Exploring the Landscape of 6d supergravity

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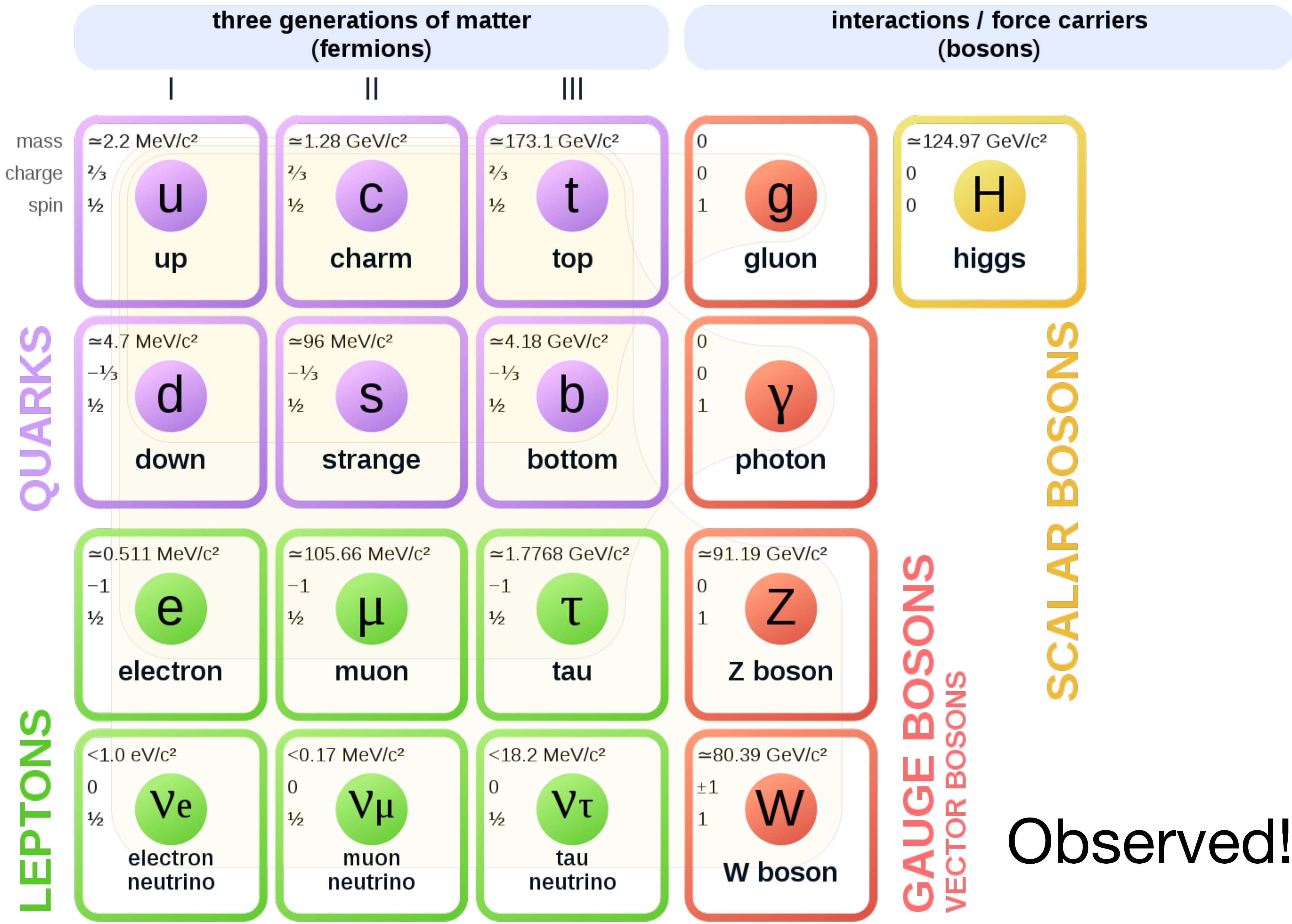
Based on the works
2309.15152 w/ Zihni Kaan Baykara, Houri-Christina Tarazi, Cumrun Vafa (Harvard)
2311.00868 and WIP w/ Gregory J. Loges (KEK)

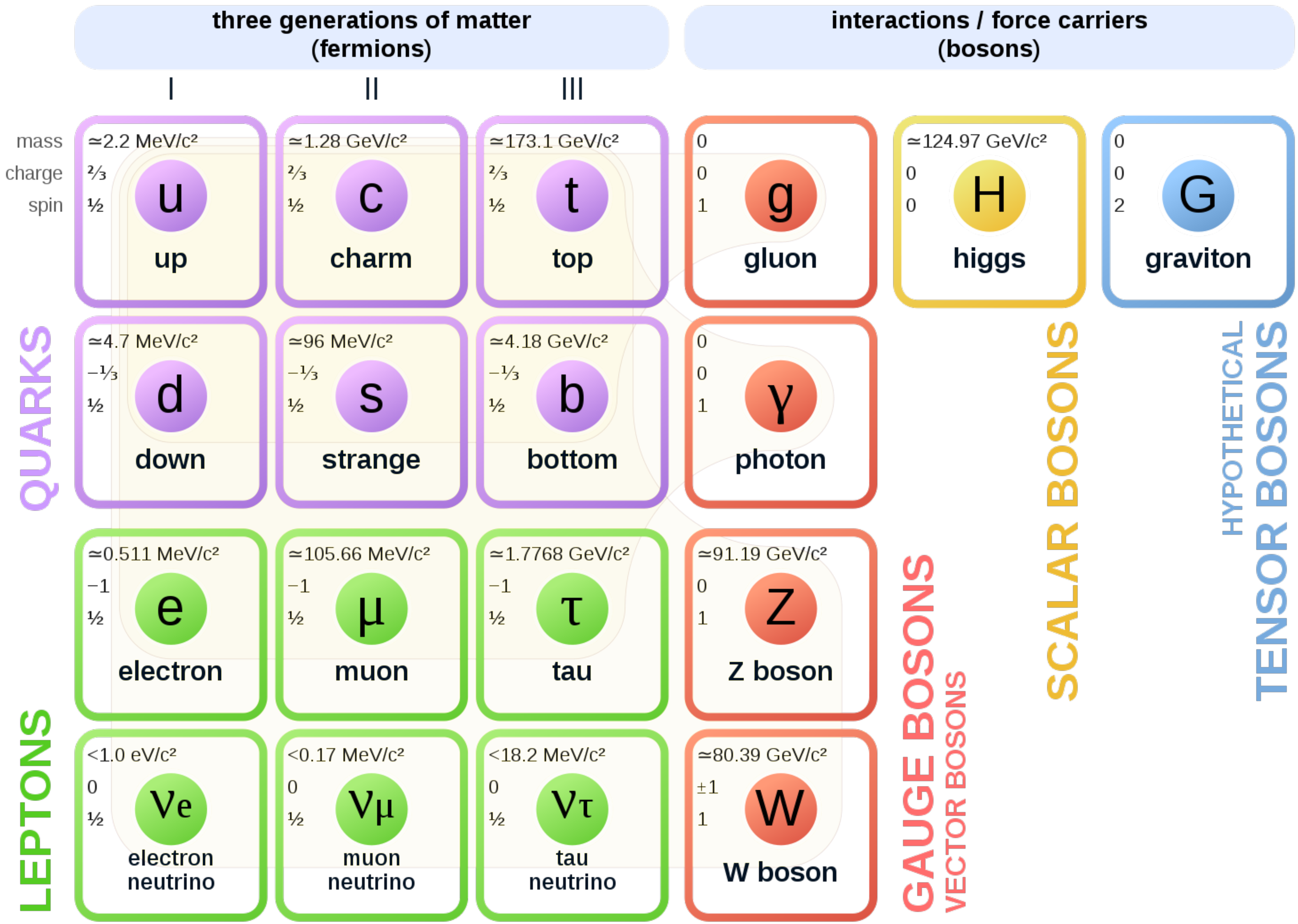
2023/11/30 KEK Theory Workshop 2023

Standard Model of Elementary Particles



Observed!

Standard Model of Elementary Particles and Gravity



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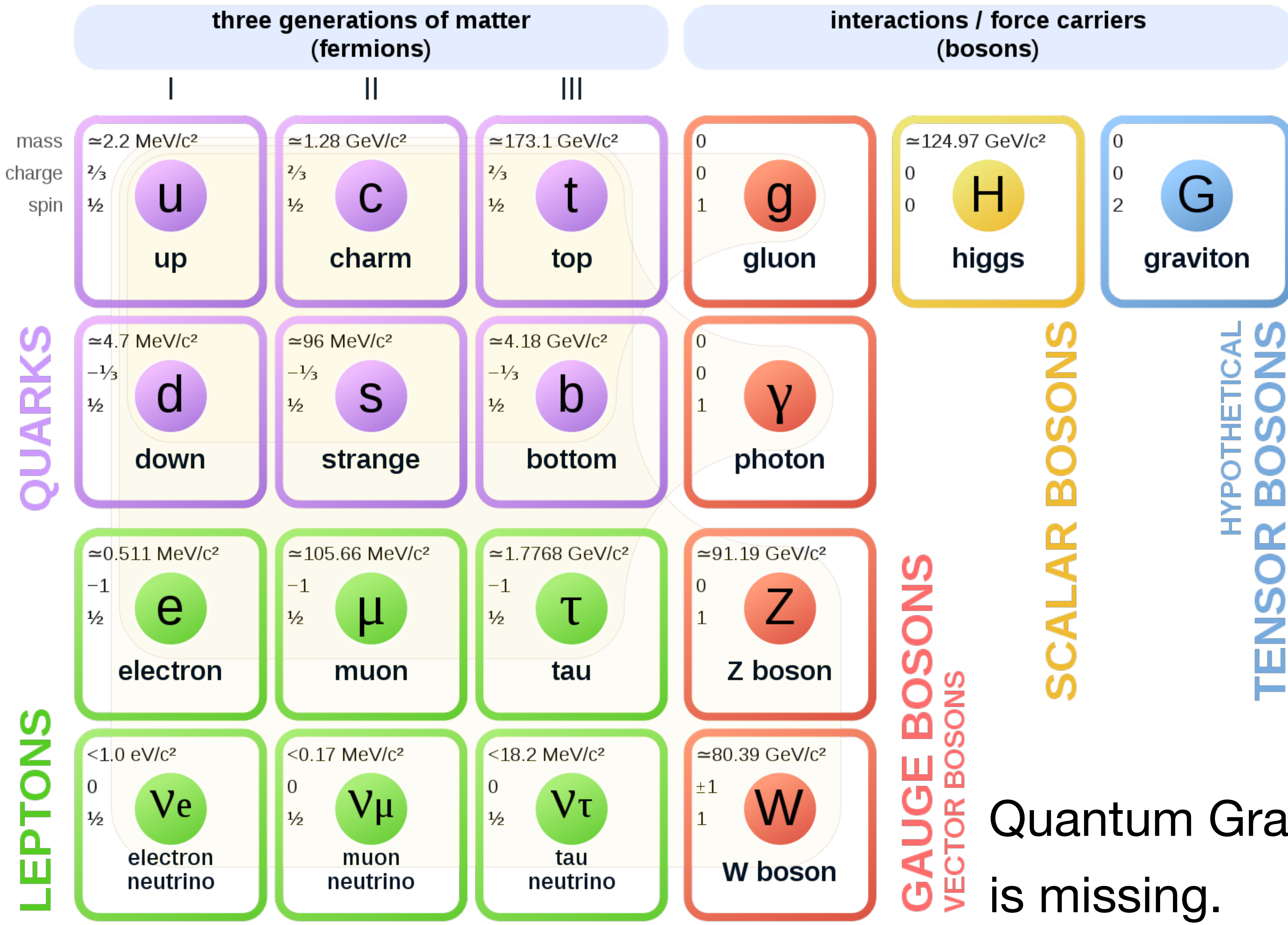

Quantum Gravity (QG) is missing.

String Theory: Good candidate for Quantum Gravity.

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Advantage:

Around the flat background, the one-loop diagram of graviton is finitely computed!

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Advantage:
Around the flat background, the one-loop diagram of graviton is finitely computed!

However, there are many vacua, and there are no predictions on low energy physics?

This is not the case!

