Non-invertible symmetries of Maxwell theory on non-spin manifold

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Based on
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[in progress]

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Introduction

☆ There are non-invertible defects in Maxwell theory.

[Kaidi-Ohmori-Zheng '21]

[Choi-Cordova-Hsin-Lam-Shao '21]

◎ Two operations are important in the construction:

- $\mathrm{SL}(2,\mathbb{Z})$ duality
- Gauging 1-form symmetries

Motivation

☆ What happens in the theory on non-spin manifold?

- $\mathrm{SL}(2,\mathbb{Z})$ action is different.
- The behavior under gauging 1-form symmetries has not been specified.

➡ What non-invertible defects can we construct?

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1-form symmetries

[Gaiotto-Kapustin-Seiberg-Willett '14]

☆ Maxwell theory on spin manifold:

$$S = \int \frac{1}{2e^2} f \wedge * f + \frac{\mathrm{i}\theta}{8\pi^2} f \wedge f$$

a : $\mathrm{U}(1)$ gauge field

$f = \mathrm{d}a$

e : coupling constant

θ : theta angle

☆ 1-form symmetries

◎ $\mathrm{U}(1)_e$ electric symmetry

- Current: $*j_e = -\frac{\mathrm{i}}{e^2} * f + \frac{\theta}{2\pi} f \left(= \frac{\mathrm{d}\tilde{a}}{2\pi} \right)$
- Charged object: Wilson loop

$$W_n = \mathrm{e}^{\mathrm{i}n \oint a}$$

◎ $\mathrm{U}(1)_m$ magnetic symmetry

- Current: $*j_m = \frac{f}{2\pi}$
- Charged object: 't Hooft loop

$$H_m = \mathrm{e}^{-\mathrm{i}m \oint \tilde{a}}$$

Duality of Maxwell theory

◎ $\mathrm{SL}(2,\mathbb{Z})$ duality: $\tau \mapsto \dfrac{a\tau+b}{c\tau+d}$

$$\tau = \frac{\theta}{2\pi} + \frac{2\pi \mathrm{i}}{e^2}$$

$$ad - bc = 1 \qquad a, b, c, d \in \mathbb{Z}$$

- The partition function is invariant under this action.

◎ Action on the line operators

- The charges of lines form a lattice by Dirac quantization condition.

$$W_n H_m \;\Longrightarrow\; (n, m) \in \mathbb{Z} \times \mathbb{Z}$$

- $S : \tau \mapsto -\dfrac{1}{\tau}$

  $(n, m) \mapsto (m, -n)$

- $T : \tau \mapsto \tau + 1$

  $(n, m) \mapsto (n - m, m)$

Gauging 1-form symmetries

[Kapustin-Seiberg '14]

- Suppose to gauge a subgroup of 1-form symmetry.

$$(\mathbb{Z}_N)_e \subset \mathrm{U}(1)_e \quad \text{or} \quad (\mathbb{Z}_N)_m \subset \mathrm{U}(1)_m$$

- This gauging procedure is explicitly performed by coupling topological field theory.

- The resulting theory is also Maxwell theory but has a different coupling.

$$(\mathbb{Z}_N)_e : \tau \mapsto \tau/N^2 \qquad (\mathbb{Z}_N)_m : \tau \mapsto N^2\tau$$

Non-invertible symmetries

[Kaidi-Ohmori-Zheng '21]
[Choi-Cordova-Hsin-Lam-Shao '21]

◎ Half gauging

- Divide spacetime into two regions.
- Gauge discrete symmetry of theory $\mathcal{T}$ on one side.

(Impose an appropriate boundary condition.)

- The resulting interface is symmetry defect, if $\mathcal{T}$ and $\mathcal{T}/\mathbb{Z}_N$ are equivalent.

e.g.) Non-invertible defect by gauging $(\mathbb{Z}_N)_e \subset \mathrm{U}(1)_e$.

$$\tau = \mathrm{i}N \xmapsto{(\mathbb{Z}_N)_e} \tau/N^2 \xmapsto{S} -N^2/\tau = \mathrm{i}N$$

➡ There is non-invertible defect at $\tau = \mathrm{i}N$.

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Choice of line operators

[Thorngren '14]...

◎ There are **three** types of anomaly free Maxwell theories on non-spin manifold.

