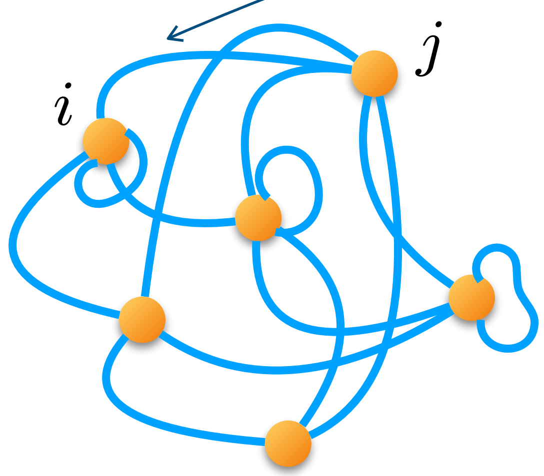
proposed as non-perturbative formulation of string theory

[Banks, Fischler, Shenker, Susskind ’96; Ishibashi, Kawai, Kitazawa, Tsuchiya ’96; …]

→ Genuinely predictable quantum gravity theory

Introduction

Matrix Model



branes connected by strings

$$X^\mu = \begin{pmatrix} X^\mu_{11} & X^\mu_{12} & \cdots & \cdots & X^\mu_{1N} \\ X^\mu_{21} & \ddots & & & \vdots \\ \vdots & & X^\mu_{ij} & & \vdots \\ \vdots & & & \ddots & \vdots \\ X^\mu_{N1} & \cdots & \cdots & \cdots & X^\mu_{NN} \end{pmatrix}$$

proposed as non-perturbative formulation of string theory

[Banks, Fischler, Shenker, Susskind '96; Ishibashi, Kawai, Kitazawa, Tsuchiya '96; …]

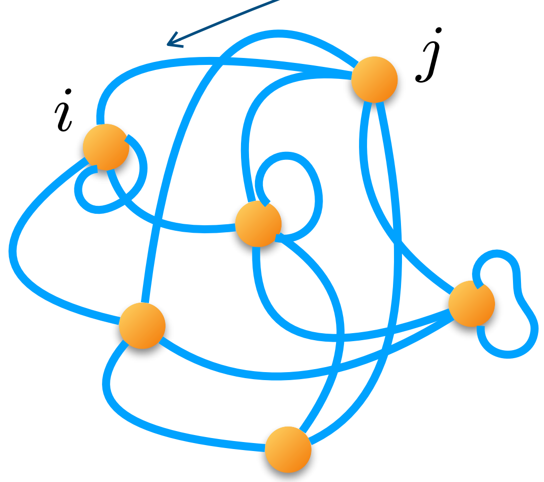
➡ Genuinely predictable quantum gravity theory

We expect (in the IKKT matrix model)

- to find a well-defined true vacuum, which we hope is 4D spacetime
  - Dynamics of the diagonal part forms 4D [Aoki, Iso Kawai, Kitazawa, Tada '98]
  - SSB to $SO(3)$ is observed [Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis '20]

Introduction

Matrix Model



branes connected by strings

$$X^\mu = \begin{pmatrix} X^\mu_{11} & X^\mu_{12} & \cdots & \cdots & X^\mu_{1N} \\ X^\mu_{21} & \ddots & & & \vdots \\ \vdots & & \ddots & X^\mu_{ij} & \vdots \\ \vdots & & & \ddots & \vdots \\ X^\mu_{N1} & \cdots & \cdots & \cdots & X^\mu_{NN} \end{pmatrix}$$

proposed as non-perturbative formulation of string theory

[Banks, Fischler, Shenker, Susskind '96; Ishibashi, Kawai, Kitazawa, Tsuchiya '96; ...]

➡ Genuinely predictable quantum gravity theory

We expect (in the IKKT matrix model)

- to find a well-defined true vacuum, which we hope is 4D spacetime
  - Dynamics of the diagonal part forms 4D [Aoki, Iso Kawai, Kitazawa, Tada '98]
  - SSB to $SO(3)$ is observed [Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis '20]
- to predict non-perturbative physics such as black holes and the early universe.

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N \times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N \times N$ matrices

$G_{\mu\nu} = \eta_{\mu\nu}$ or $\delta_{\mu\nu}$ $\left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya ’96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N \times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N \times N$ matrices

$G_{\mu\nu} = \eta_{\mu\nu}$ or $\delta_{\mu\nu}$ $\left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$

We don’t really know how the IKKT action enters in the partition fn.