[cf. Misumi-san's talk for no global symmetry conjecture]

Landscape vs Swampland

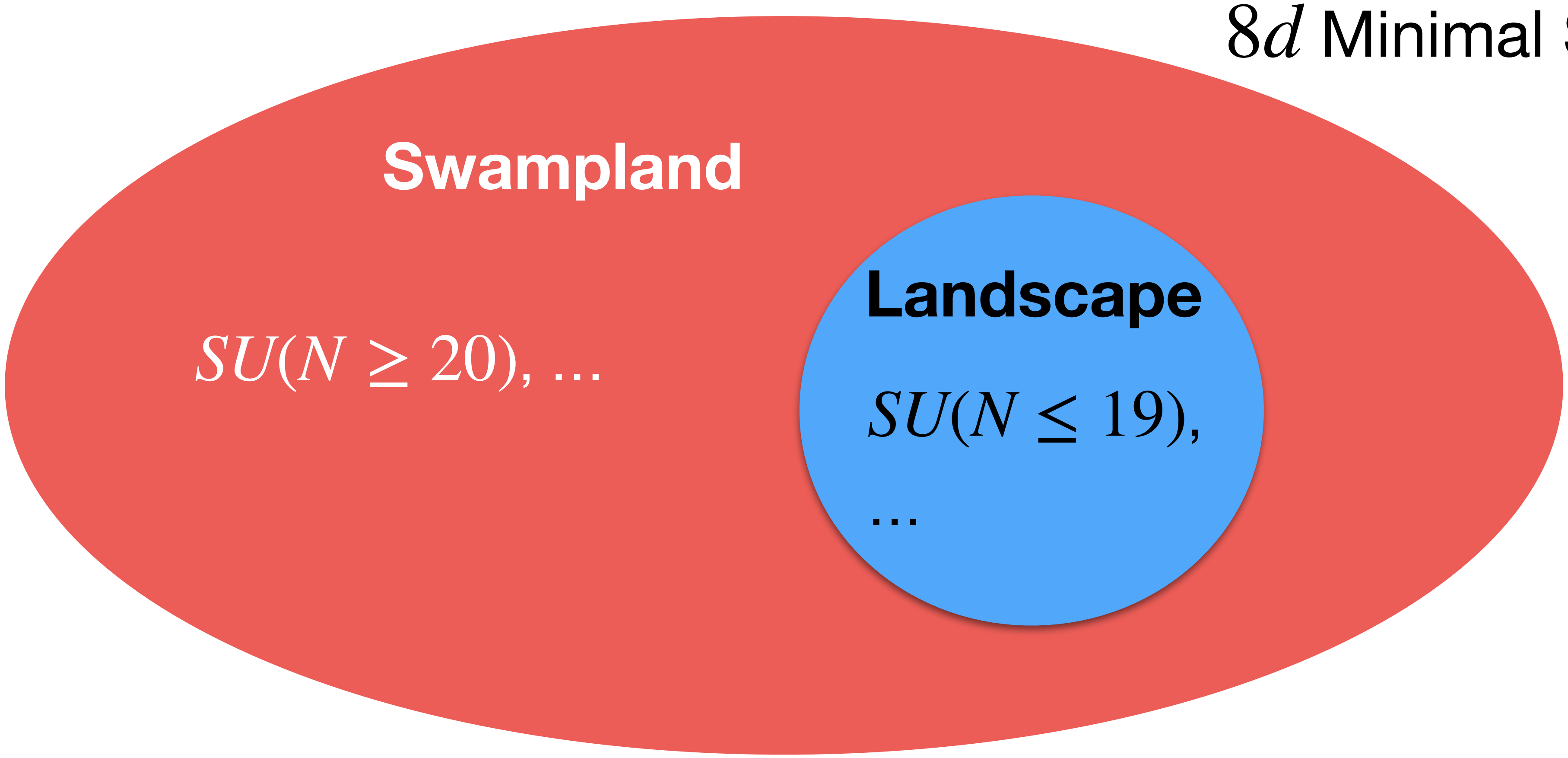
Field theory

Any anomaly-free gauge group is fine.

QG (String theory)

Very constraining.

$8d$ Minimal SUSY theories

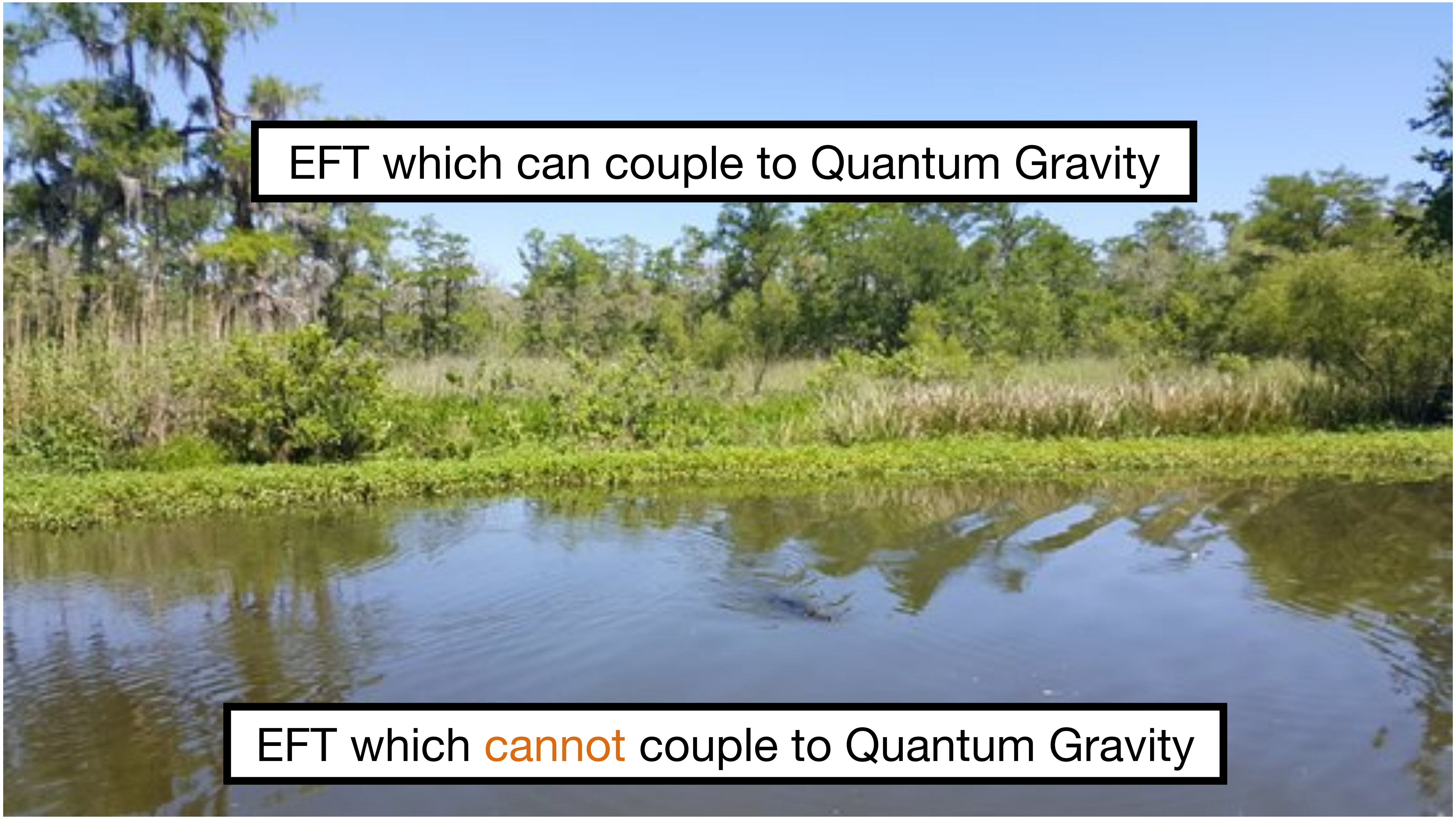


Number of theory in the Landscape is finite, while that in the Swampland is infinite.

Goal: Understanding "kinematics" of Quantum Gravity.

[cf. Dynamical approach to select one of vacua.   IKKT matrix model, string field theory.

Talks by Hatakeyama-san, Piensuk-san, Yamamori-san, Tripathi-san, Asano-san, Ando-san, Konosu-san, Sato-san]



Current Status

#(SUSY)

Dimension	8Q	16Q	32Q
11d	×	×	M-theory
10d	×	$SO(32)$, $E_8 \times E_8$	IIA/IIB
9d	×	rank $= 1,9,17$	
8d	×	rank $= 2,10,18$	
7d	×	rank $= 3,5,7,11,19$ (?)	
6d		rank $= 0,2,4,6,8,12,20$ (?) for $\mathcal{N} = (1,1)$, Unique EFT for $\mathcal{N} = (2,0)$.	

$\left(S^1\right)^d$ ↓

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$(S^1)^d$

Dimension

Why 6d supergravity?

- Less SUSY than higher dim cases.

- Particle physics models starting from 6d [e.g. Asaka, Buchmuller, Covi '01; Hall, Nomura, Okui, Tucker-Smith '01]. Advantage to break GUT gauge group.

- 8 SUSY is starting point of KKLT scenario.
  dS is realized by adding orientifold, flux, anti-brane.

Talk Plan

1. Review of 6d supergravity
2. Toward a classification of 6d supergravity

   [YH-Loges '23]
3. Extending Landscape [YH-Baykara-Tarazi-Vafa '23]

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6d Supergravity

6d gravity theories with minimal supersymmetry (8 SUSY).

Multiplets are

- Gravity multiplet $(g_{\mu\nu}, \psi_\mu, B^-_{\mu\nu})$.
- Tensor multiplet $(B^+_{\mu\nu}, \phi, \psi)$.
- Vector multiplet (A_μ, λ).
- Hyper multiplet (Φ, Ψ).

I will denote #(tensor), #(vector), #(hyper) by T, V, H.

There are $(T+1)$ B-fields, $B_\alpha,\ \alpha = 0,1,\cdots,T$.

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- Vector multiplet (A_μ, λ).
- Hyper multiplet (Φ, Ψ). ← Higgs branch.

Moduli

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There are $(T+1)$ B-fields, B_α, $\alpha = 0,1,\cdots,T$.

Anomalies

The (perturbative) anomaly is characterized by

8-form anomaly polynomial.

$I_8 = \#\mathrm{Tr}\,R^4 + \#\mathrm{Tr}\,F^4 + \#\left(\mathrm{Tr}\,R^2\right)^2 + \#\left(\mathrm{Tr}\,F^2\right)^2 + \#\mathrm{Tr}\,R^2\mathrm{Tr}\,F^2.$

The theory is anomalous if I_8 is non-vanishing as

$$I_8 = dI_7,$$

$$\delta I_7 = dI_6,$$

$$\delta S = \int I_6.$$

δI_7: gauge tr. of I_7.

δS: gauge tr. of the action.

Two Contributions

$$I_8 = \#\mathrm{Tr}\,R^4 + \#\mathrm{Tr}\,F^4 + \#\left(\mathrm{Tr}\,R^2\right)^2 + \#\left(\mathrm{Tr}\,F^2\right)^2 + \#\mathrm{Tr}\,R^2\mathrm{Tr}\,F^2.$$

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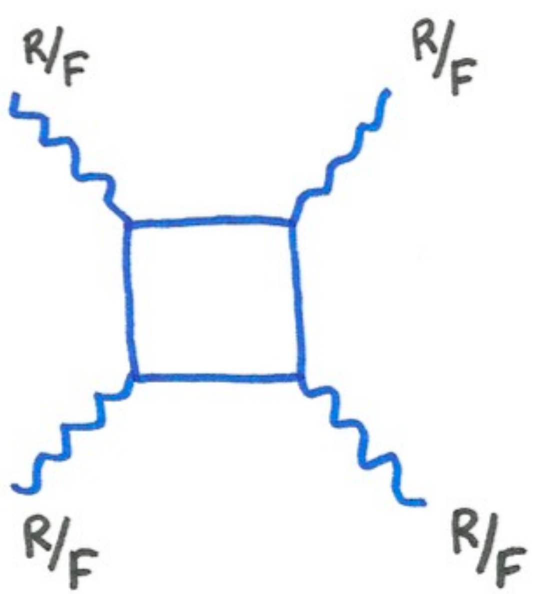
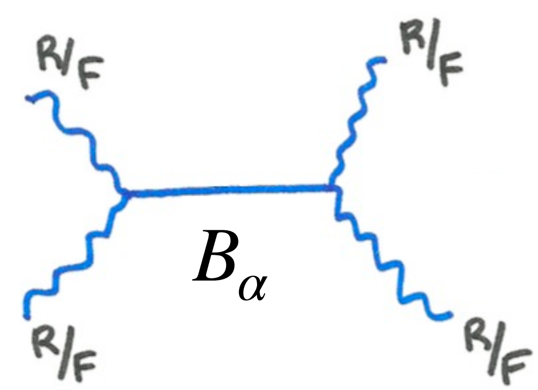


$+$

2: B fields

$$I_8 \sim \sum Y_4^2, \qquad Y_4 \sim b_0^\alpha\,\mathrm{Tr}R^2 + b_F^\alpha\,\mathrm{Tr}F^2$$



$= 0$

$$\mathscr{L}_{\mathrm{int}} \sim b_0^\alpha B_\alpha R^2 + b_F^\alpha B_\alpha F^2.$$

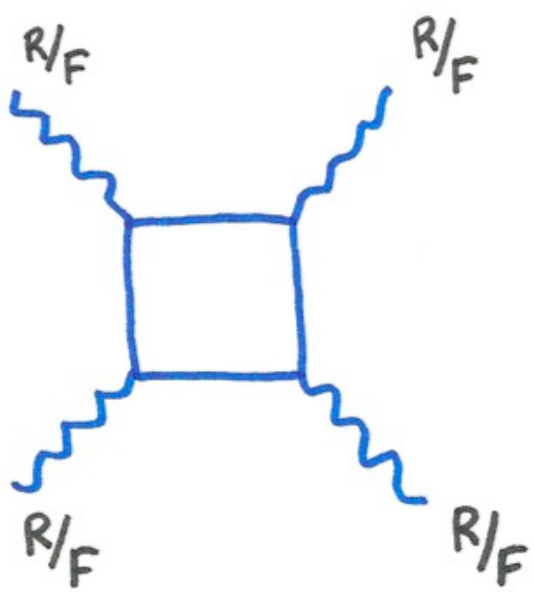
Couplings

Irreducible part

$$I_8 = \#\mathrm{Tr}\,R^4 + \#\mathrm{Tr}\,F^4 + \#\left(\mathrm{Tr}\,R^2\right)^2 + \#\left(\mathrm{Tr}\,F^2\right)^2 + \#\mathrm{Tr}\,R^2\mathrm{Tr}\,F^2.$$

Only fermion contribution.

