Grassmann tensor network study of multi-flavor gauge theory

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Based on JHEP11(2023)187, with **Jun Nishimura** (KEK) and **Kouichi Okunishi** (Niigata U)

Introduction

- Lattice fermions are known to be difficult to simulate because of its large computational cost

$$\int \exp\left[-\theta^T A\eta\right] d\theta\, d\eta = \det A$$

Dirac matrix grows with lattice volume!

- In many systems, the fermions give the Numerical Sign Problem
  ==> Hubbard model, Chiral fermion, Finite density

- Tensor network (Grassmann TRG) offers a solution

Introduction

Selected works that use ‘Grassmann tensor network’

- Lattice Schwinger model
  - Shimizu-Kuramashi (`14,`14,`18)
- 2D QCD
  - Bloch-Lohmayer (`23)
- Hubbard model
  - Akiyama-Kuramashi (`21)
  - Akiyama-Kuramashi-Yamashita (`22)
- and many others

Introduction

- Tensor size grows with the dimension of local Hilbert space

  ==> Grows exponentially with Nf !!!

  ➔ Akiyama (`23) proposes a way to reduce the CG cost based on matrix product decomposition.

- Gauge theory with multiple flavors is even more difficult because of the gauge d.o.f.

  ==> The topic of this talk

Content

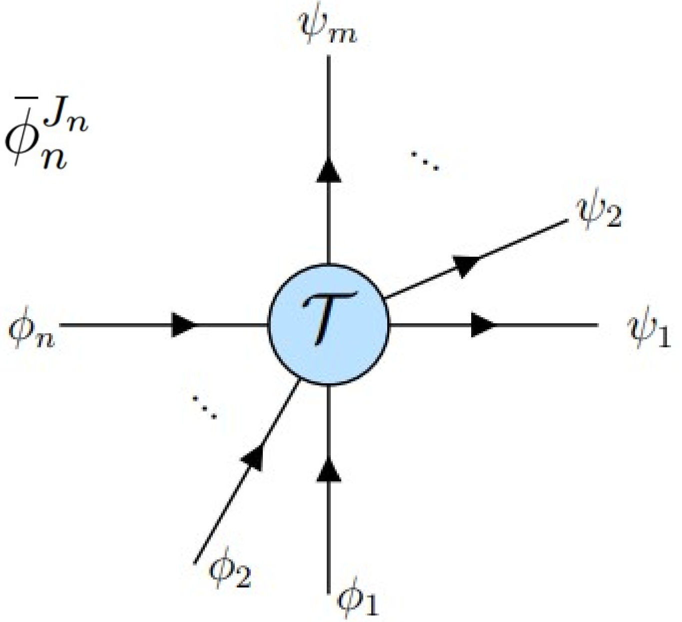
- Introduction
- Multi-layer construction
- Results
- Summary

Grassmann tensor

$$\mathcal{T}_{\psi_1\cdots\psi_m\bar{\phi}_1\cdots\bar{\phi}_n} = \sum_{I_1,\cdots,I_m,J_1,\cdots,J_n} T_{I_1\cdots I_m J_1\cdots J_n}\psi_1^{I_1}\cdots\psi_m^{I_m}\bar{\phi}_1^{J_1}\cdots\bar{\phi}_n^{J_n}$$

Coefficient tensor

Multi-component
Grassmann numbers



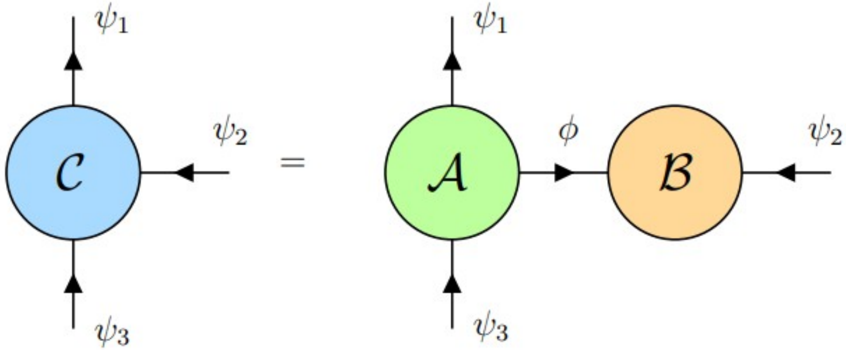
- Grassmann numbers are anti-commuting
- Non-conjugated and Conjugated fermions are distinguished !

