KEK Theory Workshop 2023, Nov.29–Dec.1, 2023

On Holography in de Sitter Space

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Grant-in-Aid for Transformative Research Areas A
Extreme Universe

Based on

dS3/CFT2

2110.03197 [PRL129(2022)041601]

2203.02852 [JHEP 05 (2022) 129]

with Yasuaki Hikida, Yusuke Taki (YITP), Tatsuma Nishioka (Osaka)

Pseudo Entropy in dS/CFT

2210.09457 [PRL130 (2023) 031601]

2302.11695 [JHEP 05 (2023) 052]

with Kazuki Doi, Jonathan Harper, Yusuke Taki (YITP), Ali Mollabashi (IPM)

Holography for a half dS

2306.07575 [JHEP10(2023)137]

with Taishi Kawamoto, Shan-Ming Ruan, Yu-ki Suzuki (YITP)

① Introduction

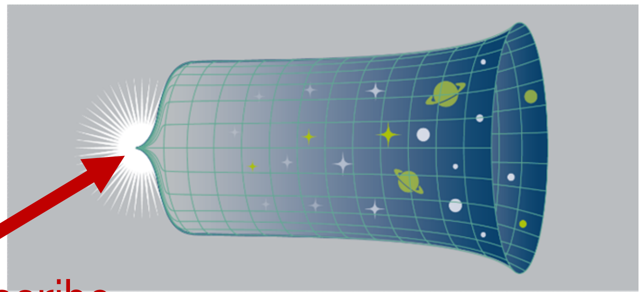
One of the biggest questions in theoretical physics is to understand how we formulate quantum gravity.

➡ One promising idea toward this goal is to apply holography.

Holography implies "Gravity = Quantum matter (e.g. QFTs)".

This has been very successful for the class of spaces which are asymptotically AdS.

A more challenging target is clearly the creation of Universe.

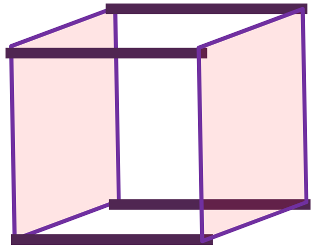


How do we describe quantum gravity ?

Quantum Gravity in Maximally Symmetric Universe

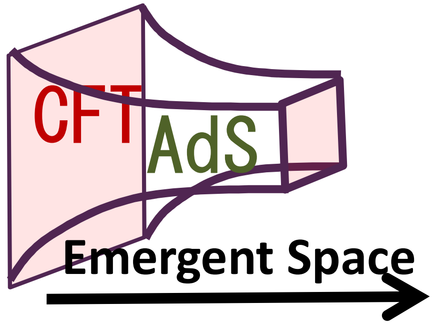
[1] $\Lambda = 0$: Flat Space

Quantum gravity is described by string theory.



[2] $\Lambda < 0$: Anti de-Sitter Space (AdS)

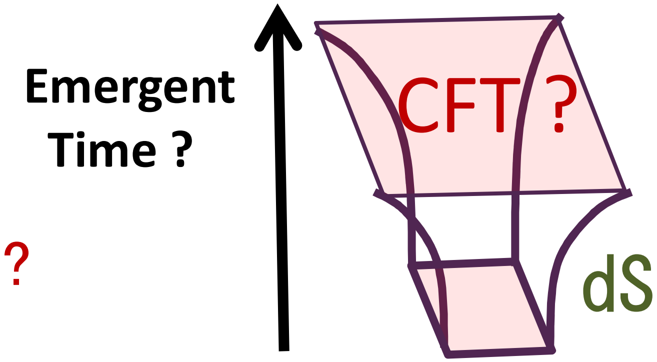
Using the AdS/CFT, we can describe quantum gravity in terms of quantum field theory.



[3] $\Lambda > 0$: de-Sitter Space (dS)

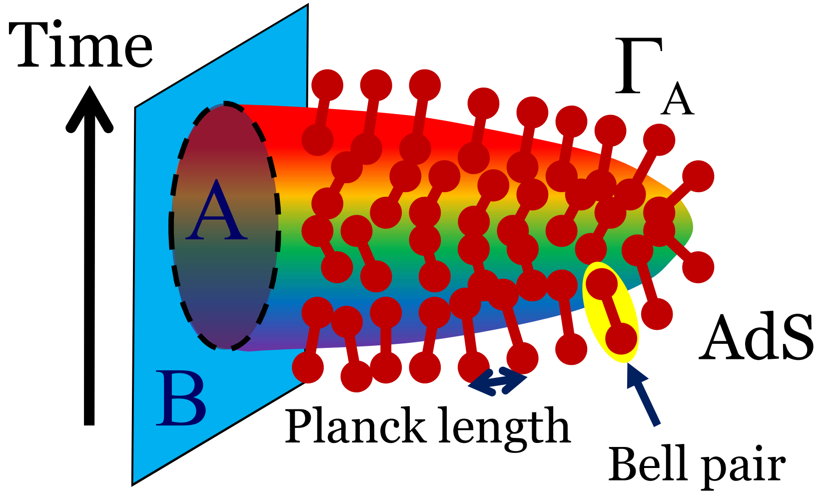
Very important to understand how the Universe begins, but very difficult !

➡ What is the holographic dual of dS ??



Recent developments suggest that AdS geometry emerges from quantum entanglement.

Can the spacetime of dS emerge from quantum information ?



	Boundary	Emergent coordinate
AdS	Time-like	Space direction emerges from QE
dS	Space-like	Time direction emerges from ??

In this talk, we provide
(i) an example of holography for 3D dS to study these questions,
(ii) another version of dS holography with a space-like boundary .

Comment

In addition to the original dS/CFT [Strominger 2001], there have recently been several different approaches :

Application of dS/dS duality and TTbar
[Alishahiha–Karch–Silverstein–Tong 2004, .., Dong–Silverstein–Torroba 2018, Gorbenko–Silvestein–Torroba 2018,…]

Surface/state duality
[Miyaji–TT 2015,…]

A half dS holography (final part of this talk)

Static patch holography (AQFT, Double Scaled SYK,..)
[Susskind 2021–now, Witten 2022, Verlinde 2023,…]

Contents

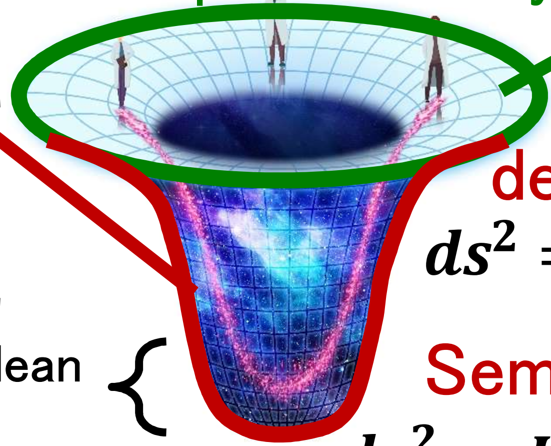
② General Properties of dS/CFT

A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,….]

Geometric sym. SO(1,d+1) **Dual** Conformal sym. SO(1,d+1)

d+1 dim. Lorentzian de-Sitter spacetime	⟺	Euclidean d. dim CFT on S^d

Space-like bdy



Time

Lorentzian time

Euclidean time

t=∞

UV cut off

$\varepsilon = e^{-t_\infty}$

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\Omega^2)$$

$$\theta = it + \frac{\pi}{2}$$

Time emerges from Euclidean CFT !

Ψ [dS gravity]=Z [CFT]

A regular Euclidean holographic CFT is dual to a Euclidean AdS.
➔ The Euclidean CFT dual to a dS should be "exotic".

Ex1. Proposed gravity dual of 4 dim. Higher spin dS gravity
➔ 3 dim. Sp(N) vector model [anti-commuting scalar fields]
[Anninos-Hartman-Strominger 2011]

Ex2. "Holographic entanglement entropy" gets complex valued.
[No space-like extreme surface ending on bdy. Narayan 2015, Sato 2015,...]

➡ We argue this is interpreted as pseudo entropy, instead !

Ex.3 Our new example in dS3/CFT2 is also non-unitary !

What we expect for dS/CFT

➔Let us assume dS Einstein gravity and extract general expectations.

d+1 dim. (Lorentzian) de-Sitter $ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$

↓

S^{d+1} (Euclidean de-Sitter) $ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$

↓ $L_{AdS} = iL_{dS},\ \rho = i\theta$

Euclidean AdS (H^{d+1}) $ds^2 = L_{AdS}^2(d\rho^2 + \mathrm{Sinh}^2\rho d\Omega^2)$

Central charge: $c \sim \frac{L_{AdS}^{d-1}}{G_N} = i^{d-1} \cdot \frac{L_{dS}^{d-1}}{G_N}$

We are interested in d=2 case in this talk !

