The Petz (lite) recovery map for scrambling channel

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Based on : 2310.18991 (hep-th)

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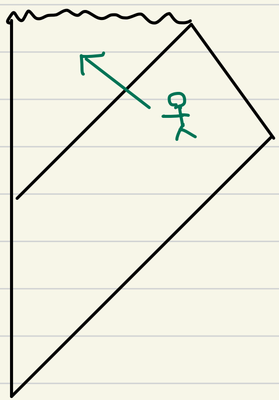
+ Work in Progress

with R. Myers (Perimeter) and S. Ruan (YITP)

BH Complimentarity : Two equivalent observers in QG.

Infalling observer



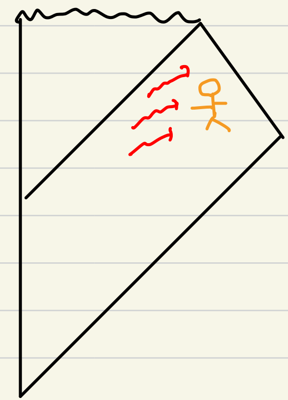
See BH interior

Interior geometry

Equivalent

$\Longleftrightarrow$

Distant observer

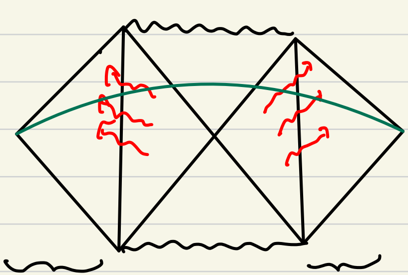


Collecting Hawking quanta

Entanglement

$$\boxed{\text{Can't see the BH interior and Hawking radiation by the same observer}}$$

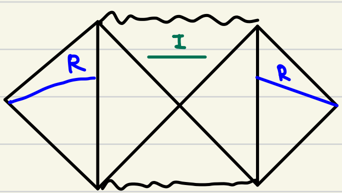
Semiclassically they appear to be independent



: AdS BH with non gravitating heat Bath

- In a Cauchy slice, BH interior looks independent from d.o.f in the ' radiation region

The island formula



$$\boxed{S(\rho_R) = \min_{I} \text{ext} \left[\frac{A(\partial I)}{4G} + S_{\substack{\text{bulk}\\ \text{QFT}}}(I \cup R) \right]}$$

$\Rightarrow$ The island region (in the BH interior) and the radiation region are not independent

$\Rightarrow$ $\boxed{\text{BH interior region is reconstractable from Hawking radiation}}$

Talk today is about

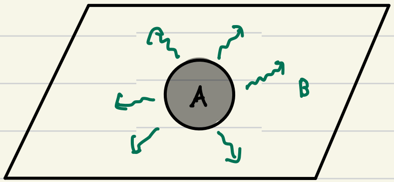
This reconstruction protocol

in (1) the SYK model

(2) a Holographic DCFT

(a doubly holographic setup)

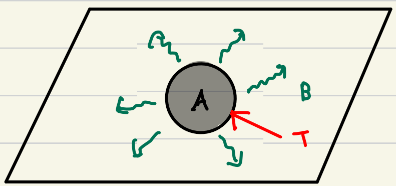
A toy model : the Hayden Preskill setup



- A BH after the Page time (old BH)
  (maximally entangled with HR)

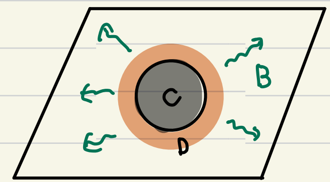
$$\boxed{|\Psi\rangle = \frac{1}{\sqrt{d_A}} \sum_{i=1}^{d_A} |\psi_i\rangle_A \otimes |i\rangle_B \equiv |EPR\rangle_{AB}}$$

