Magnetic operators in 2D compact scalar field theories on the lattice

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Magnetic operators and Witten effect

They are called monopole or ’t Hooft line

Special phenomena “Witten effect” [Witten, 1979]

➡ In the presence of θ term, monopole gain electric charge

$$\langle M(C) \rangle_{\theta+2\pi} = \langle M(C) W(C) \rangle_{\theta}\,, \quad (q, m) \xrightarrow{2\pi} (q+m, m)$$

Witten effect relates to various physics.

- Mixed ’t Hooft anomaly [Gaiotto, et al, 2017], [Honda, Tanizaki, 2020], …
- Generalized Dirac quantization [Aharony, Seiberg, Tachikawa, 2013], [Hsieh, Yonekura, Tachikawa, 2019], …
- Axion physics [Sikivie, 1984], [Yokokura, 2020], …

etc.

Motivation and Topological charge

We would like to provide fully regularized framework for monopole physics with θ term. → Lattice

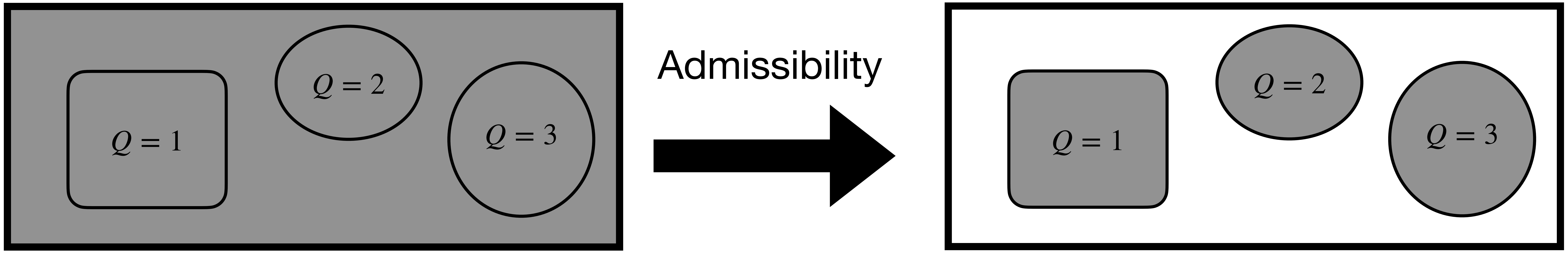
↓

Topological charge on lattice ?

Naively, no smooth deformation on discretized system ???

Answer : Admissibility condition e.g. $U_p \approx 1$ [Lüscher, 1982]

Topological charge is only defined on the "admissible" configuration.

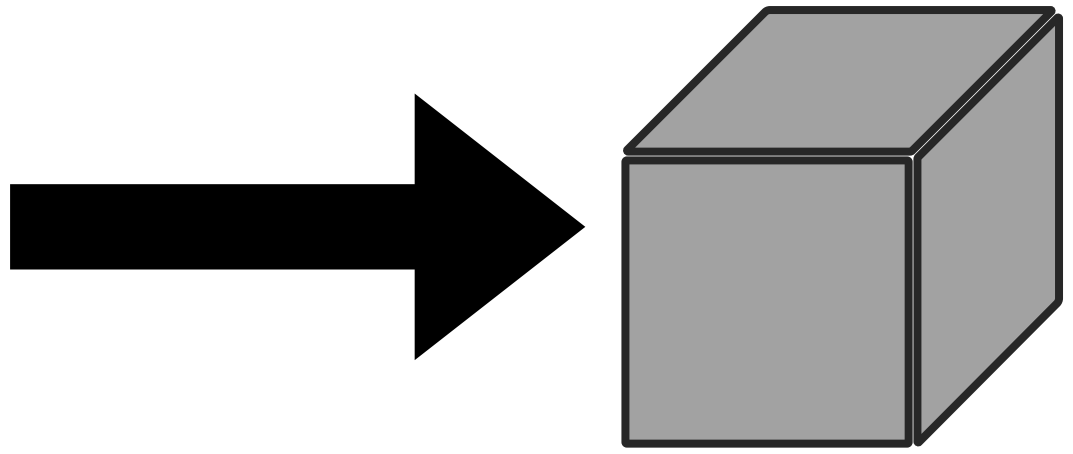


Admissibility and Magnetic operator

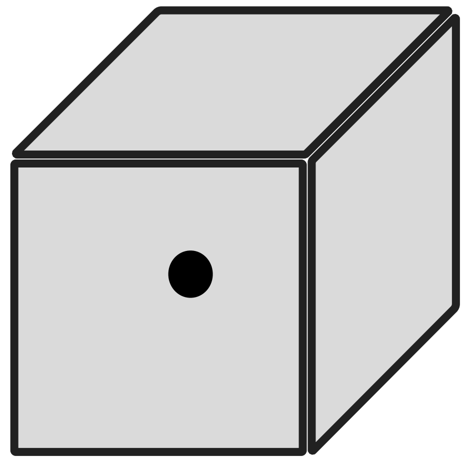
Roughly, Admissibility = Configurations must be “sufficiently smooth”

