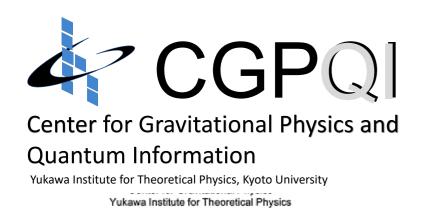
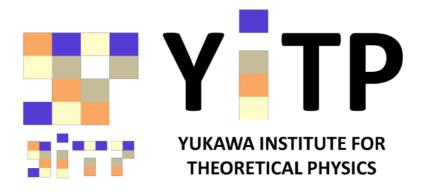
# A Recipe for bulk construction from a scalar CFT by a conformal flow

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work in progress with

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## 0. Motivation/Question

How can we map properties of QFT to a "geometry"?

In this talk, to answer this question, we exclusively consider a scalar CFT with Euclidean signature, which is quantized by the path-integral.

We will provide a prototype of AdS/CFT correspondence without using string duality (gauge/gravity correspondence). Therefore results may differ from usual understanding. We however hope that our attempt may provide deeper understanding of the standard AdS/CFT correspondence.

We can easily extend our method to a general scalar (non-conformal) QFT, which may provide an answer to the above question.

# 1. Construction of a bulk space

We consider an O(N) scalar CFT in d-dimensions. Starting point

A non-singlet primary field satisfies  $\langle 0|\hat{\varphi}^a(x)\hat{\varphi}^b(y)|0\rangle = \delta^{ab}\frac{C_0}{|x-y|^{2\Delta}}$ 

**Smearing** We smear the field by the flow equation as

$$\Delta < \frac{d}{2}$$

$$(-\alpha\eta\partial_{\eta}^{2} + \beta\partial_{\eta})\hat{\phi}^{a}(x;\eta) = \Box_{x}\hat{\phi}^{a}(x;\eta), \quad \hat{\phi}^{a}(x;0) = \hat{\varphi}^{a}(x)$$

 $\begin{array}{ll} \textbf{Conformal flow} & \text{We take } \nu := 1 + \frac{\beta}{\alpha} = \frac{d}{2} - \Delta & \text{with } \eta = \frac{\alpha}{4}z^2 \\ \text{and introduce a normalized field as } & \hat{\sigma}^a(X) := \frac{\hat{\phi}^a(x;\eta)}{\sqrt{\langle 0|\hat{\phi}^2(x;\eta)|0\rangle}} & X := (x,z) \end{array}$ 

z is an extra direction, which corresponds to an energy scale of CFT.

$$z = 0$$
 (UV) and  $z = \infty$  (IR)

QFT(CFT) in d-dimension + energy scale ——— d+1 dimensional bulk space

Holography

## Why do we call it a conformal flow?

conformal symmetry on  $\hat{\varphi}^a(x)$  generates a coordinate transformation on  $\hat{\sigma}^a(X)$  as

$$\delta^{\text{conf}} \hat{\varphi}^{a}(x) = -\delta x^{\mu} \partial_{\mu} \hat{\varphi}^{a}(x) - \frac{\Delta}{d} (\partial_{\mu} \delta x^{\mu}) \hat{\varphi}^{a}(x)$$

$$\delta x^{\mu} := a^{\mu} + w^{\mu}{}_{\nu} x^{\nu} + \lambda x^{\mu} + b^{\mu} x^{2} - 2x^{\mu} (b \cdot x)$$

$$\delta^{\text{conf}}\hat{\sigma}^a(X) = -\delta X^A \partial_A \hat{\sigma}^a(x) \qquad \qquad \delta X^\mu := \delta x^\mu + z^2 b^\mu \qquad \delta X^{d+1} := (\lambda - 2b \cdot x) z$$

This coordinate transformation is nothing but the AdS isometry.

Aoki-Balog-Onogi-Yokoyama, PTEP 2023(2023) 013B03.

AdS/CFT correspondence?