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N\times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N\times N$ matrices

$G_{\mu\nu} = \eta_{\mu\nu}$ or $\delta_{\mu\nu}$ $\left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$

We don't really know how the IKKT action enters in the partition fn.

$$Z = \int [dX][d\psi]\, e^{iS[X,\psi;\eta_{\mu\nu}]} \ ?$$

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N\times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N\times N$ matrices

$G_{\mu\nu} = \eta_{\mu\nu}$ or $\delta_{\mu\nu}$ $\left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$

We don't really know how the IKKT action enters in the partition fn.

$$Z = \int [dX][d\psi]\, e^{iS[X,\psi;\eta_{\mu\nu}]} \ ?$$

		metric in the action	
		Euclidean	Lorentzian
weight	Euclidean	$e^{-S[X,\psi;\delta_{\mu\nu}]}$	$e^{-S[X,\psi;\eta_{\mu\nu}]}$
	Lorentzian	$e^{iS[X,\psi;\delta_{\mu\nu}]}$	$e^{iS[X,\psi;\eta_{\mu\nu}]}$

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N\times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N\times N$ matrices

$G_{\mu\nu} = \eta_{\mu\nu}$ or $\delta_{\mu\nu}$ $\left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$

We don't really know how the IKKT action enters in the partition fn.

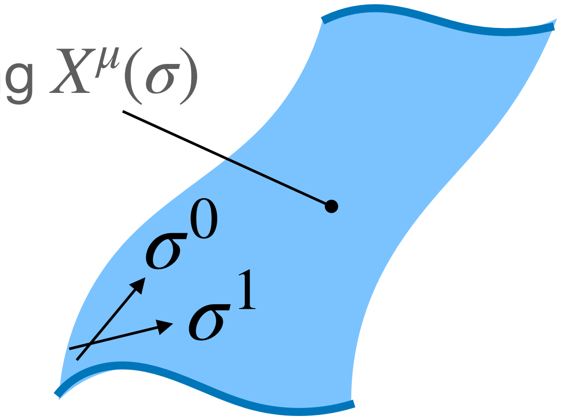
$$Z = \int [dX][d\psi]\, e^{iS[X,\psi;\eta_{\mu\nu}]} \text{ ?}$$

		metric in the action	
		Euclidean	Lorentzian
weight	Euclidean	$e^{-S[X,\psi;\delta_{\mu\nu}]}$	$e^{-S[X,\psi;\eta_{\mu\nu}]}$
	Lorentzian	$e^{iS[X,\psi;\delta_{\mu\nu}]}$	$e^{iS[X,\psi;\eta_{\mu\nu}]}$

"Euclidean IKKT model" (Euclidean weight, Euclidean metric)

"Lorentzian IKKT model" (Lorentzian weight, Lorentzian metric)

Green-Schwarz formalism



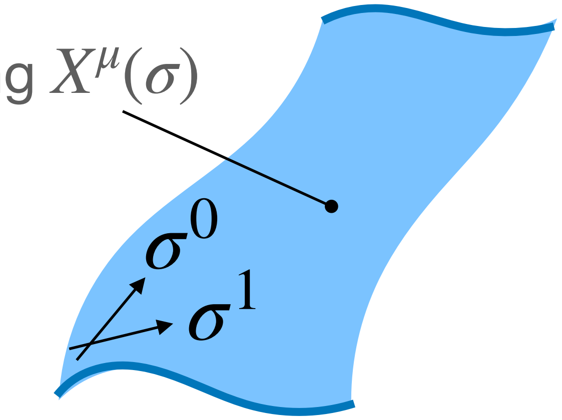
Nambu-Goto-type action

$$S_{\text{GS}} = -\frac{1}{2\pi}\int d^2\sigma \Big\{ \sqrt{-\det(\eta_{\mu\nu}\Pi_a^\mu \Pi_b^\nu)} - i\varepsilon^{ab}\partial_a X^\mu (\theta^{1T}\Gamma_\mu \partial_b \theta^1 - \theta^{2T}\Gamma_\mu \partial_b \theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu \partial_a \theta^1 \, \theta^{2T}\Gamma_\mu \partial_b \theta^2 \Big\}$$

X^μ: bosons (position of a string) θ^A: Majorana-Weyl fermions ($A = 1,2$)

$\Pi_a^\mu = \partial_a X^\mu - i(\theta^{1T}\Gamma^\mu \partial_a \theta^1 + \theta^{2T}\Gamma^\mu \partial_a \theta^2)$

