On the classification of 6d supergravities

Gregory J. Loges

[2311.00868] w/ Yuta Hamada

KEK Theory Workshop

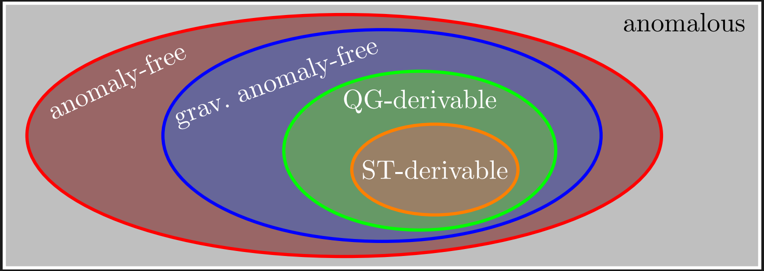
2023 – 11 – 30

The plan

- Motivation
- 6d $\mathcal{N} = (1,0)$ SUGRA & its consistency conditions
- Multigraphs, cliques & classification
- Thoughts

Motivation

- Absence of gauge anomalies is a nontrivial condition required of gauge theories with chiral matter (e.g. SM, MSSM)
- Gravitational anomaly cancellation is especially constraining in dimensions $d = 2, 6, 10$
- String universality: much recent progress in $d \geq 7$ [Kim, Tarazi, Vafa – 19]
  [Kim, Shiu, Vafa – 19] [Montero, Vafa – 21] [Bedroya, Hamada, Montero, Vafa – 22] . . .



Consistency Conditions

6d supergravity

$\mathcal{N} = (1,0)$ supermultiplets:

$$\text{gravity} : (g, \Psi, B^-) \qquad \text{tensor} : (B^+, \chi, \phi) \qquad \text{vector} : (A, \lambda) \qquad \text{hyper} : (\psi, \varphi)$$

Pick $(T, G, \mathcal{H})$:

- $T \geq 0$ (# tensor multiplets)
- Gauge group $G = \prod_{i=1}^{k} G_i$
- Hypermultiplets $\mathcal{H} = \{n \times (R_1, \ldots, R_k)\}$

Anomaly-cancellation

The Green-Schwarz mechanism requires the anomaly polynomial to factorize:

$$I_8 = I_{\text{grav}} - T\, I_{\text{tensor}} + I_{1/2}^{\text{Adj}} - \sum_R I_{1/2}^R \quad \overset{!}{=} \quad X_4 \cdot X_4$$

$$X_4^\alpha = -\frac{1}{2} b_0^\alpha \operatorname{tr} \mathcal{R}^2 + \sum_i 2 b_i^\alpha \operatorname{tr} F_i^2 \,, \qquad X_4, b_0, b_i \in \mathbb{R}^{1,T}$$

Matching terms on either side, there are irreducible terms which must vanish:

$$(H - V + 29T - 273) \operatorname{tr} \mathcal{R}^4 \,, \quad \Big(\textstyle\sum_R n_R^i B_R^i - B_{\text{Adj}}^i \Big) \operatorname{tr} F_i^4$$

The other terms determine all inner products amongst the vectors $b_I = b_0, b_i$, e.g.

$$b_0 \cdot b_0 = 9 - T \,, \quad b_i \cdot b_i = \frac{1}{3} \Big(\textstyle\sum_R n_R^i C_R^i - C_{\text{Adj}}^i \Big) \,, \; \dots$$

Collect into a Gram matrix $\boxed{\mathbb{G}_{IJ} = b_I \cdot b_J}$

$$\operatorname{tr}_R F_i^2 = A_R^i \operatorname{tr} F_i^2$$
$$\operatorname{tr}_R F_i^4 = B_R^i \operatorname{tr} F_i^4 + C_R^i (\operatorname{tr} F_i^2)^2$$

Consistency conditions

- Anomaly-cancellation:

$$\boxed{H_{\text{ch}} - V \leq 273 - 29T} \quad \boxed{\sum_R n_R^i B_R^i = B_{\text{Adj}}^i \text{ for each } G_i} \quad \boxed{n_+(\mathbb{G}) \leq 1\,,\ n_-(\mathbb{G}) \leq T}$$

gravitational bound — "B-constraint"

- Absence of global/Witten anomalies

- Positivity: from SUSY, gauge kinetic terms are $-(j \cdot b_i)\,\mathrm{tr}\,F_i^2$ for some moduli-dependent $j \in \mathbb{R}^{1,T}$. Need $\boxed{\exists\, j \text{ with } j \cdot j > 0\,,\ j \cdot b_i > 0}$

- Unimodularity: $\Lambda = \bigoplus b_I \mathbb{Z}$ is always an integer lattice, but also need $\boxed{\Lambda \hookrightarrow \Gamma_{1,T}}$ with $\Gamma_{1,T}$ unimodular (integer and self-dual)

Classification

Outline

Previously...