Throwing a diary into the BH



BH evaporation

$\Longrightarrow$



C : remaining BH : H_C

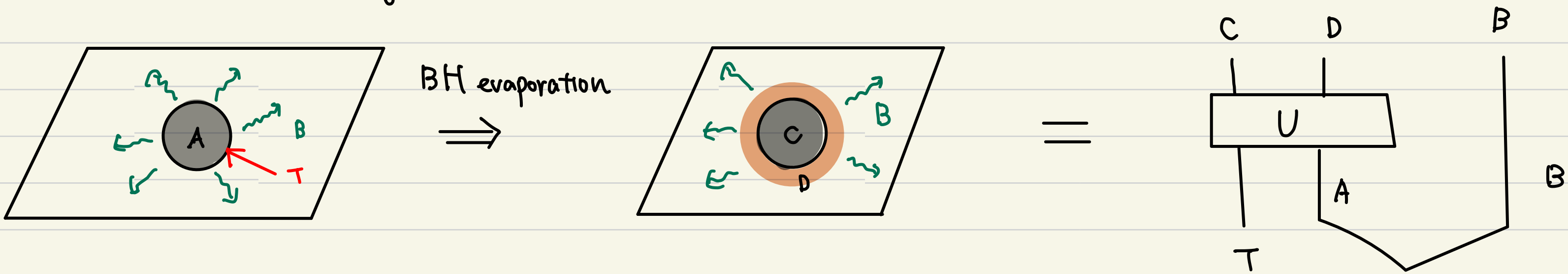
D : late radiation : H_D

B : early radiation

$$|\Psi\rangle_{CDB} = |EPR\rangle_{AB} \otimes |T\rangle_T \Rightarrow I_B \otimes U_{CD \leftarrow AT} |EPR\rangle_{AB} \otimes |T\rangle_T$$

$U_{AT \to CD}$: a Unitary matrix modeling evaporation dynamics $H_A \otimes H_T \to H_C \otimes H_D$

Circuit diagram. [Hayden Preskill]



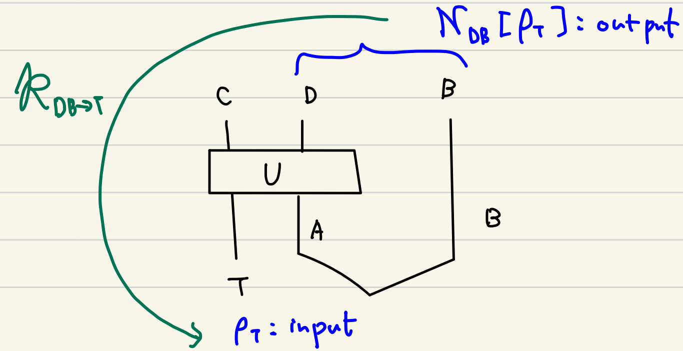
This defines a map $\mathcal{N}: T \to DB$

$$\rho_T \Rightarrow \operatorname{tr}_C \left(I_B \otimes U_{AT\to CD} \right) \left(\rho_T \otimes |EPR\rangle\langle EPR|_{AB} \right) \left(U^{\dagger}_{AT\to CD} \otimes I_B \right) \equiv \mathcal{N}_{T\to DB}[\rho_T]$$

- Reconstructing BH interior from HR

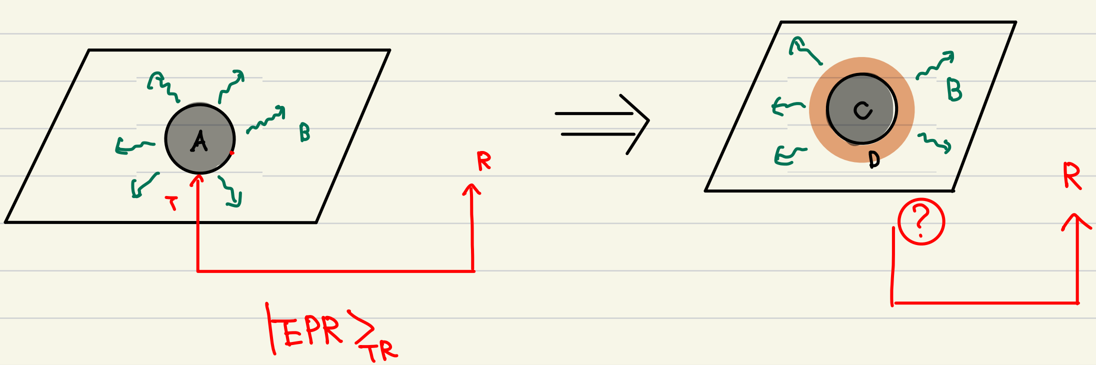
$\Leftrightarrow$ Recovery map : $DB \to T$

$$\mathcal{R}_{DB\to T} \circ \mathcal{N}_{T\to DB} = 1_T$$



Where is the diary?