1: Fermions



G: $(g_{\mu\nu}, \psi_\mu, B^-_{\mu\nu})$, T: $(B^+_{\mu\nu}, \phi, \psi)$, V: (A_μ, λ), H: (Φ, Ψ).

$$\mathrm{Tr}\,R^4: \qquad H - V = 273 - 29T.$$

$$\mathrm{Tr}\,F^4: \qquad 0 = \sum_R n_R B_R - B_{\mathrm{Adj}}.$$

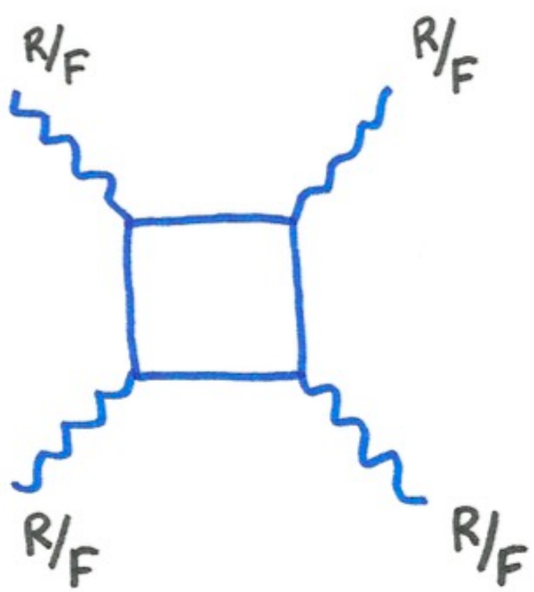
n_R: #(Rep. R hyper), $\mathrm{Tr}_R F^4 = B_R \mathrm{Tr} F^4 + C_R \left(\mathrm{Tr} F^2\right)^2$

Green-Schwarz

$$I_8 = \#\mathrm{Tr}\,R^4 + \#\mathrm{Tr}\,F^4 + \#\left(\mathrm{Tr}\,R^2\right)^2 + \#\left(\mathrm{Tr}\,F^2\right)^2 + \#\mathrm{Tr}\,R^2\mathrm{Tr}\,F^2.$$

Both contributions

1: Fermions

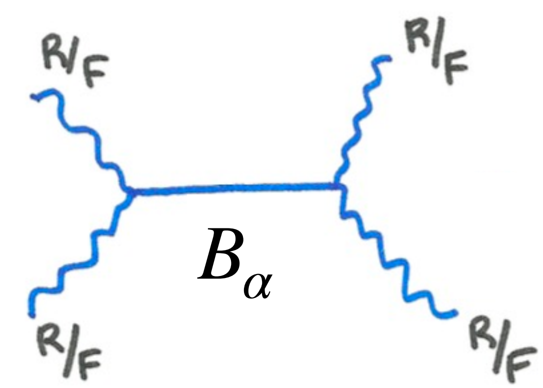


$$\left(\mathrm{Tr}\,R^2\right)^2: \qquad 9 - T = b_0 \cdot b_0.$$

$$\left(\mathrm{Tr}\,F^2\right)^2: \frac{1}{3}\left(\sum_R n_R C_R - C_{\mathrm{Adj}}\right) = b_F \cdot b_F$$

$$\mathrm{Tr}\,R^2\mathrm{Tr}\,F^2: \frac{1}{6}\left(\sum_R n_R A_R - A_{\mathrm{Adj}}\right) = b_0 \cdot b_F$$

2: B fields



$\mathscr{L}_{\mathrm{int}} \sim b_0^\alpha B_\alpha R^2 + b_F^\alpha B_\alpha F^2$

n_R: #(Rep. R hyper), $\mathrm{Tr}_R F^2 = A_R \mathrm{Tr} F^2$, $\mathrm{Tr}_R F^4 = B_R \mathrm{Tr} F^4 + C_R \left(\mathrm{Tr} F^2\right)^2$

Anomaly Cancellation

Generalization to product of Gauge group $G_1 \times \cdots \times G_k$.

$$H - V = 273 - 29T, \quad \longleftarrow \quad \mathrm{Tr}(R^4)$$

$$0 = \sum_R n_R^i B_R^i - B_{\mathrm{Adj}}^i, \quad \longleftarrow \quad \mathrm{Tr}(F_i^4)$$

$b_F \to b_i$

$$b_0 \cdot b_0 = 9 - T, \qquad b_0 \cdot b_i = \frac{1}{6}\Big(\sum_R n_R^i A_R^i - A_{\mathrm{Adj}}^i\Big),$$

$$b_i \cdot b_i = \frac{1}{3}\Big(\sum_R n_R^i C_R^i - C_{\mathrm{Adj}}^i\Big), \qquad b_i \cdot b_j = \sum_{R,S} n_{(R,S)}^{i,j} A_R^i A_S^j, \quad (i \neq j).$$

Green-Schwarz

$i = 1, \cdots, k.$ $\quad b_0, b_i \in \mathbb{R}^{1,T}, \quad n_R, A_R, B_R, C_R$: Group and representation R dependent number.

No anomaly $\leftrightarrow$ $^\exists b_0, b_i$.

The cancellation of local anomaly implies the absence of the global anomaly as $\Omega_7^{\mathrm{spin}} = 0$.

[Lee, Tachikawa '20; Davighi, Lohitsiri '20]. (See however [Basile, Leone '23] for twisted string structure)

Consistency conditions

Positivity of gauge kinetic term

Gauge kinetic terms $-j \cdot b_i \, \mathrm{Tr}(F_i^2)$

$\implies \quad {}^{\exists} j \in \mathbb{R}^{1,T}$ with $j \cdot j = 1$ such that $j \cdot b_i > 0$.

Unimodularity

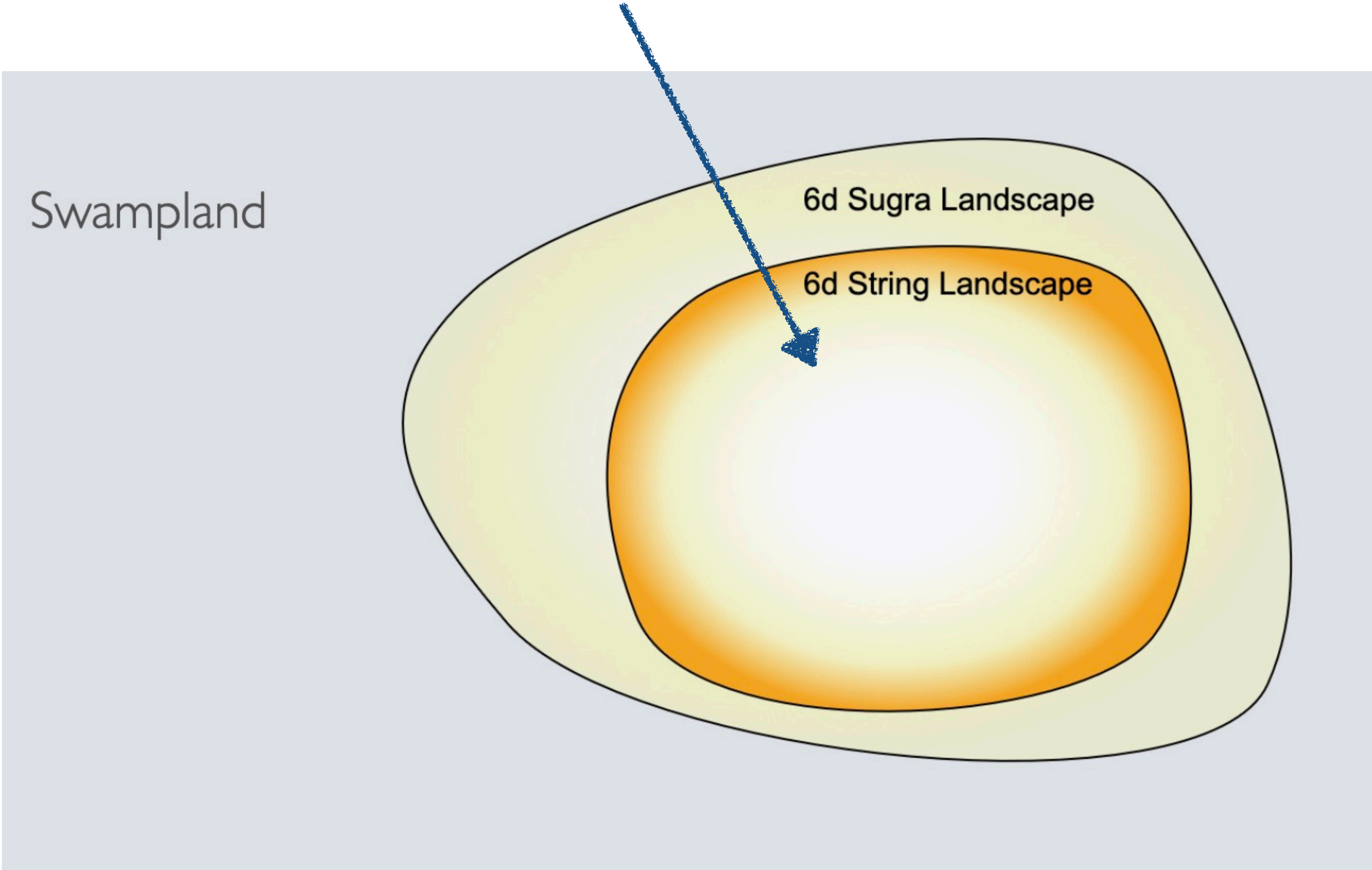
$\Lambda = \bigoplus_I \mathbb{Z}\, b_I \subset \mathbb{R}^{1,T}$ is an integral lattice [Kumar, Morrison, Taylor '10]

$\Lambda \subseteq \Gamma \subset \mathbb{R}^{1,T}$ where Γ is unimodular (integral and $|\det \Gamma| = 1$) [Seiberg, Taylor '11]

Unimodular lattices of indefinite signature $(1,T)$ have a simple classification:

$$\mathrm{I}_{1,T} \cong \mathbb{Z}^{T+1}, \qquad \mathrm{II}_{1,T} \cong \left\{ x \in \mathbb{Z}^{T+1} \cup (\mathbb{Z}+\tfrac{1}{2})^{T+1} \mid \sum x^\alpha \in 2\mathbb{Z} \right\} \quad (T \equiv 1 \mod 8)$$

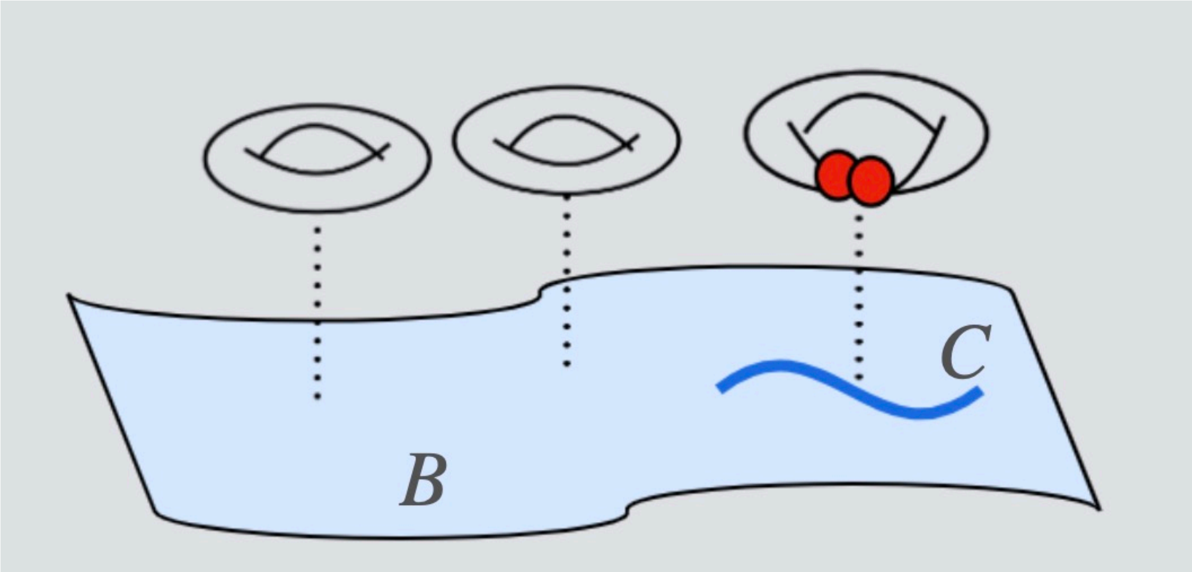
What is known for theories here?



[Vafa '96, Morrison-Vafa '96, Morrison-Vafa '96]

F-theory vacua

A large class of 6d $\mathcal{N} = (1{,}0)$ vacua is obtained from F-theory compactified on the elliptic CY_3.



Elliptic fibration over base B.

7-brane wraps curve B which supports non-Abelian gauge symmetry.

Hypers appear at intersection of 7-branes.

[Vafa '96, Morrison-Vafa '96, Morrison-Vafa '96]

F-theory vacua

The number of neutral hypermultiplets:

$$H^0 = h^{2,1}(X) + 1$$

Universal hypermultiplet

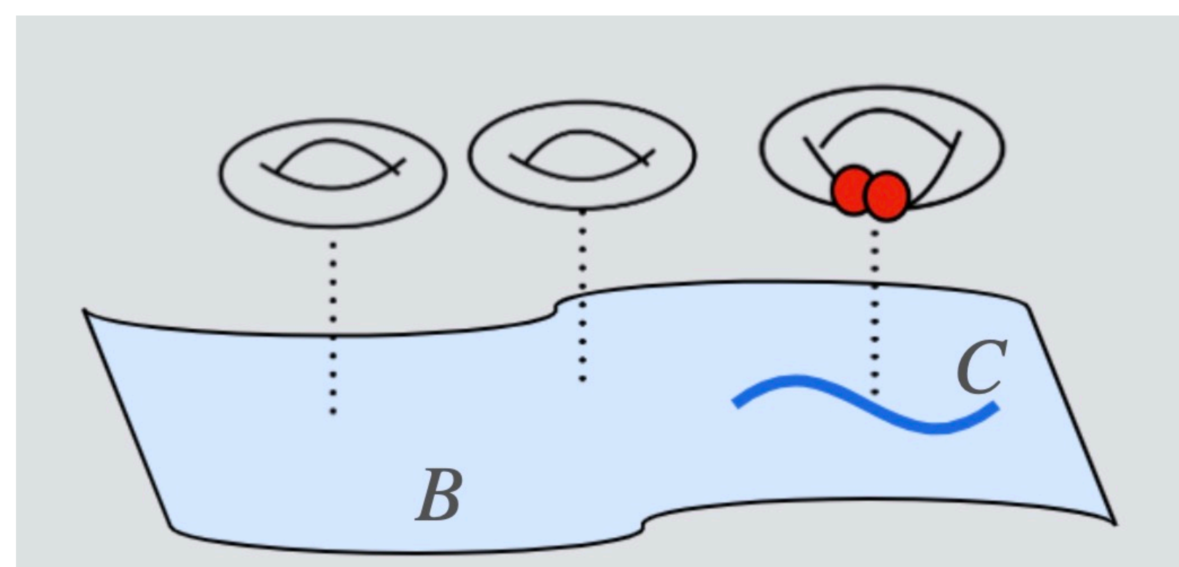
The number of tensor hypermultiplets:

$$T = h^{1,1}(B) - 1$$

The rank of gauge group is

$$r(T) = h^{1,1}(X) - h^{1,1}(B) - 1.$$

base B, total space X



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Based on work with

Gregory Loges.

