Problems and prospects of QG and string theory

KEKTH 2023

December 1, 2023

Hikaru Kawai (National Taiwan University)

Contents of this talk

- What is QG?
- Wave function of universe
- Difficulty of FT description of QG
- Problems of string theory
- Problems of MM
- QG and Naturalness

What is quantum gravity?

QG = quantum theory including gravity + SM

⇒ Should we assume Schrodinger equation literally?

Schrodinger equation might be an effective theory or an approximation of a more fundamental equation.

- Violation of the superposition principle?
- Can Schrodinger equation derived or explained from a more fundamental principle?

Violation of the superposition principle destroys the many world interpretation.

⇒ We do not consider it here.

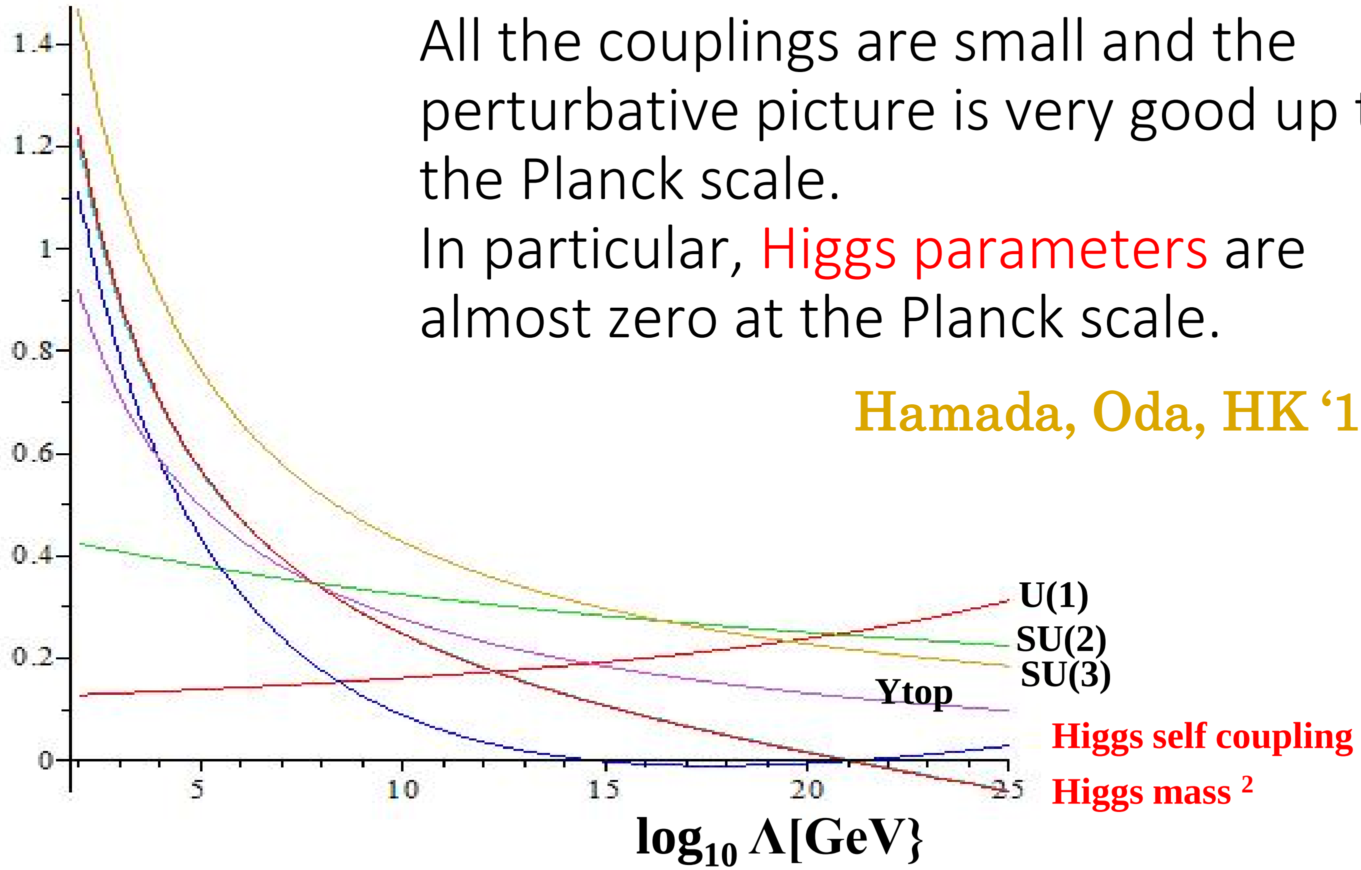
Can Schrodinger equation derived or explained from a more fundamental principle?
⇒ We will come back to this.

For a while we will assume that the ordinary path integral holds, and as a consequence Schrödinger equation is satisfied.

Does it make sense to consider QG at this time?

$\Rightarrow$ Probably Yes.

RG analysis of SM



All the couplings are small and the perturbative picture is very good up to the Planck scale.
In particular, Higgs parameters are almost zero at the Planck scale.

Hamada, Oda, HK '13

It is natural to imagine that SM is directly connected to the string theory or QG at the Planck scale without large modification.

Wave function of universe

It is natural to assume that the behavior of QG at low energies is described by the EH action, independent of the underlying microscopic theory.

Low energy = energy per particle $\ll m_P$

= curvature of space-time $\ll m_P^2$

- multiverse or single universe
- inconsistency of Euclidean gravity
- topology change of universe
- technical problems of WDW wave equation

These are related.

Canonical quantization of EH action

EH action + matter: ϕ: matter

$$S = \frac{m_P^2}{2}\left(\int_M \sqrt{g}R - \int_{\partial M} 2\sqrt{\gamma}K\right) + \int_M \mathcal{L}_\phi$$

ADM variables:

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j)$$

$$\mathrm{S} = \int d^4x\, \mathcal{L},$$

$$\mathcal{L} = \frac{m_P^2}{2}\left(\sqrt{g}R - 2\frac{\partial}{\partial t}(\sqrt{\gamma}K)\right) + \mathcal{L}_\phi$$

$$= \frac{m_P^2}{2} N\sqrt{\gamma}\left(\frac{1}{2}\gamma_{ia}\gamma_{jb}K^{ij}K^{ab} - \frac{1}{2}\left(\gamma_{ij}K^{ij}\right)^2 - R^{(3)}\right) + \mathcal{L}_\phi\ .$$

Here, $K^{ij} = \frac{D}{dt}\gamma^{ij} = \frac{1}{N}\left(\dot{\gamma}^{ij} + \mathcal{L}_{\vec{N}}\gamma^{ij}\right)$.

conjugate momentum: $\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{\gamma}^{ij}}$, $\pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$.

Hamiltonian:

$$H = \int d^3x \left(\pi_{ij}\dot{\gamma}^{ij} + \pi_\phi \dot{\phi} - \mathcal{L}\right)$$

$$= \int d^3x \left(N\mathcal{H} + N^i \mathcal{P}_i\right)$$

$$\mathcal{H} = \frac{1}{4m_P^2}\frac{1}{\sqrt{\gamma}}\left(\pi^{ij}\pi_{ij} - \frac{5}{4}\left(\pi_{ij}\gamma^{ij}\right)^2\right) + \frac{m_P^2}{4}\sqrt{\gamma}R^{(3)} + \mathcal{H}_\phi \ ,$$

$$\mathcal{P}_i = \pi_{ab}\partial_i\gamma^{ab} + 2\partial_a\left(\pi_{ib}\gamma^{ab}\right) + \mathcal{P}_{\phi\, i} \ .$$

GR as constraint

$$S' = \int d^4x \; \left(\pi_{ij}\dot{\gamma}^{ij} + \pi_\phi \dot{\phi} - N\mathcal{H} - N^i \mathcal{P}_i\right).$$

$$\left[\mathcal{P}_i(\boldsymbol{x}), \mathcal{P}_j(\boldsymbol{y})\right] = i\left(\partial_j(\mathcal{P}_i(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y})) - (\boldsymbol{x} \leftrightarrow \boldsymbol{y}, i \leftrightarrow j)\right)$$

$$[\mathcal{P}_i(\boldsymbol{x}), \mathcal{H}(\boldsymbol{y})] = i\, \partial_j \delta(\boldsymbol{x}-\boldsymbol{y})\, \mathcal{H}(\boldsymbol{y})$$

$$[\mathcal{H}(\boldsymbol{x}), \mathcal{H}(\boldsymbol{y})] = i\, \partial_i \delta(\boldsymbol{x}-\boldsymbol{y}) \frac{1}{2}\left(\gamma^{ij}(\boldsymbol{x})\mathcal{P}_j(\boldsymbol{x}) + \gamma^{ij}(\boldsymbol{y})\mathcal{P}_j(\boldsymbol{y})\right)$$

No time evolution, Only constraint: WDW eq.

$$\mathcal{P}_i(\boldsymbol{x})|\Psi\rangle = 0,\; \mathcal{H}(\boldsymbol{x})|\Psi\rangle = 0 \;\Leftrightarrow\; \mathcal{H}(\boldsymbol{x})|\Psi\rangle = 0$$

$$\because \int \mathcal{D}N \;\Rightarrow \prod_x \delta\left(\mathcal{H}(\boldsymbol{x})\right),\; \int \mathcal{D}N^i \;\Rightarrow \prod_x \delta\left(\mathcal{P}_i(\boldsymbol{x})\right)$$

comment on time evolution

The fact that WDW eq. has no time seems profound and reminiscent of holography, but it should not be so surprising.