- We try to formulate the theory on oriented **non-spin** manifold.
- To this end, we must choose **the statistics of line defects.**

	$W_b T_b$	$W_b T_f$	$W_f T_b$
Wilson loop	bosonic	bosonic	fermionic
't Hooft loop	bosonic	fermionic	bosonic

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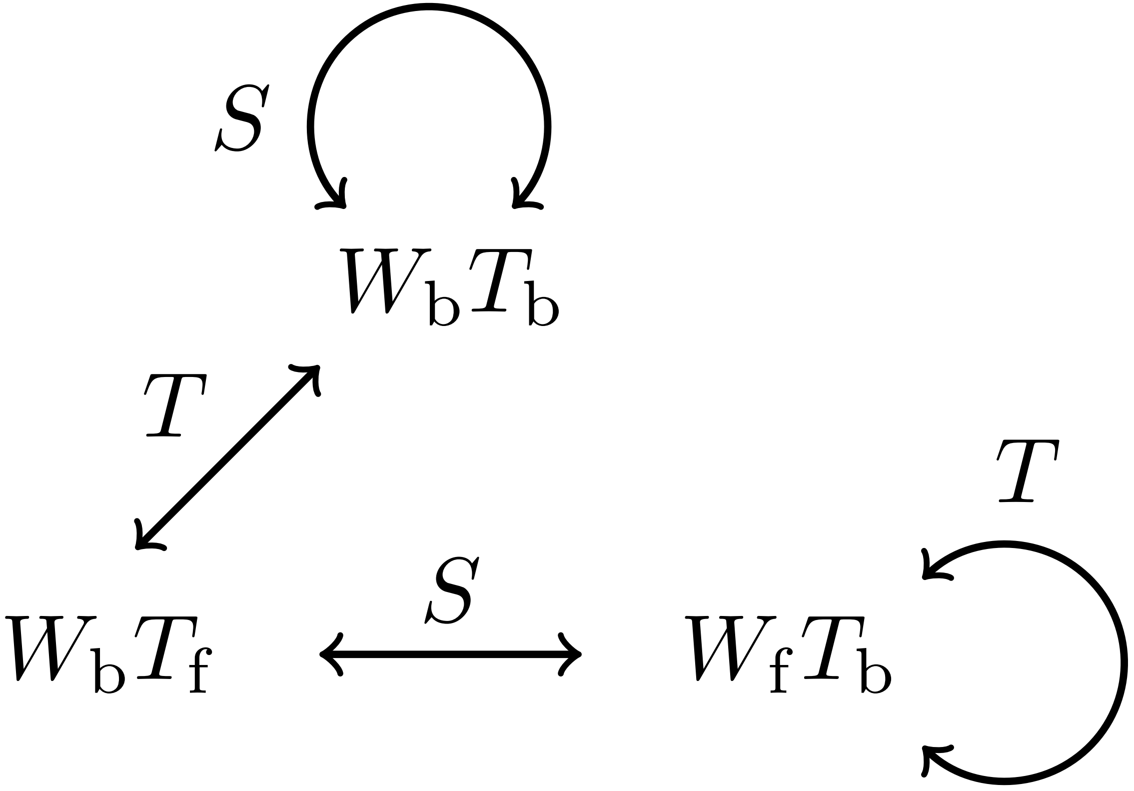
Duality on non-spin manifold

[Metlitski '15]