➔
(i) Central charge becomes imaginary for d=even !
(ii) Central charge gets larger in classical gravity limit.

③ dS3/CFT2 example

(3-1) Two well-known facts on Chern-Simons theory

[1] The Einstein gravity on 3d de Sitter space can be rewritten as the 3d CS gauge theory with gauge group G=SU(2)×SU(2):

$\boldsymbol{A}$ $\qquad$ $\bar{\boldsymbol{A}}$

$$I_{\text{dS gravity}} = i\,(I_{CS}[A] - I_{CS}[\bar{A}]),$$

$$I_{CS}[A] = \frac{k}{4\pi}\int_{S^3} \mathrm{Tr}\left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right]$$

k = level

[Witten 1988,⋯
for recent analysis, refer to e.g.
Castro, Sabella-Garnier, Zukowski 2020.]

$$Z_{CS(\text{dS})} = \int DA D\bar{A}\, e^{-I_{\text{dS gravity}}[A,\bar{A}]}$$

➡ Einstein gravity on S^3

$$k = i \cdot \frac{L_{dS}}{4G_N}$$

$$A = e + \omega, \quad \bar{A} = e - \omega$$

[2] We also remember "CS holography" :

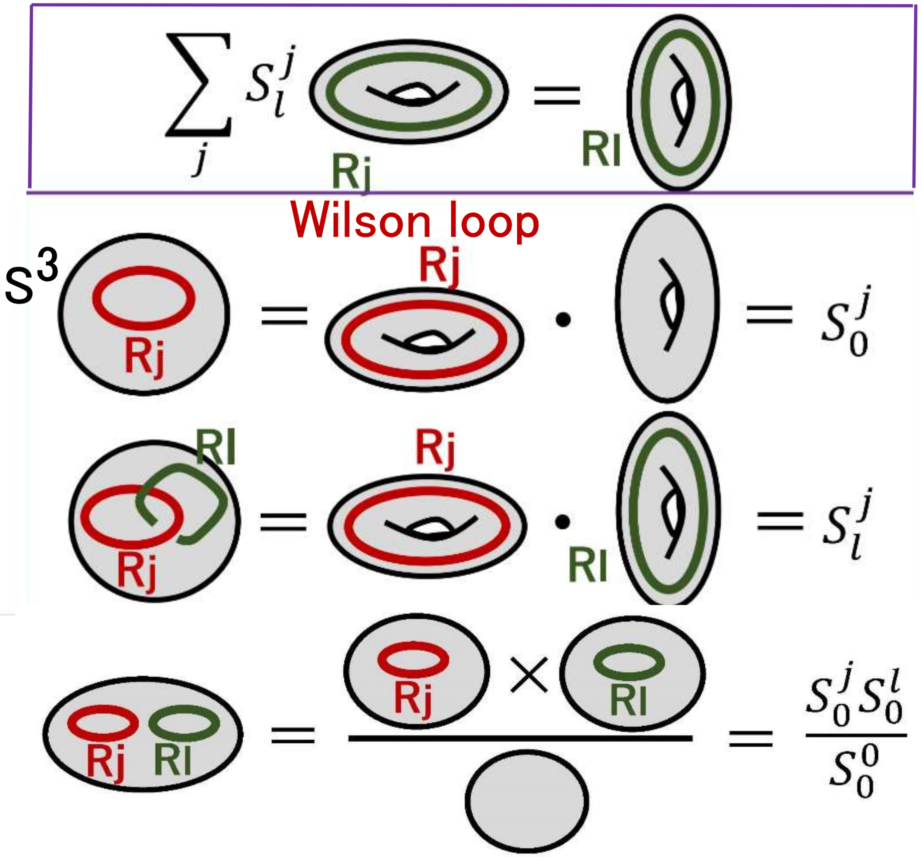
[Witten 1989]

SU(2) CS gauge theory at level k

= conformal block of SU(2) WZW model at level k

$$\sum_j S_j^l Z_j(\tau) = Z_l(-\frac{1}{\tau}) \longrightarrow S_j^l = \sqrt{\frac{2}{k+2}}\mathrm{Sin}\left[\frac{\pi(2j+1)(2l+1)}{k+2}\right]$$

Modular S-matrix



$$Z_{CS} = \int DAD\bar{A}\, e^{iI_{CS}[A]} W(R_j)\cdots$$

$$Z_{CS}[S^3, R_j] = S_0^j$$

$$Z_{CS}[S^3, L(R_j, R_l)] = S_l^j$$

$$Z_{CS}[S^3, R_j, R_l] = \frac{S_0^j S_0^l}{S_0^0}$$

(3-3) Our formulation of dS3/CFT2

A puzzle about dS3/CFT2

By employing the facts explained, one may suspect

3d de Sitter gravity $\stackrel{?}{=}$ SU(2) × SU(2) CS gauge theory
$\stackrel{?}{=}$ SU(2) WZW model ➡ Is this CFT dual ?

However, this does not seem to work because

Einstein gravity limit: $k = i \cdot \dfrac{L_{dS}}{4G_N} \to i\infty$

leads to $c_{SU(2)} = \dfrac{3k}{k+2} \to 3$. This is not the large c limit , expected from the dS/CFT !

Our claim

Instead, we argue that "k→−2 limit" realizes the dS/CFT duality:

$$k \approx -2 + \frac{4iG_N}{L_{dS}} \quad \Rightarrow \quad c_{SU(2)} = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N} \equiv ic_{ds}$$

This is what we expect from dS/CFT.

Duality relation for excitations

We identify excitations in dS with primary operators in WZW CFT:

$$\Delta_j = \frac{2j(j+1)}{k+2} = iL_{dS}E_j$$

Conformal dim. → Δ_j; Energy in dS → E_j

The spin j is continuous and can be complex valued.

We have in mind a non-rational version of SU(2) WZW CFT ∼ Liouville CFT.

(3-4) Evidence of dS3/CFT2: Free Energy Computation

dS3/CFT2 example [Hikida-Nishioka-Taki-TT 2021]

k→−2 limit of SU(2) WZW model (a 2dim. CFT) $c_{SU(2)} \to i\infty$

⇕ Dual

Einstein Gravity on a 3D de Sitter space (radius $L_{ds} \to \infty$)

One may wonder if our limit can correctly reproduce the Einstein gravity on a 3d de Sitter :

$$I_G = -\frac{1}{16\pi G_N}\int \sqrt{g}(R - 2\Lambda), \qquad \left(\Lambda \equiv \frac{1}{L_{dS}^2}\right)$$

➔ Below we will compare both partition functions.

Partition Functions with a Single Wilson loop

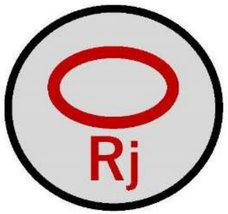
Consider partition functions with Wilson loops inserted.

Useful relation: $\mathbf{1 - 8G_N E_j = 1 - \frac{12\Delta_j}{C_{SU(2)}} \approx (2j+1)^2}$.

$$S_j^l = \sqrt{\frac{2}{k+2}}\,\mathrm{Sin}\left[\frac{\pi(2j+1)(2l+1)}{k+2}\right] \approx e^{\frac{\pi i(2j+1)(2l+1)}{k+2}}.$$

k➔−2

CFT Prediction: partition function with (i) a single Wilson loop



$$Z_{CS(dS)}\left[S^3, R_j\right] = \left|S_j^0\right|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E_j}}$$

In particular, when E=0, we obtain the de Sitter entropy $\frac{\pi L_{ds}}{2G_N}$!

Gravity dual: 3 dim. de Sitter `black hole' with energy Ej

$$ds^2 = L_{ds}^2\left[\left(1-8G_N E_j - r^2\right)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right].$$

The regularity at the horizon requires the periodicity of τ :

$$\tau \sim \tau + \frac{2\pi}{\sqrt{1-8G_N E_j}}.$$

The on-shell action for this solution is evaluated as

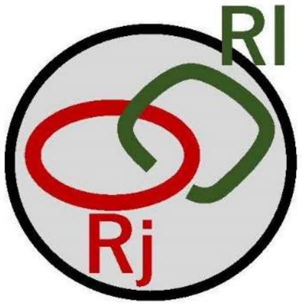
$$I_G = -\frac{1}{16\pi G_N}\int \sqrt{g}(R-2\Lambda) = -\underbrace{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E_j}}_{\text{Black hole entropy}}.$$

This reproduces the CS result: $Z_{CS(dS)}[S^3, R_j] = e^{-I_G} = e^{S_{\mathrm{BH}}}$.