Grassmann tensor contraction

$$\mathcal{C}_{\psi_1\bar{\psi}_2\bar{\psi}_3} = \int_{\bar{\phi}\phi} \mathcal{A}_{\psi_1\phi\bar{\psi}_3}\mathcal{B}_{\bar{\psi}_2\bar{\phi}}$$

$$\int_{\bar{\eta}\eta} \equiv \int d\bar{\eta}d\eta e^{-\bar{\eta}\eta}$$

Symbolic



Diagram

$$C_{IJK} = \sum A_{ILK}B_{JL}s_{JKL}$$

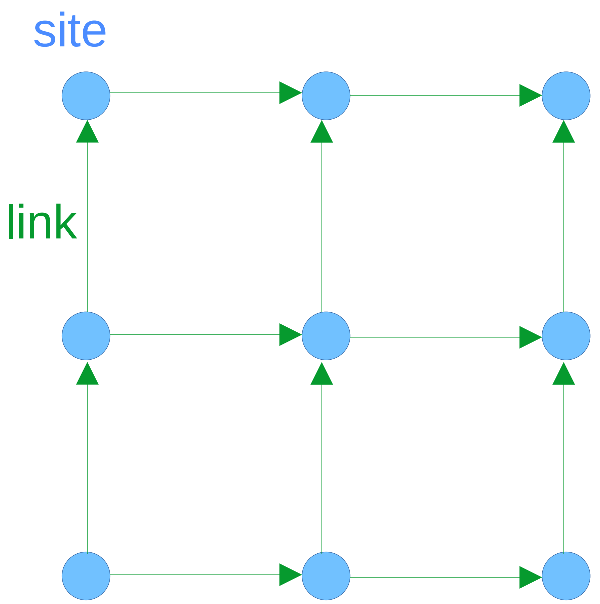
$$s_{JKL} = \sigma_L \times (-)^{p(L)(p(J)+p(K))+p(J)p(K)}$$

Numerical coefficient tensor

Lattice gauge theory

$$S = S_{\text{gauge}}[\varphi] + \sum_{x\in\Lambda_2}\sum_{\alpha=1}^{N_f} \bar{\psi}_x^{(\alpha)} \not{D}^{(\alpha)} \psi_x^{(\alpha)} \ ,$$

$$S_{\text{gauge}}[\varphi] = \beta \sum_{x\in\Lambda_2} \{1 - \cos(\varphi_{x,1} + \varphi_{x+\hat{1},2} - \varphi_{x+\hat{2},1} - \varphi_{x,2})\}$$



a 2D lattice

- Gauge fields live on the links
- Fermions live on the sites
- All (Wilson) fermion flavors live on the same site

Local Hilbert space is very big for large Nf

Multi-layer construction

1) Separate the action for different flavors

$$S^{(\alpha)} = \frac{1}{N_{\mathrm{f}}} S_{\mathrm{gauge}}[\varphi^{(\alpha)}] + \sum_{x \in \Lambda_2} \bar{\psi}_x^{(\alpha)} \not{D}^{(\alpha)} \psi_x^{(\alpha)}$$