For simplicity, consider one time and integrate over it:

$$|\Psi\rangle = \int_{-\infty}^{\infty} dt\, |\Psi(t)\rangle = \int_{-\infty}^{\infty} dt\, \exp(-iHt)|\Psi(0)\rangle = \delta(H)|\Psi(0)\rangle.$$

If the system contains the entire universe, we can reconstruct $|\Psi(t)\rangle$ from $|\Psi\rangle$ by considering the projection onto the eigenspace such that some clock in the universe indicates time t.

In this sense, $|\Psi\rangle$ has information about time evolution.

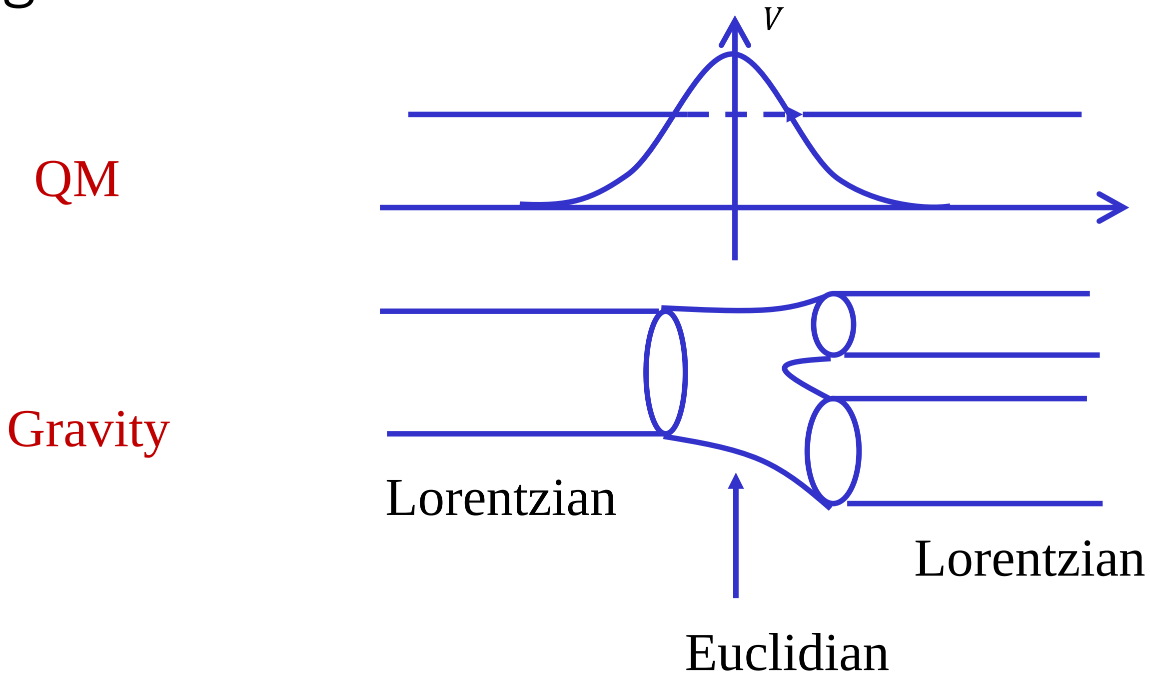
WDW equation is not so different from the ordinary gauge fixing such as the temporal gauge $N(\boldsymbol{x}) = 1$.

Some of the open questions on the wave function of universe

topology change, multiverse and baby universes

Universe arises from matrices. By considering block diagonal configuration, multiverse appears naturally.

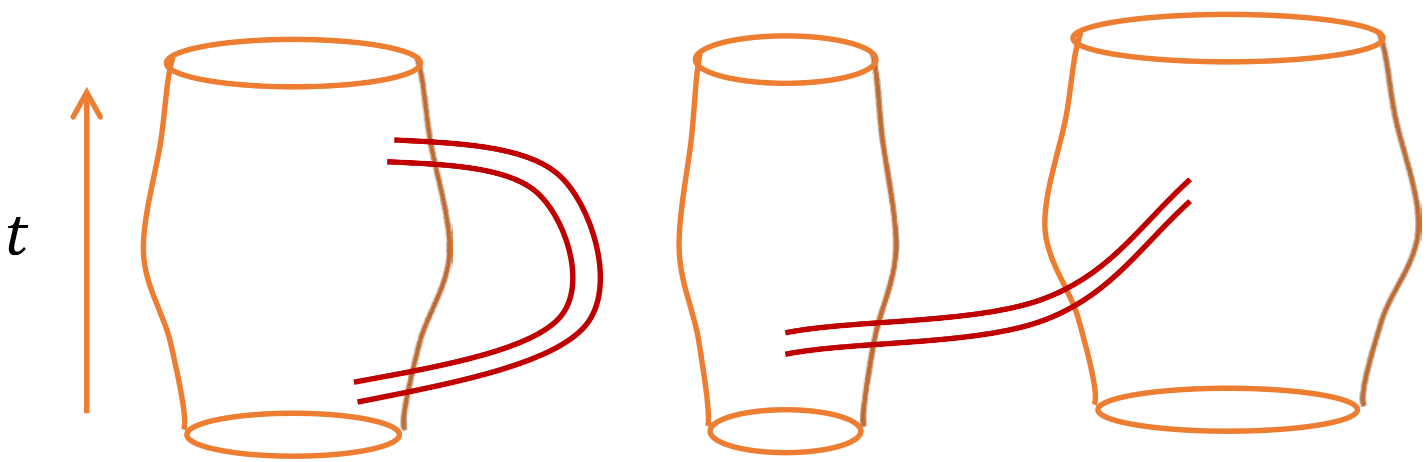
In terms of the WKB approximation of GR, topology change can be described by connecting Lorentzian and Euclidian signature.



For macroscopic universes topology change is highly suppressed, because the transition probability is proportional to $\exp(-\text{classical Euclidean action})$.

On the other hand, if one of the universes is small (baby universe), topology change becomes important.

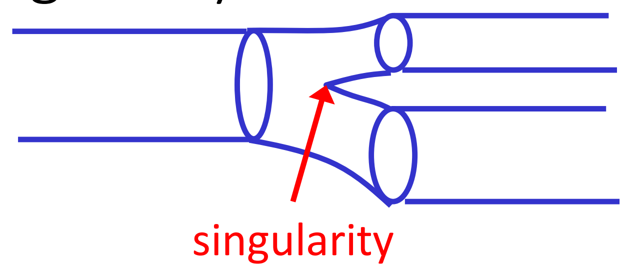
What we should consider is emission and absorption of BU's by multiverse.



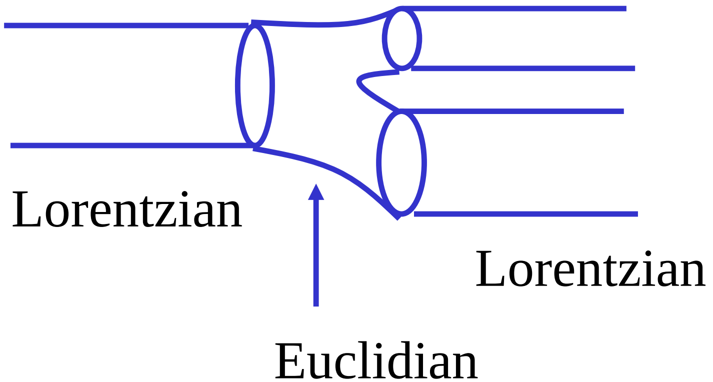
problem: Lorentzian gravity with topology change

Probably, the picture above is true.
But we do not have a good definition of path integral including topology change in the Lorentzian gravity.

In fact, in the Lorentzian gravity topology change does not occur without singularity.



We can consider tunneling through Euclidean signature.
But Euclidean gravity is not completely well defined.