# 2. (Quantum) bulk space

Operators in the boundary and the bulk enjoy these symmetries as

$$\langle 0 | \prod_{i=1}^{m} G_{A_{1}^{i} \cdots A_{n_{i}}^{i}}^{i} (\tilde{X}_{i}) \prod_{j=1}^{s} O_{\mu_{1}^{j} \cdots \mu_{l_{j}}^{j}}^{j} (\tilde{y}_{j}) | 0 \rangle = \prod_{i=1}^{m} \frac{\partial X_{i}^{B_{1}^{i}}}{\partial \tilde{X}_{i}^{A_{1}^{i}}} \cdots \frac{\partial X_{i}^{B_{n_{i}}^{i}}}{\partial \tilde{X}_{i}^{A_{n_{i}}^{i}}} \prod_{j=1}^{s} J(y_{j})^{-\Delta_{j}} \frac{\partial y_{j}^{\nu_{1}^{j}}}{\partial \tilde{y}_{j}^{\nu_{1}^{j}}} \cdots \frac{\partial y_{j}^{\nu_{l_{j}}^{j}}}{\partial \tilde{y}_{j}^{\nu_{j}^{j}}} \times \langle 0 | \prod_{i=1}^{m} G_{B_{1}^{i} \cdots B_{n_{i}}^{i}}^{i} (X_{i}) \prod_{j=1}^{s} O_{\nu_{1}^{j} \cdots \nu_{l_{j}}^{j}}^{j} (y_{j}) | 0 \rangle$$

**bulk operator** (with an arbitrary spin)  $G^i_{A^i_1\cdots A^i_{n_i}}(X_i)$  coordinate transformation  $X\to \tilde{X}$ 

boundary operator (with an arbitrary spin)  $O^j_{\mu^j_1\cdots\mu^j_{l_j}}(y_j)$  conformal transformation  $y o \tilde{y}$ 

Correlation functions including all quantum corrections are defined in the bulk.

## Quantum bulk space

This is different from the standard AdS/CFT correspondence.

**Example** Bulk-boundary (scalar-scalar) propagator is given by

$$\langle 0|\hat{\Phi}(X)\hat{S}(y)|0\rangle_c = C_S \left(\frac{z}{(x-y)^2+z^2}\right)^{\Delta_S}$$
  $\Delta_S$ : conformal dimension of  $\hat{S}$ 

where  $\hat{\Phi}(X)$  and  $\hat{S}(y)$  are bulk and boundary scalar operators, respectively.

While a geometry of the bulk space is not defined so far, the above propagator satisfies a free Klein-Gordon equation in the AdS with the mass given by

$$m^2 = \frac{\Delta_S(\Delta_S - d)}{L_{\text{AdS}}^2}$$

We add a small source at the boundary as

$$\Phi_J(X) := \langle 0|\hat{\Phi}(X) \exp\left[\int d^d y J(y) \hat{S}(y)\right] |0\rangle_c \simeq \int d^d y J(y) \langle 0|\hat{\Phi}(X) \hat{S}(y)|0\rangle_c + O(J^2)$$

In the  $z \rightarrow 0$  limit, we obtain **GKP-Witten relation** as

$$\lim_{z \to 0} \Phi_J(X) = z^{d - \Delta_S} \left[ \tilde{C}_s J(x) + O(z^2) \right] + z^{\Delta_S} \left[ \langle 0 | \hat{S}(x) | 0 \rangle_J + O(z^2) \right]$$

$$\langle 0|\hat{S}(x)|0\rangle_J := \langle 0|\hat{S}(x)\exp\left[\int d^dy J(y)\hat{S}(y)\right]|0\rangle \simeq C_S \int d^dy \frac{J(y)}{|x-y|^{2\Delta_S}} + O(J^2)$$

# 3. Geometry of the bulk space

How can we determine a geometrical structure of the bulk space?

#### **Possibilities**

- a geometry which makes the (scalar) propagator solution to a free Klein-Gordon equation.
- a geometry which makes the (boundary) entanglement entropy equal to the minimal surface in the bulk.
- others

They are rather complicated. We instead consider a more direct method.

We determine a bulk geometry using Bures information metric.