Partition Functions with Two Wilson loops

CFT prediction

Partition function with (ii) Two Linked Wilson loop

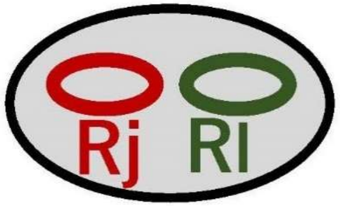


$$Z_{CS(dS)}\left[S^3, L(R_j\,, R_l)\right] = \left|S_j^l\right|^2$$

$$\approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E_j}\sqrt{1-8G_N E_l}}$$

Agree with dS gravity !

Partition function with (iii) Un-linked Wilson loop

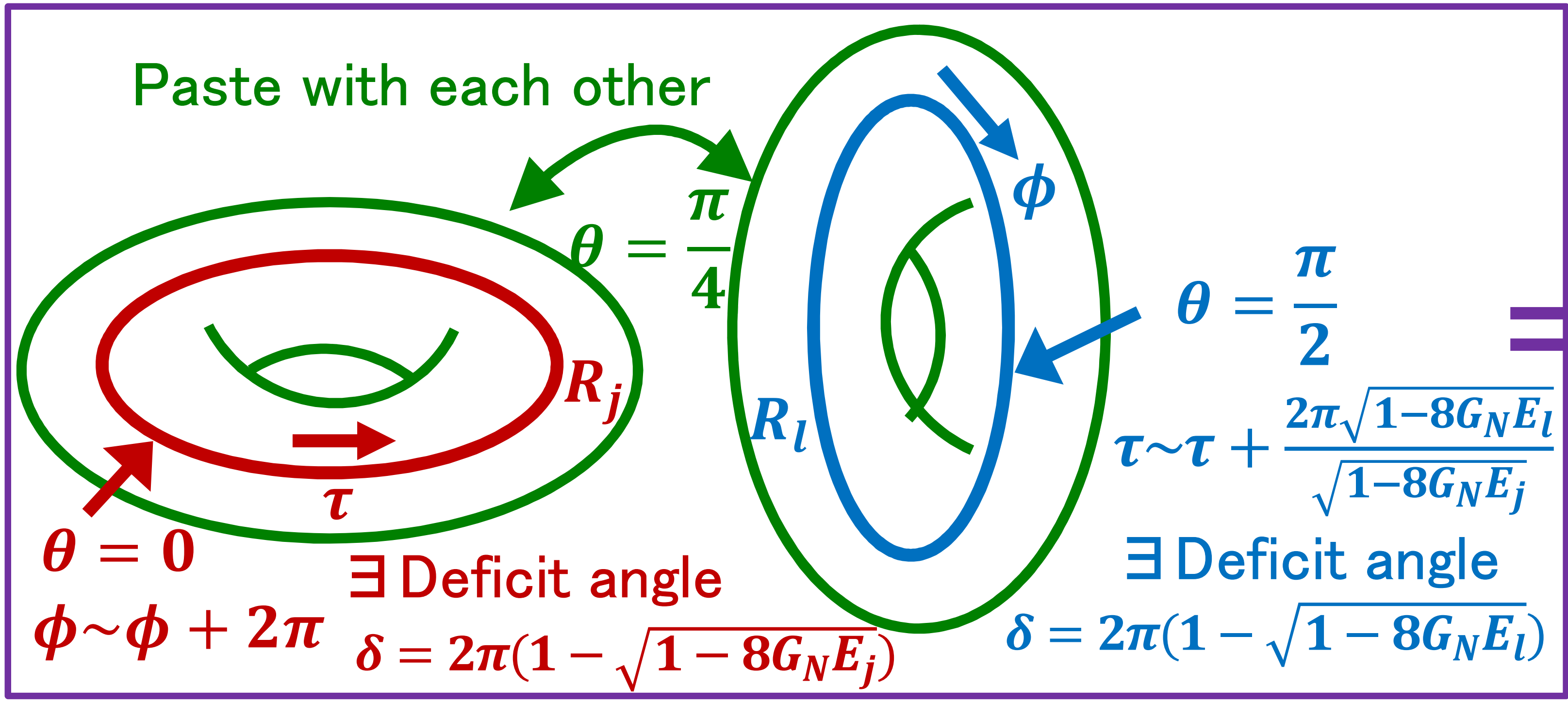


$$Z_{CS(dS)}\left[S^3, R_j\,, R_l\right] = \left|\frac{S_0^j S_0^l}{S_0^0}\right|^2$$

$$\approx e^{\frac{\pi L_{ds}}{2G_N}\left(\sqrt{1-8G_N E_j}+\sqrt{1-8G_N E_l}-1\right)}$$

Gravity dual of (ii): Linked Wilson loops

$$ds^2 = L_{dS}^2\left[d\theta^2 + \left(1 - 8G_N E_j\right)\left(\mathrm{Cos}^2\theta d\tau^2 + \mathrm{Sin}^2\theta d\phi^2\right)\right].$$



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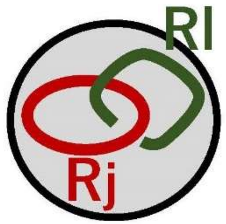


$$I_G = -\frac{\pi L_{ds}}{2G_N}\sqrt{1 - 8G_N E_j\sqrt{1 - 8G_N E_l}}.$$

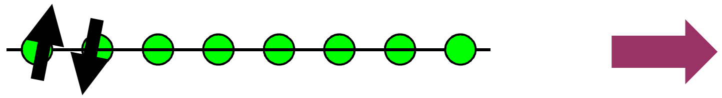
Agree with the CS result !

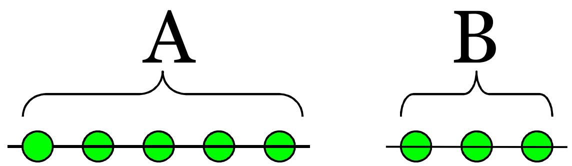
④ Holographic Pseudo Entropy (HPE)

(4-1) Entanglement Entropy (EE) and Holography

We decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain





Introduce the reduced density matrix $\rho_A = \mathrm{Tr}_B\left[\left|\Psi_{tot}\right\rangle\left\langle\Psi_{tot}\right|\right]$.

The entanglement entropy (EE) S_A is defined by

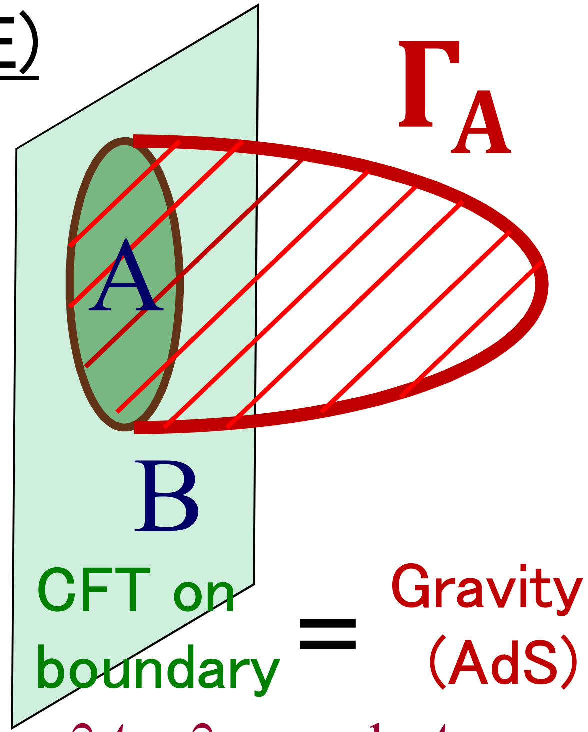
$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]} \propto \text{\# of Bell Pairs between A and B}$$

Holographic Entanglement Entropy (HEE)

[Case 1: Static States] [Ryu-TT 06]

In AdS/CFT, S_A can be computed from the minimal area surface Γ_A:

$$S_A = \underset{\Gamma_A}{\mathrm{Min}}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$



$\partial A = \partial \gamma_A$ and $A \sim \gamma_A$.

[Case 2: Time-dependent States] [Hubeny-Rangamani-TT 07]

S_A can be computed from the area of extremal surface Γ_A:

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|]$$

$$\Downarrow$$

$$S_A(t)$$

$$S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

(4-2) Holographic Pseudo Entropy

Question

Minimal areas in ***Euclidean time dependent*** asymptotically AdS spaces

= What kind of QI quantity (Entropy ?) in CFT ?