2) Identify all the gauge field from different layers in the partition function

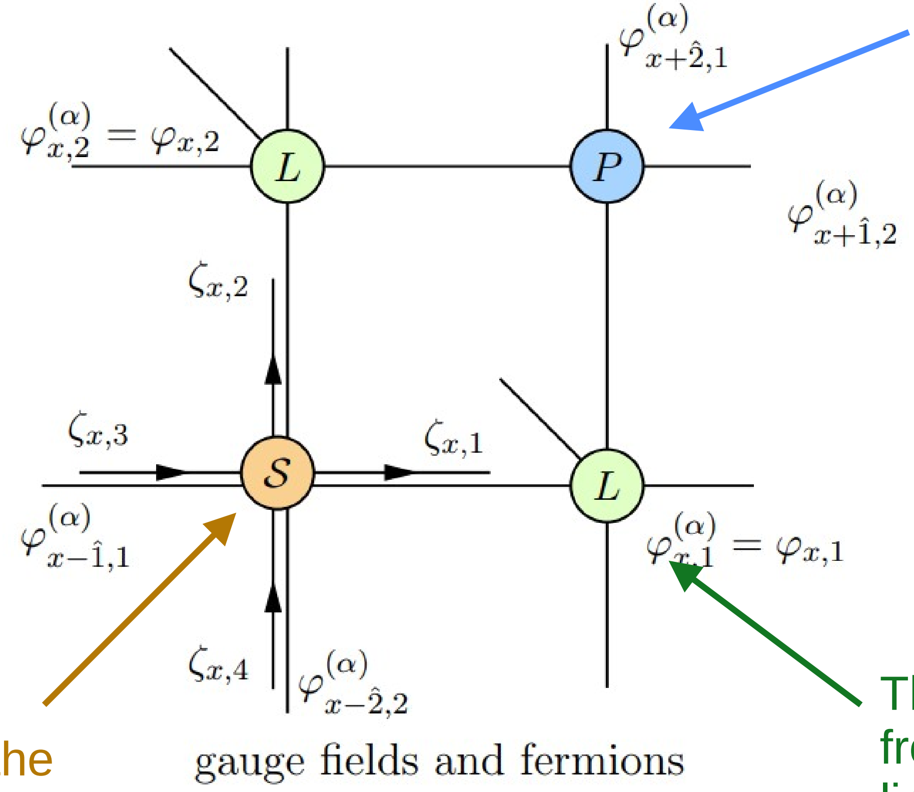
$$Z = \int D\varphi \prod_{\alpha=1}^{N_{\mathrm{f}}} \left(D\varphi^{(\alpha)} D\psi^{(\alpha)} D\bar{\psi}^{(\alpha)} \right) \delta(\varphi^{(\alpha)} - \varphi) e^{-\sum_\alpha S^{(\alpha)}}$$

In this way, the flavor index acts as an extra dimension.

Inspiration: Domain wall fermion

The tensor network (one site, one layer)

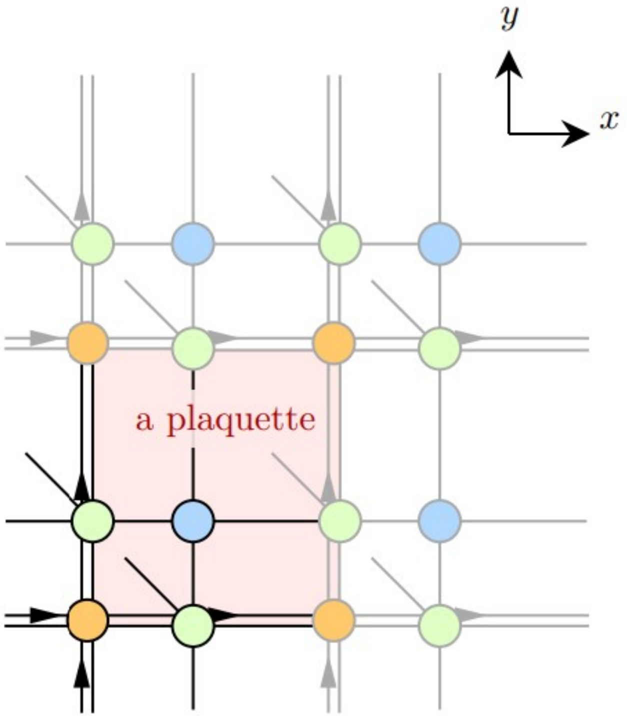
From the plaquette action



This Grassmann tensor comes from integrating the site fermions.