• Entangling T with a reference system R



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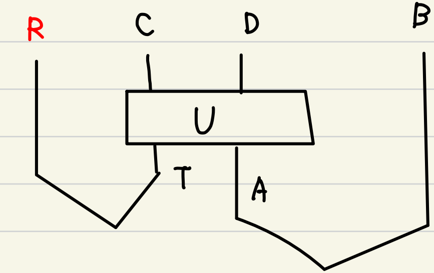


• Information of the diary is in the Hilbert space R is entangled with

Decoupling thm

$$\int dU \, \| \rho_{RC} - \rho_R \otimes \rho_C \|_1 \le \left(\frac{d_T}{d_D} \right)^2$$

⇒ If $d_T < d_D$ then, $\rho_{RC} = \rho_R \otimes \rho_C$

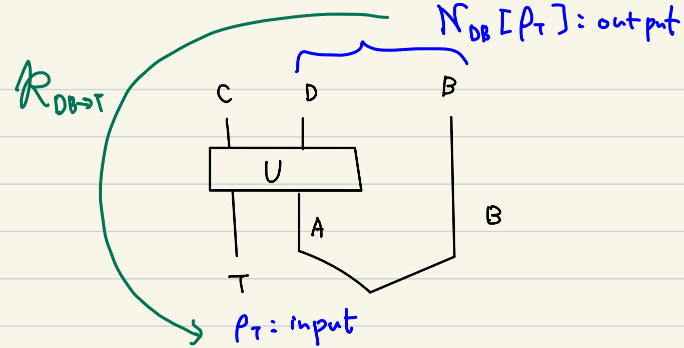
⇒ R is mostly entangled with

DB : Hawking radiation !!

Petz recovery map

For a given $\mathcal{N}$ which is recoverable,

The recovery map is



$$\boxed{\mathcal{R}_\sigma[\rho_{DB}] = \sigma_T^{1/2}\, \mathcal{N}^{\dagger}\left[\mathcal{N}^{-1/2}[\sigma]\, \rho_{DB}\, \mathcal{N}^{-1/2}[\sigma]\right] \sigma_T^{1/2}}$$

Petz recovery map

σ_T : a full rank density matrix on T. $\sigma_T = \dfrac{\mathbb{1}}{d_T}$

Naturally appear by regarding the setup as a

quantum error correcting code (Knill Laflame)

Petz gets Simplified in a chaotic system

- Scrambling : Information of O_T spread over entire BD after the scrambling time,

$$\mathcal{N}[O_T] \sim \frac{1}{d_D d_B} I_{DB}$$

$$\boxed{\mathcal{R}_\sigma[\rho_{DB}] = O_T^{1/2}\, \mathcal{N}^\dagger\left[\mathcal{N}^{-1/2}[\sigma]\, \rho_{DB}\, \mathcal{N}^{-1/2}[\sigma]\right] O_T^{1/2} \sim \mathcal{N}^\dagger[\rho_{DB}]}$$

We check this in
1. HP setup
2. the SYK model

$$S(\mathcal{N}^\dagger \mathcal{N}[\rho] \| \rho) = 0$$

$$\mathcal{N}^\dagger \mathcal{N} = \mathbb{1} \text{ on } H_{code}$$

the Recovery map in the SYK model

The SYK model

0+1 dimensional theory of N Majorana fermions

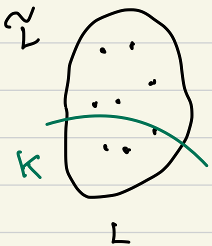
$\#(\cdot) = N$

$$\boxed{H = (i)^{q/2} \sum_{1 \le i_1 \le i_2 \cdots \le N} J_{i_1 \cdots i_N} \psi_{i_1} \cdots \psi_{i_q}}$$

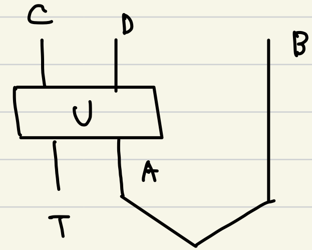
$J_{i_1 \cdots i_q}$: Random coupling drawn from Gaussian $\quad \langle J_{i_1 \cdots i_q}^2 \rangle = \dfrac{J^2}{(q-1)!\, N^{q-2}}$