Difficulty : “Monopole” configurations are NOT admissible

The “sufficiently smoothness” prohibit “discontinuity” like monopole.



= 0 (Bianchi identity)



Monopole

$\neq 0$ (Not admissible)

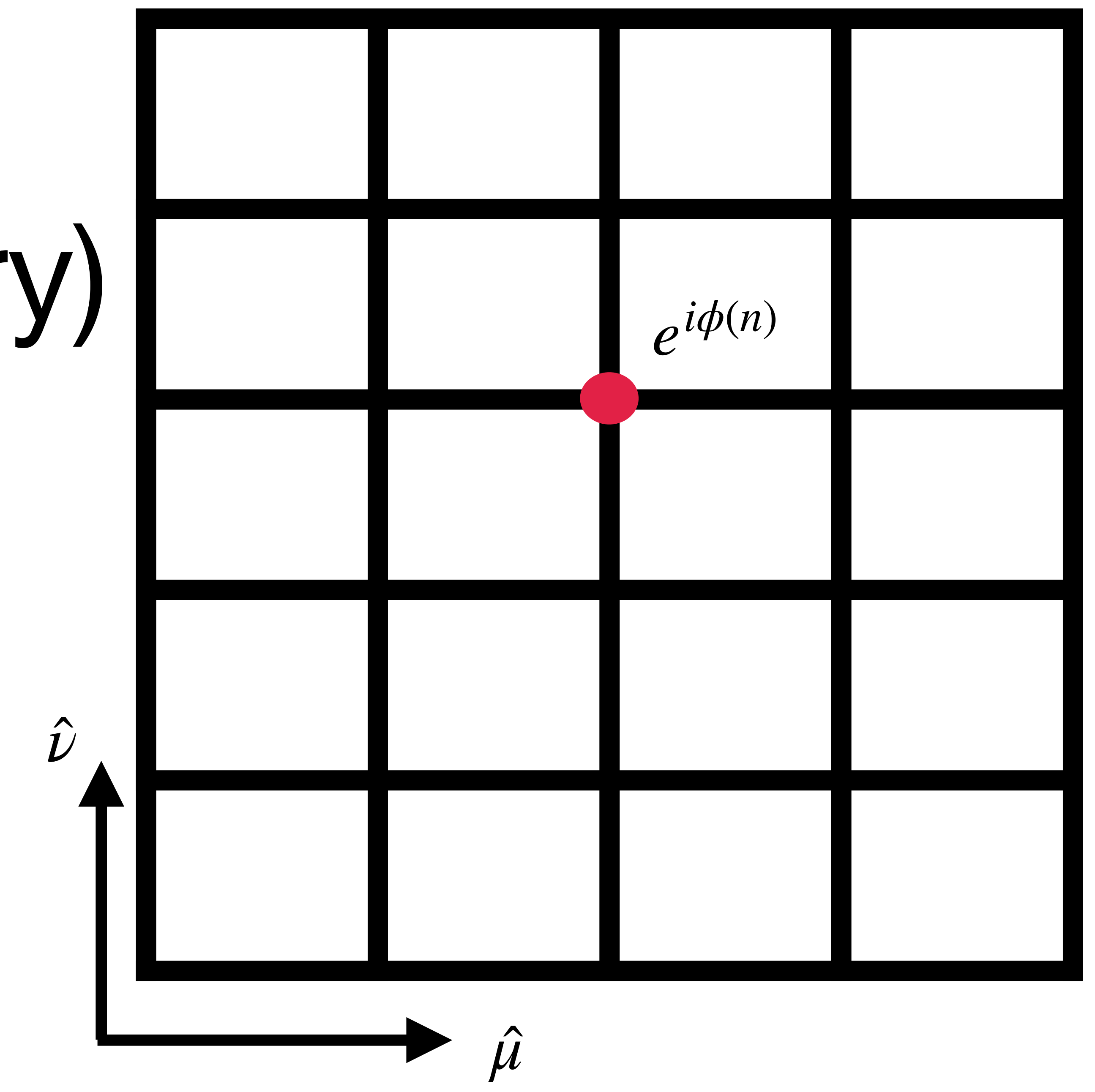
How construct we magnetic operators on lattice consistent with admissibility condition ?