Parallel talk today.

Toward a classification

Consistency conditions on theories put strong constraints.

However, classification of consistent supergravities is not yet done.

Previously:

- With $T < 9$, the number of anomaly-free 6d, $\mathcal{N} = (1,0)$ supergravities are known to be finite [Kumar, Morrison, Taylor '10].
- Given specific gauge groups, enumeration for $T = 0,1$ theories [Avramis, Kehagias '05, Kumar, Park, Taylor '10].

What's new

New:

- Classify in a T-agnostic way.
- Gauge groups with any number of simple factors.

Technical limitations:

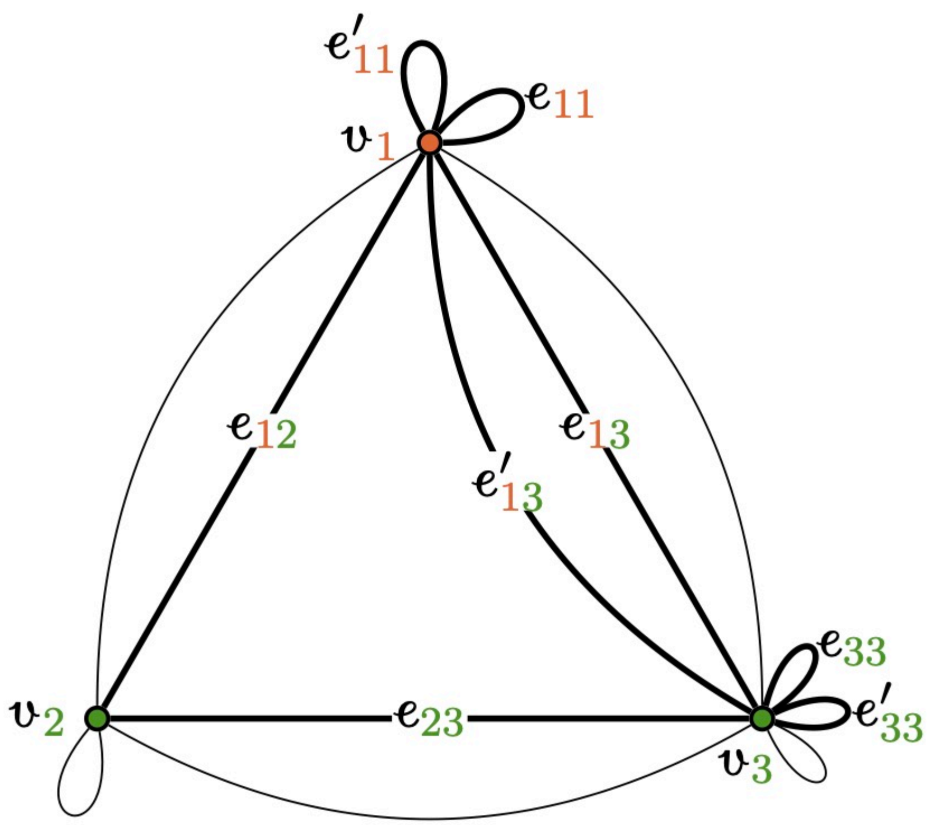
- no $U(1)$, $SU(2)$ or $SU(3)$ simple factors.
- no $(3+)$-charged hypers.

[YH-Loges '23]

Idea

Express theories as graphs (vertexes and edges).

Example



[cf. Misumi-san's and Ohta-san's talks]

Vertices: gauge group G_i & charged matter H_i

v_i	G_i	H_i	Δ_i
v_1	SU(8)	$16 \times \underline{\mathbf{8}} + 3 \times \underline{\mathbf{28}}$	149
v_2	SU(16)	$8 \times \underline{\mathbf{16}} + \underline{\mathbf{136}}$	9
v_3	Sp(4)	$16 \times \underline{\mathbf{8}}$	92

$\Delta_i := H_i - V_i$

Edges: bi-charged matter H_{ij}

e_{ij}	$\hbar(e_{ij})$	H_{ij}
e_{11}	(v_1, v_1)	$(\underline{\mathbf{8}}, \underline{\mathbf{8}})$
e'_{11}	(v_1, v_1)	$2 \times (\underline{\mathbf{8}}, \underline{\mathbf{8}})$
e_{12}	(v_1, v_2)	$(\underline{\mathbf{8}}, \underline{\mathbf{16}})$
e_{13}	(v_1, v_3)	$(\underline{\mathbf{8}}, \underline{\mathbf{8}})$
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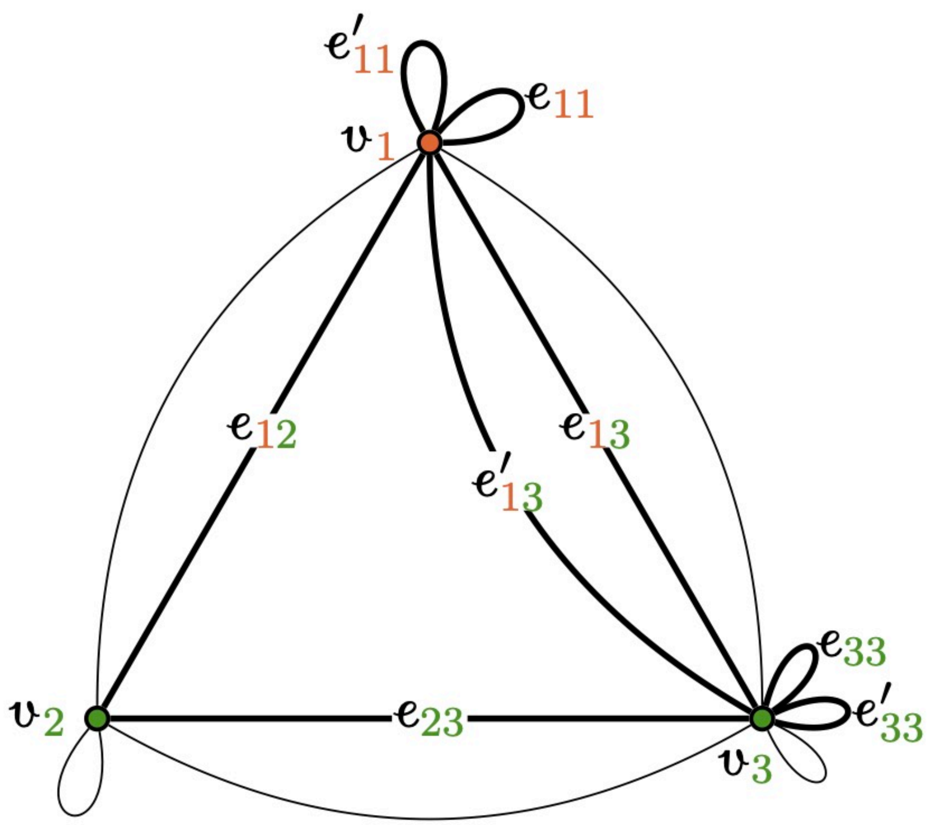

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$\mathrm{Tr}(R^4)$ anomaly

[cf. Misumi-san's and Ohta-san's talks]

[YH-Loges '23]

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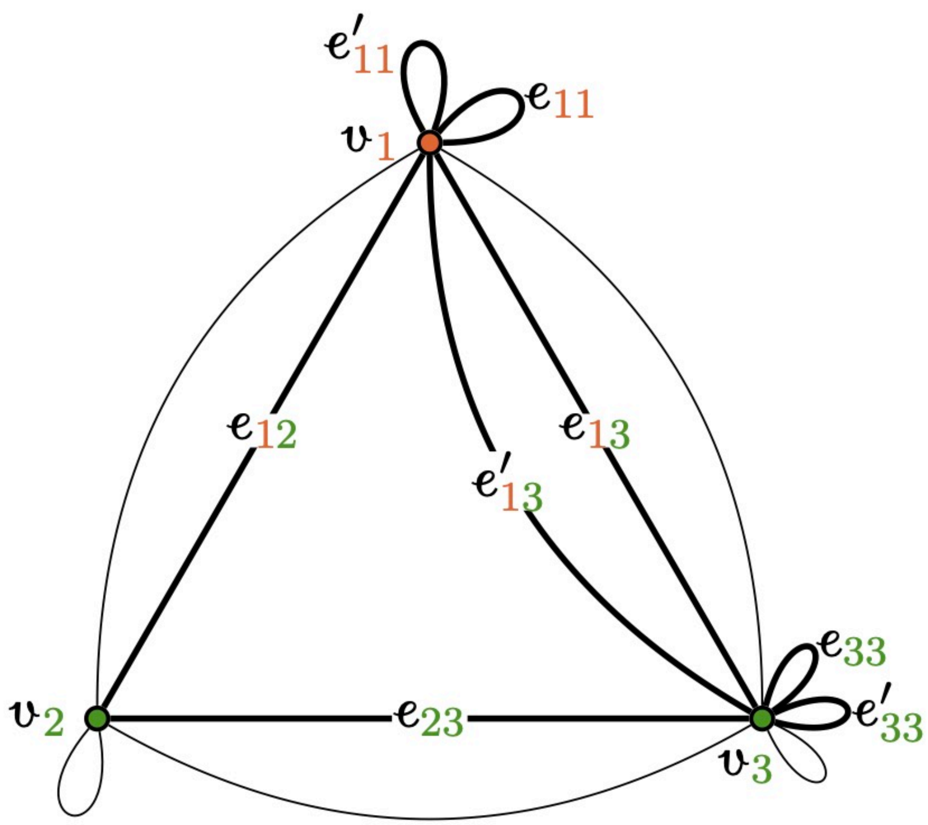

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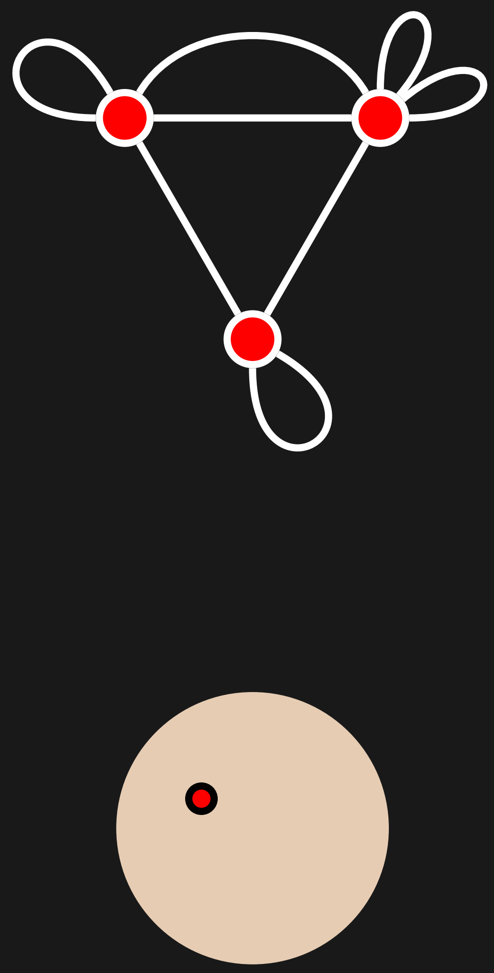
Clique construction: branch & prune



[Slide by G. Loges]

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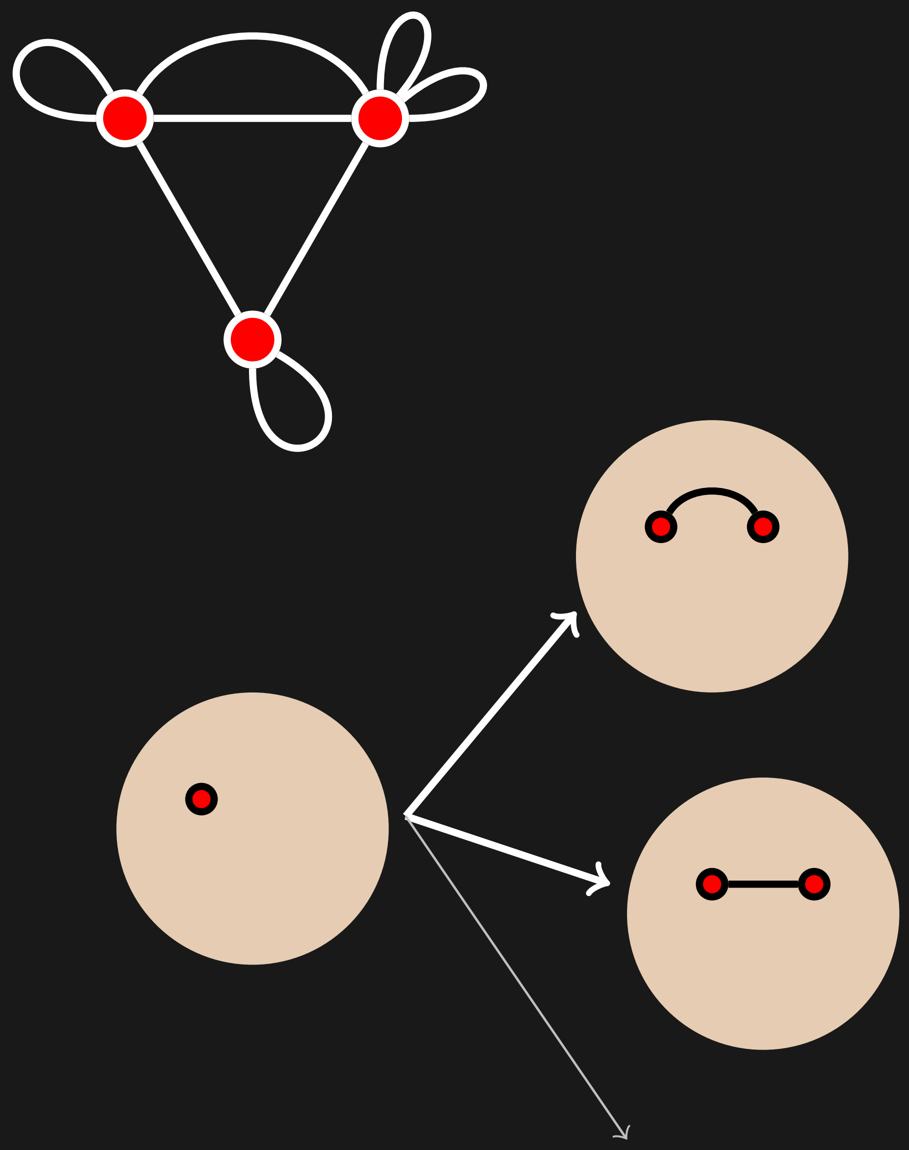