- Maximally chaotic (OTOCs grow fastest)
- Solvable in the large N limit
- Low energy effective action coincides with the boundary action of JT gravity

Hayden Preskill in the SYK



Two copies of the SYK



the HP setup

$L \iff$ Original BH A

$R \iff$ Early radiation B

$|TFD\rangle_{LR} = \frac{1}{\sqrt{2}} \sum e^{-\beta E/2} |E_n\rangle |E_n\rangle \iff |EPR\rangle_{AB}$

K (ψ_i $i = 1 \cdots K$) $\iff$ Late radiation

$\tilde{L}$ (ψ_i $i = K, \cdots N$) $\iff$ Remaining BH

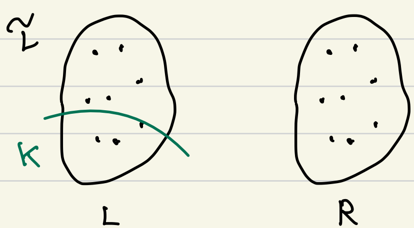
$$\boxed{\text{Subspace: } \mathrm{Span}\{ |\beta\rangle, \psi_{i\tilde{L}} |\beta\rangle \} \iff H_T = \mathrm{Span}\{ |0\rangle, |1\rangle \}}$$

We expect : $\boxed{\mathcal{R}^{\rm stk}_{RK\to T} = a \, N^{\dagger \, \rm stk}_{RK\to T}}$ a : constant to ensure $\mathrm{tr}[\mathcal{R}^{\rm stk}]=1$

check : $\boxed{\langle T^1 | \, \mathcal{R} \circ N\left[|T^2\rangle\langle T^3| \right] |T^4\rangle = \langle T^1 | T^2\rangle \langle T^3 | T^4 \rangle}$

We do this by writing the LHS in terms of a correlator and compute it in the large N limit.

$$\langle 1 | N^{\dagger}_{KR\to T} \, N_{T\to KR}\left[|0\rangle\langle 0|_T \right] |1\rangle \propto \underbrace{\langle TFD | \psi_{i,L}(t - i\delta) \left(I_{\tilde{L}} \otimes \rho_{KR} \right) \psi_{iL}(t + i\delta) | TFD \rangle}_{\text{Modular flowed correlator}}$$



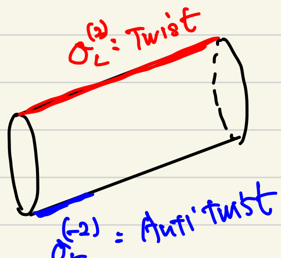
$$\rho_{KR} = \mathrm{tr}_{\tilde{L}}\left[|TFD\rangle\langle TFD|_{LR} \right]$$

The result

$$\langle TFD | \psi_{iL}(t-i\delta) \left(I_{\tilde{L}} \otimes \rho_{KR} \right) \psi_{iL}(t+i\delta) | TFD \rangle$$

$$= \frac{G_{T2\beta}(2\beta - 2\delta)}{G_{T2\beta}(2\delta)} \left[1 - \frac{G_{T2\beta}(\beta)}{\sin\left(\frac{\pi\delta}{\beta}\right)} \left(\frac{2\beta J}{q^2 C} \right) \frac{K}{N} e^{\frac{\pi}{\beta} t} \right]$$

Similar to OTOC : $\langle \psi(\tau_1)\psi(\tau_2)\psi(\tau_3)\psi(\tau_4) \rangle$ with $\tau_1 > \tau_3 > \tau_2 > \tau_4$

$\langle \beta | \psi_{iL} \, \rho_{KR} \, \psi_{iL} | \beta \rangle =$ 

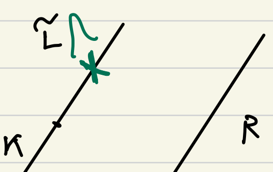
$$\sigma_K^{(-2)} = 1 + \frac{K}{N} \sum_{i=1}^{K} \psi_{iK}\psi_{iK} + \cdots\cdots$$

$$\langle \beta^{\otimes 2} | \psi_{iL} \, \sigma_L^{(2)} \, \sigma_K^{(-2)} \, \psi_{iL} | \beta^{\otimes 2} \rangle \Rightarrow \langle \beta^{\otimes 2} | \psi_{iL} \, \psi_{jK} \, \psi_{jK} \, \psi_{iL} | \beta^{\otimes 2} \rangle$$