- With $T < 9$ ($b_0 \cdot b_0 > 0$) the number of anomaly-free 6d, $\mathcal{N} = (1, 0)$ supergravities is known to be finite [Kumar, Taylor – 09] [Kumar, Morrison, Taylor – 10]
- Classification for $T = 0, 1$ and/or simple gauge groups is well understood [Avramis, Kehagias – 05] [Kumar, Park, Taylor – 10] ...

What's new:

- Classify in a T–agnostic way
- Any hyper representations
- Gauge groups with any number of simple factors

Technical limitations:

- no U(1), SU(2) or SU(3) simple factors
- no (3+)-charged hypers
- eight choices of hypers for gauge groups E_n, F_4 are removed (*more on this later)

Outline

- Gauge/grav/mixed anomalies
- Positivity
- Unimodularity

"Decomposition"
and pair-wise compatibility
of simple theories

<u>Classification</u>
(multi)graphs and cliques
via "branch and prune"

Main idea: “decomposition”

All of the consistency conditions except for the gravitational bound behave nicely upon “decomposition”

$$G = G_1 \times G_2 \times \cdots \times G_k,$$
$$\Lambda = \langle b_0, b_1, \ldots, b_k \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_i > 0, \text{ etc}$$

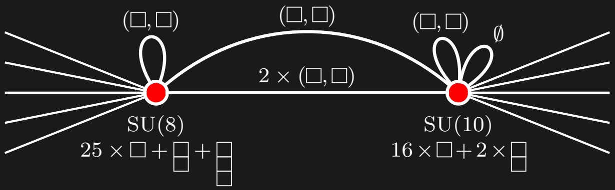
$$G' = G_1 \times G_2 \times \cdots \times G_m,$$
$$\Lambda' = \langle b_0, b_1, \ldots, b_m \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_{i \leq m} > 0, \text{ etc}$$

$$G'' = G_{m+1} \times G_{m+2} \times \cdots \times G_k,$$
$$\Lambda'' = \langle b_0, b_{m+1}, \ldots, b_k \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_{i > m} > 0, \text{ etc}$$

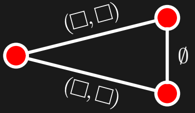
Strategy: reverse this procedure by understanding the theories with simple gauge group and how they can be recombined

Multigraphs and cliques

- Vertex: solution of B-constraint for some simple group
- Edge: "admissible" choice of bi-charged hypers



- Clique: collection of k vertices, all pair-wise connected by edges



$$\Longleftrightarrow$$

$$G = \mathrm{SU}(8) \times \mathrm{SU}(10) \times \mathrm{SU}(10)$$

$$\mathcal{H} = (\square, \square, \bullet) + (\square, \bullet, \square) + \ldots$$

$$\mathbb{G} = \begin{pmatrix} 9-T & 5 & 2 & 2 \\ 5 & 3 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{pmatrix} \qquad H_{\mathrm{ch}} - V = 363 \qquad T \geq 1$$

Classification

An anomaly-free clique either...

- ...has one or more copies of

$$E_6 + \emptyset\,, \quad E_6 + \underline{\mathbf{27}}\,, \quad E_7 + \emptyset\,, \quad E_7 + \tfrac{1}{2}\underline{\mathbf{56}}\,,$$
$$E_7 + \underline{\mathbf{56}}\,, \quad E_7 + \tfrac{3}{2}\underline{\mathbf{56}} \quad E_8 + \emptyset\,, \quad F_4 + \emptyset$$

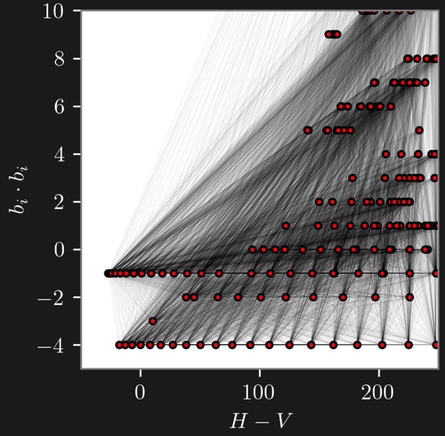
  and is part of a (large!) infinite family, or

- ...is composed of irreducible cliques which are individually bounded as

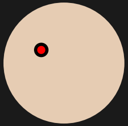
$$0 \;\leq\; (\Delta + 28n_-) \;\leq\; (273 - T) \;\leq\; 273$$

  and joined together by trivial edges.