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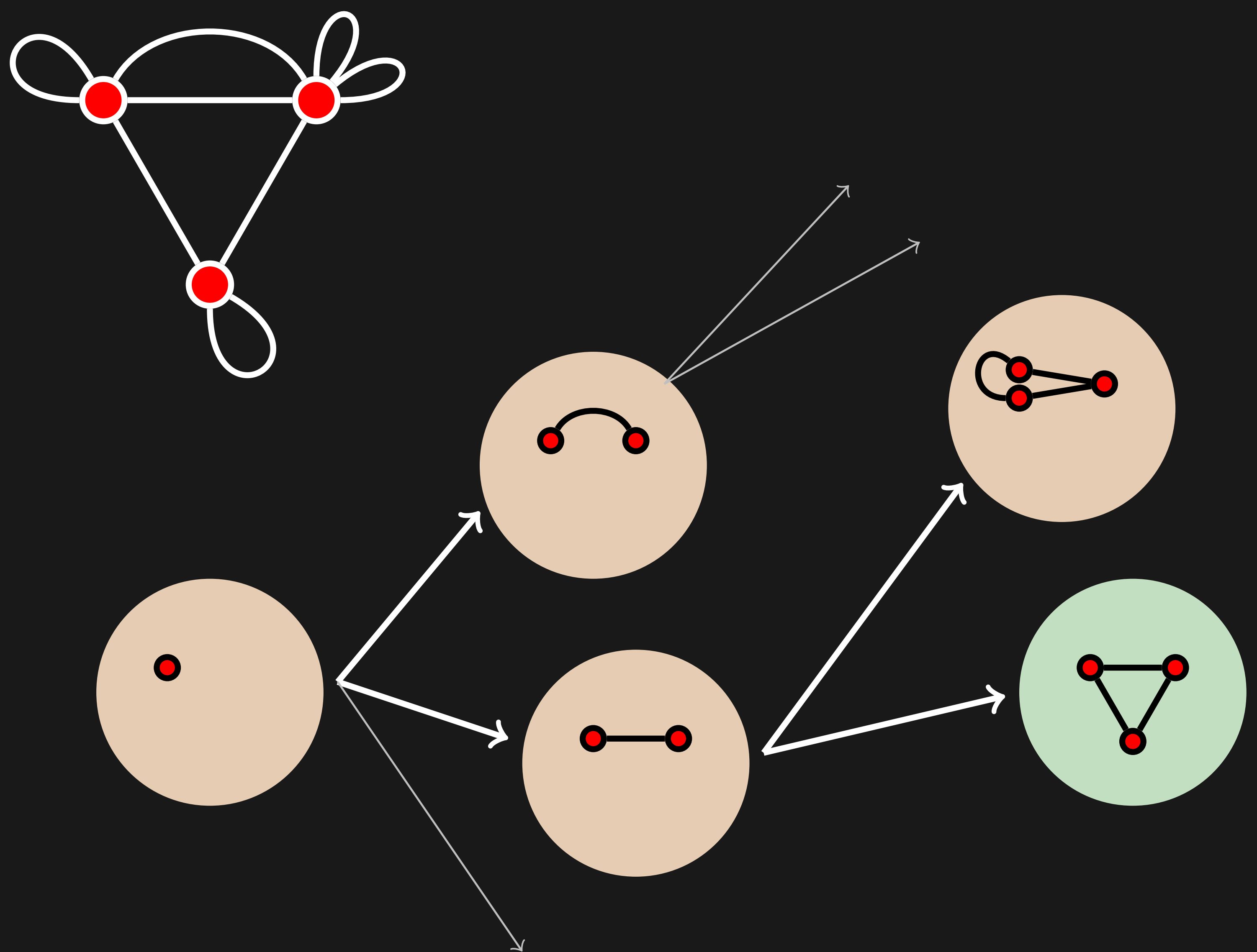

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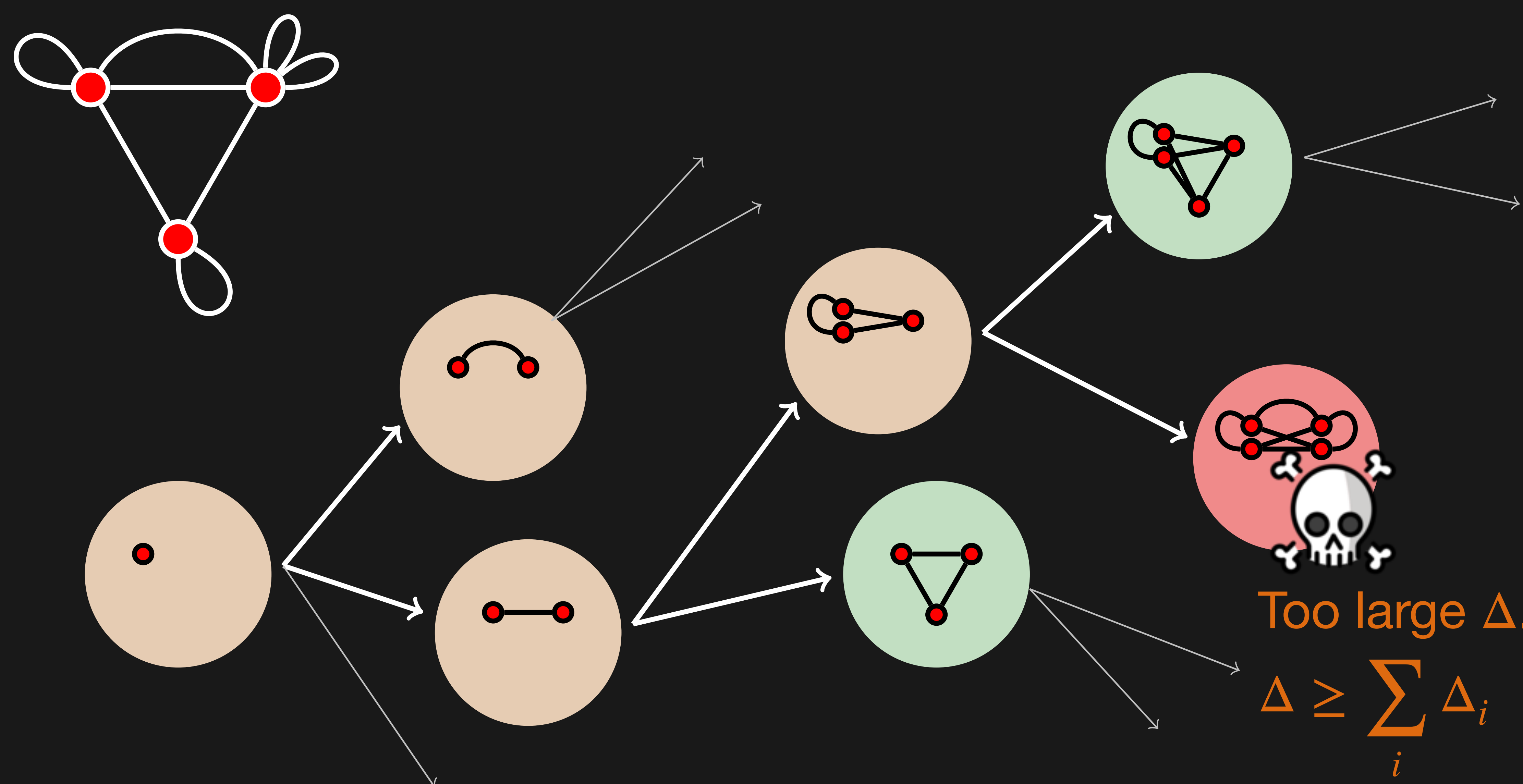

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Results

$T = 0$ results:

All $G = \prod_{i=1}^{k} G_i$ composed of $SU(N \geq 4)$, $SO(N \geq 7)$, $Sp(N \geq 2)$, $E_{6,7,8}$, F_4, G_2.

Any T results:

G_i is $A_{4\to 24}$, $B_{3\to 16}$, $C_{3\to 16}$, $D_{4\to 16}$, $E_{6,7,8}$, F_4, G_2.

Eight choices of hypers for gauge groups E_n, F_4 are removed.

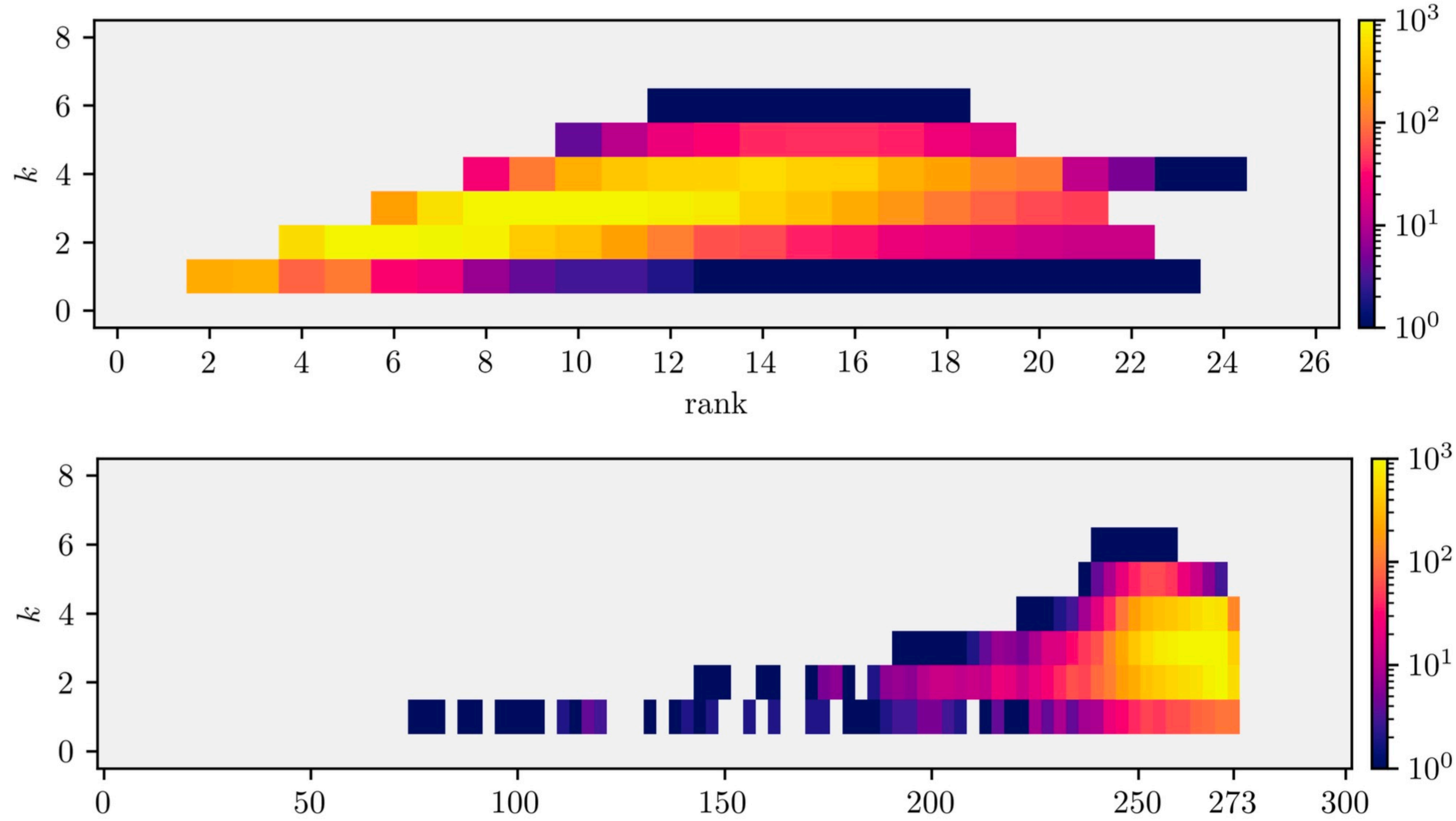
Generates an ensemble of $\mathcal{O}(10^7)$ theories.

In the following, I will show $T = 0$ results [Talk by Gregory for general T].