an Interpretation

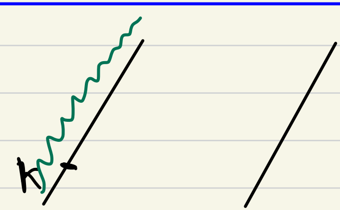
$$\boxed{\langle 1 | N^{T}_{KR\to T}\, N_{T\to KR}\left[\,|0\rangle\langle 0|_T\,\right] |1\rangle = \frac{G_{2\beta}(2\beta-2\delta)}{G_{2\beta}(2\delta)}\left[1-\frac{G_{2\beta}(\beta)}{\sin\left(\frac{\pi\delta}{\beta}\right)}\left(\frac{2\beta J}{q^2 C}\right)\frac{K}{N}\, e^{\frac{\pi}{\beta}t}\right]}$$

$$= \begin{cases} t \sim 0 \quad \neq 0 \Rightarrow N^{\dagger}N \neq 1 : \text{recovery is not possible} \\ t \sim \dfrac{\beta}{\pi}\log\dfrac{N}{K} = 2\,t_{\text{scrambling}} \Rightarrow N^{\dagger}N = 1 : \text{recovery is possible} \end{cases}$$



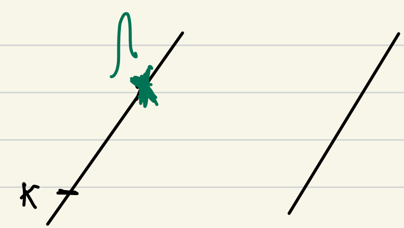
$t=0$ ψ_i Still localized

Spreading $\Rightarrow$



$t = t_{\text{scrambling}}$: $\psi_{i,L}$ spread entire L

recovery $\Rightarrow$

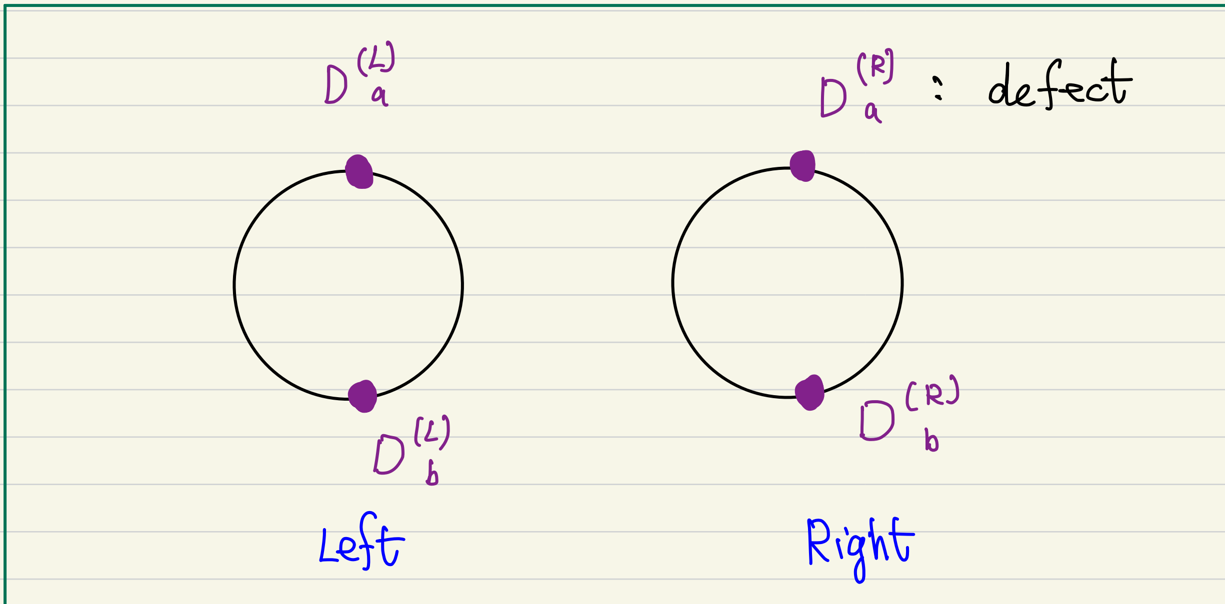