Hamiltonian for the universe

The space is assumed to be S^3.

temporal gauge: $ds^2 = -dt^2 + \gamma_{ij}(t,x)dx^i dx^j$

x^i: coordinates of S^3

mini-superspace: $ds^2 = -dt^2 + a(t)^2 d\Omega^2$

$d\Omega^2$: metric of unit S^3

Hamiltonian derived from EH action is

$m_P = 1$

$$H_{\text{uiverse}} = -\frac{p_a^2}{2\,a} + \frac{a^3}{6}\lambda(a),\ p_a = a\,\dot{a}\,.$$

←**wrong sign**

$$\lambda(a) = -\frac{1}{a^2} + \rho(a)$$

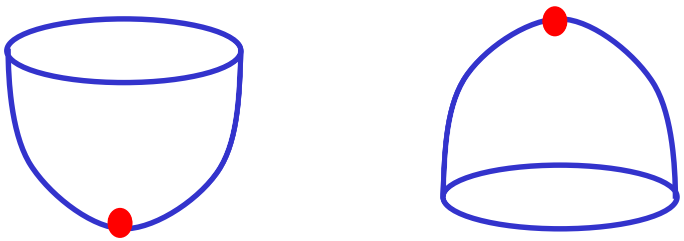
$$\rho(a) = \lambda + \varepsilon(a)$$

λ: cosmological constant

ε: energy density of matter and gravitons

problem: creation and annihilation of universe

EH action does not tell anything about creation and annihilation of the infinitesimally small universe. Once it is created its time evolution is described by the Hamiltonian before it is annihilated.



This is a special case of topology change. But in this case we may be able to introduce the transition amplitude by hand.

At any rate, the ultimate answer for topology change would be obtained by an constructive definition of string such as IIB MM.

Problem of Euclidean gravity and flow of time

WDW eq. $$H_{\text{total}}\left|\Psi\right\rangle = 0$$

$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}} + \cdots$$

$$H_{\text{universe}} = -\left(\frac{1}{2} p_a{}^2 + \cdots\right)$$

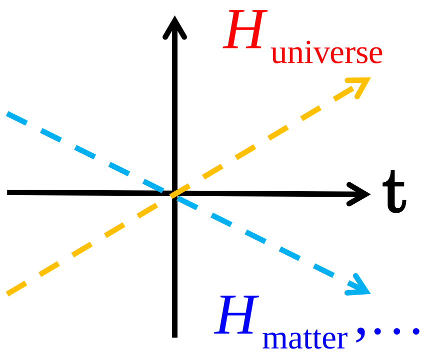
$\Downarrow$

"Ground state" does not exist.

Wick rotation is not well defined.

H_{matter} **is bounded from below.**

H_{universe} **is bounded from above.**



Evolution of universe:

$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}}$$

$$H_{\text{universe}} = -\frac{1}{2a} p_a^2 + \cdots \quad \leftarrow \textbf{wrong sign}$$

Analogy: $H = -a^\dagger a + b^\dagger b + \epsilon \cdot (\text{int. betwen } a \text{ and } b)$

$$\psi = |0\rangle \quad \underset{\text{time evolution}}{\Rightarrow} \quad \psi = \text{const.} \left(a^\dagger\right)^n \left(b^\dagger\right)^n |0\rangle + \cdots$$

$t = 0$ $\quad$ $H = \text{const.}$

A simple universe evolves into a rich universe.

The flow of time emerges from evolution of universe.

Anomaly of WDW eq. (technical or essential?)

WDW eq. $\mathcal{H}(\boldsymbol{x})|\Psi\rangle = 0 \ \Rightarrow \ \mathcal{P}_i(\boldsymbol{x})|\Psi\rangle = 0$

Schwinger term causes a contradiction.

(ex.) $1+1$ dimensional CFT $\boldsymbol{H_n = L_n + \bar{L}_{-n}}$, $\boldsymbol{P_n = L_n - \bar{L}_{-n}}$

$$\begin{aligned}
[H_n, H_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n,m} \\
+(-n+m)L_{-n-m} + \frac{c}{12}(-n)(n^2-1)\delta_{-n,-m} \\
= (n-m)P_{n+m}
\end{aligned}$$

$$[P_n, P_m] = (n-m)P_{n+m}$$

$$\begin{aligned}
[H_n, P_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n,m} \\
-(-n+m)L_{-n-m} - \frac{c}{12}(-n)(n^2-1)\delta_{-n,-m} \\
= (n-m)H_{n+m} + \frac{c}{6}n(n^2-1)\delta_{n,m}
\end{aligned}$$

If $c \neq 0$, $H_n|\Psi\rangle = 0 \ \Rightarrow \ |\Psi\rangle = 0$.

Yoneya '84, …

Related to Liouville theory $c_{tot} = 0$.

Difficulty of FT description of QG

Basic question:

Can QG be described by a local field theory?

EH action is not renormalizable.

In the ordinary field theory (without gravity), if the theory is non-renormalizable we had three cases:

1. There is a renormalizable theory whose low energy theory is the theory we are considering.
   (ex.) 4-fermi interaction ⇒ SM
2. The theory is well-defined non-perturbatively.
   (ex.) 3D non-linear σ model
3. The theory is not well defined as a QFT
   (ex.) 5D ϕ^4

Renormalizable theory of gravity?

EH action is not renormalizable.

$$S = \frac{m_P^2}{2}\int d^4x\sqrt{g}R + SM + \cdots$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{m_P}h_{\mu\nu}$$

$$= \int d^4x\,\{(\partial h)^2 + \cdots + SM_{\mathrm{free}}$$

$$+\frac{1}{m_P}h\partial h\partial h + \frac{1}{m_P^2}hh\partial h\partial h + \cdots$$

$$+\frac{1}{m_P}hFF + \cdots\}$$

$\Rightarrow$ each virtual graviton $\propto \left(\frac{\Lambda}{m_P}\right)^2$

$\Rightarrow$ need higher order counter terms

R^2 Lagrangian is renormalizable but not unitary.

$$S = \int d^4x \left(\frac{m_P^2}{2} \sqrt{g} R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} \right) + SM + \cdots$$

α, β dimensionless ⇒ similar to YM: $\alpha, \beta \sim \frac{1}{g_{YM}^2}$

⇒ renormalizable

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

propagator: $\frac{1}{\mathbf{const.}(k^2)^2 + m_P^2 k^2} = \frac{c_1}{k^2} + \frac{c_2}{k^2 + m^2}$

$c_1 + c_2 = 0$ ⇒ either c_1 or c_2 <0

⇒ ∃ negative norm state (ghost)

Some arguments to save this, but hard to accept:

1. unstable ghost (Lee-Wick)
2. confined ghost

Non-perturbative definition of EH?

Sometimes QFT is well-defined non-perturbatively even if the action is non-renormalizable.

(ex.) 3D O(N) non-linear σ model

$$S = \int d^3x \frac{1}{2g^2}\left(\partial_\mu \vec{s}\right)^2, \ \vec{s}^2 = 1$$

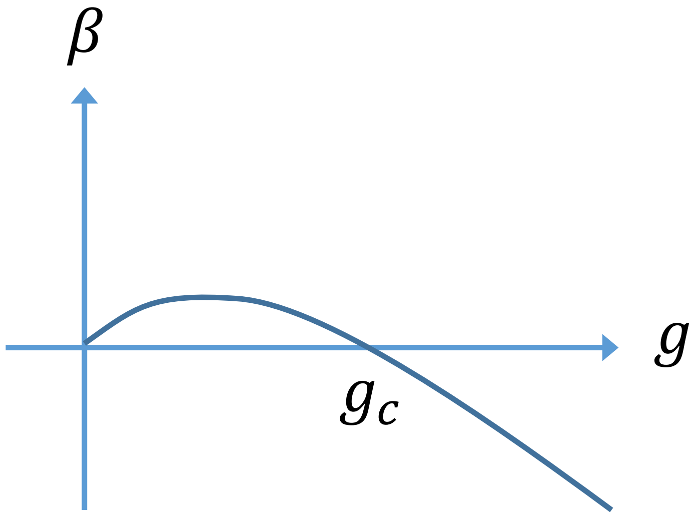
Constructive definition gives non trivial theory:
O(N) spin system on the lattice

$$S = \sum_{\langle i,j \rangle} \frac{1}{g_0^2} \vec{s}_i \cdot \vec{s}_j$$

continuum limit: $g_0^2 = g_c^2 + const.\, a^{3-d_\varepsilon}\ (a \to 0)$

The essence is that we have a CFT at $g = g_c$.
Then we perturb around it with a relevant operator to make a theory with mass scale.