Compact scalar case in 2 dimension

[2304.14815]

2D compact scalar on lattice

Compact scalar : $\phi(n) \sim \phi(n) + 2\pi$ (Gauge symmetry)



➡ Fundamental DoF : $e^{i\phi(n)} \in U(1)$

Directional (gauge inv) derivative

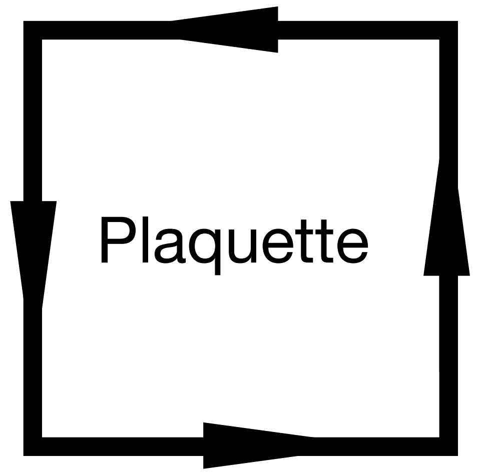
$$\partial\phi(n,\mu) \equiv \frac{1}{i}\ln[e^{-i\phi(n)}e^{i\phi(n+\hat{\mu})}], \ -\pi < \partial\phi(n,\mu) \leq \pi\,.$$

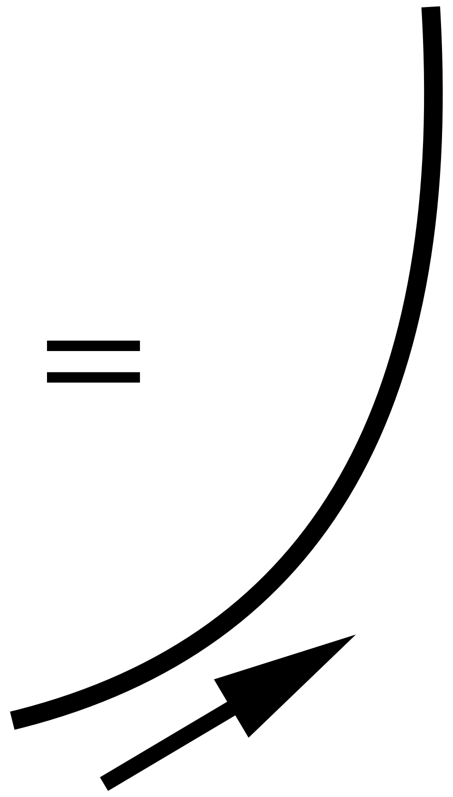
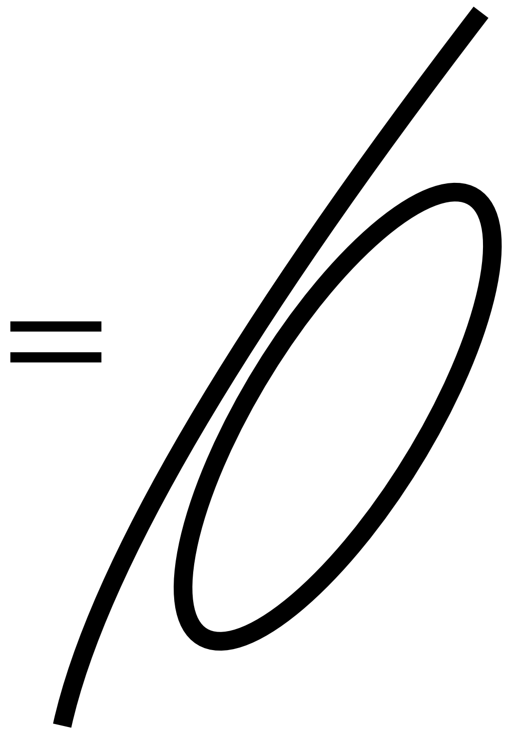
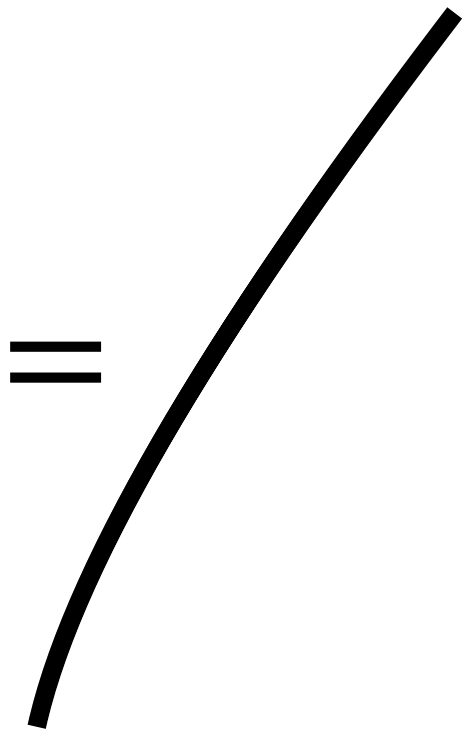
$$\partial\phi(n,\mu) = \underbrace{\phi(n+\hat{\mu}) - \phi(n)}_{\equiv \Delta_\mu\phi(n)} + 2\pi\ell_\mu(n), \quad \ell_\mu(n) \in \mathbb{Z}$$