➡ Pseudo Entropy !

[Nakata-Taki-Tamaoka-Wei-TT, 2020]

Definition of Pseudo Entropy

For two quantum states $|\psi\rangle$ and $|\varphi\rangle$, define the transition matrix:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

Decomposing the Hilbert space as $H_{tot} = H_A \otimes H_B$, we introduce

the reduced transition matrix: $\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right].$

The pseudo entropy is defined by

$$S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi} \log \tau_A^{\psi|\varphi}\right].$$

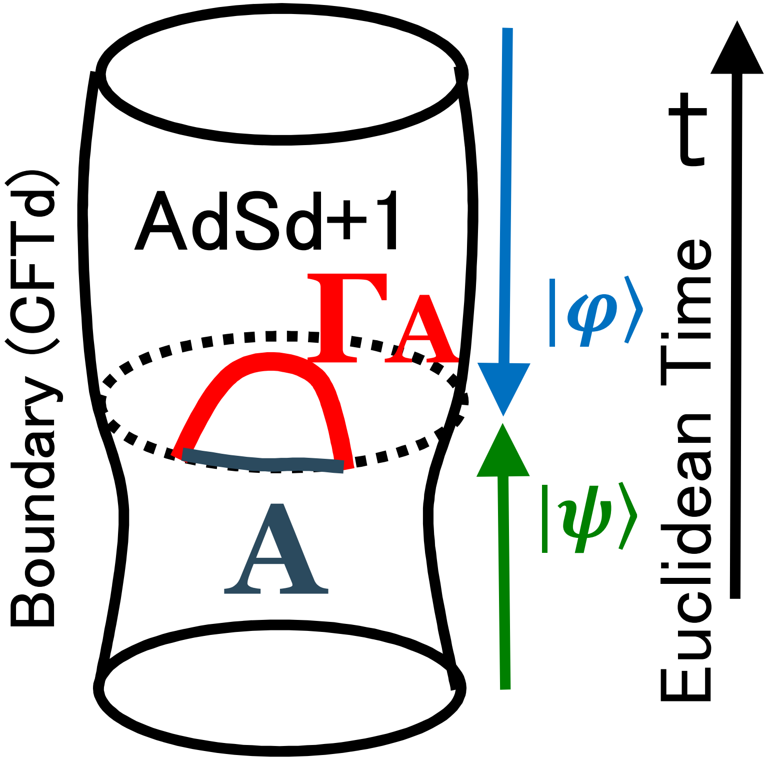
Note: This quantity is complex valued in general.

Holographic Pseudo Entropy (Case 3)

In Euclidean time dependent background,
the mininal surface area coincides with
the pseudo entropy.

$$S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

[Nakata-Taki-Tamaoka-Wei-TT, 2020]



Comment

In quantum theory, transition matrices arise when we consider post-selection.

$$\frac{\langle\varphi|O_A|\psi\rangle}{\langle\varphi|\psi\rangle} = \mathrm{Tr}[O_A \tau_A^{\psi|\varphi}]$$

Final state after post-selection　　　Initial State

This quantity is called **weak value** and is complex valued in general. [Aharanov-Albert-Vaidman 1988,⋯]

Thus "Hol. pseudo entropy = weak value of area operator":

$$S\left(\tau_A^{\psi|\varphi}\right) = \frac{\langle\varphi|\boldsymbol{A}|\psi\rangle}{\langle\varphi|\psi\rangle},$$

(4-3) Pseudo Entropy and Quantum Phase Transitions

[Mollabashi-Shiba-Tamaoka-Wei-TT 20, 21]

Basic Properties of Pseudo entropy in QFTs

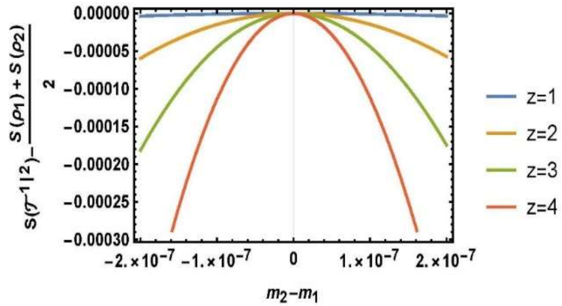
[1] Area law

$$S_A \sim \frac{\text{Area}(\partial A)}{\varepsilon^{d-1}} + (\text{subleading terms}),$$

[2] The difference

$$\Delta S = S\left(\tau_A^{1|2}\right) + S\left(\tau_A^{1|2}\right) - S(\rho_A^1) - S(\rho_A^2)$$

is negative if $|\psi_1\rangle$ and $|\psi_2\rangle$ are in a same phase.



PE in a 2 dim. free scalar when we change its mass.

What happen if they belong to different phases ?
Can ΔS be positive ?

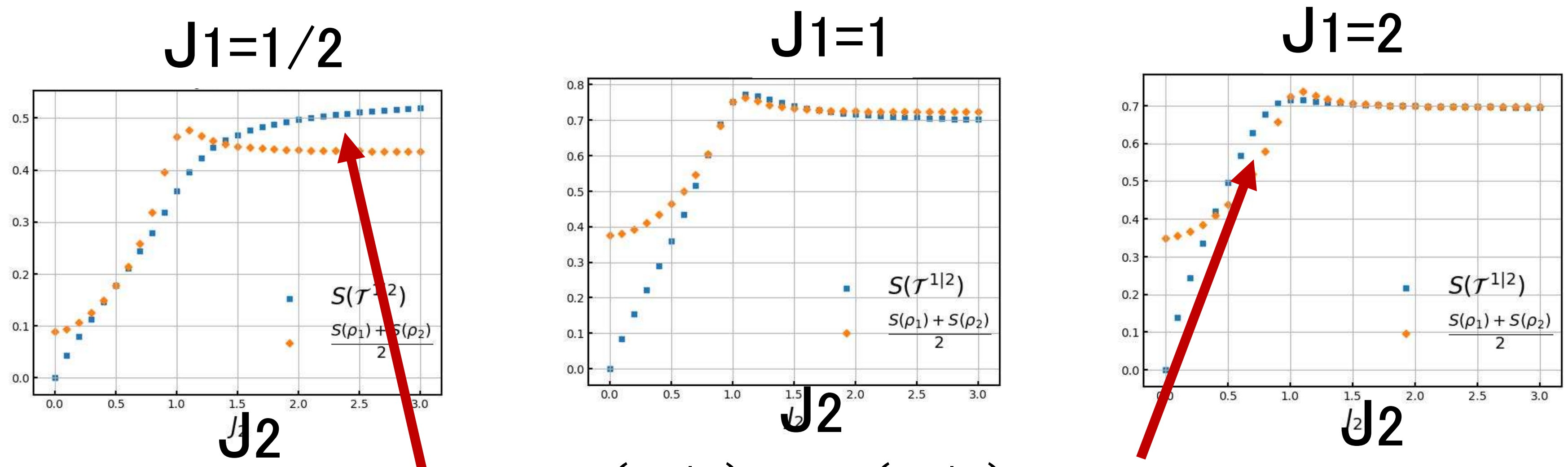
Example: Quantum Ising spin chain with a transverse magnetic field

$$H = -J\sum_{i=0}^{N-1}\sigma_i^z\sigma_{i+1}^z - h\sum_{i=0}^{N-1}\sigma_i^x,$$

Ψ1→ vacuum of H(J1)
Ψ2→ vacuum of H(J2)
(We always set h=1)

J<1 Paramagnetic Phase
J>1 Ferromagnetic Phase

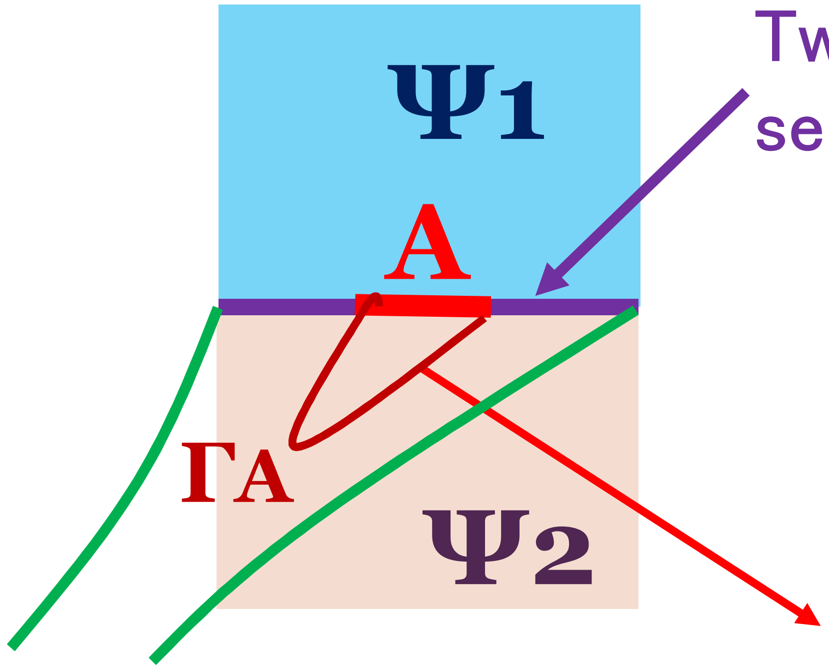
N=16, NA=8



We find $\Delta S = S\left(\tau_A^{1|2}\right) + S\left(\tau_A^{1|2}\right) - S(\rho_A^1) - S(\rho_A^2) > 0$

when Ψ1 and Ψ2 are in different phases !