One way to see the fixed point (CFT) is ϵ-expansion around 2 dimensions.



$$D = 2 + \epsilon$$

$$\beta = \epsilon g - b g^3 + \cdots$$

$$\Rightarrow g_c^2 = \frac{\epsilon}{b} + O(\epsilon^2)$$

In general, If a D_0-dimensional theory is asymptotically free, $D_0 + \epsilon$ -dimensional theory has a fixed point at least for sufficiently small ϵ.

In fact for the spin system (scalar field), the critical point is non-trivial (not a free field) for $2 < D < 4$.

The above example in 3D is not so surprising.
The theory is equivalent to ϕ^4.

In 3D, sometimes it happens that
a non-renormalizable theory is equivalent to a super renormalizable theory.

The real question is whether we have a non-trivial QFT when we have no description based on a (super) renormalizable theory.

Questions:

1. Does 5D YM exist as QFT.? $\Rightarrow$ Probably No.
2. How about 5D super YM.? $\Rightarrow$ Some people believe so.
3. Does 4D EH exist as QFT? $\Rightarrow$ Hard to exist.

Difficulty of QG as local FT

There are several attempts, but not successful:

- ϵ-expansion around two dimensions
- dynamical triangulation (DT)
- functional renormalization group (FRG)

ϵ-expansion

Weinberg, HK-Ninomiya, Aida-Kitazawa, …

It is possible to consider ϵ-expansion for EH action around D=2.

There are extra poles in ϵ because the EH action vanished in 2D.

Nevertheless, a systematic ϵ-expansion is possible. However, D=4 is too far from D=2 to draw a clear conclusion for 4D QG.

DT and FRG

HK-Ohta arXiv: 2305.10591

Several results have been reported where non-trivial fixed points are obtained by DT and FRG.

The major problem is that there is no guarantee that the obtained fix points are not that of the R^2 Lagrangian.

In the case of spin system, the lattice theory manifestly satisfies unitarity. Therefore it is guaranteed that the fixed point is a unitary theory.

On the other hand, unitarity is not guaranteed in DT or FRG. Therefore it is not clear whether the fixed point is unitary or not, and nothing prevents the RG flow from converging to the fixed point of R^2 Lagrangian.

Problems of string theory

Probably, the only possibility of QG is string theory.

The good points of string theory:

1. Critical string automatically contains gravity. Yoneya, Scherk-Schwarz
2. There is no UV divergence.
3. Matter and gauge fields are contained.
4. There are many 4D tachyon free perturbative vacua which look like SM.

The bad point:

Because there are too many perturbative vacua, we cannot make meaningful predictions from perturbative formulation.

We need a non-perturbative formulation of string.

Basic question:
Is the degeneracy of the perturbative vacua resolved, when we consider non-pertutbative dynamics?

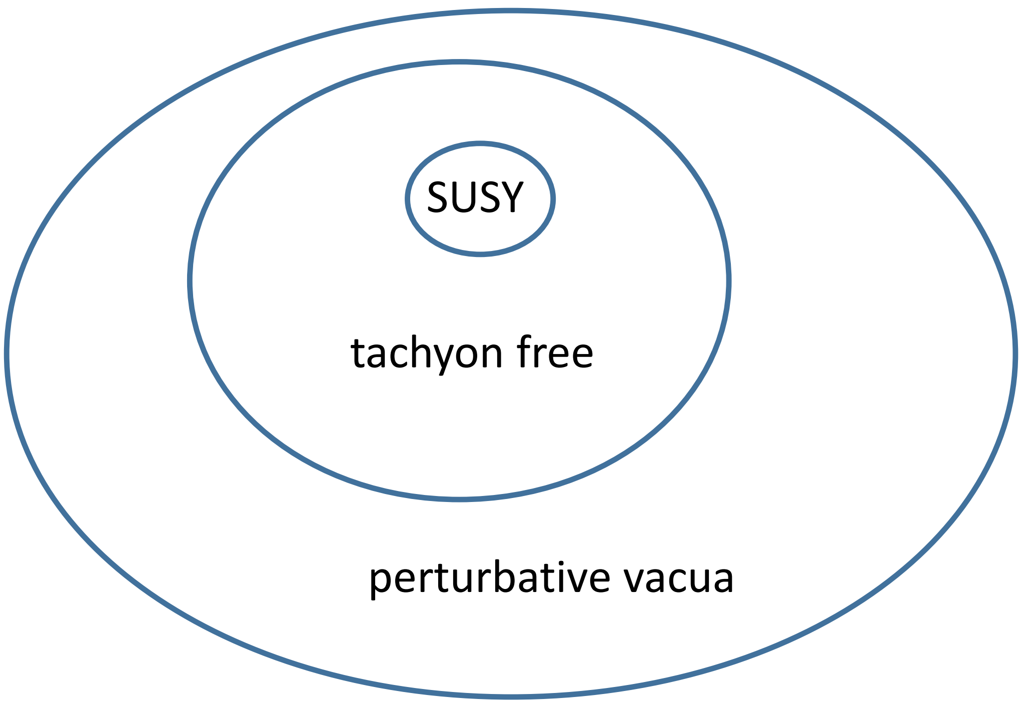
If the space-time SUSY remains, the answer is probably no because the degeneracy of vacua is protected by SUSY. So we may still have a complicated "landscape" even after taking account of non-perturbative dynamics.

Fortunately, there is no space-time SUSY in our nature. Therefore, nothing prevents the system from leaving the degenerate vacua and finding the true vacuum.

String vacua without space-time SUSY

It is important not to confuse the existence of space-time SUSY with the non-existence of tachyons.

Space-time SUSY ⇒ No tachyons. true
No tachyons ⇒ Space-time SUSY false



Heterotic string

Space-time SUSY : tachyon free non SUSY

10D space-time

2 : 1

4D space-time

<<

(randomly generated)

HK-Lewellen-Tye, Dienes,

We cannot classify all possible perturbative vacua with 4D space-time.

But we can randomly construct 4D tachyon free vacua by free fermionic construction or orbifold.

Then we observe

# of vacua with space-time SUSY << that without SUSY.

Note Because of the modular invariance, absence of tachyon indicates that asymptotically

# of bosonic states ~ # of fermionic states .

In other words,

tachyon free ⇔ SUSY at Planck scale .

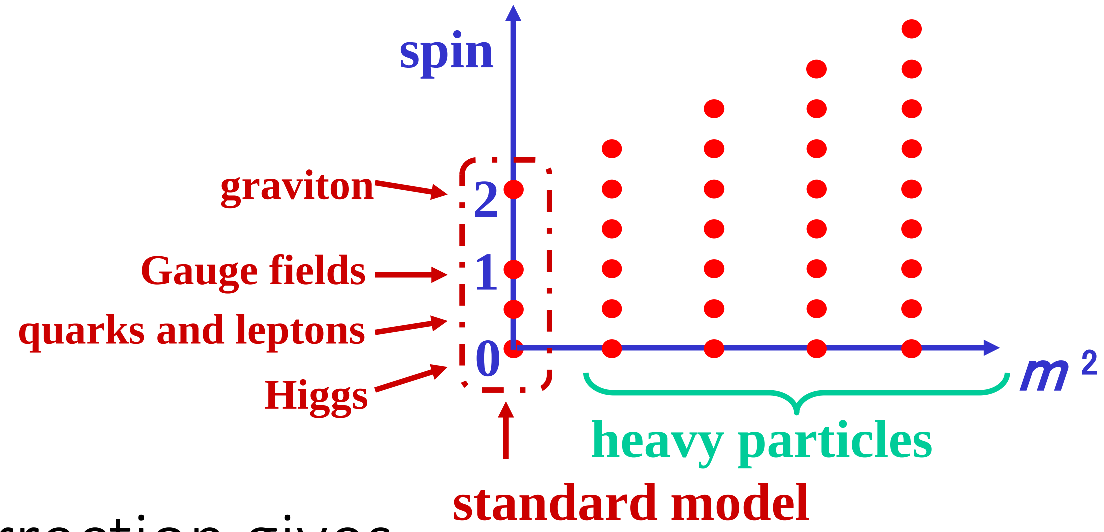
It is natural to expect that SUSY is broken at Planck scale.

Naturalness problem in non-SUSY vacua

It is natural to expect that the true vacuum is close to a perturbative vacuum without space-time SUSY.

Then we need to resolve the naturalness (hierarchy) problem for the Higgs mass and cosmological constant.

Suppose that in the tree level vacuum energy and Higgs mass are zero.



Then the radiative correction gives

$$\lambda \sim m_s^4(0/g_s^2 + 1 + \cdots),\ m_H^2 \sim m_s^2(0 + g_s^2 + \cdots).$$

We need some mechanism to tune λ and m_H^2 close to zero compared with m_s^4 and m_s^2.

Space-time SUSY was a candidate for resolving the problem for m_H^2, although it does not solve the problem for λ.