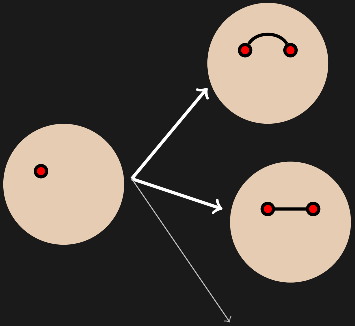
$\Longrightarrow$ These irreducible cliques can be systematically enumerated.



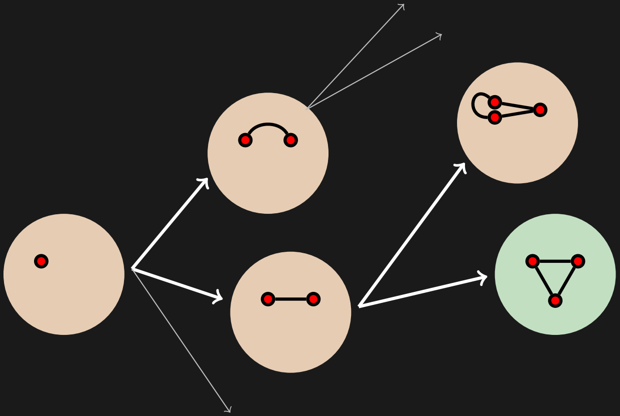
Clique construction: branch & prune



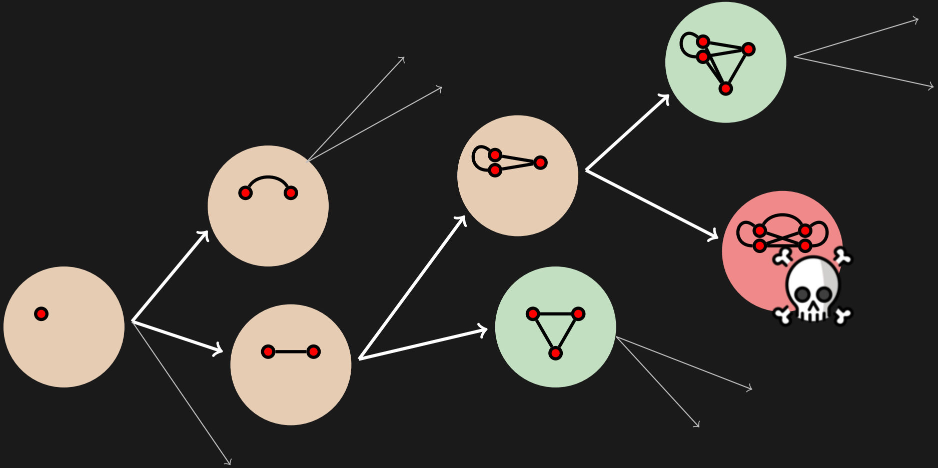
Clique construction: branch & prune



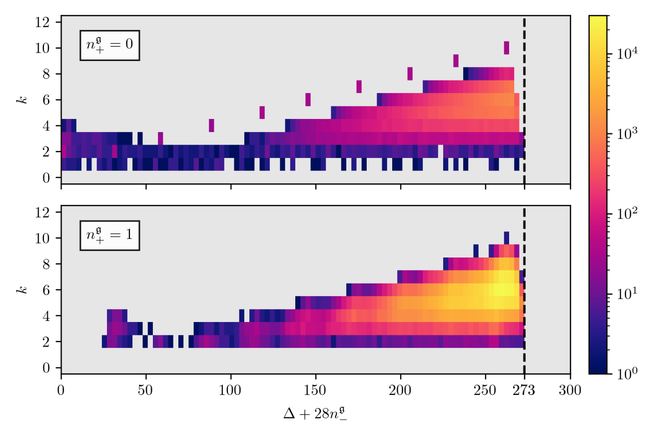
Clique construction: branch & prune



Clique construction: branch & prune



Results: irreducible cliques



Cliques split into irreducible pieces,

$$\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \cdots$$

for which $H_{\text{ch}} - V$ and $n_\pm$ add nicely:

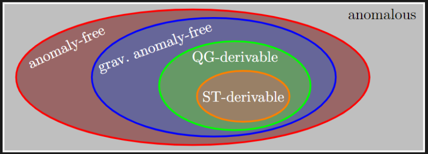
$$H_{\text{ch}} - V = \sum_j (H_{\text{ch}} - V)_j$$

$$n_\pm = \sum_j (n_\pm)_j$$

For all combinations of SU(5–25), SO(7–33), Sp(3–16), $E_{6,7,8}$, F_4, G_2 there are $\mathcal{O}(10^8)$ irreducible cliques

Summary & Thoughts

- Anomaly cancellation and the Green-Schwarz mechanism
- 6d $\mathcal{N} = 1$ supergravity and its consistency conditions
- "Decomposition" & pair-wise compatibility $\implies$ multigraph and algorithmically constructing k-cliques
- Characterization of infinite families w/ eight problematic vertices
- Two sided bounds and T-independent enumeration of irreducible cliques
- We now have a large ensemble of consistent theories: which appear in string theory?
    - to have a geometric F-theory realization need to satisfy Kodaira condition
    - additional conditions from BPS strings (e.g. see [Kim, Shiu, Vafa – 19])



Thanks!

- There are three non-trivial homotopy groups:

$$\pi_6\big(\mathrm{SU}(2)\big) \cong \mathbb{Z}_{12}\,, \qquad \pi_6\big(\mathrm{SU}(3)\big) \cong \mathbb{Z}_6\,, \qquad \pi_6\big(G_2\big) \cong \mathbb{Z}_3$$

  These groups are subject to an additional constraint:

$$\sum\nolimits_R n_R^i C_R^i - C_{\mathrm{Adj}}^i \equiv 0 \mod 12\,,\ 6\,,\ 3 \qquad \text{i.e.} \qquad b_i \cdot b_i \in 4\mathbb{Z}\,,\ 2\mathbb{Z}\,,\ \mathbb{Z}$$

- There can be half-hypermultiplets for quaternionic representations, but only if A_R is even. An odd number of half-hypermultiplets is anomalous if A_R is odd (e.g. fundamentals of $\mathrm{Sp}(N)$ must occur as full-hypermultiplets)

$$G = \mathrm{SU}(10) \qquad \left(\sum n_R B_R = B_{\mathrm{Adj}}\right)$$

$$\mathcal{H}_{\mathrm{SU}(10)} = 7 \times \underline{\mathbf{10}} + 5 \times \underline{\mathbf{45}} + 8 \times \underline{\mathbf{120}} + 3 \times \underline{\mathbf{210}} + \tfrac{1}{2} \times \underline{\mathbf{252}} + 2 \times \underline{\mathbf{825}} + \underline{\mathbf{990}} + 2 \times \underline{\mathbf{1848}}$$

$$H_{\mathrm{ch}} - V = 8248$$

$$\mathbb{G} = \begin{bmatrix} 9-3 & 473 \\ 473 & 895 \end{bmatrix} \qquad \begin{aligned} b_0 &= (3, 1, 1, 1) \\ b_1 &= (103, -77, -44, -43) \quad = j \end{aligned}$$

$$G' = \mathrm{SU}(10) \times E_8^m \qquad \left(\sum n_R B_R = B_{\mathrm{Adj}}\right)$$

$$\mathcal{H}_{\mathrm{SU}(10)} = 7 \times \underline{\mathbf{10}} + 5 \times \underline{\mathbf{45}} + 8 \times \underline{\mathbf{120}} + 3 \times \underline{\mathbf{210}} + \tfrac{1}{2} \times \underline{\mathbf{252}} + 2 \times \underline{\mathbf{825}} + \underline{\mathbf{990}} + 2 \times \underline{\mathbf{1848}}$$

$$(H_{\mathrm{ch}} - V)' = 8248 - 248m$$

$$\mathbb{G}' = \begin{bmatrix} 9 - T' & 473 & -10_m \\ 473 & 895 & 0_m \\ -10_m & 0_m & -12\,\mathbb{I}_{m\times m} \end{bmatrix} \qquad \frac{10^2}{12}\, m + 3 \le T' \le \frac{273 - 8248 + 248m}{29}$$

and take $j = j^1 b_1 - (b_2 + b_3 + \cdots + b_{m+1})$ with $j^1 > \sqrt{12m/895}$.

The first T', m which work are $\boxed{T' = 32{,}251 \text{ and } m = 12{,}093}$