Green-Schwarz formalism



Nambu-Goto-type action

$$S_{\mathrm{GS}} = -\frac{1}{2\pi}\int d^2\sigma \left\{ \sqrt{-\det(\eta_{\mu\nu}\Pi^\mu_a \Pi^\nu_b)} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu \partial_b \theta^1 - \theta^{2T}\Gamma_\mu \partial_b \theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu \partial_a \theta^1\, \theta^{2T}\Gamma_\mu \partial_b \theta^2 \right\}$$

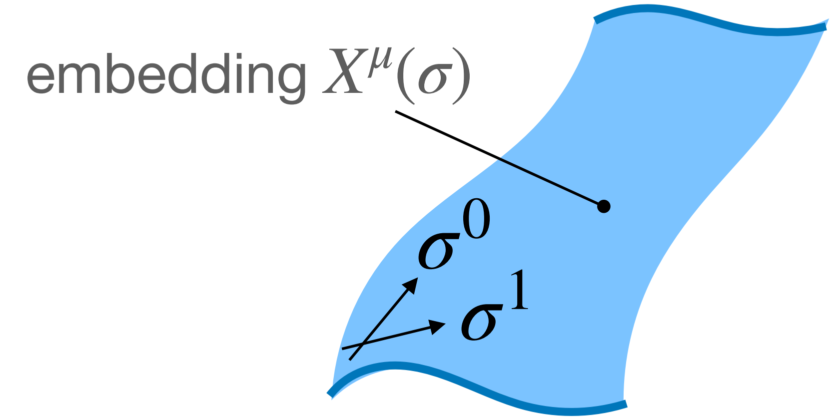
X^μ: bosons (position of a string) θ^A: Majorana-Weyl fermions ($A = 1,2$)

$\Pi^\mu_a = \partial_a X^\mu - i(\theta^{1T}\Gamma^\mu \partial_a \theta^1 + \theta^{2T}\Gamma^\mu \partial_a \theta^2)$

"κ symmetry" (local fermionic symmetry):

$$\delta^{\mathrm{f}}\theta^1 = (\mathbf{1} + \tilde{\Gamma})\kappa^1 \quad \delta^{\mathrm{f}}\theta^2 = (\mathbf{1} - \tilde{\Gamma})\kappa^2 \quad \delta^{\mathrm{f}} X^\mu = -i(\delta^{\mathrm{f}}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\mathrm{f}}\theta^{2T}\Gamma^\mu\theta^2)$$

Green-Schwarz formalism



Nambu-Goto-type action

$$S_{\rm GS} = -\frac{1}{2\pi}\int d^2\sigma\Big\{\underbrace{\sqrt{-\det(\underbrace{\eta_{\mu\nu}\Pi^\mu_a\Pi^\nu_b}_{h_{ab}})}}_{h} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2\Big\}$$

X^μ: bosons (position of a string) θ^A: Majorana-Weyl fermions $(A = 1{,}2)$

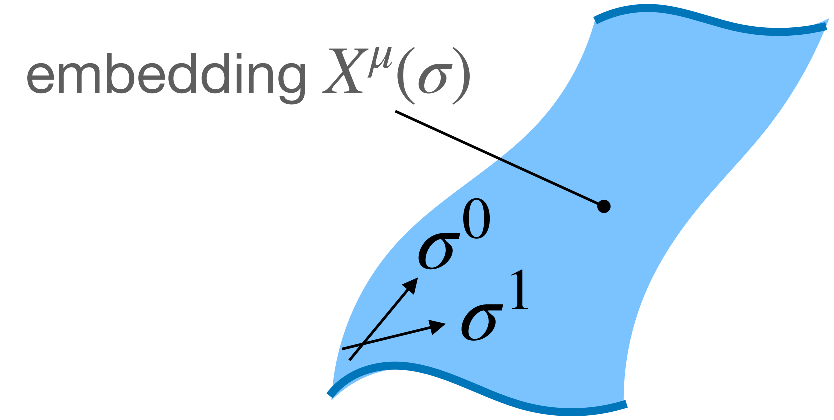
$$\Pi^\mu_a = \partial_a X^\mu - i(\theta^{1T}\Gamma^\mu\partial_a\theta^1 + \theta^{2T}\Gamma^\mu\partial_a\theta^2)$$