Actually, it is very difficult to resolve the naturalness problem for λ in the context of the ordinary local field theory.

Later we will discuss the possibility that the non-perturbative dynamics in string theory could solve this problem.

Problems of MM

Constructive (non-perturbative) definition of string

The aims of non-perturbative string theory:

1. Find the true vacuum.
   good approximation, numerical method, …
2. Investigate stringy non-perturbative effects.
   mechanism of fine tunings, …

So far, there are two directions:

1. String field theory
   It is difficult to realize the modular invariance for closed string.
2. Matrix models
   It is not completely proved that the perturbative string is recovered.

IIB matrix model

Ishibashi-HK-Kitazawa-Tsuchiya

Schild action of type IIB string

$$S_S = \alpha \int \omega \left(\frac{1}{4} \{X^\mu, X^\nu\}^2 - \frac{i}{2} \bar{\psi} \gamma_\mu \{X^\mu, \psi\} \right) - \beta \int \omega$$

The worldsheet M can be regarded as a phase space:

$$\{A, B\} = \frac{\epsilon^{ab}}{\sqrt{g}} \partial_a A \partial_b B, \ \ \omega = \frac{\sqrt{g}}{2\pi} d\xi^1 \wedge d\xi^2.$$

We can regularize the path integral by “quantization”.

phase space M	$\Leftrightarrow$	vector space V
function $A \in \mathcal{C}(M)$	$\Leftrightarrow$	matrix $\hat{A} \in \mathrm{End}(V)$
$\{A, B\}$	$\Leftrightarrow$	$\frac{1}{i}[\hat{A}, \hat{B}]$
$\int \omega\, A$	$\Leftrightarrow$	$Tr(\hat{A})$
$\int \frac{\mathcal{D}g\, \mathcal{D}X}{\mathrm{vol(Diff)}}$	$\Leftrightarrow$	$\sum_{\dim(V)} \int d\hat{X}$

Thus the path integral for the Schild action

$$Z = \int \frac{\mathcal{D}g\,\mathcal{D}X}{\mathrm{vol(Diff)}}\, e^{iS_{\mathrm{S}}[X]},$$

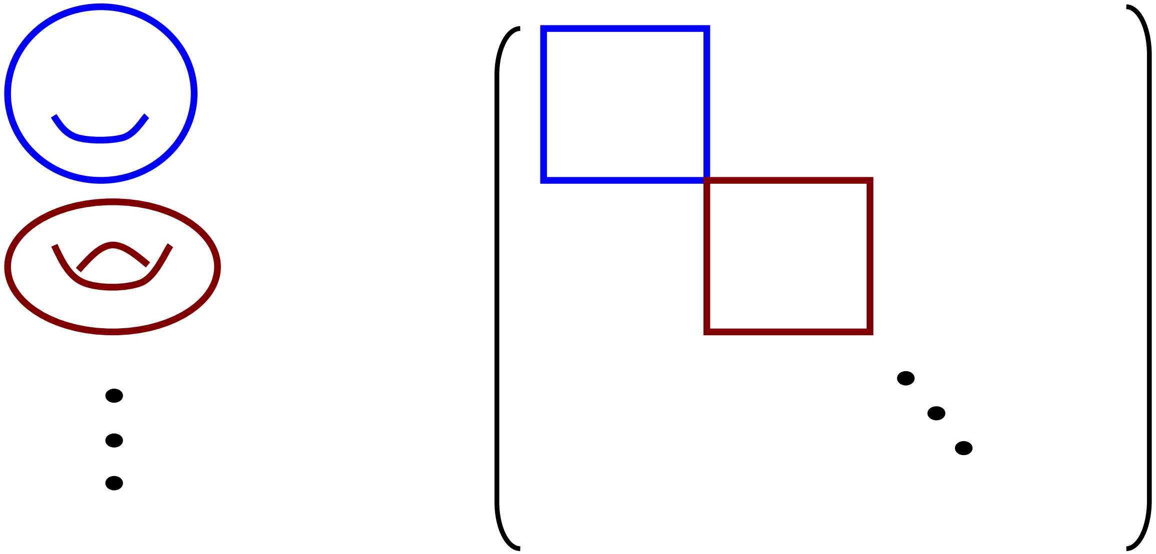
$$S_S = \alpha \int \omega \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 - \frac{i}{2}\bar{\psi}\gamma_\mu\{X^\mu, \psi\}\right) - \beta \int \omega$$

is regularized as

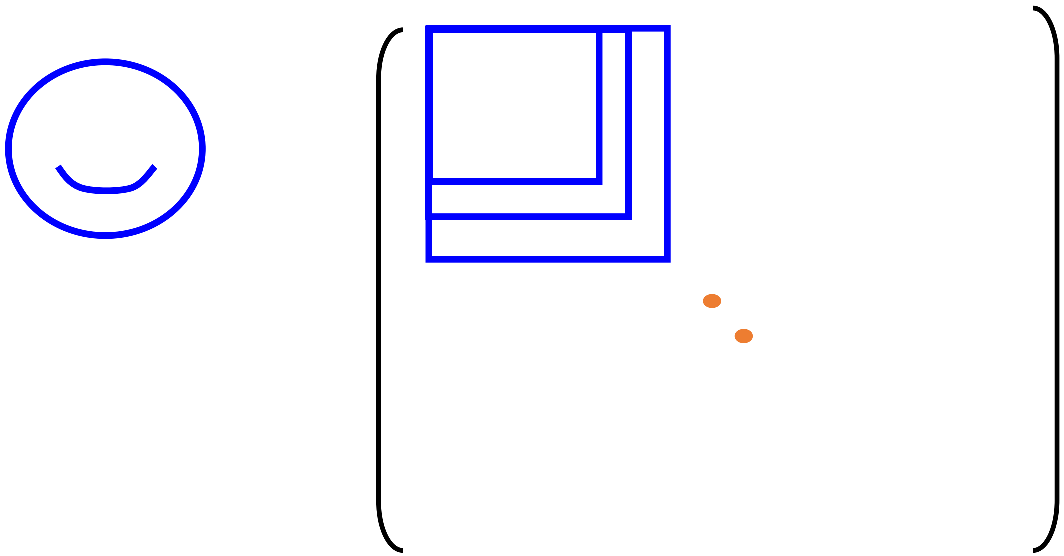
$$Z = \sum\nolimits_{\mathrm{dim}(V)} \int dA\, e^{iS_M[A]},$$

$$S_M = \alpha\, \mathrm{Tr}\left(-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{1}{2}\bar{\psi}\gamma_\mu\left[A_\mu, \psi\right]\right) - \beta\, \mathrm{Tr}1\ .$$

All topologies of the worldsheet are automatically included in the matrix integral.
Disconnected worldsheets are also included as block diagonal configurations.



Furthermore the sum over the size of the matrix is automatically included, if it is imbedded in a larger matrix as a sub matrix.



Then the second term of

$$S_M = \alpha\,\mathrm{Tr}\left(-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{1}{2}\bar{\psi}\gamma_\mu\left[A_\mu, \psi\right]\right) - \beta\,\mathrm{Tr}1$$

can be regarded as describing the chemical potential for the block size.

Thus we expect that the whole universe is described by a large matrix that obeys

$$S_{IIB} = \mathrm{Tr}\left(-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{1}{2}\bar{\psi}\gamma_\mu\left[A_\mu, \psi\right]\right).$$

This is nothing but the dimensional reduction of the 10D super YM theory to 0D.

Reductions to other dimensions are also considered:

0D ⇒ IIB MM

1D ⇒ Matrix theory — {de Witt–Hoppe–Nicolai, Banks–Fischler–Shenker–Susskind}

2D ⇒ Matrix string — Motl, Dijkgraaf–Verlinde–Verlinde

4D ⇒ AdS/CFT

As in what is known as quenched reduced model, they are equivalent if the diagonal elements of the matrices are quenched.

The lower dimensional theories have the more degrees of freedom:

$$0\mathrm{D} \supset 1\mathrm{D} \supset 2\mathrm{D} \supset 4\mathrm{D}$$

⇒ IIB MM is the maximum theory.

Unresolved issues with IIB MM

$$S_{IIB} = \frac{1}{g^2}\mathrm{Tr}\left(-\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{1}{2}\bar{\psi}\gamma_\mu[A_\mu, \psi]\right)$$

(1) Is an infrared cutoff necessary?

$$-L < \mathrm{Eigen}(A_\mu) < L$$

(2) How the large-N limit should be taken?

$$l_s = \mathrm{const.}\, g^a N^b L^c$$

(3) How does the space-time emerge?
Are A_μ coordinates of space-time ,
momentum space, NC space, or … ?