$t = 2t_{\text{scrambling}}$ $\psi_{i,L}$ localized again

HP decoupling in 1+1 D DCFT

1+1 D DCFT setup

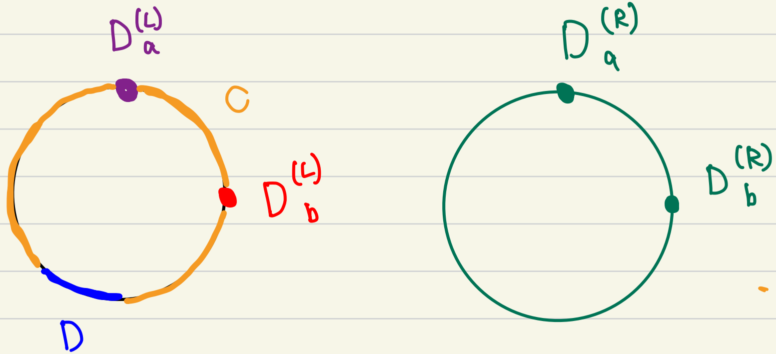
Two copies of 2d CFT on $S^1 \times \mathbb{R}$ with two defects



- Each defect is interacting with ambient d.o.f
- L and R form a TFD state

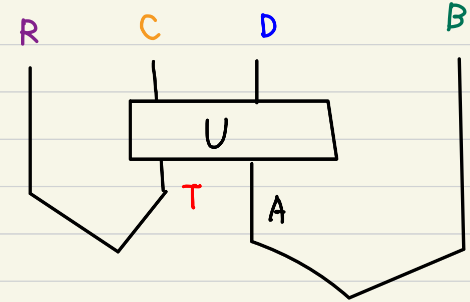
Identification with the HP setup

DCFT



$\longleftrightarrow$

HP



The right system $\longleftrightarrow$ Early Hawking radiation

$D_a^{(L)}$ $\longleftrightarrow$ The reference system

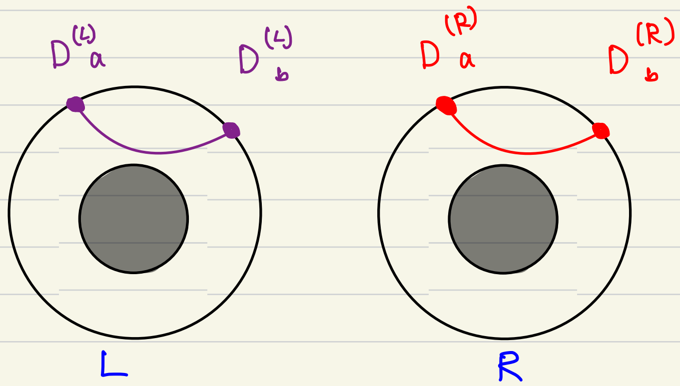
$D_b^{(L)}$ $\longleftrightarrow$ The diary thrown into the BH

C : the ambient d.o.f $\longleftrightarrow$ The remaining BH

the Holographic description in AdS_3

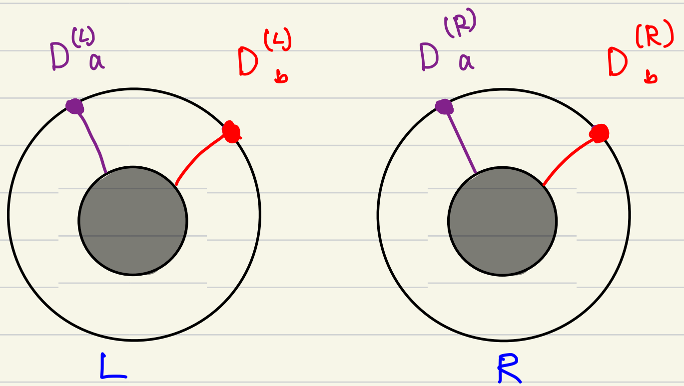
Two defects are connected by a brane in the bulk.