[YH-Loges '23]

$T = 0$ Results: 19847 theories

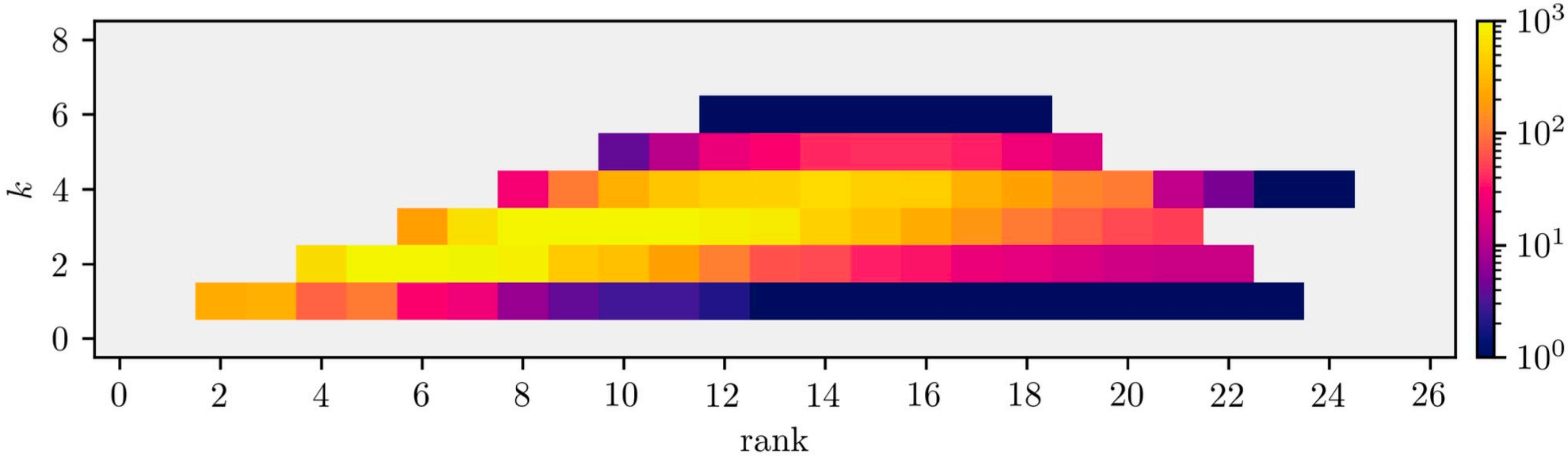
All $G = \prod_{i=1}^{k} G_i$ composed of $SU(N \geq 4)$, $SO(N \geq 7)$, $Sp(N \geq 2)$, $E_{6,7,8}$, F_4, G_2.



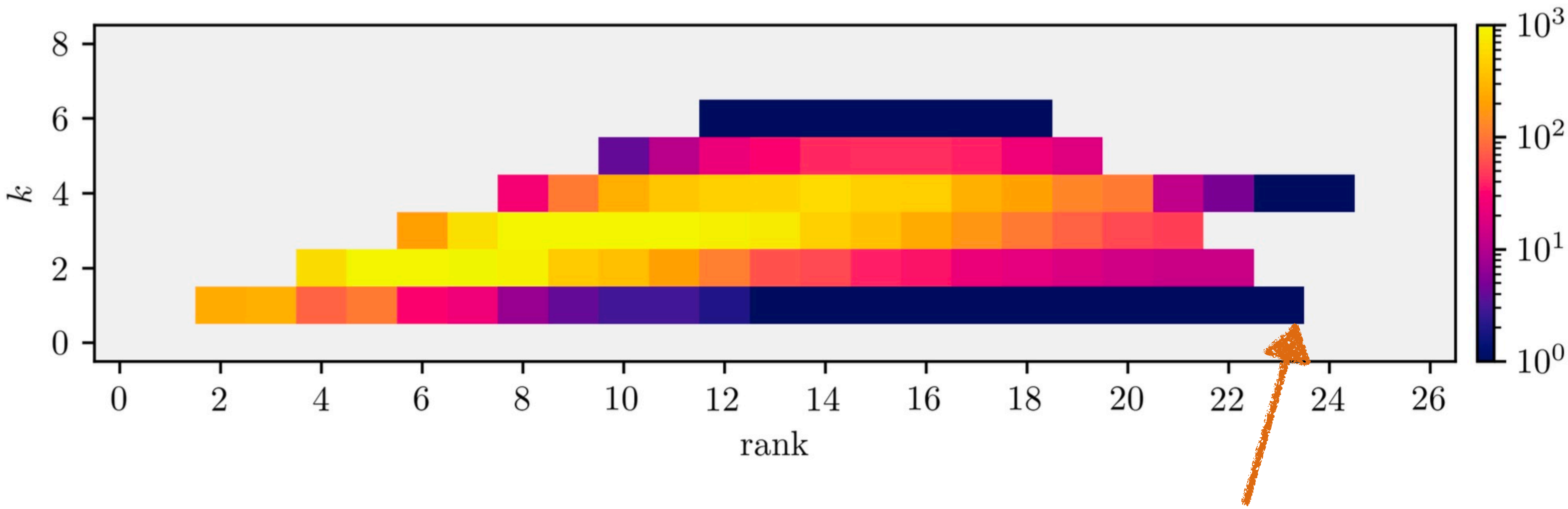
$\Delta = H_{\text{ch}} - V = 273 - H_{\text{neutral}}$

[YH-Loges '23]

$T = 0$ Results

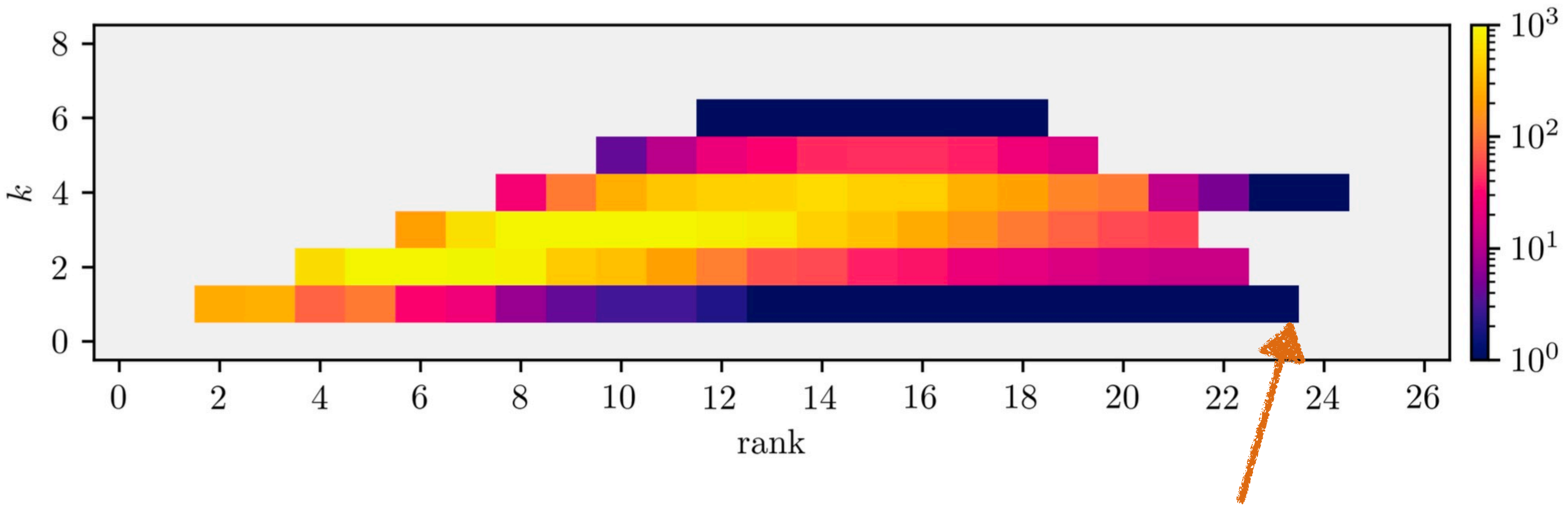


$T = 0$ Results



$G = SU(24),\ H_{\text{ch}} = 3(\mathbf{276}).$

$T = 0$ Results



$G = SU(24),\ H_{\text{ch}} = 3(\mathbf{276}).$

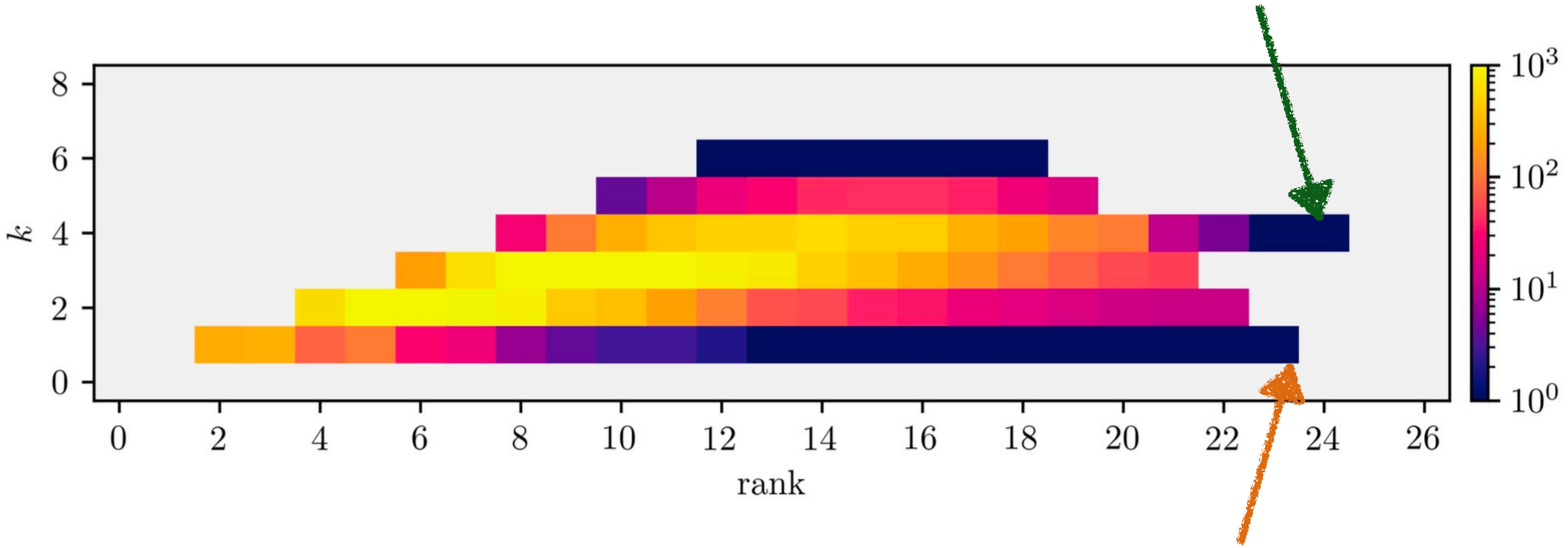
cf.

$G = SO(18),\ H_{\text{ch}} = (\mathbf{170}) + (\mathbf{256}).$

$G = Sp(12),\ H_{\text{ch}} = 2(\mathbf{275}).$

$T = 0$ Results

Four copies of $[G = SU(7),\ H_{\rm ch} = 22(\mathbf{7}) + (\mathbf{35})]$.



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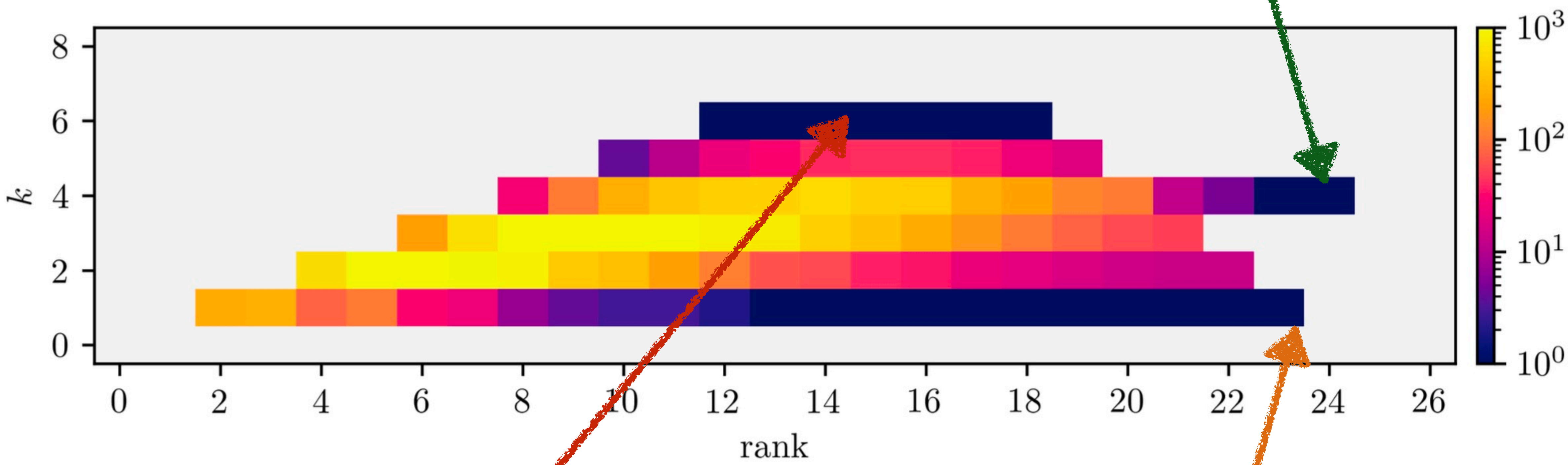


r copies of [$G = SU(4),\ H_{\text{ch}} = 20(\mathbf{4}) + 3(\mathbf{6})$],

$+\ 6 - r$ copies of

[$G = Sp(2),\ H_{\text{ch}} = 20(\mathbf{4}) + 2(\mathbf{5})$].

$r = 0,1,\cdots,6.$

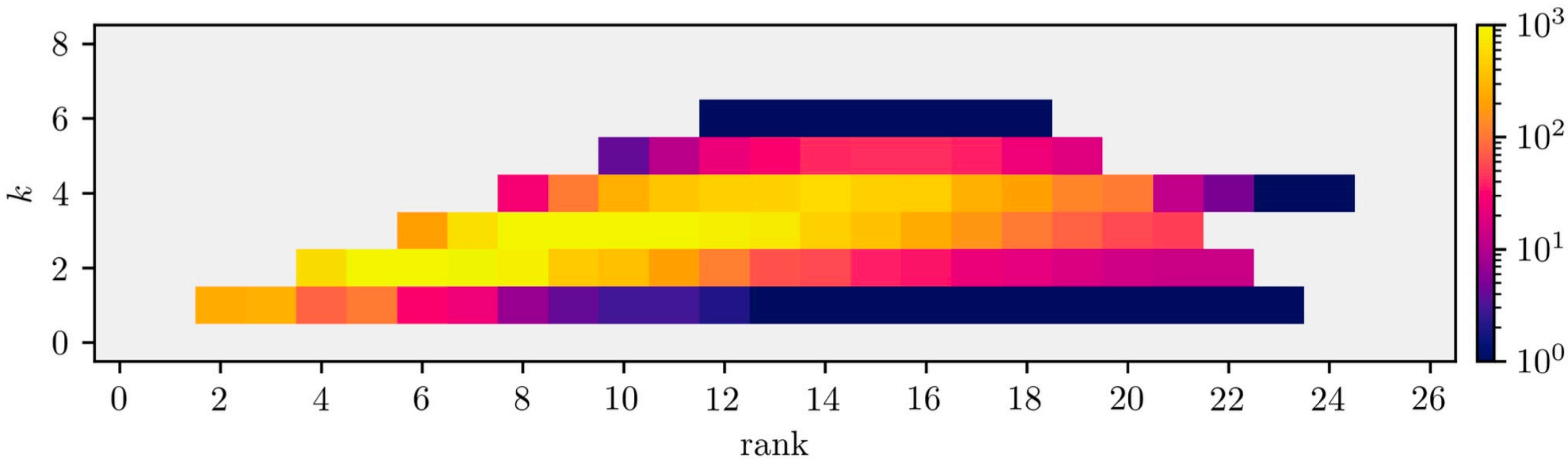
$G = SU(24),\ H_{\text{ch}} = 3(\mathbf{276}).$

cf.