"κ symmetry" (local fermionic symmetry):

$$\delta^{\rm f}\theta^1 = (\mathbf{1}+\tilde\Gamma)\kappa^1 \quad \delta^{\rm f}\theta^2 = (\mathbf{1}-\tilde\Gamma)\kappa^2 \quad \delta^{\rm f}X^\mu = -i(\delta^{\rm f}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\rm f}\theta^{2T}\Gamma^\mu\theta^2)$$

$$\left(\tilde\Gamma = \frac{\varepsilon^{ab}}{2\sqrt{-h}}\Gamma_{\mu\nu}\Pi^\mu_a\Pi^\nu_b, \quad \tilde\Gamma^2 = \mathbf{1}\right)$$

Green-Schwarz formalism



Nambu-Goto-type action

$$S_{\rm GS} = -\frac{1}{2\pi}\int d^2\sigma \Big\{ \underbrace{\sqrt{-\underbrace{\det(\eta_{\mu\nu}\Pi_a^\mu \Pi_b^\nu)}_{h_{ab}}}}_{h} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \Big\}$$

X^μ: bosons (position of a string) θ^A: Majorana-Weyl fermions $(A = 1,2)$

$\Pi_a^\mu = \partial_a X^\mu - i(\theta^{1T}\Gamma^\mu\partial_a\theta^1 + \theta^{2T}\Gamma^\mu\partial_a\theta^2)$

"κ symmetry" (local fermionic symmetry):

$$\delta^{\rm f}\theta^1 = (\mathbf{1}+\tilde\Gamma)\kappa^1 \quad \delta^{\rm f}\theta^2 = (\mathbf{1}-\tilde\Gamma)\kappa^2 \quad \delta^{\rm f}X^\mu = -i(\delta^{\rm f}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\rm f}\theta^{2T}\Gamma^\mu\theta^2)$$

$$\left(\tilde\Gamma = \frac{\varepsilon^{ab}}{2\sqrt{-h}}\Gamma_{\mu\nu}\Pi_a^\mu\Pi_b^\nu, \quad \tilde\Gamma^2 = \mathbf{1}\right)$$

Obstacles of quantisation in the G-S formalism:

- κ symmetry has an infinite series of gauge symmetry
- κ symmetry is not closed off-shell $[\delta^{\rm f}_{\kappa_1}, \delta^{\rm f}_{\kappa_2}] = \delta^{\rm b}_{v_3} + \delta^{\rm f}_{\kappa_3} + \delta^{\lambda}_{\lambda_3}$ + (E.o.M.)

Schild-type action

The Nambu-Goto-type action is equivalent to the following Schild-type action:

$$S_{\text{Schild}} = -\frac{1}{2\pi}\int d^2\sigma\left\{ \underline{-\frac{1}{2}\left(\frac{h}{e_g} - e_g\right)} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \right\}$$

e_g: a Lagrange multiplier or “gauge field”

Schild-type action

The Nambu-Goto-type action is equivalent to the following Schild-type action:

$$S_{\text{Schild}} = -\frac{1}{2\pi}\int d^2\sigma \Bigg\{ \underline{-\frac{1}{2}\left(\frac{h}{e_g} - e_g\right)} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \Bigg\}$$

e_g: a Lagrange multiplier or "gauge field"

E.o.M. for e_g: $e_g^2 = -h$

※ Integrating out e_g brings this back to the Nambu-Goto action.

Schild-type action

The Nambu-Goto-type action is equivalent to the following Schild-type action:

$$S_{\text{Schild}} = -\frac{1}{2\pi}\int d^2\sigma\left\{ \underline{-\frac{1}{2}\left(\frac{h}{e_g} - e_g\right)} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \right\}$$

e_g: a Lagrange multiplier or "gauge field"

E.o.M. for e_g: $e_g^2 = -h$

※ Integrating out e_g brings this back to the Nambu-Goto action.

Remarkably, the fermionic gauge symmetry is formally enhanced:

$$\delta^{\mathrm{f}} X^\mu = -i(\delta^{\mathrm{f}}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\mathrm{f}}\theta^{2T}\Gamma^\mu\theta^2)$$

$$\delta^{\mathrm{f}} e_g = \frac{4ie_g^2}{e_g^2 + h}\sum_{A=1}^{2}\left(\frac{-h}{e_g}h^{ab} + (-1)^{A+1}\varepsilon^{ab}\right)\delta^{\mathrm{f}}\theta^{AT}\Gamma_\mu\Pi_a^\mu\partial_b\theta^A$$

$\delta^{\mathrm{f}}\theta^A$ is not projected by $\frac{1}{2}(\mathbf{1} \pm \tilde{\Gamma})$.