Heuristic Interpretation



Two gapped phases are separated by a gapless phase.

CFT !

PE is enhanced ! $\Delta S > 0$

AdS Dual of Gapless Interface

The gapless interface (edge state) also occurs in topological orders.

➔Topological pseudo entropy [Nishioka-Taki-TT 2021].

(4-4) Holographic pseudo entropy in dS3/CFT2

Consider dS3/CFT2. As in AdS3/CFT2, we may expect "EE in CFT= geodesic length" is a good probe to dual geometry.

For the global dS3 $ds^2 = -dt^2 + \cosh^2 t(d\theta^2 + \sin^2\theta d\varphi^2)$,

the geodesic length D_{12} between $(t_1, \underbrace{\theta_1, \varphi_1}_{\overrightarrow{\Omega_1}})$ and $(t_2, \underbrace{\theta_2, \varphi_2}_{\overrightarrow{\Omega_2}})$ is

$$\boxed{\cos D_{12} = \left(\overrightarrow{\Omega_1} \cdot \overrightarrow{\Omega_2}\right)\cosh t_1 \cosh t_2 - \sinh t_1 \sinh t_2}$$

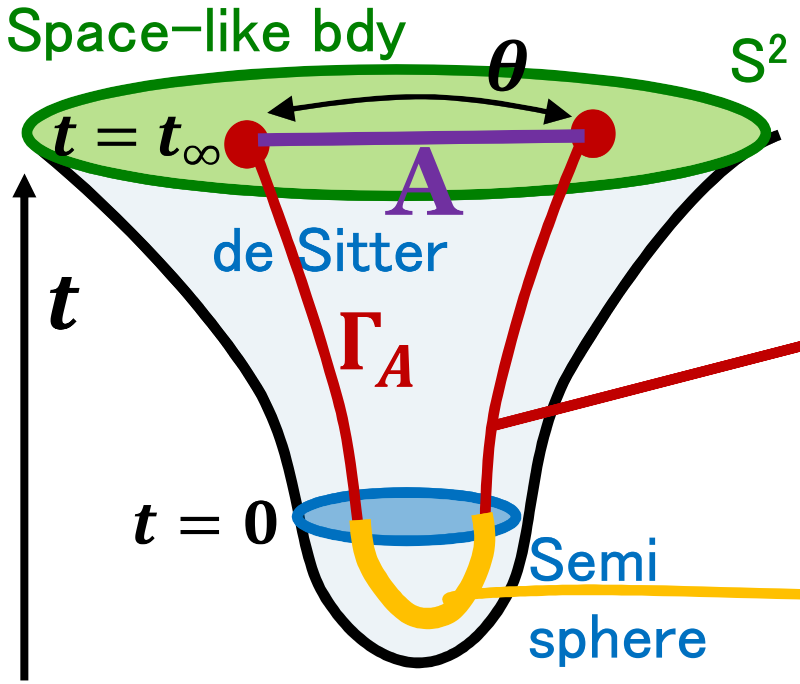
↓

When the two points are near the dS boundary $t_1, t_2 \to \infty$, we have $\cos D_{12} < -1$ and thus D_{12} become complex valued !

Entropy becomes complex valued !

Computing holographic pseudo entropy in dS3/CFT2

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\,(d\theta^2 + Sin^2\theta d\varphi^2))$$



$$\mathrm{L}(\Gamma_A) = 2it_\infty + i\log(\mathrm{Sin}^2\frac{\theta}{2}) + \pi$$

Length of time-like geodesics
➔imaginary value

Space-like geodesic (Semi circle)

$$S_A = \frac{\mathrm{L}(\Gamma_A)}{4G_N} = i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \frac{c_{dS}}{6}\pi.$$

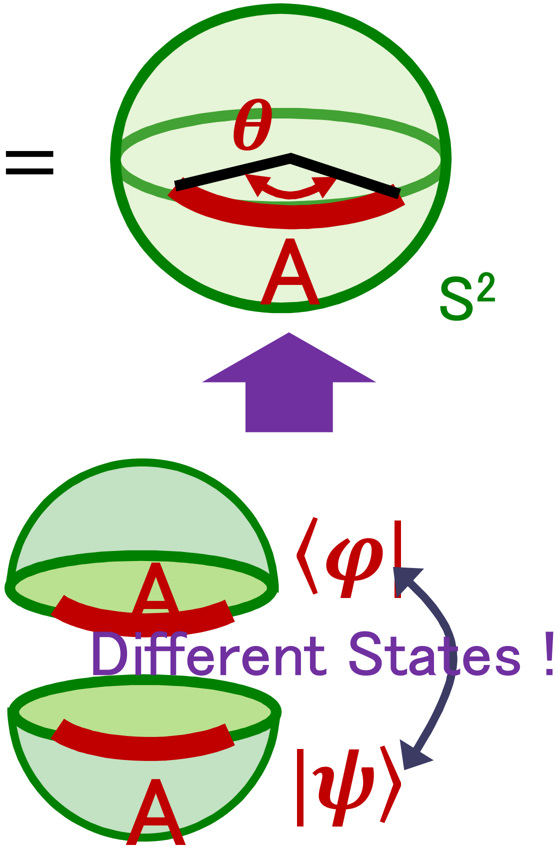
SdS/2

This nicely reproduces the 2d CFT result as follows:

$$S_A = \frac{c_{CFT}}{6}\log\left[\frac{\mathrm{Sin}^2\frac{\theta}{2}}{\tilde{\varepsilon}^2}\right],$$ by setting

$$c_{CFT} = i c_{dS}$$ and $$\tilde{\varepsilon} = i\varepsilon = i e^{-t_\infty}.$$

$\rho_A =$



Why is the EE complex valued ?

➔ It should be regarded as pseudo entropy because ρ_A is not Hermitian !

[Doi-Harper-Mollabashi-Taki-TT 2022, 2023]

Entanglement entropy ➔ Emergent space in AdS
Imaginary part of Pseudo entropy ➔ Emergent time in dS !

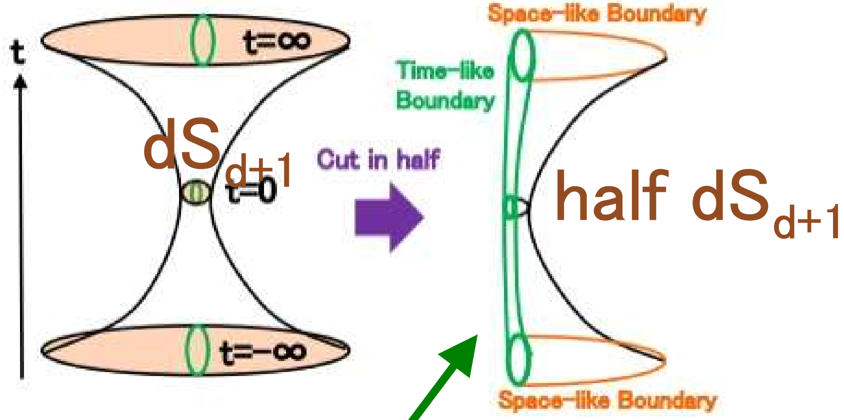
⑤ Holography for a half de Sitter space

[Kawamoto-Ruan-Suzuki-TT 2023]

(5-1) Two setups of a half dS holography

The main difficulty of dS is that its boundary is space-like.

➡ By cutting a dS into a half, we create a time-like boundary.