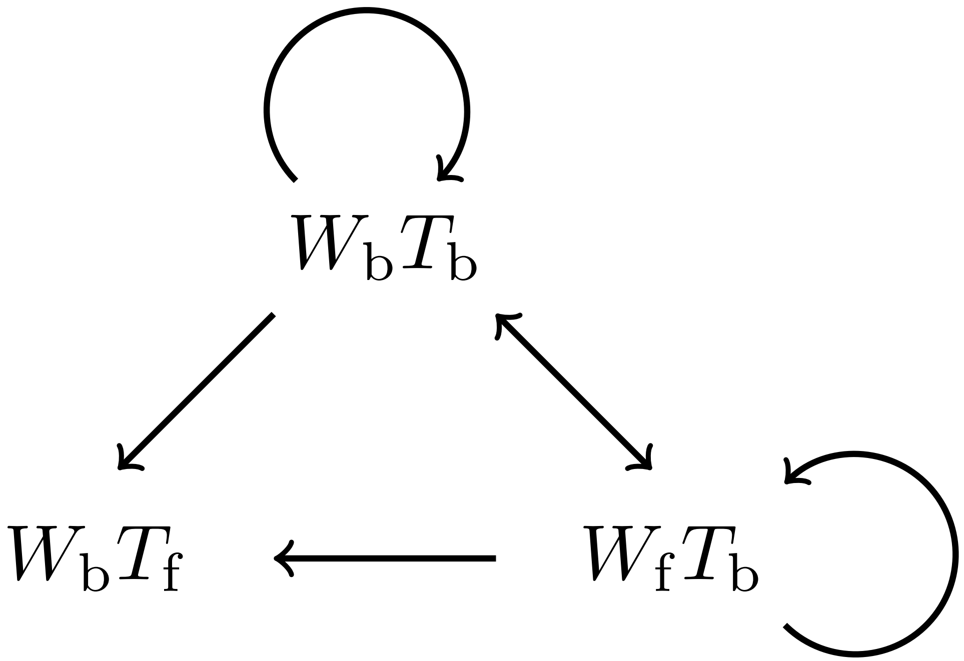
◎ $\mathrm{SL}(2,\mathbb{Z})$ action may map the theory to another one.



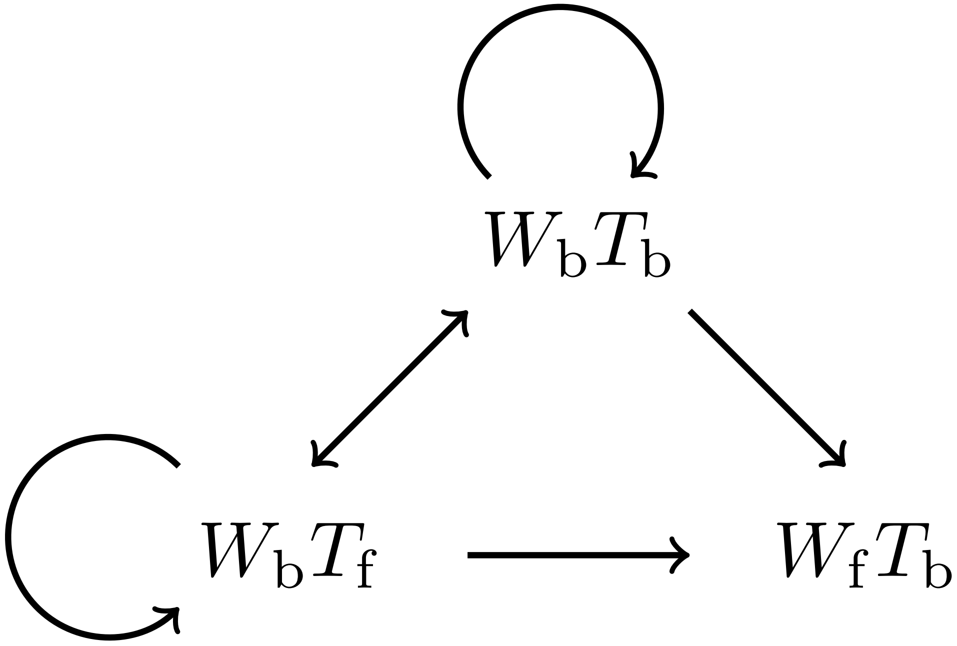
Gauging 1-form symmetries

◎ The theory after gauging depends on coupled TQFT.

- Gauge $(\mathbb{Z}_2)_e \subset \mathrm{U}(1)_e$
- Gauge $(\mathbb{Z}_2)_m \subset \mathrm{U}(1)_m$