[Y.A. to appear]

Enhanced kappa symmetry

Algebra of the enhanced kappa symmetry

Each sector of the “enhanced” κ transformation

$$\delta^{\mathrm{f}} = \delta^{\mathrm{f}\varphi} + \delta^{\mathrm{f}\psi} \qquad \text{with} \qquad \varphi = \frac{1}{2}(\theta_1 + i\theta_2) \qquad \psi = \frac{1}{2}(\theta_1 - i\theta_2)$$

is closed off-shell, with a “trivial” gauge trf. $\delta^{\mathrm{g}}_{\mu} e_g = \frac{e_g^2}{e_g^2 + h}\partial_a(e_g \mu^a)$

Enhanced kappa symmetry

Algebra of the enhanced kappa symmetry

Each sector of the "enhanced" κ transformation

$$\delta^{\mathrm{f}} = \delta^{\mathrm{f}\varphi} + \delta^{\mathrm{f}\psi} \qquad \text{with} \qquad \varphi = \frac{1}{2}(\theta_1 + i\theta_2) \qquad \psi = \frac{1}{2}(\theta_1 - i\theta_2)$$

is closed off-shell, with a "trivial" gauge trf. $\delta^{\mathrm{g}}_{\mu} e_g = \dfrac{e_g^2}{e_g^2 + h}\partial_a(e_g \mu^a)$

$$[\delta^{\mathrm{f}\varphi}_{\kappa_1}, \delta^{\mathrm{f}\varphi}_{\kappa_2}] = \delta^{\mathrm{f}\varphi}_{\kappa_3} + \delta^{\mathrm{g}}_{\mu_3}$$

$$[\delta^{\mathrm{b}}_{\nu_1}, \delta^{\mathrm{b}}_{\nu_2}] = \delta^{\mathrm{b}}_{\nu_3}, \qquad [\delta^{\mathrm{b}}_{\nu}, \delta^{\mathrm{f}\varphi}_{\kappa}] = \delta^{\mathrm{f}\varphi}_{\kappa'} - \delta^{\mathrm{b}}_{\delta_\kappa \nu},$$

(δ^{b}: reparametrisation trf.)

$$[\delta^{\mathrm{b}}_{\nu}, \delta^{\mathrm{g}}_{\mu}] = \delta^{\mathrm{g}}_{\mu'} - \delta^{\mathrm{b}}_{\delta_\mu \nu}, \qquad [\delta^{\mathrm{f}\varphi}_{\kappa}, \delta^{\mathrm{g}}_{\mu}] = \delta^{\mathrm{f}\varphi}_{\kappa''} + \delta^{\mathrm{g}}_{\mu''}, \qquad [\delta^{\mathrm{g}}_{\mu_1}, \delta^{\mathrm{g}}_{\mu_2}] = \delta^{\mathrm{g}}_{\mu_3'},$$

[Y.A. to appear]

Enhanced kappa symmetry

Algebra of the enhanced kappa symmetry

Each sector of the "enhanced" κ transformation

$$\delta^{\rm f} = \delta^{\rm f\varphi} + \delta^{\rm f\psi} \qquad \text{with} \qquad \varphi = \frac{1}{2}(\theta_1 + i\theta_2) \qquad \psi = \frac{1}{2}(\theta_1 - i\theta_2)$$

is closed off-shell, with a "trivial" gauge trf. $\delta^{\rm g}_\mu e_g = \dfrac{e_g^2}{e_g^2 + h}\partial_a(e_g \mu^a)$

$$[\delta^{\rm f\varphi}_{\kappa_1}, \delta^{\rm f\varphi}_{\kappa_2}] = \delta^{\rm f\varphi}_{\kappa_3} + \delta^{\rm g}_{\mu_3}$$

$$[\delta^{\rm b}_{v_1}, \delta^{\rm b}_{v_2}] = \delta^{\rm b}_{v_3}, \qquad [\delta^{\rm b}_{v}, \delta^{\rm f\varphi}_{\kappa}] = \delta^{\rm f\varphi}_{\kappa'} - \delta^{\rm b}_{\delta_\kappa v},$$

($\delta^{\rm b}$: reparametrisation trf.)