Aoki-Iso-HK-Kitazawa-Tada
Steinacker
Kim-Nishimura-Tsuchiya

(4) Express the diff. invariance explicitly.

(5) Find or construct multiverse.

QG and Naturalness

Hamada, Kawana, HK arXiv: 2210.05134
Kawana, HK, Oda, Yagyu arXiv:2307.11420

Probably, constructive formulation of string theory gives the right (ultimate) description of nature, and its low energy effective theory is given by SM (with modification) + EH action.

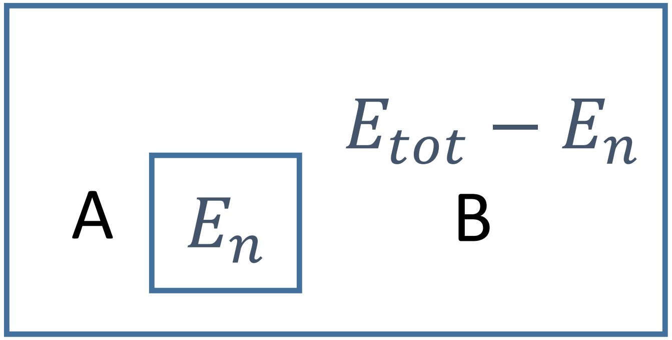
However, such low energy theory has a few unnatural parameters, that is, the cosmological constant, Higgs mass and θ-parameter.

In fact, the process by which low-energy effective theories emerge from constructive string theory is not simple.

There is a possibility that the ordinary field theory appears as an approximation to a more general theory without the problem of naturalness.

Analogy: Canonical ensemble

Canonical ensemble is an effective theory for a small subsystem.



"total system" very large DOF

E_{tot}: total energy, fixed

micricanonical ensemble

$$\rho \propto \delta(H - E_{tot})$$

A : subsystem, small compared with the total system

B:= (total system) – A

basic assumption $E_{tot} = E_A + E_B$

P_n: probability that A takes a microstate n with energy E_n

$P_n \propto$ # of the microstates of B with energy $E_{tot} - E_n$

$\propto e^{S_B(E_{tot}-E_n)}$

$$S_B(E_{tot} - E_n) = S_B(E_{tot}) - \frac{\partial S_B}{\partial E}(E_{tot})E_n + \frac{1}{2}\frac{\partial^2 S_B}{\partial E^2}(E_{tot})E_n^2 + \cdots$$

$\uparrow$ $O(V^1)$ $\quad$ $\uparrow$ $O(V^0)$ $\quad$ $\uparrow$ $O(V^{-1})$

As a function of E_n , we have

$$\frac{1}{T} = \frac{\partial S_E}{\partial E}(E_{tot})$$

$$S_B(E_{tot} - E_n) = const. - \frac{E_n}{T} + O\left(\frac{1}{V}\right).$$

Thus we have $P_n = \mathbf{const.}\, e^{-\frac{E_n}{T}}$ in the large V limit.

The canonical ensemble is an effective theory of small subsystems of a large system.

The FT counterpart:

The ordinary path integral (Schrödinger equation) may be an effective theory for subsystems of larger systems.

Generalized QFT (or QM)?

Nambu, …

ordinary QFT

$$\langle t_2, q_2 | t_1, q_1 \rangle = \int_{\substack{q(t_2) = q_2 \\ q(t_1) = q_1}} \mathcal{D}q \; e^{\frac{i}{\hbar} S[q]} \sim \sum_n e^{-\frac{E_n}{T}}$$

↑ tempting to imagine

microcanonical QFT $\displaystyle \int \mathcal{D}q \, \delta(S[q] - A) \sim \sum_n \delta(E_n - E)$

generalized QFT $\displaystyle \int \mathcal{D}q \, f(S[q])$

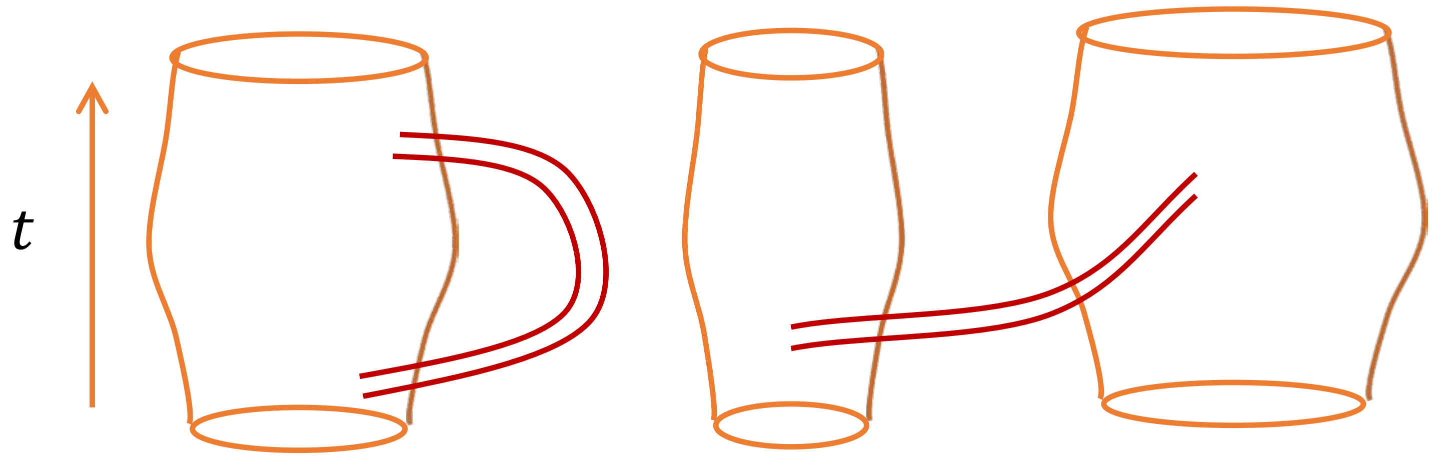
We will show that under some circumstances they are equivalent, and that the naturalness problem is resolved in the generalized QFT.

Emergence of Generalized QFT in QG

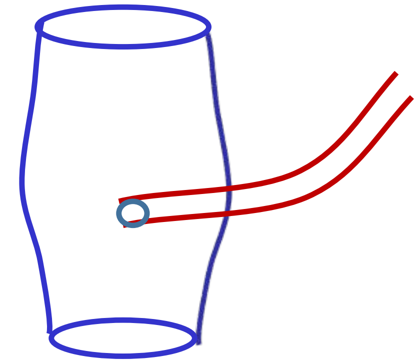
multiverse and baby universes in MM

Universe arises from matrices. By considering block diagonal configuration, multiverse appears naturally.

We consider emission and absorption of BU's by multiverse.

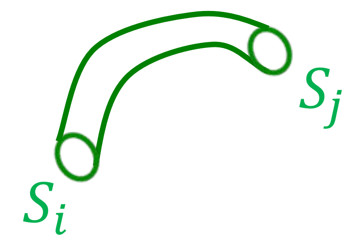


For the large universe, emission or absorption of BU looks like an insertion of a local operator.



Therefore, the emission and subsequent absorption of a BU modify the effective action as

$$S \to S + \sum_{i,j} c_{ij} S_i S_j \ ,$$



where S_i is a space-time integral

$$S_i = \int d^4x \sqrt{-g(x)}\, O_i(x)$$

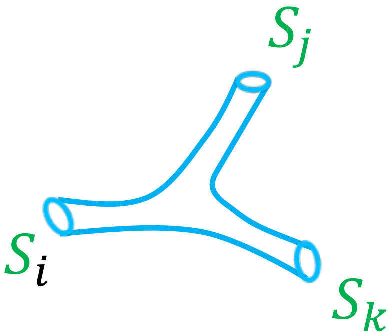
of a scalar operator O_i such as

$$1\ ,R\ ,R_{\mu\nu}R^{\mu\nu}\ ,F_{\mu\nu}\ F^{\mu\nu}, \overline{\psi}\gamma^{\mu}D_{\mu}\psi\ , \cdots .$$

Each S_i has a form of local action.

Furthermore, bifurcated BU's contribute to the effective action as

$$S \to S + \sum_{i,j,k} c_{ijk} S_i S_j S_k \ .$$



Thus the low energy effective theory of QG / MM is not a simple local action but a generalized QFT:

$$S_{\rm eff} = f\left(S_1, S_2, \cdots\right).$$

Coleman '89
Asano-HK-Tsuchiya

Other possibilities of generalized QFT

(1) microcanonical QFT

ordinary matrix model

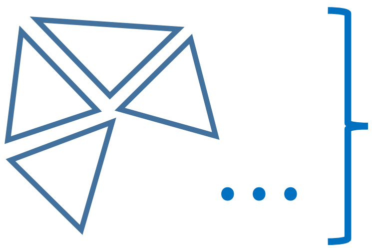
$$Z = \int dA d\psi \ e^{iS_{IIB}[A,\psi]}$$

microcanonical matrix model

$$Z = \int dA d\psi \ \delta(S_{IIB}[A,\psi] - 1)$$

A priori, we don't know which is more fundamental.