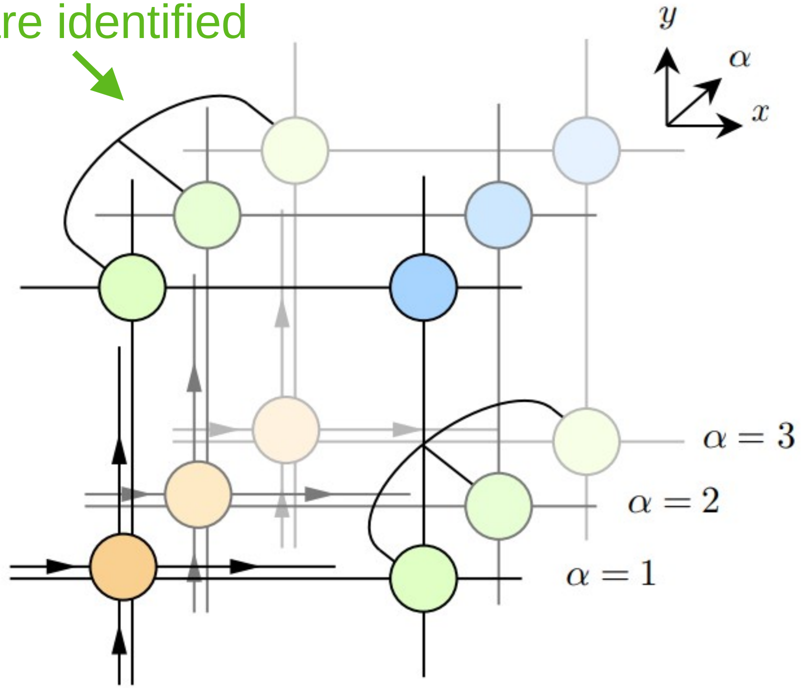
This bosonic tensor comes from integrating the bosonic link variables.

The tensor network



2x2 lattice, 1 layer

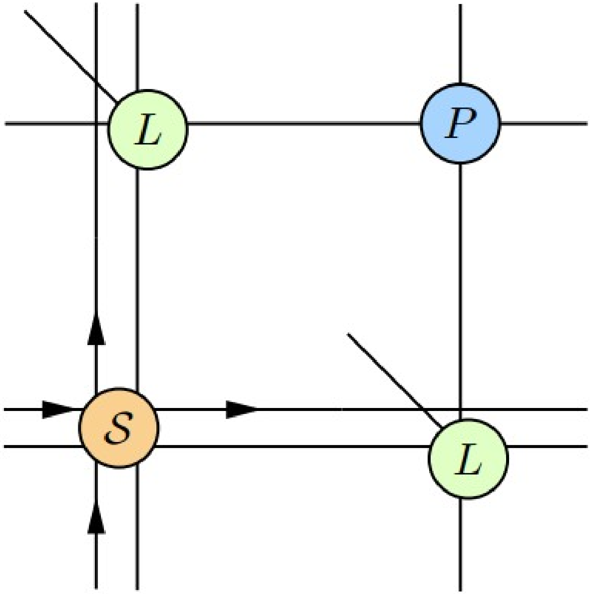
Gauge fields from different layers are identified