$$(\mathbb{Z}_2)_e : \tau \mapsto \tau/2^2$$



$$(\mathbb{Z}_2)_m : \tau \mapsto 2^2\tau$$

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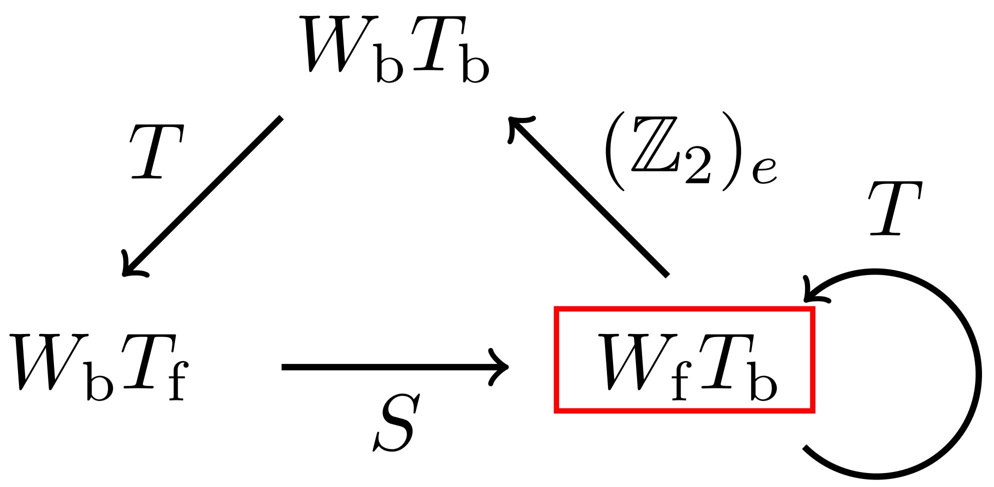
Non-invertible defects on non-spin manifold

☆ Non-invertible defects can be constructed by usual half gauging.

◎ example 1

- There is a defect in $W_f T_b$ at $\tau = \frac{1}{2}(5+\sqrt{7}\mathrm{i})$.

$$\tau \xmapsto{(\mathbb{Z}_2)_e} \tau/2^2 \xmapsto{T} \tau/2^2+1 \xmapsto{S} -\frac{1}{\tau/2^2+1} \xmapsto{T} -\frac{1}{\tau/2^2+1}+1$$



Non-invertible defects on non-spin manifold

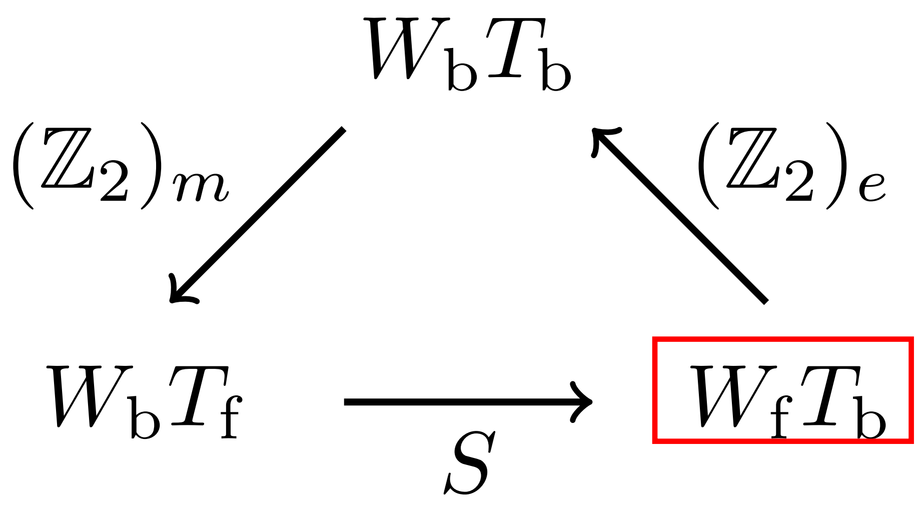
☆ Non-invertible defects can be constructed by usual half gauging.

◎ example 2 (S dual-like defect)

- S trans. does not map $W_f T_b$ to itself.

➡ But we can construct a defect similar to S dual at $\tau = \mathrm{i}$ by composite gauging 1-form symmetries.

$$\tau = \mathrm{i} \overset{(\mathbb{Z}_2)_e}{\longmapsto} \tau/2^2 \overset{(\mathbb{Z}_2)_m}{\longmapsto} \tau \overset{S}{\longmapsto} -\frac{1}{\tau} = \mathrm{i}$$



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Summary

☆ We consider non-invertible defects in Maxwell theory on non-spin manifold.

◎ Key idea: Half gauging [Kaidi-Ohmori-Zheng '21] [Choi-Cordova-Hsin-Lam-Shao '21]

- $\mathrm{SL}(2,\mathbb{Z})$ action and gauging 1-form sym. play important role.
- On non-spin manifold, there are three different Maxwell theories.
- We identify the behavior under the gauging 1-form symmetries.