(2) M.C. simulation of dynamical triangulation of QG



\# of D-simplexes $= N_D$: fixed $\leftrightarrow \delta(\int d^D x \sqrt{g} - N_D)$

discretized version of Microcanonical C.C.

Equivalent to the ordinary QFT?

For the generalized QFT

$$Z = \int \mathcal{D}q\, f(S_i[q])\,, \qquad S_i = \int d^4x \sqrt{-g(x)}\, O_i(x)$$

we express f as

$$f(x) = \int \prod_i d\alpha_i\, w(\alpha)\, e^{i \sum_i \alpha_i x_i}.$$

Then

$$Z = \int \mathcal{D}q\, f(S_i[q])$$

$$= \int \mathcal{D}q \int \prod_i d\alpha_i\, w(\alpha)\, e^{i \sum_i \alpha_i S_i[q]}$$

$$= \int \prod_i d\alpha_i\, w(\alpha) Z(\alpha)\,,$$

where

$$Z(\alpha) = \int \mathcal{D}q\, e^{i \sum_i \alpha_i S_i[q]}.$$

Ordinary FT with coupling constants α_i .

Generalized QFT = "superposition" of ordinary QFT

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

Basic question:

(1) Does one point in the α space, $\alpha_i = \alpha_i^{(0)}$, dominate the integral? Then the theory is equivalent to the ordinary QFT with coupling constants $\alpha_i^{(0)}$.

(2) If it is the case, are $\alpha_i^{(0)}$ good values so as to solve the naturalness problem?

Static vacuum

We first consider static vacuum to evaluate the path integral:

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

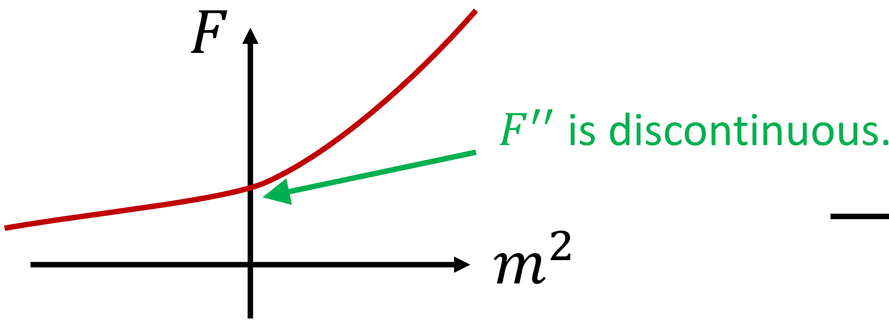
In this case, we can regard

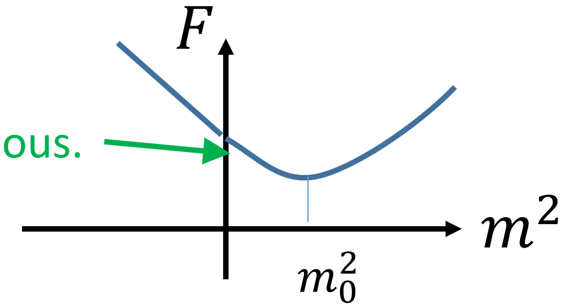
$$Z(\alpha) = e^{-i\, VF(\alpha)}$$

where $F(\alpha)$ is the vacuum energy density.

As a simplest example we take the physical m^2 of a scalar field as α .

In general, if $m^2 = 0$ is the 2nd order phase transition point, F'' is discontinuous. Then we have two cases:





F is monotonic on each side $m^2 \lessgtr 0$.

F has other extrema at $m^2 = m_0^2, \cdots$.

The m^2 integral

$$Z = \int dm^2\, w(m^2) \exp(-iVF(m^2))$$

can be evaluated as follows.

The crucial point is that V appears only in the exp:

$$Z = \int d\, w(m^2) \exp\big(-iVF(m^2)\big) .$$

Therefor, when V is large, the integral is dominated by the extrema or singularities of $F(m^2)$ as long as the function $w(m^2)$ is smooth.

This is reminiscent of the RG analysis, where the fixed point Hamiltonian has no singular behavior. Similarly, $w(\alpha)$ itself is expected to be a smooth function of α.

Under this assumption we can estimate the integral as

$$Z \sim \frac{1}{V^2} w(0) \exp\big(-iVF(0)\big) + o(\frac{1}{V^3}), \text{ and}$$

$$Z \sim \frac{1}{\sqrt{V}} w(m_0^2) \exp\Big(-iVF(m_0^2)\Big) + \cdots + o(\frac{1}{\sqrt{V}})$$

for the two cases.

The coupling constants are automatically tuned to either a phase transition point or the minimum of F.
The same is true for phase transitions of any orders.

remark If there are more than one extrema, the system is no longer equivalent to ordinary field theory.

Time evolution of universe

We have considered static vacuum to evaluate the path integral:

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

In that case, we can regard

$$Z(\alpha) = e^{i\, VF(\alpha)}$$

where F is the vacuum energy density.

If we consider the time evolution of universe, the notion of critical point should be generalized to critical point of the history of universe, which means coupling constants that significantly change the time evolution of universe when they are changed.

Generalize MPP:

The coupling constants of the low energy effective canonical FT of MM or quantum gravity are automatically adjusted either to minimize the vacuum energy density or to one of the critical points of the history of universe.

Examples

1. QCD θ-parameter

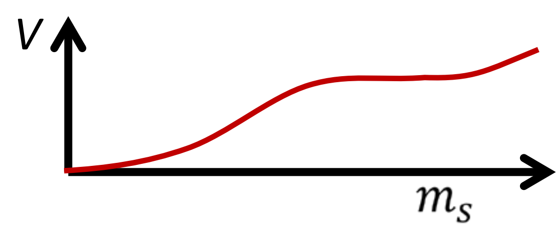
   $\theta = 0$ minimizes the vacuum energy.

2. Cosmological constant

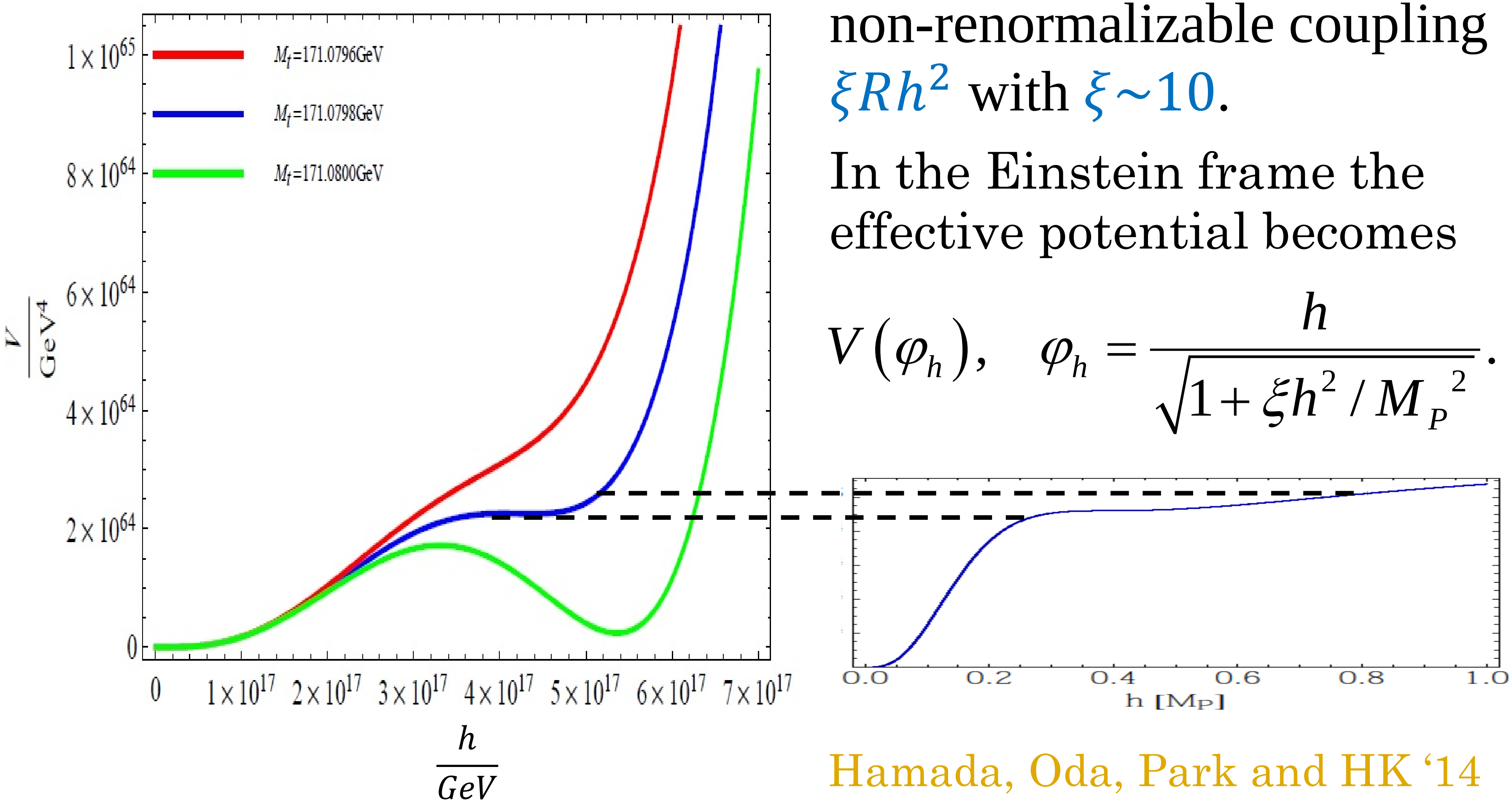
   $\lambda = 0$ is the critical point.