1 site, 3 layers

Tensor compression

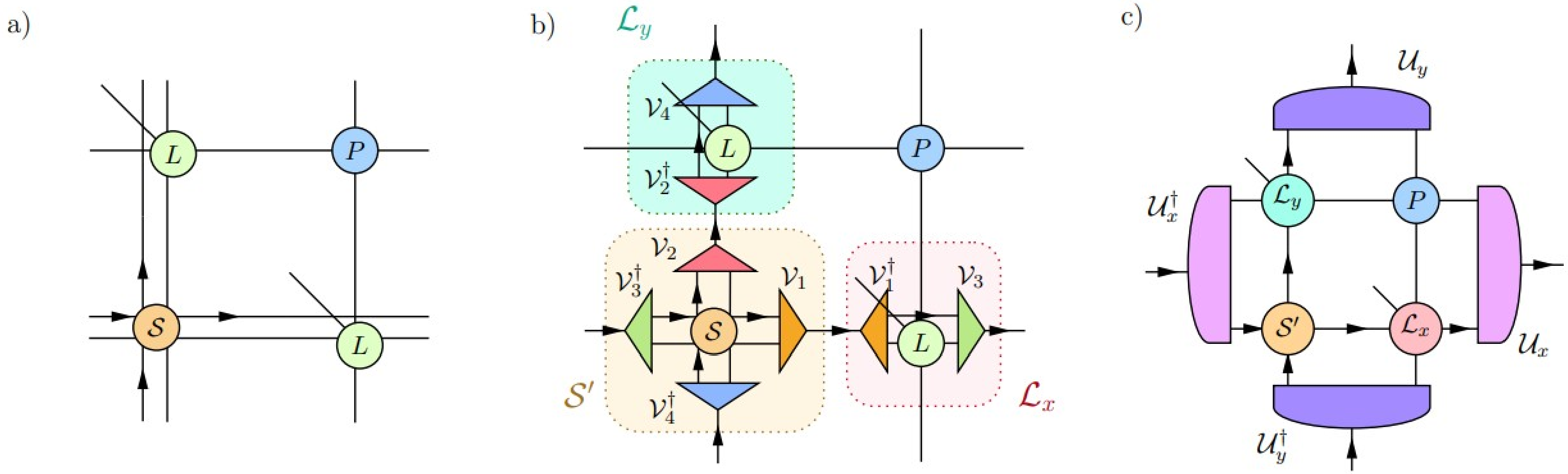
The initial tensor is still too big



10 bosonic legs & 4 fermionic legs
= K^{10} 16^4 components

Some compression is needed
to reduce the tensor size first

Tensor compression



Isometries are first applied around the Grassmann tensor *S*:   a → b
Then another set is applied around the whole tensor: b → c