Admissibility Condition and Magnetic sym

$$\boxed{\sup_{n,\mu} |\,\partial\phi(n,\mu)\,| < \epsilon, \quad 0 < \epsilon < \frac{\pi}{2}} \qquad \left|\sum_{(n,\mu)\in p} \ell_\mu(n)\right| = \frac{1}{2\pi}\left|\sum_{(n,\mu)\in p} \partial\phi(n,\mu)\right| < \frac{1}{2\pi}\times 4\epsilon < 1$$

$$\Rightarrow \sum_{(n,\mu)\in p} \partial\phi(n,\mu) = \sum_{\mu,\nu} \varepsilon_{\mu\nu}\Delta_\mu \partial\phi(n,\nu) = 0 \ \text{(Bianchi identity)}$$

$\Rightarrow$ 

$= 0 \Rightarrow \sum_{(n,\mu)\in \text{line}} \partial\phi(n,\mu) =$  $=$  $=$ 

$$= \sum_{(n,\mu)\in \text{line}'} \partial\phi(n,\mu)$$

Admissibility makes the line integral(sum) topological.

$\Rightarrow$ Magnetic symmetry on lattice

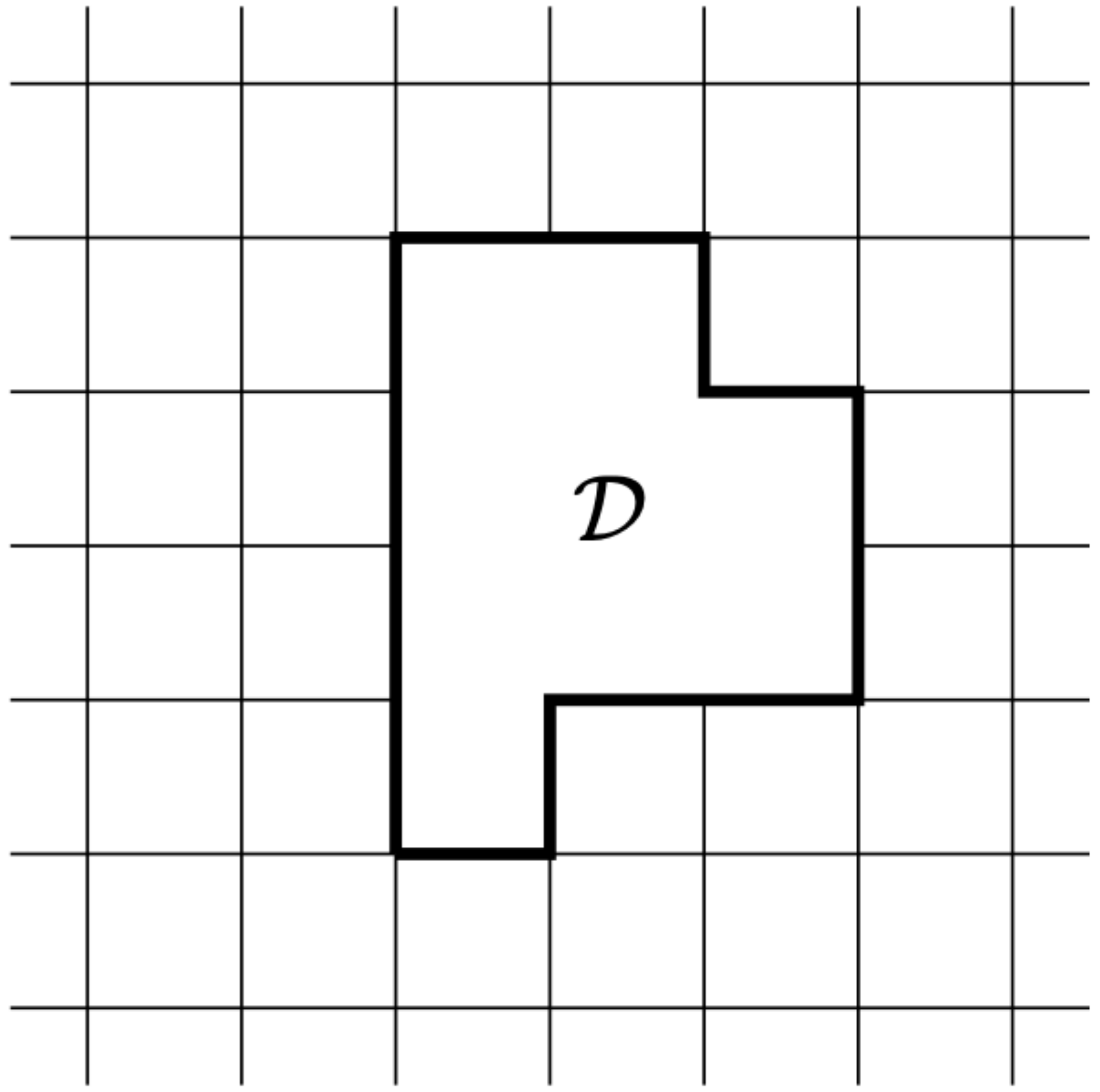
Monopole

Monopole is charged object corresponded to magnetic symmetry.

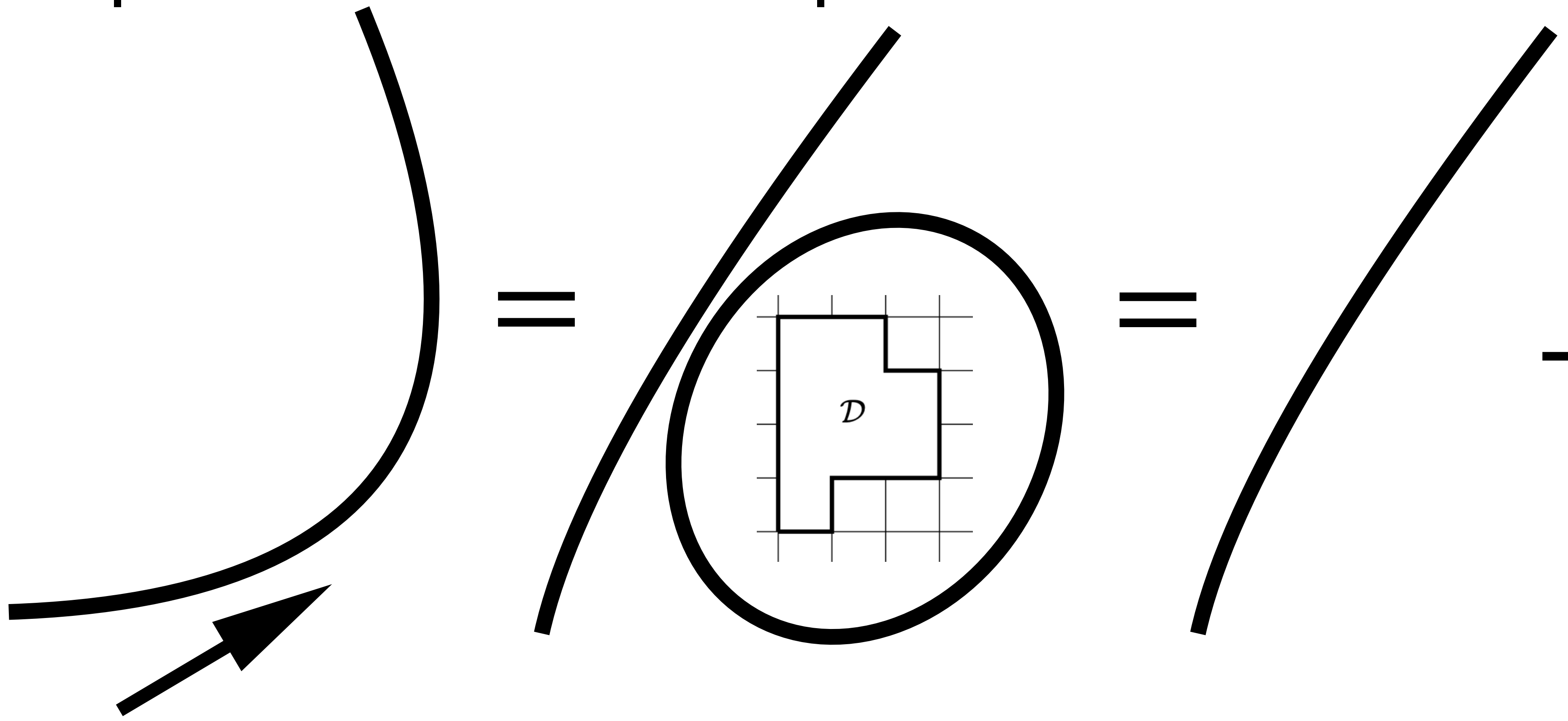
= Monopole is obstruction of topo deformation of the line integral.