$G = SO(18),\ H_{\text{ch}} = (\mathbf{170}) + (\mathbf{256}).$

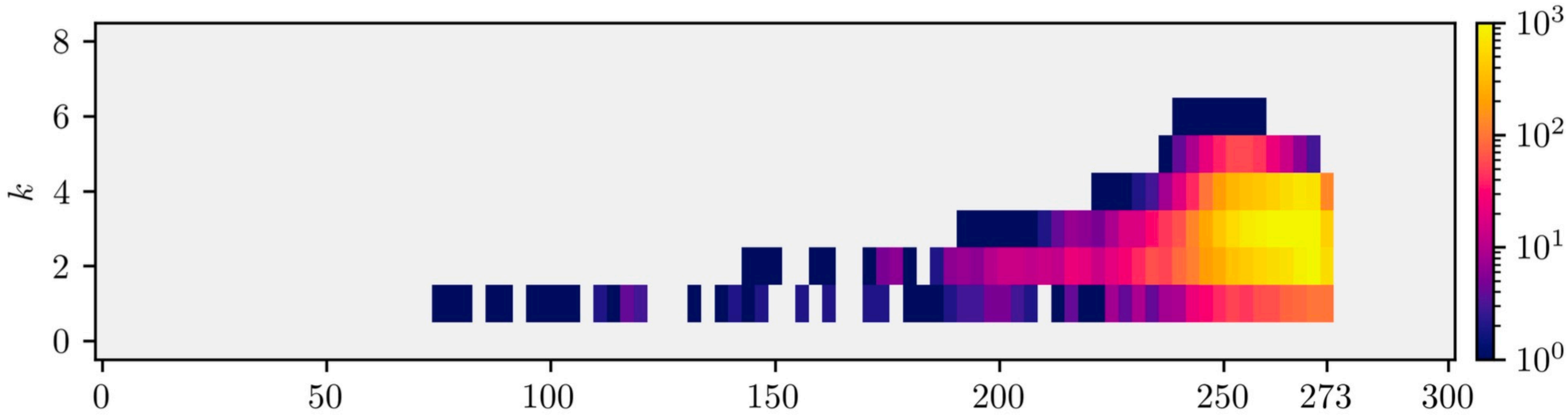
$G = Sp(12),\ H_{\text{ch}} = 2(\mathbf{275}).$

$T = 0$ Results



#(simple gauge factors)	1	2	3	4	5	6	7+
#(anomaly-free theories)	783	6130	8644	4004	279	7	0

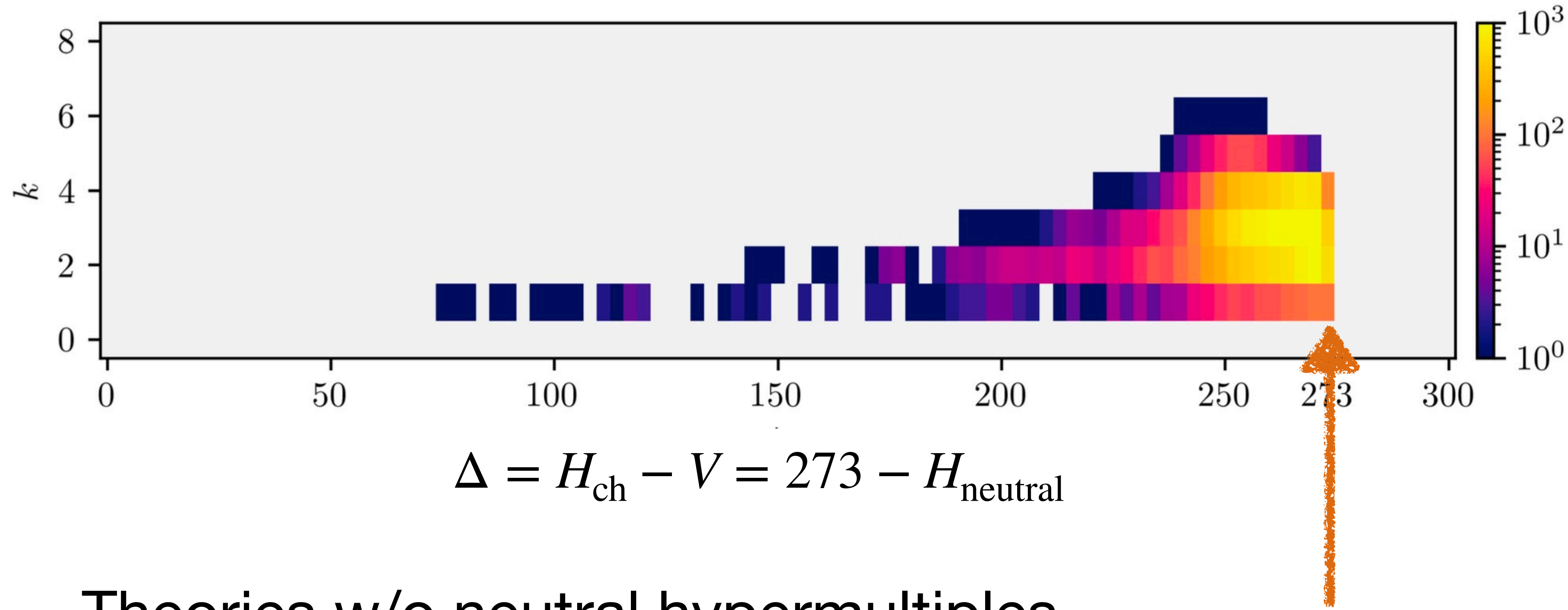
$T = 0$ Results



$\Delta = H_{\text{ch}} - V = 273 - H_{\text{neutral}}$

[YH-Loges '23]

$T = 0$ Results



$\Delta = H_{\text{ch}} - V = 273 - H_{\text{neutral}}$

Theories w/o neutral hypermultiples.

Cannot be realized in geometric phase of F-theory.

e.g.

- $G = SO(18),\ H_{\text{ch}} = (\mathbf{170}) + (\mathbf{256})$.
- $G = Sp(2),\ H_{\text{ch}} = (\mathbf{10}) + (\mathbf{35}) + (\mathbf{84}) + (\mathbf{154})$.
- $G = SU(7) \times E_6, H_{\text{ch}} = (\mathbf{210}, \mathbf{1}) + (\mathbf{7}, \mathbf{27})$.

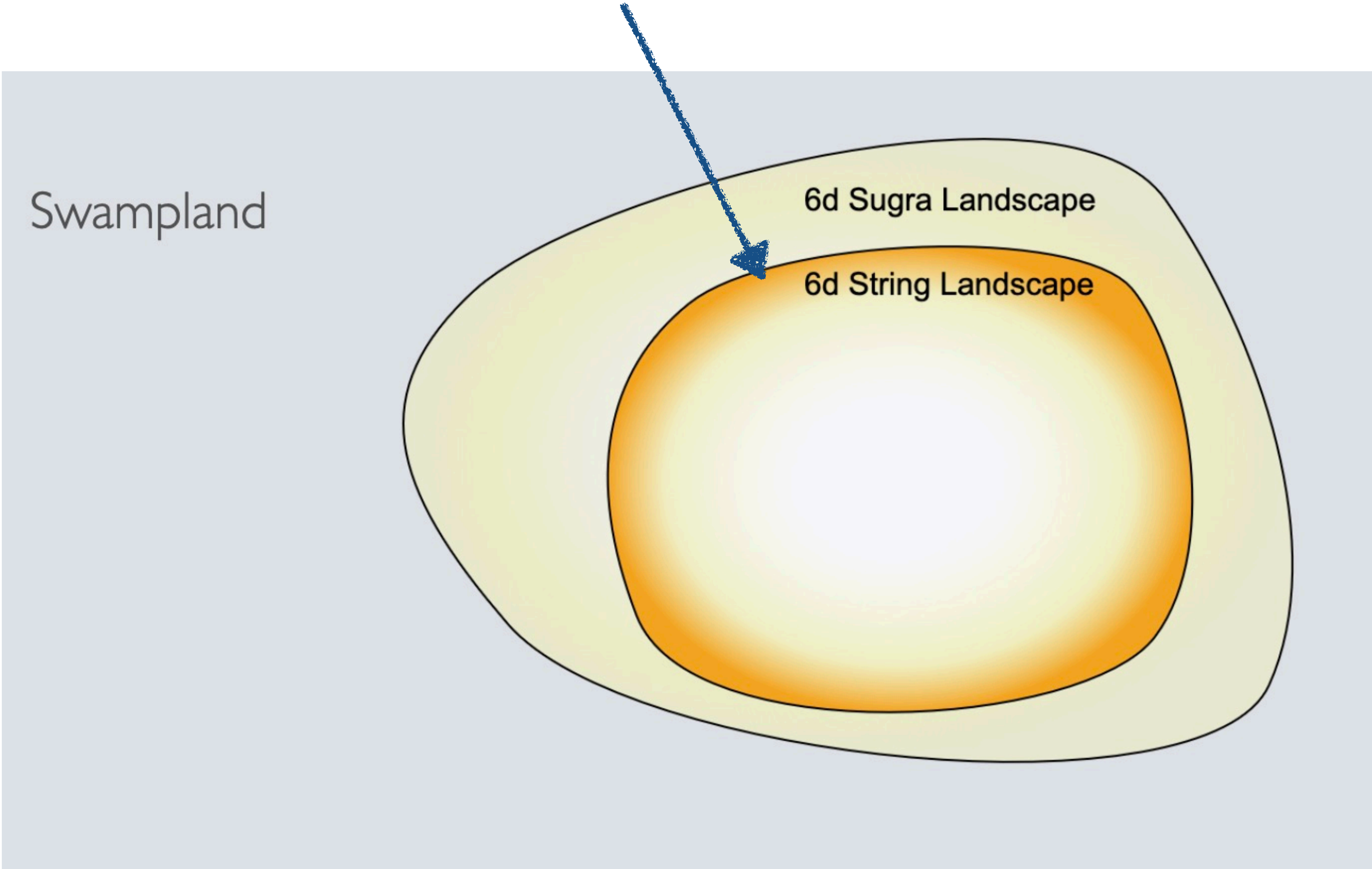
Talk Plan

1. Review of 6d supergravity
2. Toward a classification of 6d supergravity

[YH-Loges '23]

3. Extending Landscape [YH-Baykara-Tarazi-Vafa '23]

Do we understand the boundary here?



[Vafa ‘96, Morrison-Vafa '96, Morrison-Vafa '96]

F-theory vacua

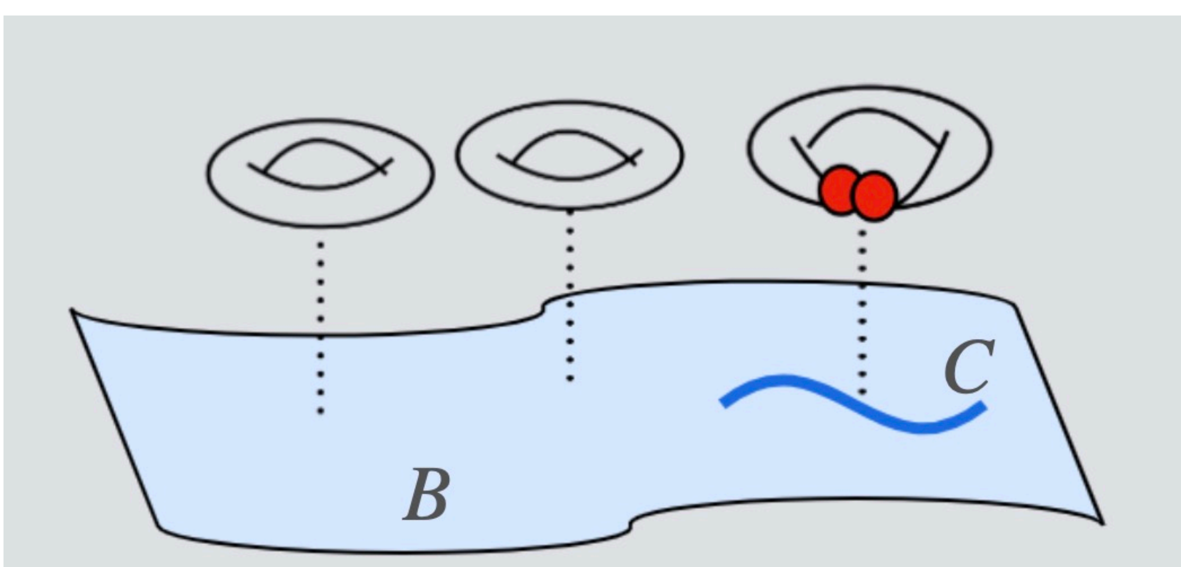
A large class of 6d $\mathcal{N} = (1{,}0)$ vacua is obtained from F-theory compactified on the elliptic CY_3. Two universal features that are not obvious from EFT.