$$[\delta^{\rm b}_{v}, \delta^{\rm g}_{\mu}] = \delta^{\rm g}_{\mu'} - \delta^{\rm b}_{\delta_\mu v}, \qquad [\delta^{\rm f\varphi}_{\kappa}, \delta^{\rm g}_{\mu}] = \delta^{\rm f\varphi}_{\kappa''} + \delta^{\rm g}_{\mu''}, \qquad [\delta^{\rm g}_{\mu_1}, \delta^{\rm g}_{\mu_2}] = \delta^{\rm g}_{\mu_3'},$$

[Y.A. to appear]

➡ BRST quantisation w/o ghosts of ghosts

“Derivation” of the IKKT model

To obtain the IKKT model by the matrix regularisation,
we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

$$\sigma^0 = e^{-i\theta}\sigma^2, \qquad X^0 = e^{-i\theta}X^{10}, \qquad \psi = e^{i\theta/2}\psi^{(\mathrm{E})}$$

“Derivation” of the IKKT model

To obtain the IKKT model by the matrix regularisation,
we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

$$\sigma^0 = e^{-i\theta}\sigma^2, \qquad X^0 = e^{-i\theta}X^{10}, \qquad \psi = e^{i\theta/2}\psi^{(\mathrm{E})}$$

Then, $$\{f_1, f_2\}_{\hat{\mathrm{P}}} = \sum_{a,b=0}^{1} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 = -e^{i\theta}\sum_{a,b=1}^{2} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 =: -e^{i\theta}\{f_1, f_2\}_{\hat{\mathrm{P}}}^{(\mathrm{E})}$$

“Derivation” of the IKKT model

To obtain the IKKT model by the matrix regularisation, we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

$$\sigma^0 = e^{-i\theta}\sigma^2, \qquad X^0 = e^{-i\theta}X^{10}, \qquad \psi = e^{i\theta/2}\psi^{(\mathrm{E})}$$

Then, $\{f_1, f_2\}_{\hat{\mathrm{P}}} = \sum_{a,b=0}^{1} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 = -e^{i\theta}\sum_{a,b=1}^{2} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 =: -e^{i\theta}\{f_1, f_2\}^{(\mathrm{E})}_{\hat{\mathrm{P}}}$

Fixing the gauge by $e_g = \hat{e}_g(\sigma) = 1 \quad \varphi = 0$

$$\exp[iS_{\mathrm{Schild}}] = \exp\left[\frac{i}{2\pi}\int d\sigma^0 d\sigma^1\left(\frac{1}{4}\{X^i, X^j\}^2_{\hat{\mathrm{P}}} - \frac{1}{2}\{X^0, X^i\}^2_{\hat{\mathrm{P}}} - \frac{1}{2} + 2i\psi^T\Gamma_i\{X^i, \psi\}_{\hat{\mathrm{P}}} + 2i\psi^T\Gamma_0\{X^0, \psi\}_{\hat{\mathrm{P}}}\right)\right]$$

“Derivation” of the IKKT model

To obtain the IKKT model by the matrix regularisation,
we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

$$\sigma^0 = e^{-i\theta}\sigma^2, \qquad X^0 = e^{-i\theta}X^{10}, \qquad \psi = e^{i\theta/2}\psi^{(\mathrm{E})}$$

Then, $\{f_1, f_2\}_{\hat{\mathrm{P}}} = \sum_{a,b=0}^{1} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 = -e^{i\theta}\sum_{a,b=1}^{2} \varepsilon^{ab}\partial_a f_1 \partial_b f_2 =: -e^{i\theta}\{f_1, f_2\}_{\hat{\mathrm{P}}}^{(\mathrm{E})}$

Fixing the gauge by $e_g = \hat{e}_g(\sigma) = 1 \qquad \varphi = 0$

$$\exp[iS_{\mathrm{Schild}}] = \exp\left[\frac{i}{2\pi}\int d\sigma^0 d\sigma^1 \left(\frac{1}{4}\{X^i, X^j\}_{\hat{\mathrm{P}}}^2 - \frac{1}{2}\{X^0, X^i\}_{\hat{\mathrm{P}}}^2 - \frac{1}{2} + 2i\psi^T\Gamma_i\{X^i, \psi\}_{\hat{\mathrm{P}}} + 2i\psi^T\Gamma_0\{X^0, \psi\}_{\hat{\mathrm{P}}}\right)\right]$$

$$\xrightarrow{\theta \to \frac{\pi}{2}} \exp\left[-\frac{1}{2\pi}\int d\sigma^1 d\sigma^2 \left(\frac{1}{4}\{X^m, X^n\}_{\hat{\mathrm{P}}}^{(\mathrm{E})\,2} + 2\psi^{(\mathrm{E})T}\Gamma_m\{X^m, \psi^{(\mathrm{E})}\}_{\hat{\mathrm{P}}} + \frac{1}{2}\right)\right] \qquad (m = 1,\cdots,9,10)$$