However, admissibility prohibit the obstruction ???

Our answer : Excision method (monopole ~ boundary)



$$\frac{1}{2\pi}\left|\sum_{(n,\mu)\in\partial D}\partial\phi(n,\mu)\right| < \frac{1}{2\pi}\times(\partial D)\epsilon \qquad \text{If } 4 < \frac{2\pi}{\epsilon} \lessapprox (\partial D), \text{ then } \neq 0$$



$+2\pi m \;\; , m \in \mathbb{Z}$

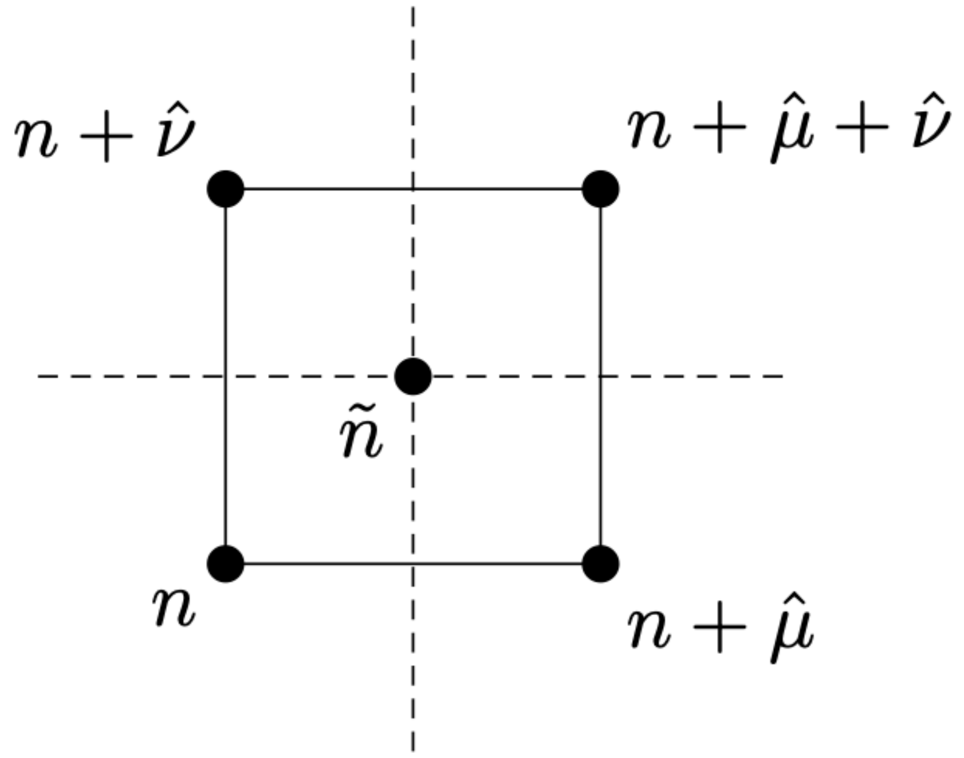
(Dirac quantization)

Witten effect

We consider two compact scalar case

$\phi_1(n)$ (original lattice), $\phi_2(\tilde{n})$ (dual lattice)

θ term : $S_\theta \equiv \dfrac{i\theta}{4\pi^2}\sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu}\partial\phi_2(\tilde{n},\mu)\partial\phi_1(n+\hat{\mu},\nu)$