3. Higgs inflation at criticality

   Flat potential is the critical point of the history of universe.



SM Higgs is close to MPP.



non-renormalizable coupling $\xi R h^2$ with $\xi \sim 10$.

In the Einstein frame the effective potential becomes

$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2/M_P{}^2}}.$$

Hamada, Oda, Park and HK ‘14
Bezrukov,Shaposhnikov

Some of the parameters of the (modified) standard model may be fixed by GMPP, independent of the detailed dynamics.

Summary and Conclusion

Possible next steps of QG and string theory:

Find the multiverse in constructive string theory and confirm that low-energy physics is described by the SM (with some modifications) + EH action with the right coupling constants.

Thank you.

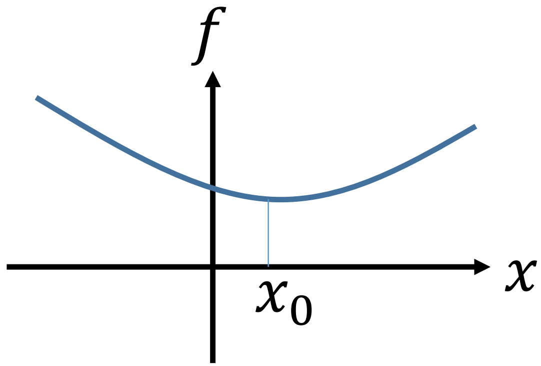
Appendix

Formulas on $e^{i\,V f(x)}$ for large V

$e^{iVf(x)}$ for large V

$f(x)$: real function

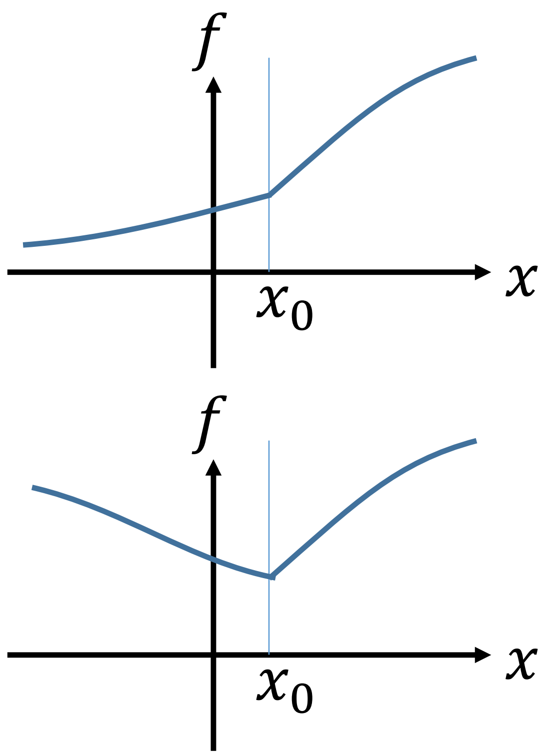
(1)



smooth and one extremum

$$e^{iVf(x)} \sim \frac{1}{\sqrt{V}} e^{i\,Vf(x_0)} \delta(x - x_0)$$

(2)



f is continuous

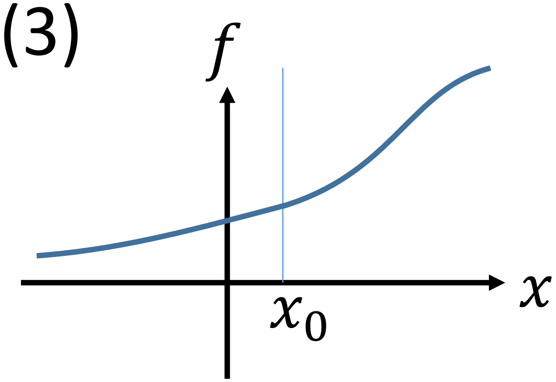
f' is discontinuous at x_0

x_0 need not be extremum

monotonic on each side $x \lesseqgtr x_0$

$$e^{iVf(x)} \sim \frac{1}{V}\left(\frac{1}{f'(x_0+0)} - \frac{1}{f'(x_0-0)}\right) \cdot e^{i\,Vf(x_0)}\, \delta(x - x_0)$$

(3)

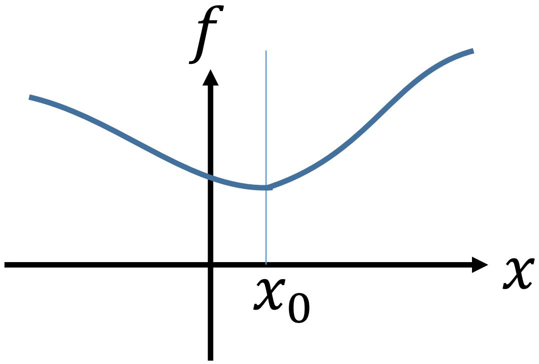


f, f' are continuous

f'' is discontinuous at x_0

x_0 need not be extremum

monotonic on each side $x \lessgtr x_0$



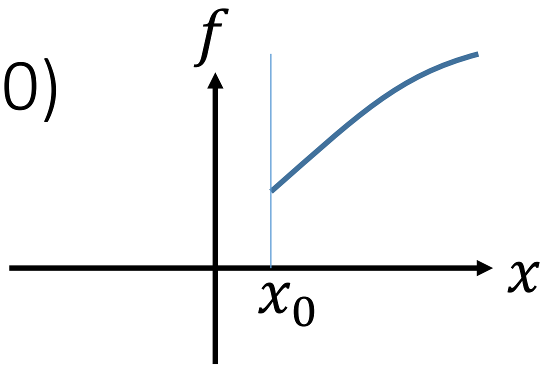
$$e^{iVf(x)}$$

$$\sim \frac{1}{V^2} \frac{1}{f'(x_0)^3} \left(f''(x_0+0) - f''(x_0-0)\right)$$

$$\cdot\, e^{i\,Vf(x_0)}\, \delta(x - x_0)$$

proof of (2), (3)

(0)



$f: [x_0, \infty] \to \mathbb{R}$
smooth
monotonically increasing

For any test function,
$\varphi \in C^\infty$, finite support,

$$\int_{x_0}^{\infty} dx \; e^{iVf(x)} \varphi(x)$$

$\leftarrow y = f(x), y_0 = f(x_0), y_\infty = f(\infty)$

$$= \int_{y_0}^{y_\infty} dy \frac{dx}{dy} \; e^{iVy} \varphi\left(f^{-1}(y)\right)$$

$\leftarrow g(y) = \frac{dx}{dy} \varphi(f^{-1}(y))$

$$= \int_{y_0}^{y_\infty} dy \, e^{iVy} g(y)$$

$$= \left[\frac{1}{iV} e^{iVy} g(y)\right]_{y_0}^{y_\infty} - \int_{y_0}^{\infty} dy \frac{1}{iV} e^{iVy} g'(y)$$

$$= \frac{i}{V} e^{iVy_0} g(y_0) + O\left(\frac{1}{V^2}\right)$$

$\leftarrow \varphi$ has finite support $\Rightarrow g(y_\infty) = 0$

$$= \frac{i}{V} e^{iVf(x_0)} \frac{1}{f'(x_0)} \; \varphi(x_0) + O\left(\frac{1}{V^2}\right)$$