"Derivation" of the IKKT model

To obtain the IKKT model by the matrix regularisation,
we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

Th

$\{f_1, f_2\}^{(\mathrm{E})}_{\hat{\mathrm{P}}}$

Fixing th

Matrix Regularisation by a map from a function

$$X(\sigma) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \tilde{X}_{lm} Y_{lm}(\sigma) \longrightarrow \sum_{l=0}^{N-1}\sum_{m=-l}^{l} \tilde{X}_{lm}(Y_{lm})_{ij} = X_{ij}$$

spherical harmonics ($Y_{lm}(\sigma)$)

matrix (X_{ij})

$$\exp[iS_{\mathrm{Schild}}] = \exp\left[\frac{i}{2\pi}\int d\sigma^0 d\sigma^1 \left(\frac{1}{4}\{X^i, X^j\}^2_{\hat{\mathrm{P}}} - \frac{1}{2}\{X^0, X^i\}^2_{\hat{\mathrm{P}}} - \frac{1}{2} + 2i\psi^T\Gamma_i\{X^i, \psi\}_{\hat{\mathrm{P}}} + 2i\psi^T\Gamma_0\{X^0, \psi\}_{\hat{\mathrm{P}}}\right)\right]$$

$$\xrightarrow{\theta \to \frac{\pi}{2}} \exp\left[-\frac{1}{2\pi}\int d\sigma^1 d\sigma^2 \left(\frac{1}{4}\{X^m, X^n\}^{(\mathrm{E})\,2}_{\hat{\mathrm{P}}} + 2\psi^{(\mathrm{E})T}\Gamma_m\{X^m, \psi^{(\mathrm{E})}\}_{\hat{\mathrm{P}}} + \frac{1}{2}\right)\right] \qquad (m = 1, \cdots, 9, 10)$$

$$\xrightarrow{\text{Mat. Reg.}} \exp\left[-N\,\mathrm{tr}\left(-\frac{1}{4}[X^m, X^n]^2 - \frac{i}{2}\psi^{(\mathrm{E})T}\Gamma_m[X^m, \psi^{(\mathrm{E})}] + \frac{1}{4N}\right)\right]$$

"Derivation" of the IKKT model

To obtain the IKKT model by the matrix regularisation,
we need to make the worldsheet Euclidean.
Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

Th $\{f_1, f_2\}^{(E)}_{\hat{P}}$

Matrix Regularisation by a map from a function

$$X(\sigma) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \tilde{X}_{lm} Y_{lm}(\sigma) \longrightarrow \sum_{l=0}^{N-1} \sum_{m=-l}^{l} \tilde{X}_{lm} (Y_{lm})_{ij} = X_{ij}$$

spherical harmonics

matrix

Fixing th

$$\exp[iS_{\text{Schild}}] = \exp\left[\frac{i}{2\pi}\int d\sigma^0 d\sigma^1 \left(\frac{1}{4}\{X^i, X^j\}^2_{\hat{P}} - \frac{1}{2}\{X^0, X^i\}^2_{\hat{P}} - \frac{1}{2} + 2i\psi^T\Gamma_i\{X^i, \psi\}_{\hat{P}} + 2i\psi^T\Gamma_0\{X^0, \psi\}_{\hat{P}}\right)\right]$$

$$\xrightarrow{\theta \to \frac{\pi}{2}} \exp\left[-\frac{1}{2\pi}\int d\sigma^1 d\sigma^2 \left(\frac{1}{4}\{X^m, X^n\}^{(E)\,2}_{\hat{P}} + 2\psi^{(E)T}\Gamma_m\{X^m, \psi^{(E)}\}_{\hat{P}} + \frac{1}{2}\right)\right] \qquad (m = 1,\cdots,9,10)$$

$$\xrightarrow{\text{Mat. Reg.}} \boxed{\exp\left[-N\,\mathrm{tr}\left(-\frac{1}{4}[X^m, X^n]^2 - \frac{i}{2}\psi^{(E)T}\Gamma_m[X^m, \psi^{(E)}] + \frac{1}{4N}\right)\right]}$$

We have "derived" the Euclidean weight w/ the Euclidean metric IKKT action from the perturbative superstring theory.