If the system has no boundary, then

$S_\theta = i\theta \sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu}\ell_2(\tilde{n},\mu)\ell_1(n+\hat{\mu},\nu)$ (using Bianchi identity)

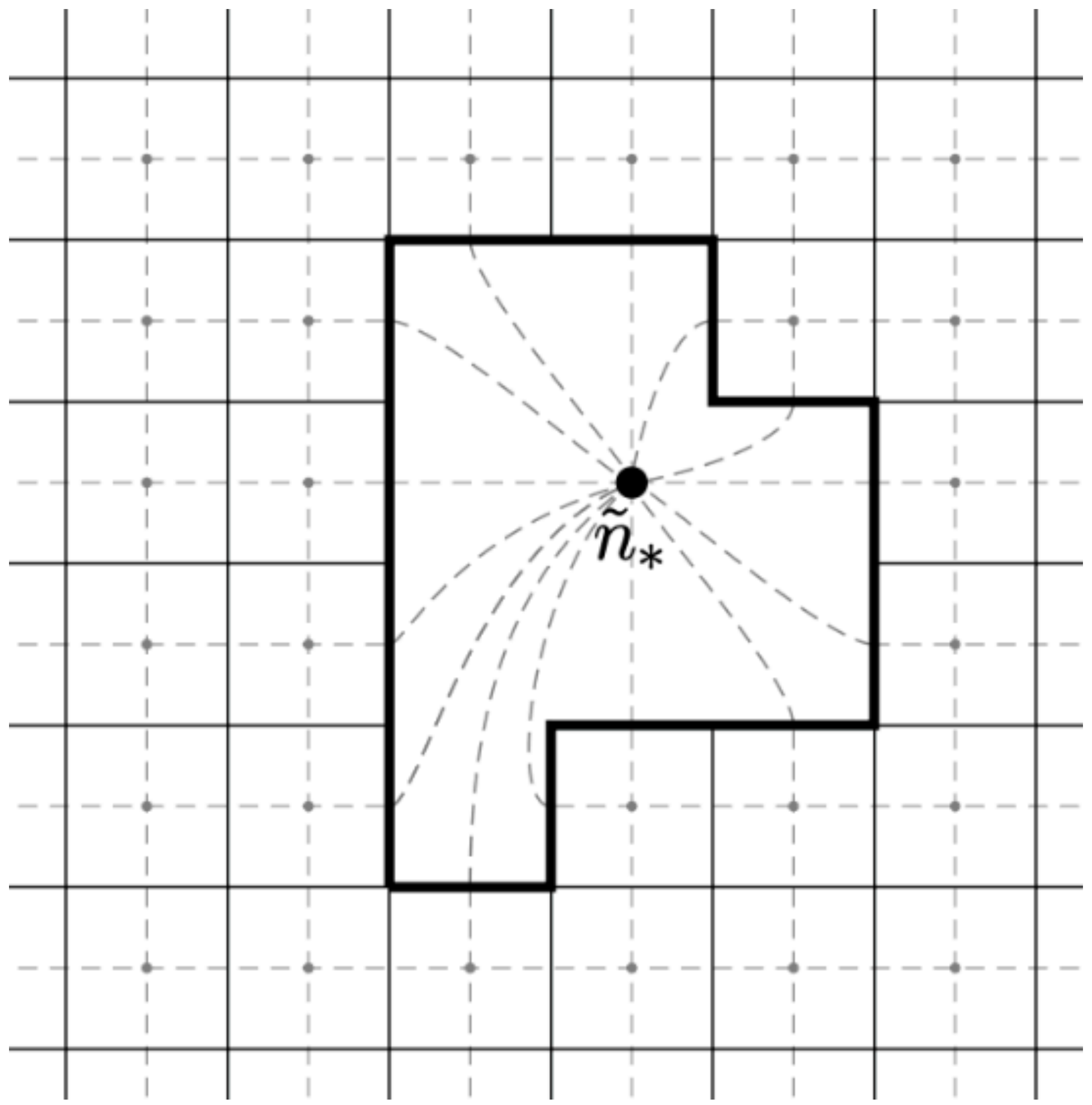
Topological charge become an integer. (due to admissibility)

Next, we put on a monopole . (by using excision method)

$$S_\theta = \frac{i\theta}{4\pi^2} \sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu} \partial\phi_2(\tilde{n},\mu)\partial\phi_1(n+\hat{\mu},\nu)$$

$$= i\theta(-\phi_2(\tilde{n}_*)\frac{1}{2\pi} \sum_{(n,\mu)\in\partial\mathscr{D}} \ell_{1,\mu}(n) + \mathbb{Z})$$

$$= i\theta(-\frac{m}{2\pi}\phi_2(\tilde{n}_*) + \mathbb{Z})$$



Key point : Total derivative term contribute to the boundary.