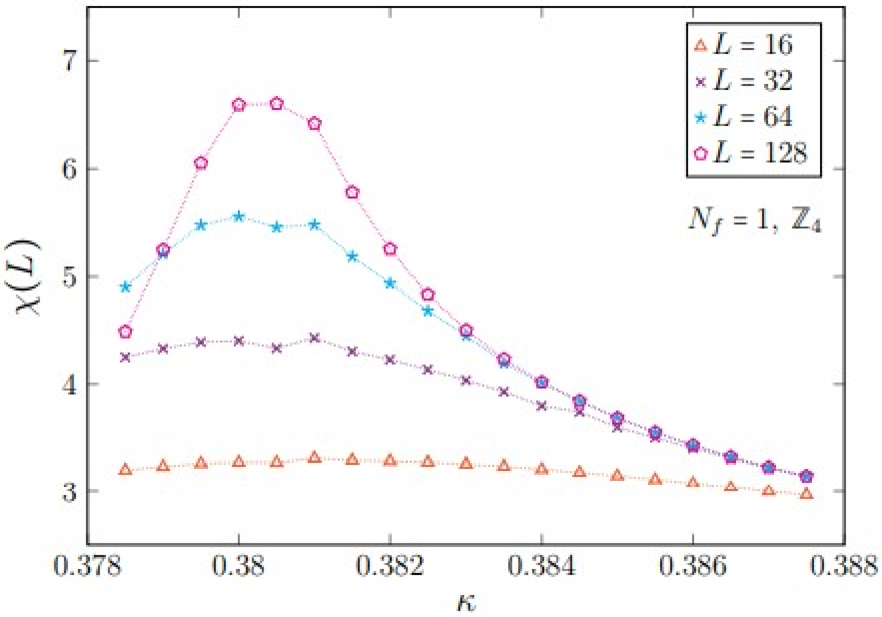
Tensor compression

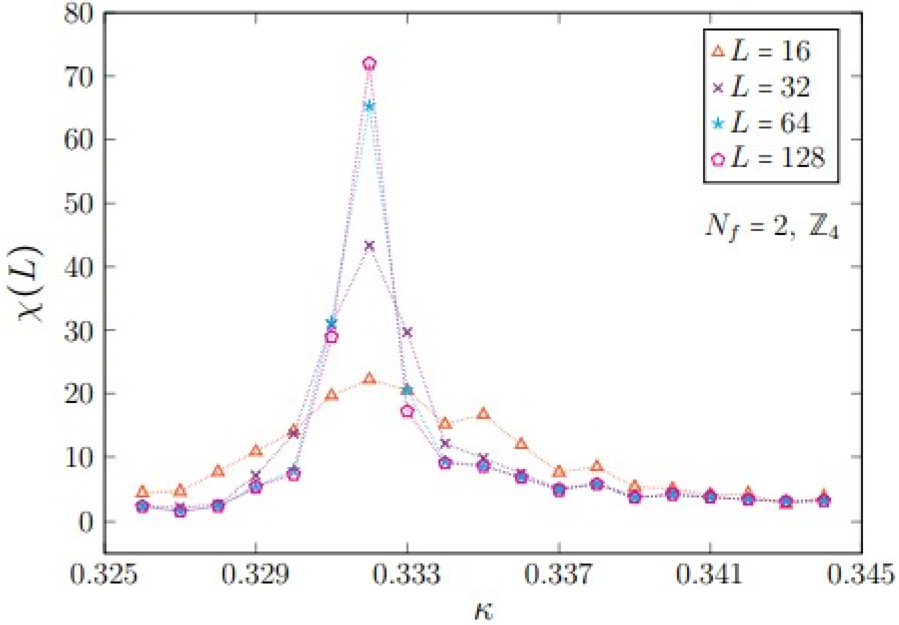
β	$\tilde{\mu}$	N_f	K	original size	compressed size	compression ratio	D_x	D_y
0.0	0.0	1	2	67108864	1024	1.53×10^{-5}	4	4
0.0	0.0	1	3	3869835264	2304	5.95×10^{-7}	4	4
0.0	0.0	1	4	68719476736	4096	5.96×10^{-8}	4	4
0.0	0.0	1	5	640000000000	6400	1.00×10^{-9}	4	4
2.0	0.0	1	2	67108864	16384	2.44×10^{-4}	8	8
2.0	0.0	2	2	67108864	16384	2.44×10^{-4}	8	8
2.0	3.0	1	2	67108864	16384	2.44×10^{-4}	8	8
2.0	3.0	2	2	67108864	16384	2.44×10^{-4}	8	8

Performance of the compression

Results

- Chiral symmetry breaking

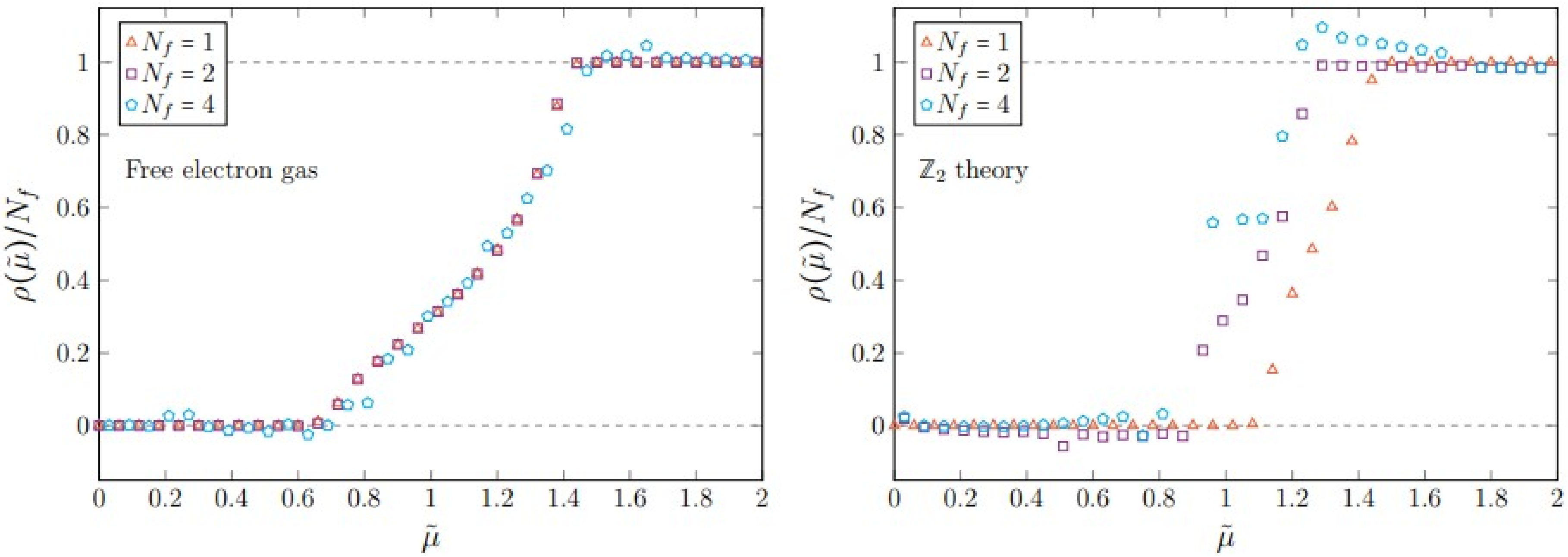




Critical behavior is observed near the chiral sym breaking point.
The critical point is in agreement with the literature.