1: Kodaira condition:

$$-j \cdot \left(-12 b_0 + \sum_i \nu_i b_i \right) \geq 0.$$

$b_0 = -a$ in literatures. ν_i: group dependent number.

$b_0, b_i \in \mathbb{R}^{1,T}$ are determined from anomaly cancellation.



2: Universal hyper.

At least one neutral hyper corresponds to $\mathrm{Vol}(B)$.

1: Kodaira condition:

$$-j \cdot \left(-12 b_0 + \sum_i \nu_i b_i \right) \geq 0.$$

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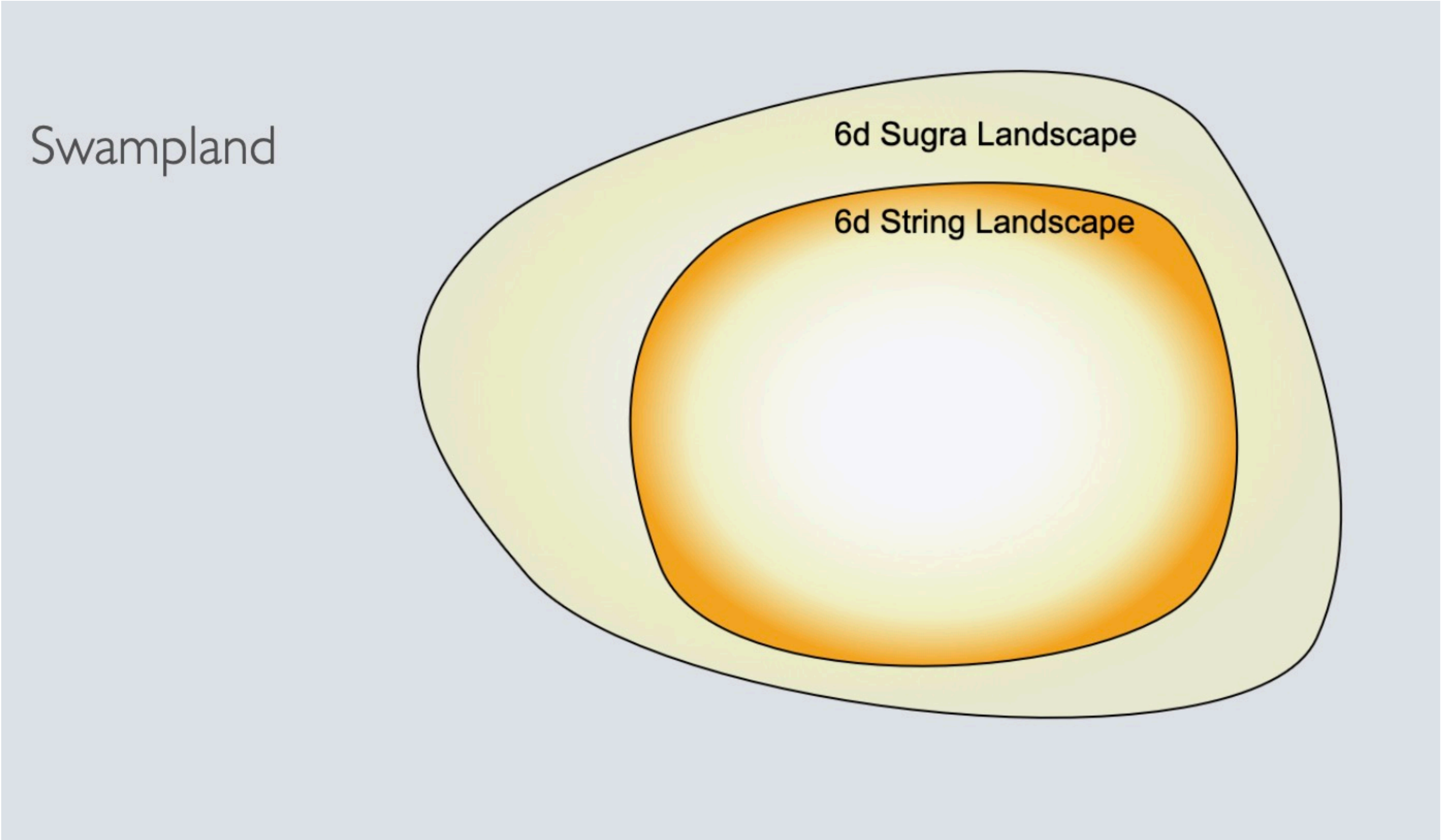
Are these Swampland conditions for 6d supergravity?

(See e.g. [Taylor ‘11] for speculation of Kodaira condition as Swampland condition).



Answer: No

Non-geometric construction leads to the violation of features of geometric F-theory [YH-Baykara-Tarazi-Vafa '23].

[Slide by H. Tarazi]

Abelian Orbifolds

- Choose the starting point: IIA, IIB, Heterotic
- Choose even self-dual lattice: $\Gamma^{D,D}(\mathfrak{g}) = \{(p_L, p_R) \,|\, p_L \in \Lambda_W(\mathfrak{g}), p_R \in \Lambda_W(\mathfrak{g}), p_L - p_R \in \Lambda_R(\mathfrak{g})\}$

$$\Gamma^{D,D}(\mathfrak{g}) + \Gamma^{16,0}(E_8 \times E_8)$$

- Choose the twist: $[g_L, g_R] = [\exp(2\pi i \phi_L), \exp(2\pi i \phi_R)]$
- Choose the shift: (v_L, v_R)

$$\left.\begin{array}{l} [g_L, g_R] = [\exp(2\pi i \phi_L), \exp(2\pi i \phi_R)] \\ (v_L, v_R) \end{array}\right\} \; |p_L, p_R> \,\to\, e^{2\pi i (p_L \cdot v_L - p_R \cdot v_R)} \,|\, g_L \cdot p_L, g_R \cdot p_R >$$

$R = L$	$R \neq L$
Symmetric Orbifolds	**Asymmetric Orbifolds**
[Dixon, Harvey, Vafa, Witten 85'/86']	[Narain, Sarmadi, Vafa 87']

Fermion picture: Free fermionic construction [Kawai-Lewellen-Tye ‘87]

$E_8 \times E_8$ Heterotic on $T^4/\mathbb{Z}_2$

Heterotic string on T^4. The special point in the Narain moduli space, the Lie algebra lattice is realized as a momentum lattice:

$\Gamma^{20,4} = 2\Gamma^{8,0}(E_8) + \Gamma^{4,4}(D_4)$.

$\Gamma^{4,4}(D_4) = \{(p_L, p_R) \,|\, p_L \in \Lambda_W(D_4), p_R \in \Lambda_W(D_4), p_L - p_R \in \Lambda_R(D_4)\}$.

$(n_1, \cdots, n_4)$, or $\left(n_1 + \frac{1}{2}, \cdots, n_4 + \frac{1}{2}\right)$.

$(n_1, \cdots, n_4)$, $\sum n_i \in 2\mathbb{Z}$

$\mathbb{Z}_2$ twist: $p_L \to p_L, \quad p_R \to -p_R$.

Shift vector in $E_8 \times E_8$: $V_L = \frac{1}{2}(1,1,0,0,0,0,0,0; 1,1,0,0,0,0,0,0)$.

[YH-Baykara-Tarazi-Vafa '23]

Spectrum

From $E_8 \times E_8$ heterotic asymmetric orbifold on $T^4/\mathbb{Z}_2$, we obtain the spectrum

$$G + T + 300V + 544H.$$

The gauge group is

$$E_7 \times SU(2) \times E_7 \times SU(2) \times SO(8).$$

All hypers are charged:

$$2(\mathbf{56}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}) + 2(\mathbf{1}, \mathbf{1}, \mathbf{56}, \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{8}_V) + (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{8}_S) + (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{8}_C).$$

No neutral hypers.

Kodaira

Anomaly cancellation condition is ($G = E_7 \times SU(2) \times E_7' \times SU(2)' \times SO(8)$)

$$\begin{pmatrix} b_0 \cdot b_0 & b_0 \cdot b_{E_7} & b_0 \cdot b_{SU(2)} & b_0 \cdot b_{E_7'} & b_0 \cdot b_{SU(2)'} & b_0 \cdot b_{SO(8)} \\ b_0 \cdot b_{E_7} & b_{E_7} \cdot b_{E_7} & b_{E_7} \cdot b_{SU(2)} & b_{E_7} \cdot b_{E_7'} & b_{E_7} \cdot b_{SU(2)'} & b_{E_7} \cdot b_{SO(8)} \\ b_0 \cdot b_{SU(2)} & b_{E_7} \cdot b_{SU(2)} & b_{SU(2)} \cdot b_{SU(2)} & b_{SU(2)} \cdot b_{E_7'} & b_{SU(2)} \cdot b_{SU(2)'} & b_{SU(2)} \cdot b_{SO(8)} \\ b_0 \cdot b_{E_7'} & b_{E_7} \cdot b_{E_7'} & b_{SU(2)} \cdot b_{E_7'} & b_{E_7'} \cdot b_{E_7'} & b_{E_7'} \cdot b_{SU(2)'} & b_3 \cdot b_{SO(8)} \\ b_0 \cdot b_{SU(2)'} & b_{E_7} \cdot b_{SU(2)'} & b_{SU(2)} \cdot b_{SU(2)'} & b_{E_7'} \cdot b_{SU(2)'} & b_{SU(2)'} \cdot b_{SU(2)'} & b_{SU(2)'} \cdot b_{SO(8)} \\ b_0 \cdot b_{SO(8)} & b_1 \cdot b_{SO(8)} & b_{SU(2)} \cdot b_{SO(8)} & b_{E_7'} \cdot b_{SO(8)} & b_{SU(2)'} \cdot b_{SO(8)} & b_{SO(8)} \cdot b_{SO(8)} \end{pmatrix} = \begin{pmatrix} 8 & 2 & 26 & 2 & 26 & 2 \\ 2 & 0 & 12 & 0 & 12 & 0 \\ 26 & 12 & 24 & 12 & 24 & 12 \\ 2 & 0 & 12 & 0 & 12 & 0 \\ 26 & 12 & 24 & 12 & 24 & 12 \\ 2 & 0 & 12 & 0 & 12 & 0 \end{pmatrix}.$$

Kodaira condition is

$$j \cdot (12b_0 - 9b_{E_7} - 9b_{E_7'} - 6b_{SO(8)} - 2b_{SU(2)} - 2b_{SU(2)'}) > 0$$

No solution of $j \in \mathbb{R}^{1,T=1}$ w/ $j^2 = 1, j \cdot b_i > 0$.

Violation of Kodaira condition!

6d $\mathcal{N} = (1{,}0)$ supergravity theories without neutral hypers/Kodaira condition are in the Landscape.

Type II on $T^4/\mathbb{Z}_2$

Take IIA/B on T^4. The special point in the Narain moduli space, the Lie algebra lattice is realized:

$$\Gamma^{4,4}(D_4) = \{(p_L, p_R)\,|\,p_L \in \Lambda_W(D_4), p_R \in \Lambda_W(D_4), p_L - p_R \in \Lambda_R(D_4)\}.$$

$\mathbb{Z}_2$ twist: Left $p_L \to -p_L$, Right $(-1)^{F_R}$.

The spectrum is

$$G + 9T + 12V + 24H.$$

All vectors are $U(1)$ gauge bosons, and charges of hyper are

$$\left(\underline{\pm 1,0,0,0},0^8\right)+\left(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},0^8\right)+\left(\underline{\pm\frac{1}{2},\mp\frac{1}{2},\mp\frac{1}{2},\mp\frac{1}{2}},0^8\right)+\left(\underline{-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}},0^8\right).$$

Without neutral H_0	**Model** 3	**Model** 4
Type	Heterotic	Heterotic
Lattice	$\Gamma^{4,4}(A_2 \oplus A_2) + 2\Gamma^{8,0}(E_8)$	$\Gamma^{4,4}(D_4) + 2\Gamma^{8,0}(E_8)$
Twist	$\phi_L = (0,0)$, $\phi_R = (\frac{2}{3}, \frac{2}{3})$	$\phi_L = (0,0)$, $E_8 \leftrightarrow E_8$, $\phi_R = (\frac{1}{2}, \frac{1}{2})$
Shift	$V_L = \frac{1}{3}(1^6, 0^2; 0^8)$	No shift
Gauge	$E_6 \times SU(3) \times E_8 \times SU(3)^2$	$E_8 \times SO(8)$
Representation	$2(\mathbf{27}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{27}, \mathbf{1}, \mathbf{1}, \underline{\mathbf{3}, \mathbf{1}})$ $+(\mathbf{27}, \mathbf{1}, \mathbf{1}, \underline{\overline{\mathbf{3}}, \mathbf{1}}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{3})$ $+(\mathbf{1}, \mathbf{3}, \mathbf{1}, \underline{\mathbf{3}, \overline{\mathbf{3}}}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}, \overline{\mathbf{3}}, \overline{\mathbf{3}})$	$2(\mathbf{248}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}_v)$ $(\mathbf{1}, \mathbf{8}_s) + (\mathbf{1}, \mathbf{8}_c)$
Spectrum	$G + T + 350V + 594H_c$	$G + T + 276V + 520H_c$
Kodaira Condition	No	No

[YH-Baykara-Tarazi-Vafa '23]

Theories w/o hypers

The construction of the theory so far can be used to construct 5d/4d theories w/o hypers.

5d rank 2, 4, 6, 8, 12, 14, 20, 22 theories w/o hypers.
4d rank 3, 5, 7, 9, 13, 15, 21, 23 theories w/o hypers.

A few number of moduli field.
Good model for moduli stabilization?

Summary

- For $T = 0$, complete classification only omitting $U(1)$, $SU(2)$, and $SU(3)$ simple factors, 19847 theories, $\mathrm{rank}G \leq 24$.
- For any T, complete classification after dropping eight problematic vertices and also omitting $SU(4)$ and $Sp(2)$ to keep computationally tractable.
- Non-geometric compactifications provide models without universal hypers and violating Kodaira condition.

Open questions

- Identifications of models that do not have F-theory nor non-geometric descriptions.
- Bottom-up argument for the theories in the Swampland.
- Relation between F-theory and non-geometric models.