So, $\left\langle M_1(\tilde{n}_*)\right\rangle_{\theta+2\pi} = \left\langle M_1(\tilde{n}_*)e^{im\phi_2(\tilde{n}_*)}\right\rangle_\theta$

Witten effect on lattice

’t Hooft anomaly

We consider the lattice action : $S \equiv \frac{R^2}{2\pi} \sum_n \sum_\mu \left\{ 1 - \cos[\partial\phi(n,\mu)] \right\}$

There are global sym : $U(1)_{(e)} \times U(1)_{(m)}$ $e^{iq\phi(n)} \mapsto e^{iq\phi(n)} e^{iq\lambda_e}$ $M(n) \mapsto M(n)\, e^{im\lambda_m}$

Background gauging : $\partial\phi(n,\mu) \to D\phi(n,\mu) \equiv \frac{1}{i} \ln e^{i\phi(n+\mu)} U^{(e)}(n,\mu) e^{-i\phi(n)}$

$$S \to S + \frac{i}{2\pi} \sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu} A_\mu^{(m)}(\tilde{n}) D\phi(n+\hat{\mu},\nu)$$

B.g. gauge fields : $U^{(e/m)}(n/\tilde{n},\mu) = e^{iA_\mu^{(e/m)}}$

Admissibility condition : $|D\phi(n,\mu)| < \epsilon$, $|F_{\mu\nu}^{(e/m)}(n/\tilde{n})| < \delta$

$(0 < \delta < \min(\pi, 2\pi - 4\epsilon))$

➡ impose $F_{\mu\nu}^{(e)}(n) = \Delta_\mu D\phi(n,\nu) - \Delta_\nu D\phi(n,\mu)$

Then, we can derive mixed 't Hooft anomaly.

Under electric and magnetic gauge transformation,

$$Z[A_\mu^{(e/m)}] \to Z[A_\mu^{(e/m)}] \exp\left\{ \frac{i}{2\pi} \sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu} \left[\frac{1}{2} \Lambda^{(m)}(\tilde{n}) F_{\mu\nu}^{(e)}(n) - 2\pi L_\mu^{(m)}(\tilde{n}) A_\nu^{(e)}(n+\hat{\mu}) \right] \right\}$$

In the presence of monopoles, the shift has additional factor $e^{im\Lambda^{(m)}(\tilde{n}_*)}$.

In the case of two compact scalar with θ term,
there is a 't Hooft anomaly related to Witten effect like shift.

$$\theta \to \theta + 2\pi,\ F_{\mu\nu}^{(m,1)}(\tilde{n}) \to F_{\mu\nu}^{(m,1)}(\tilde{n}) - F_{\mu\nu}^{(e,2)}(\tilde{n}),\ F_{\mu\nu}^{(m,2)}(n) \to F_{\mu\nu}^{(m,2)}(n) + F_{\mu\nu}^{(e,1)}(n)$$

$$Z[A_\mu^{(e/m),1}, A_\mu^{(e/m),2}] \to Z[A_\mu^{(e/m),1}, A^{(e/m),2}] \exp\left\{ -\frac{i}{2\pi} \sum_n \sum_{\mu,\nu} \varepsilon_{\mu\nu} A_\mu^{(e,2)}(\tilde{n}) A_\nu^{(e,1)}(n+\hat{\mu}) \right\}$$

Conclusion

How construct we magnetic operators on lattice consistent with admissibility condition ?

・In the case of 2D compact scalar theory, we can define magnetic operator (monopole) by using “Excision method”.

・In this case, Witten effect can be derived on lattice.

The technical key point : Using dual lattice.

・We can derive ’t Hooft anomaly related to electric and magnetic symmetry.

The technical key point : Imposing admissibility condition.

Future Direction

・Extending to 4D gauge theory.
U(1) Maxwell (Work in progress), SU(N) pure Yang-Mills, …
<u>We should construct ’t Hooft line.</u>

・Lattice description of non invertible symmetry. (Work in progress)
non invertibility ~ Projection to specific monopole sector.
(Analogue of ABJ anomaly case) [Choi, Lam, Shao, 2022] [karasik, 2022]

<u>Lattice provide good description ?</u>

・Understanding statistics of line operators in our construction.