Warm phase



Two defects on the same boundary are connected in the bulk

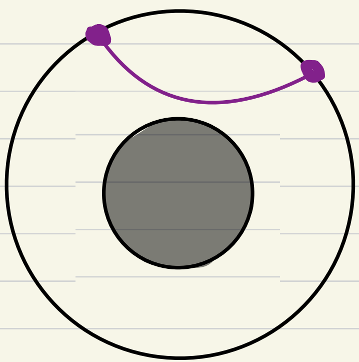
Hot phase



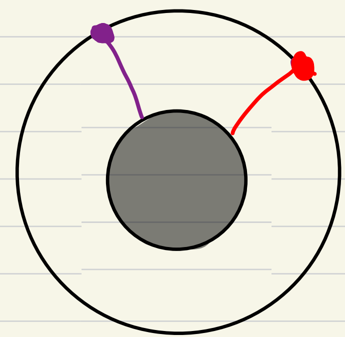
Two defects on the opposite boundary are connected in the bulk

The phase structure

The dimensionless parameter : $\tau = \underbrace{T}_{\text{temperature}} \times \underbrace{L_2}_{\text{The distance between two defects}}$



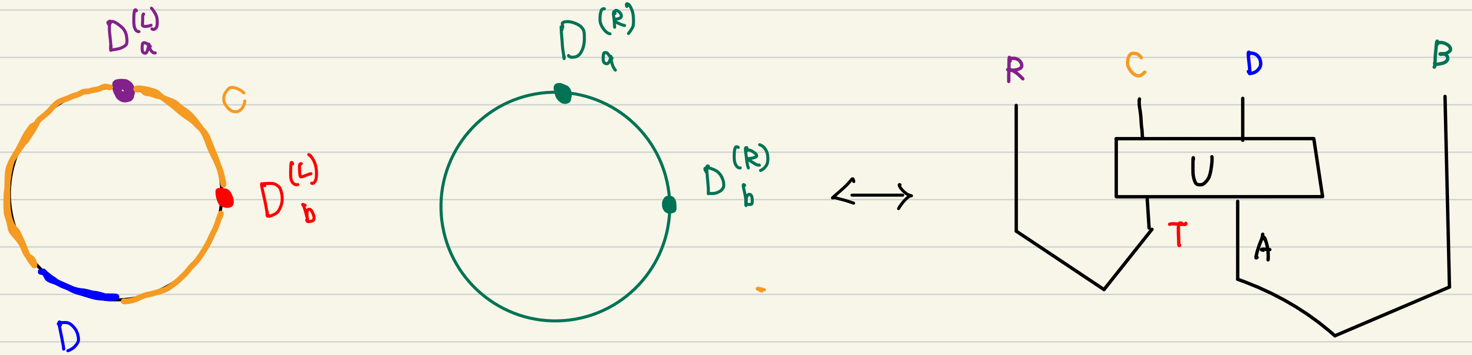
$\Longrightarrow$



$\tau \ll 1$: the Warm phase dominates.

$\tau \gg 1$: the Hot phase dominates

Decoupling in the DCFT setup



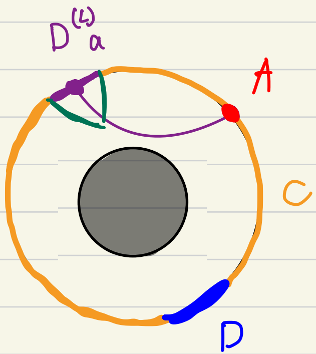
$$I_{RC} = S_R + S_C - S_{RC} = 0 \iff \rho_{RC} = \rho_R \otimes \rho_C$$