Vertex operators

The BRST transformation on the worldsheet is

$$\delta^{\mathrm{BRST}} X^\mu = -2i\epsilon\gamma^T\Gamma^\mu\psi + \epsilon\{c, X^\mu\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\psi = \epsilon\{c,\psi\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\varphi = \epsilon\gamma + \epsilon\{c,\varphi\}_{\hat{\mathrm{P}}},$$

A BRST inv. vertex: $\displaystyle\int d^2\sigma\, e^{ik_\mu(X^\mu + 2i\varphi^T\Gamma^\mu\psi)} \to \int d^2\sigma\, e^{ik_\mu X^\mu}$ (momentum-k_μ mode)

Vertex operators

The BRST transformation on the worldsheet is

$$\delta^{\mathrm{BRST}} X^\mu = -2i\epsilon\gamma^T\Gamma^\mu\psi + \epsilon\{c, X^\mu\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\psi = \epsilon\{c,\psi\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\varphi = \epsilon\gamma + \epsilon\{c,\varphi\}_{\hat{\mathrm{P}}},$$

A BRST inv. vertex: $\int d^2\sigma\, e^{ik_\mu(X^\mu + 2i\varphi^T\Gamma^\mu\psi)} \to \int d^2\sigma\, e^{ik_\mu X^\mu}$ (momentum-k_μ mode)

➡ In the matrix model, $V^\Phi = \mathrm{tr}\, e^{ik_\mu X^\mu}$

Vertex operators

The BRST transformation on the worldsheet is

$$\delta^{\mathrm{BRST}} X^\mu = -2i\epsilon\gamma^T\Gamma^\mu\psi + \epsilon\{c, X^\mu\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\psi = \epsilon\{c,\psi\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\varphi = \epsilon\gamma + \epsilon\{c,\varphi\}_{\hat{\mathrm{P}}},$$

A BRST inv. vertex: $\int d^2\sigma\, e^{ik_\mu(X^\mu + 2i\varphi^T\Gamma^\mu\psi)} \to \int d^2\sigma\, e^{ik_\mu X^\mu}$ (momentum-k_μ mode)

➡ In the matrix model, $V^\Phi = \mathrm{tr}\, e^{ik_\mu X^\mu}$

This forms a massless multiplet of type-IIB SUGRA
by acting the supercharge operator Q onto this vertex.

Perturbative String states	⟷	Matrix model Operators
massless multiplet of type-IIB SUGRA	⟷	massless multiplet of type-IIB SUGRA

[Kitazawa ’02; Iso, Terachi, Umetsu ’04; Kitazawa, Mizoguchi, Saito ’07]

Summary

- We find there is a gauge trf. that closes algebra off-shell for the string action in the G-S formalism by rewriting the action to the Schild type.
  It allows us to quantise the theory without an infinite tower of ghosts.

- If we assume the IKKT matrix model is derived by the matrix regularisation of the Schild-type action, it is the Euclidean action w/ e^{-S}.
  ※ doesn't mean the IKKT model should be Euclidean

- The matrix regularised massless-mode vertex $\int d^2\sigma \, e^{ik_\mu(X^\mu + 2i\varphi^T \Gamma^\mu \psi)}$, invariant under the BRST trf., is the suggested matrix-model vertex operator $\mathrm{tr}\, e^{ik_\mu X^\mu}$ of a string (or D1), which forms a massless multiplet of type IIB SUGRA by acting the supercharge operator.

Summary

- We find there is a gauge trf. that closes algebra off-shell for the string action in the G-S formalism by rewriting the action to the Schild type.
  It allows us to quantise the theory without an infinite tower of ghosts.

- If we assume the IKKT matrix model is derived by the matrix regularisation of the Schild-type action, it is the Euclidean action w/ e^{-S}.

  ※ doesn't mean the IKKT model should be Euclidean

- The matrix regularised massless-mode vertex $\int d^2\sigma\, e^{ik_\mu(X^\mu + 2i\varphi^T\Gamma^\mu\psi)}$, invariant under the BRST trf., is the suggested matrix-model vertex operator $\mathrm{tr}\, e^{ik_\mu X^\mu}$ of a string (or D1), which forms a massless multiplet of type IIB SUGRA by acting the supercharge operator.

- I plan to find out the exact relationship between perturbative superstring d.o.f. and matrix d.o.f., starting with this identification for the vertex operator.

  [cf. Fukuma, Kawai, Kitazawa, Tsuchiya '97]