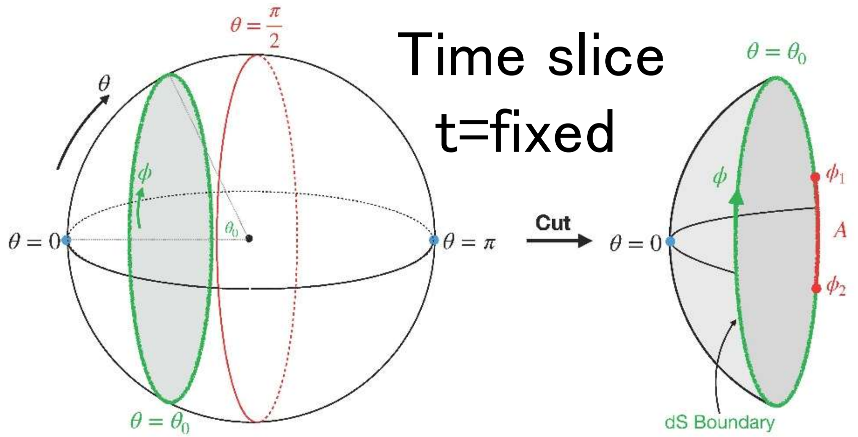
The dual theory lives on dSd

<u>A half dS3 space</u>

$$ds^2 = -dt^2 + \cosh^2 t(d\theta^2 + \sin^2\theta d\varphi^2)$$

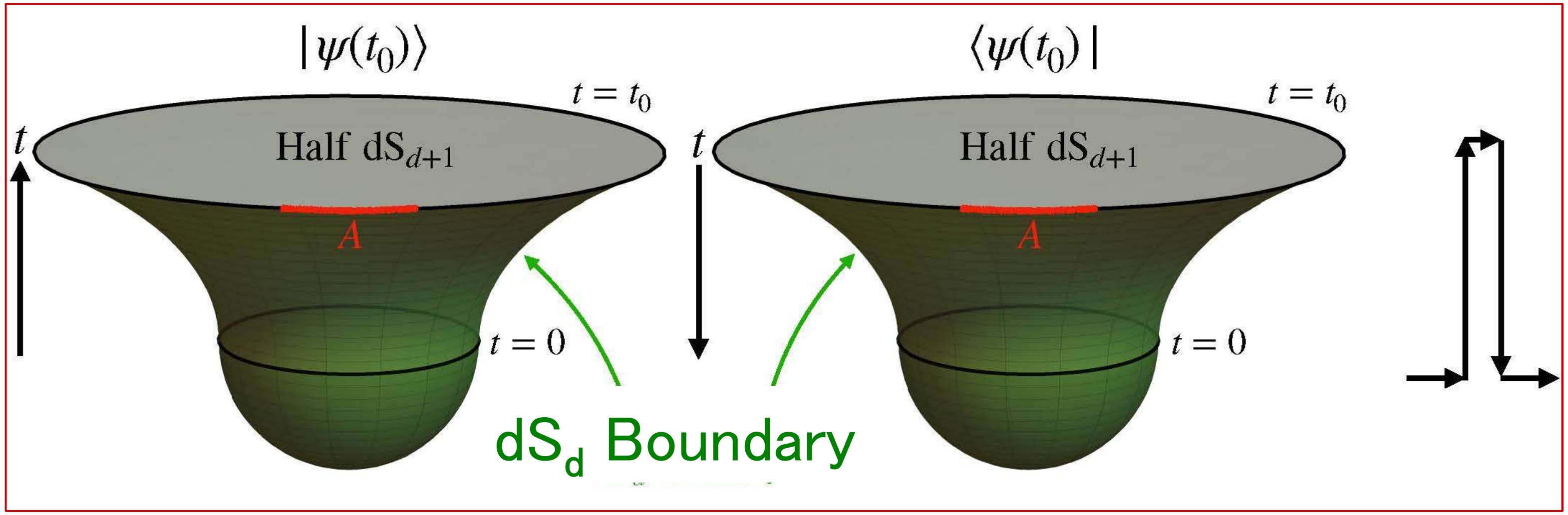
$$0 \leq \theta \leq \theta_0$$

Time slice t=fixed

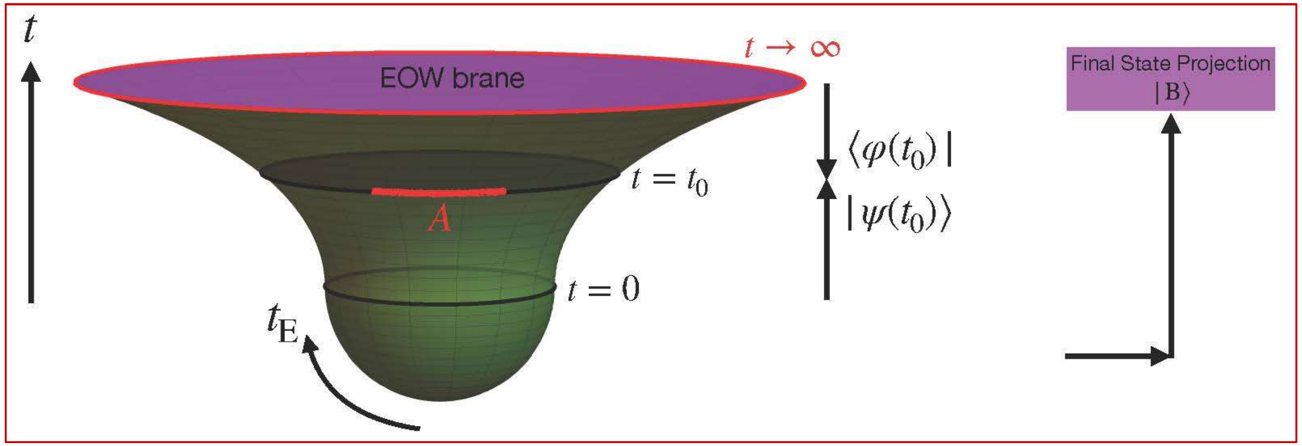


Two setups

(i) Schwinger–Keldysh: Pure State➔Entanglement Entropy



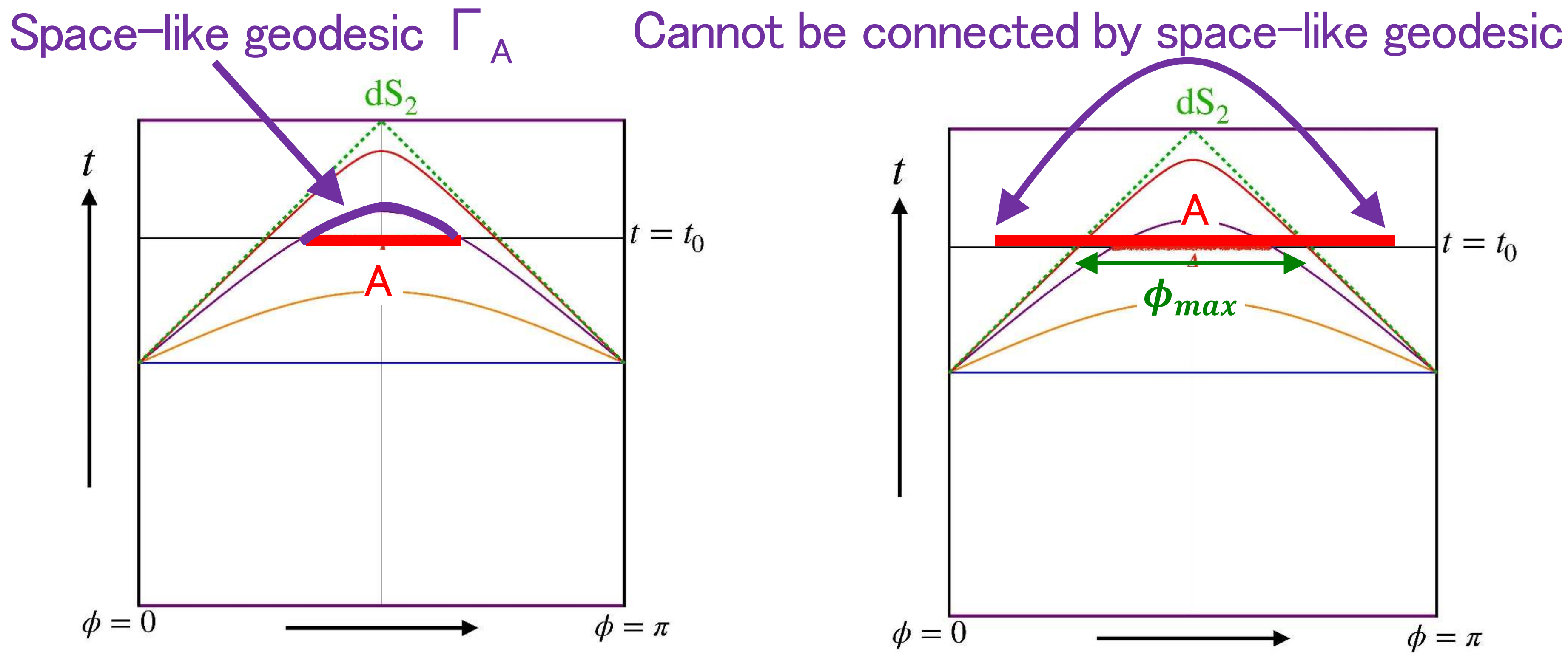
(ii) Final State Projection: Transition Matrix➔Pseudo entropy



(5-2) Holographic Entropy in a half dS holography

Let us focus on (i) Schwinger-Keldysh setup in a half dS3.