#### **Bures information metric**

A distance between a density matrix  $\rho$  and  $\rho + d\rho$  is defined as

$$d^2(\rho, \rho + d\rho) := \frac{1}{2} \operatorname{Tr} \left( d\rho \hat{G} \right)$$

where  $\hat{G}$  satisfies

$$\rho \hat{G} + \hat{G}\rho = d\rho$$

## State dependent density matrix

 $|S\rangle$ : some state in CFT

$$ho_S(X) := \sum_{a=1}^N \hat{\sigma}_S^a(X) |S\rangle\langle S|\hat{\sigma}_S^a(X)$$
 N entangled pairs (mixed state)  $\operatorname{tr} \rho_S(X) = 1$ 

$$\hat{\sigma}^a_S(X) := \frac{\hat{\sigma}^a(X)}{\sqrt{\langle S | \hat{\sigma}^2(X) | S \rangle}} = \frac{\hat{\phi}^a(x; \eta)}{\sqrt{\langle S | \hat{\phi}^2(x; \eta) | S \rangle}} \quad \text{normalized for the state } |S\rangle$$

since  $\hat{G} = Nd\rho_S(X) = NdX^A \partial_A \rho_S(X)$ , we obtain

$$\frac{1}{2}\operatorname{Tr}\left[d\rho_S(X)\hat{G}\right] = \frac{N}{2}\operatorname{Tr}\left[\partial_A\rho_S(X)\partial_B\rho_S(X)\right]dX^AdX^B$$

$$\qquad \qquad \Rightarrow \quad g_{AB}^S(X) := \ell^2 \sum_{a=1}^N \langle S | \partial_A \sigma_S^a(X) \partial_B \sigma_S^a(X) | S \rangle \quad \text{ The metric is state dependent.}$$

# 4. State dependent metric

$$g_{AB}^{S}(X) := \ell^{2} \sum_{a=1}^{N} \langle S | \partial_{A} \sigma_{S}^{a}(X) \partial_{B} \sigma_{S}^{a}(X) | S \rangle$$

Bulk geometry is state dependent. The bulk geometry is not unique.

Vacuum case 
$$|0\rangle$$
 
$$\hat{\sigma}_0^a(X) = \hat{\sigma}^a(X) = \frac{\hat{\phi}^a(x;\eta)}{\sqrt{\langle 0|\hat{\phi}^2(x;\eta)|0\rangle}}$$

$$g_{AB}^{\mathrm{vac}}(X) = \ell^2 \frac{\Delta(d-\Delta)}{d+1} \frac{\delta_{AB}}{z^2}$$
 AdS metric in the Poincare coordinate

A vacuum state in an arbitrary CFT —— AdS metric

## AdS/CFT correspondence

Aoki-Yokoyama, PTEP 2018(2018) 031B01.

What is a metric for excited states?

## Finite temperature (Thermo field double)

$$|\text{TFD}\rangle := \frac{1}{\sqrt{Z_T}} \sum_n e^{-E_n/2T} |E_n\rangle \otimes |\widetilde{E_n}\rangle$$
  $Z_T := \sum_n e^{-E_n/T}$ 

If the O(N) model is free, the metric becomes the asymptotic AdS.

In the UV region  $(z \to 0)$ , the metric is a classical solution to f(R) gravity.

Aoki-Shimada-Balog-Kawana, arXiv:2308.01076[hep-th]

**Scalar state**  $|S\rangle = |\Phi\rangle$ : scalar state with a conformal dimension  $\Delta_{\Phi}$ 

In general, the metric becomes the asymptotic AdS.

In the case of free O(N) model with  $\Delta_{\phi} = 2\Delta$ , we have

$$g_{AB}^{\Phi}(X) = g_{AB}^{\mathrm{vac}}(X) + \frac{1}{N}\delta g_{AB}(X) + O(1/N^2)$$
 Asymptotic AdS with 1/N correction

Aoki-Balog-Shimada, work in progress

# 5. Summary

CFT bulk (quantum) space with symmetry conformal flow state dependent normalization

state dependent metric/geometry

vacuum -> AdS

excited state -> asymptotic AdS

metric becomes classical in the large N limit

Is a concept of the state-dependent geometry compatible with the conventional AdS/CFT correspondence?

We are also working on the metric for excited states from the standard point of view.

Stay tuned!