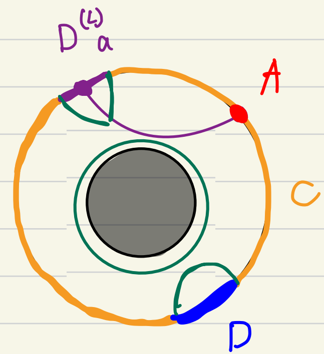
Vanishing of the mutual information

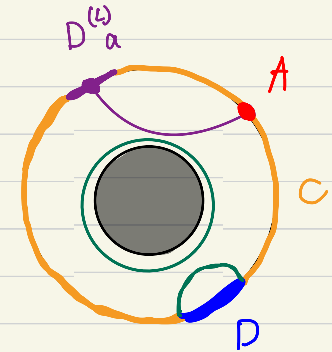
Decoupling

Holographic calculation of I_{RC} : Warm phase

Each entropy is computed by the appriate geodesic length in the bulk







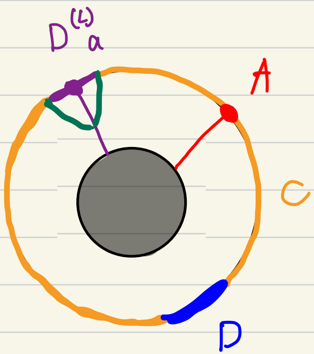
$$S_R = 2\log g$$

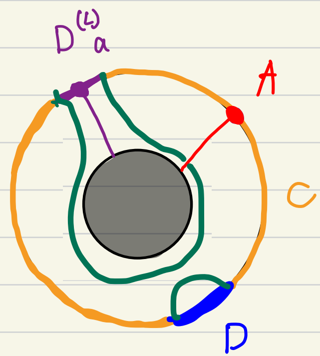
$$S_C = 2\log g + S_{th} + S_{EE,\beta}(D)$$

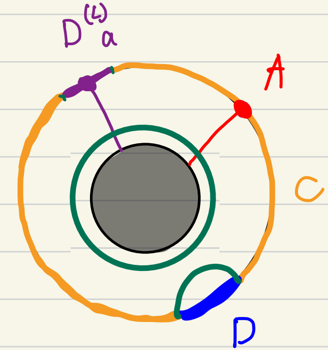
$$S_{RC} = S_{th} + S_{EE,\beta}(D)$$

$$\Rightarrow \boxed{I_{RC} = 4\log[g] \ : \ \text{Decoupling is not archived in the Warmphase}}$$

Holographic calculation of I_{RC} : Hot phase







$$S_R = 2\log g$$

$$S_C = S_{th} + 2\log g + S_{\beta,EE}(D)$$

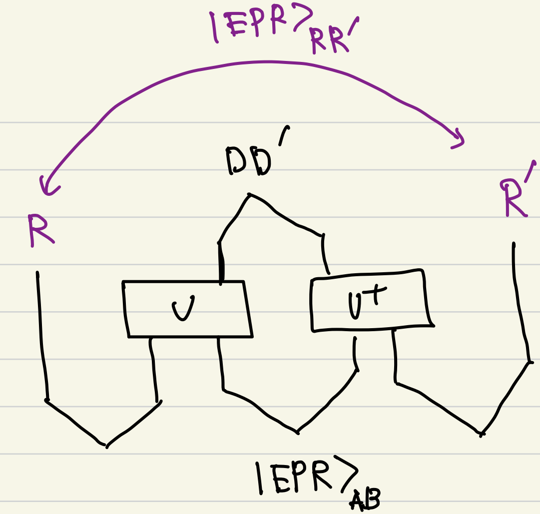
$$S_{RC} = S_{th} + 4\log g + S_{\beta,EE}(D)$$

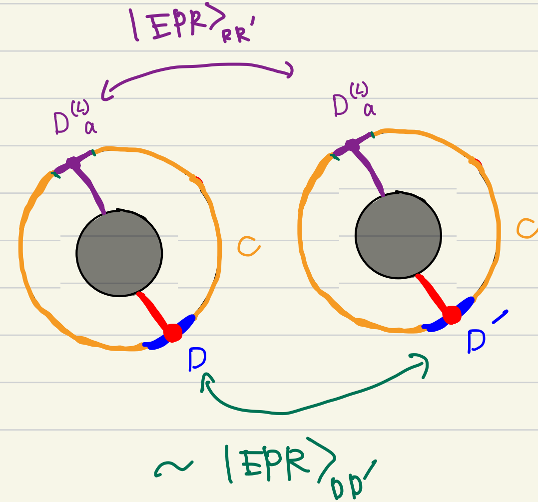
$I_{RC} = 0$: Decoupling is archived in the Hot phase

Recovery map in the DCFT setup

The Petz recovery map is equivalent to the Yoshida-Kitaev protol

- Prepare a copy of $|EPR\rangle_{RT}$
- Simulate the BH dynamics
- Post select to $|EPR\rangle_{DD'}$





- The Yoshida Kitaev has a natural DCFT realization when $D_b^{(L)} \in D$

Summary

- We studied the Petz recovery map in chaotic system
  - → First step to reconstruct BH interior from Hawking radiation
- We found the expression of the recovery map significantly gets simplified in chaotic system
  - the Hayden Preskill model
  - the SYK model

Future work

- Generalizations to higher D (Brane world setup)
- de Sitter (Preliminary result : 2308.09748)

with Balasubramanian, Nomura