We choose $\theta_0 = \frac{\pi}{2}$ and compute Hol EE = Geodesic length.



Space-like geodesic does not exist when the size of A $=\Delta\phi > \phi_{\max}$!

Computing HEE for dS

If we apply the Hartle-Hawking state preparation, we can find an appropriate ΓA by connecting time-like and space-like geodesic.

HEE in 3 dim.

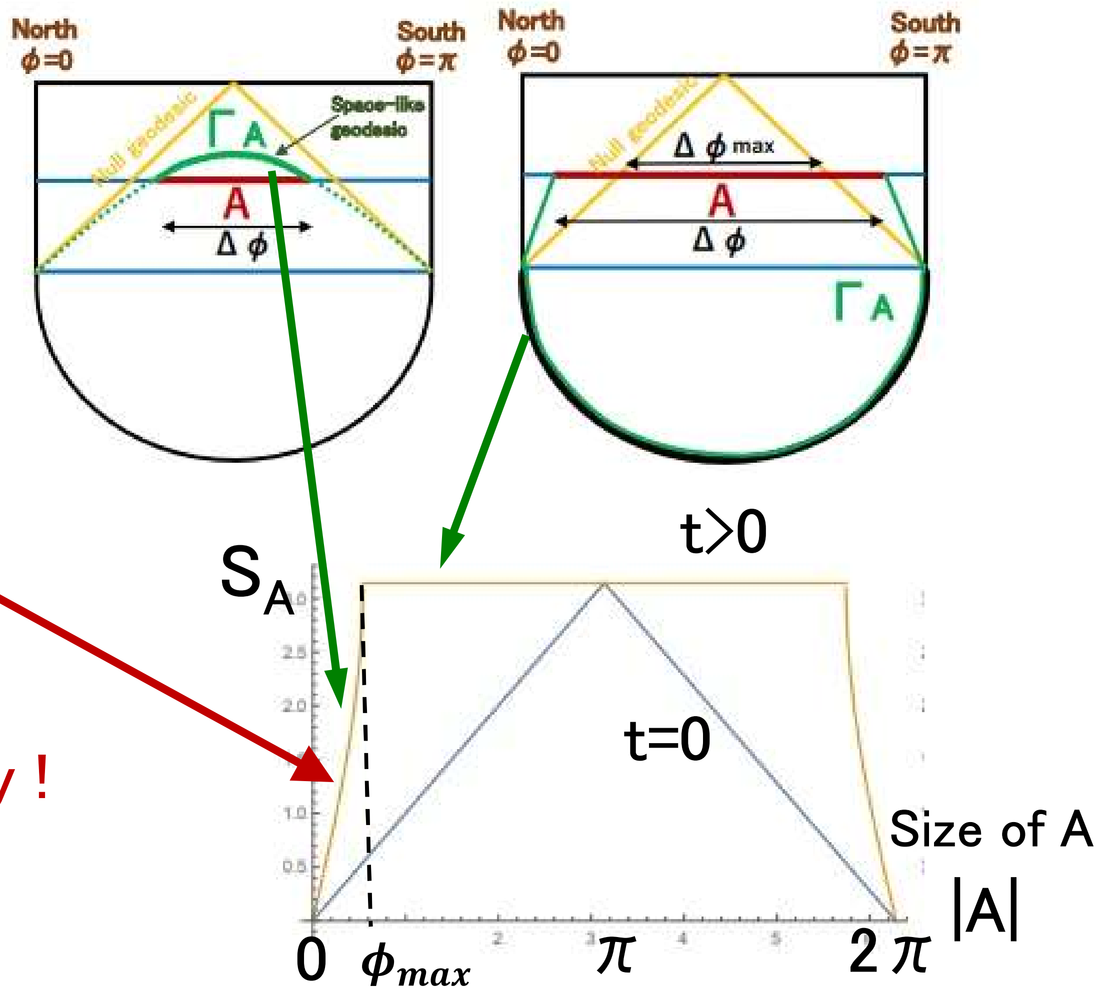
$$S_A = \frac{\mathrm{Length}(\Gamma_A)}{4G_N}$$

$$S_A \propto |A|^{\alpha} \quad (\alpha > 1)$$

Super volume law !

Violation of subadditivity !

$$S_A + S_B \geq S_{A \cup B}$$



(5-3) Non-local QFTs and Super-volume law

From the EE behavior, we argue the following holographic duality:

A half dS Holography

Gravity on a d+1 dim. half dS = A non-local QFT on d dim. dS

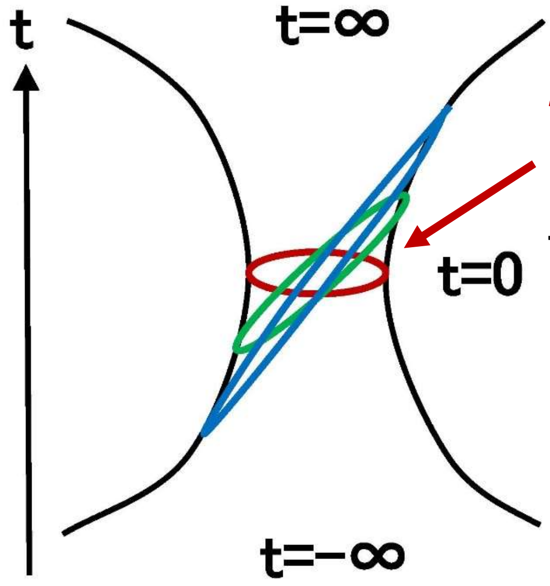
<u>Toy example of non-local QFT</u>

$$I_{\rm nc} = \int dx^d \phi(x) e^{(-\partial^2)^d} \phi(x) \quad \Rightarrow \quad S_A \propto \left(\frac{|A|}{\epsilon}\right)^{d-2+2q}$$

Super volume law if $q > \frac{1}{2}$

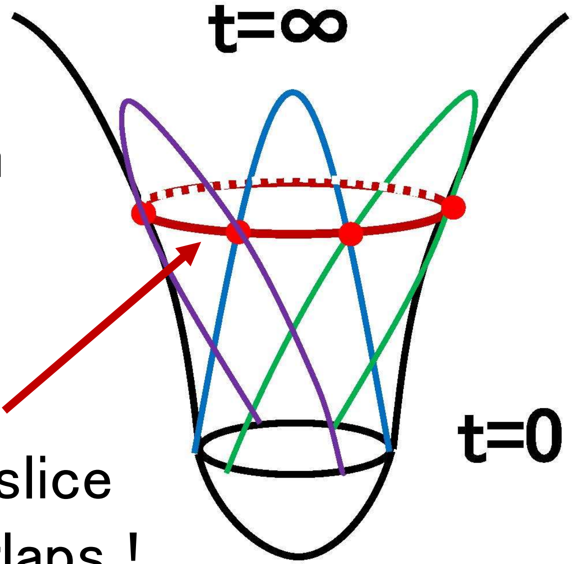
(5-4) Comments on Hilbert Space Structure

Time slices on the boundary dSd, related by SO(1,d) isometry



$S_A = |A|$

Maximally entangled in the Hilbert space H_{dS}



\`Hilbert space’ at a constant t slice has mutual overlaps !

The constant t slices overestimate the size of Hilbert space.

➔ Subadditivity violation, non-local QFTs, ···

⑥ Conclusions

- First we presented an example of a CFT dual of dS3 in the Einstein gravity. This also has higher spin extension:

2d CFT: k→−N limit of SU(N) WZW × [MCFT]

⇕

Classical Spin N Gravity on a 3D de Sitter space (radius $L_{ds} \to \infty$)

➔ Partition functions and hol pseudo entropy are reproduced.

- This analysis implies that the time direction of dS may emerge from the imaginary part of pseudo entropy.