Results

- Finite density and Silver Blaze phenomena



This cannot be observed in traditional Monte Carlo simulation because of the sign problem!

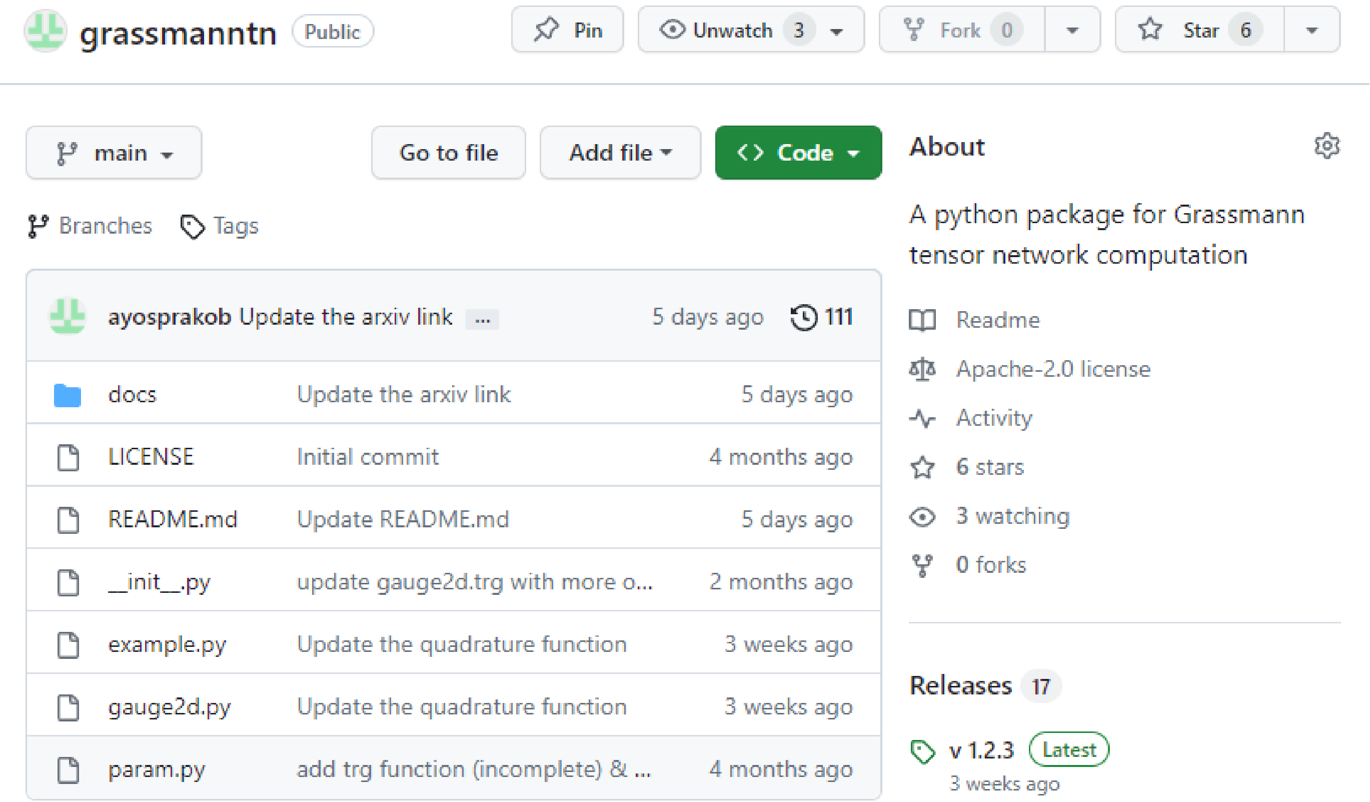
Summary and Outlooks

- We propose a way to incorporate multiple flavors for gauge theory
- Compression on the initial tensor is essential
- Chiral symmetry breaking and Silver Blaze phenomenon are demonstrated

Future works:

- Increase Dcut (new workstation)
- Non-abelian theory
- Domain-wall fermion

GrassmannTN:
a python package for Grassmann tensor networks



https://github.com/ayosprakob/grassmanntn