➡ Non-invertible defects are constructed by using this behavior.

Back up

Generalized symmetry

[Gaiotto-Kapustin-Seiberg-Willet '14]

☆ What is symmetry?

➡ The existence of topological defects.

◎ Ordinary symmetry (0-form symmetry)

・For continuous symmetry, we can define unitary operators from Noether current.

e. g.) $\mathrm{U}(1)$ symmetry
$$U_\alpha(V) = \exp\left(\mathrm{i}\alpha \int_V \star j\right)$$
j : Noether current

・These operators have following properties.

(1) Topological (2) Codimension 1 (3) Group structure

Generalized symmetry

[Gaiotto-Kapustin-Seiberg-Willet '14]

◎ Properties of an ordinary symmetry defects

(1) Topological

We can deform the defects topologically.

$$\langle U_\alpha(V)\ldots\rangle = \langle U_\alpha(V')\ldots\rangle$$

(2) Codimension 1

The symmetry defects are defined on codimension 1 submanifolds.

(3) Group structure

The algebraic structure is characterized by a group.

$$\langle U_\alpha(V)U_\beta(V)\ldots\rangle = \langle U_{\alpha+\beta}(V)\ldots\rangle \longleftrightarrow \mathrm{e}^{\mathrm{i}\alpha}\mathrm{e}^{\mathrm{i}\beta} = \mathrm{e}^{\mathrm{i}(\alpha+\beta)}$$

Generalized symmetry

[Gaiotto-Kapustin-Seiberg-Willet '14]

Which property is most important?

(1) Topological　　(2) Codimension 1　　(3) Group structure

The property (1) is most important.

- Conservation law
- Commutability with Hamiltonian
- Ward-Takahashi identity

➡ We can define generalized symmetries by relaxing the property (2) and/ or (3).

Generalized symmetry

◎ p-form symmetry (higher form symmetry)

Symmetries acting on p-dimensional operators

(1) Topological (2) Codimension (p+1) (3) Group structure

◎ Higher group symmetry

Symmetry consisting of several codimension defects

(1) Topological (2) Several Codimension (3) Higher group

◎ Non-invertible symmetry

Symmetries without inverse.

(1) Topological (2) Codimension (p+1) (3) Fusion category

Action of Maxwell theory on non-spin manifold

◎ $W_b T_b$:

$$S = \int \frac{1}{2e^2} f \wedge * f + \frac{\mathrm{i}\theta}{8\pi^2} f \wedge f$$

a: $\mathrm{U}(1)$ gauge field

$f = \mathrm{d}a$

e : coupling constant

θ: theta angle

w_2 : Stiefil-Whitney class

◎ $W_f T_b$:

$$S = \int \frac{1}{2e^2} (f + \pi w_2) \wedge * (f + \pi w_2) + \frac{\mathrm{i}\theta}{8\pi^2} (f + \pi w_2) \wedge (f + \pi w_2)$$

◎ $W_b T_f$:

$$S = \int \frac{1}{2e^2} f \wedge * f + \frac{\mathrm{i}\theta}{8\pi^2} f \wedge f + \frac{\mathrm{i}}{2} f \wedge w_2$$

Action of BF theory on non-spin manifold

a: $\mathrm{U}(1)$ 1-form gauge field

b: $\mathrm{U}(1)$ 2-form gauge field

w_2 : Stiefel-Whitney class

◎ $L_b S_b$:

$$S = \int \frac{\mathrm{i}}{\pi} b \wedge \mathrm{d}a$$

◎ $L_f S_b$:

$$S = \int \frac{\mathrm{i}}{\pi} b \wedge \mathrm{d}a + \mathrm{i} b \wedge w_2$$

◎ $L_b S_f$:

$$S = \int \frac{\mathrm{i}}{\pi} b \wedge \mathrm{d}a + \frac{\mathrm{i}}{2} w_2 \wedge \mathrm{d}a$$

Gauging 1-form symmetries

◎ The theory after gauging depends on coupled TQFT.

- Gauge $(\mathbb{Z}_2)_e \subset \mathrm{U}(1)_e$
- Gauge $(\mathbb{Z}_2)_m \subset \mathrm{U}(1)_m$