- Finally, we present another formulation of dS holography by introducing the time-like boundary (i.e. a half dS holography).

➡ Our analysis suggests its dual theory is highly non-local.

We also studied the structure of Hilbert space using HEE.

This suggests:

∗ A static patch of dS may be described by a Hilbert space H_{dS}.

∗ The global dS may not correspond to a larger Hilbert space, but it seems to be described by H_{dS} .

➡ ❖ Global dS (Expanding Universe) may be obtained from some fake degrees of freedom ?

Thank you very much !

Relation to Higher Spin Holography

We can extend the previous duality to that in higher spin gravity.

➔ $\mathbf{hs}[\boldsymbol{\lambda}]$: gauge theory of Spin 2, 3, $\cdots \lambda$ fields.

[For higher spin gravity on dS3, refer to Anninos−Denef−Law−Sun 2020]

For this, consider SU(N) CS gauge theory at level k, related to $SU(N)_k$ WZW model and take the limit:

$$k \approx -N + i\frac{N(N^2-1)}{c_{dS}}$$

This leads to

$$c_{SU(N)} = \frac{k(N^2-1)}{k+N} \approx iC_{dS} \gg 1$$

➔ dS/CFT

In this limit, the conformal dimension looks like

$$\Delta_\lambda = \frac{(\lambda, \lambda+2\rho)}{k+N} \approx -i\frac{C_{dS}}{12} \cdot \frac{(\lambda, \lambda+2\rho)}{(\rho,\rho)}$$

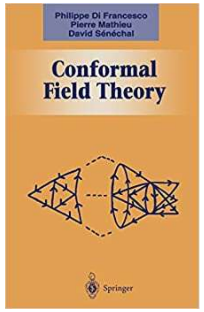
λ: Weight vector of a rep.

ρ: Weyl vector

Partition function in SU(N) CS theory with two linked Wilson loops

$$Z_{CS(dS)}\left[S^3, L(R_\lambda, R_\mu)\right] = \left|S_\lambda^\mu\right|^2 \approx e^{\frac{\pi C_{ds}}{3}\frac{(\lambda+\rho,\mu+\rho)}{(\rho,\rho)}}$$

Refer to e.g.



Dual higher spin gravity calculation

$$A = (h\,b^2\,\bar{h})^{-1}\,\mathrm{d}(h\,b^2\,\bar{h}) \;, \qquad \bar{A}=0$$

with parameters:

$$b = \prod_{i=1}^{N} \exp\left[\rho_i\, e_{i,i}\right] \qquad \left(\rho_i \equiv \frac{N+1}{2} - i\right) \;,$$

$$h = \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[-(e_{2i-1,2i} - e_{2i-1,2i})\,(n_i\,\phi + \tilde{n}_i\,\tau)\right] \;,$$

$$\bar{h} = \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[(e_{2i-1,2i} - e_{2i-1,2i})\,(n_i\,\phi - \tilde{n}_i\,\tau)\right] \;.$$

Here $e_{i,j}$ are $N \times N$ matrices with elements $(e_{i,j})_k{}^l = \delta_{ik}\,\delta_j{}^l$.

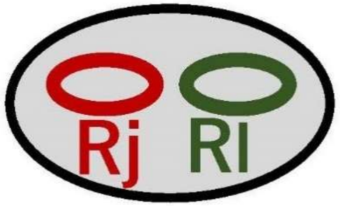
The on-shell action for the gauge configuration can be evaluated as

$$I_{\mathrm{CSG}} = -\frac{\pi}{G_N}\,\frac{\sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} n_i \tilde{n}_i}{(\rho,\rho)} \;,$$

$$I_G = -\frac{\pi C_{ds}}{3}\,\frac{(\lambda+\rho,\mu+\rho)}{(\rho,\rho)}$$

Perfectly matching !

This analysis also explains the result for (iii) unlinked two loops:



$$Z_{CS(dS)}\left[S^3, R_j\,, R_l\right] = \left|\frac{S_0^j S_0^l}{S_0^0}\right|^2$$

$$\approx e^{\frac{\pi L_{ds}}{2G_N}\left(\sqrt{1-8G_N E_j}+\sqrt{1-8G_N E_l}-1\right)}$$

by setting $\lambda = \lambda_j + \lambda_l, \quad \mu = 0$.

Perfect match !

Indeed, we find

$$I_G = -\frac{\pi C_{ds}}{3}\frac{(\lambda_j + \lambda_l + \rho, \rho)}{(\rho, \rho)}$$

$$= -\frac{\pi C_{ds}}{3}\cdot\frac{(\lambda_j+\rho,\rho)+(\lambda_l+\rho,\rho)-(\rho,\rho)}{(\rho,\rho)}.$$

Interpretation from Higher Spin Holography

$$W_N - \textbf{minimal model} \quad \frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$$

Dominant
in our limit k→−N

= **3D Higher Spin Gravity hs[λ]** [Gaberdiel–Gopakumar 2010]

Asymptotic sym.=W∞[λ]

$$N \to \infty, k \to \infty, \lambda \equiv \frac{N}{N+k} = \text{fixed}$$

Triality: $$(N', \lambda') = (\lambda, N)$$ [Gaberdiel–Gopakumar 2012]

$N \to \infty$

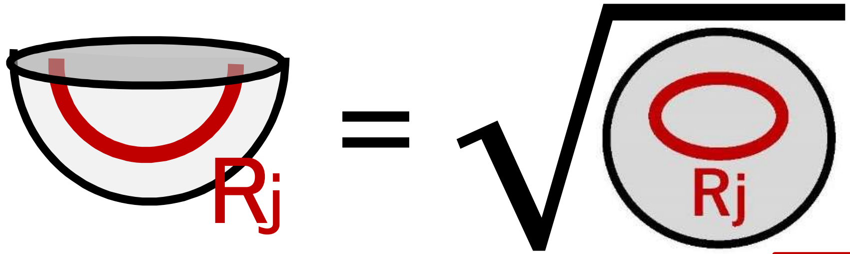
Our limit

$$k \to -N \quad = \quad \lambda' = N, \quad N' \to -i\infty$$

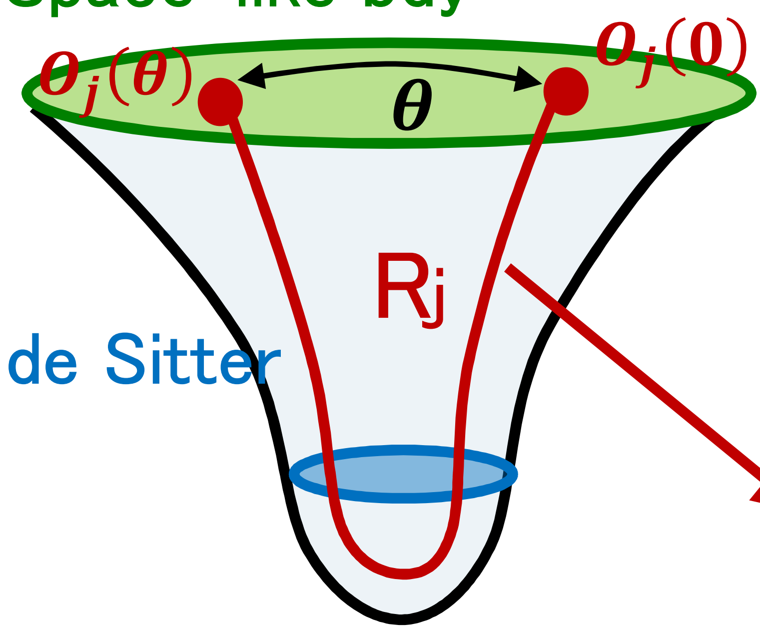
Classical spin N gravity

For N=2, Einstein gravity on dS3 !

Two Point Functions



$$= e^{\frac{\pi L_{ds}}{4G_N}\left(\sqrt{1-8G_N E_j}-1\right)} \approx e^{\pi i \Delta_j}$$



$$\langle O_j(\theta) O_j(0) \rangle$$

$$\approx e^{-L_{ds}\cdot E_j \cdot L[\theta]} = e^{\pi i \Delta_j} \cdot \left(\frac{\epsilon^2}{\mathrm{Sin}^2 \frac{\theta}{2}} \right)^{\Delta_j}$$

$$\mathrm{L}[\theta] = \pi + 2it_{\infty} + i\log(\mathrm{Sin}^2 \frac{\theta}{2})$$

Note: We can regard this as a CFT 2-pt function with an imaginary UV cut off $i\varepsilon = i\, e^{-t_{\infty}}$.