Gradient Flow Exact Renormalization Group for Scalar Quantum Electrodynamics

Junichi Haruna
Osaka University, Center for Quantum Information and Quantum Biology

2023/11/29 @ KEK-TH 2023
arXiv:2312.XXXXX with M.Yamada (Jilin)

Summary

- Gradient Flow Exact Renormalization Group defines the Wilsonian effective action based on diffusion equations
- Scalar Quantum Electrodynamics with off-shell BRST invariance is studied
- The modified BRST invariance reduces to the ordinary Ward-Takahashi identity under some assumption
- The RG flow equation is perturbatively solved up to second order of the gauge coupling,
- The consistent results with the perturbation theory

Content

Content

Exact Renormalization Group

- Framework to study the change of physics under variation of the energy scale

- Wilsonian effective action S_τ is intuitively defined by integrating out the higher momentum mode:

$$e^{-S_\tau} := \int D\phi_{p>\Lambda} e^{-S_0}$$

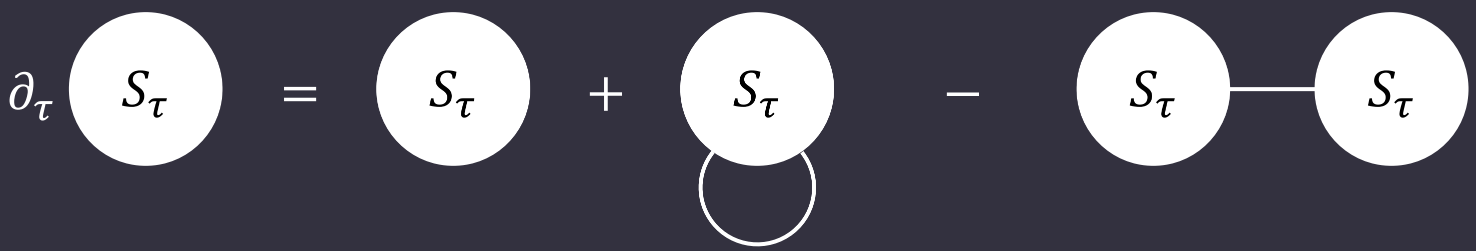
($\Lambda := \Lambda_0 e^{-\tau}$, Λ_0: UV cutoff)

- τ-depenedence of S_τ is described by a functional deferential equation → "ERG equation"

Wilson-Polchinski equation

- Typical example of ERG equation:

$$\partial_\tau S_\tau = \int_p \left\{ \left[\left(2p^2 + \frac{D+2-2\gamma_\tau}{2} \right) + p_\mu \frac{\partial}{\partial p_\mu} \right] \phi_i(p) \frac{\delta S_\tau}{\delta \phi_i(p)} + \left(2p^2 + 1 - \gamma_\tau\right) \left(\frac{\delta^2 S_\tau}{\delta\phi_i(p)\delta\phi_i(-p)} - \frac{\delta S_\tau}{\delta\phi_i(p)} \frac{\delta S_\tau}{\delta\phi_i(-p)} \right) \right\}$$



- ERG equation nonperturbatively defines an RG flow

(γ_τ: anomalous dimension) [J.Polchinski Nucl.Phys.B 231 (1984) 269-295]

(We use dimensionless notations on D-dimensional Euclidean spacetime)

Gauge Invariance in ERG

- Wilsonian effective action with a naïve UV cutoff is NOT consistent with gauge invariance

- Gauge transformation mixes the various momentum modes

$$A^a_\mu(p) \to A^a_\mu(p) - p_\mu \omega^a(p) - i f^{abc} \int_q \omega^b(p-q) A^c(q)$$

- Can we define the Wilsonian effective action in a manifestly gauge-invariant way?

Gradient Flow-ERG (GF-ERG)

- Framework to define the RG flow of the Wilsonian effective action based on coarse-graining along diffusion equations [H. Sonoda and H. Suzuki 2012.03568]

$$e^{-S_\tau[A_\mu^a]} \coloneqq \hat{s}_A^{-1} \int [DA'^a_\mu] \prod_{x',a,\mu} \delta\left(A_\mu^a(x) - e^{\int_\tau (D-2+2\gamma_\tau)/2} B'^a_\mu(t, x' e^\tau)\right) \hat{s}_{A'} e^{-S_{\tau=0}[A'^a_\mu]}$$

($\hat{s}_A \coloneqq \exp\left[-\frac{1}{2}\int_x \frac{\delta^2}{\delta A_\mu^a(x) \delta A_\mu^a(x)}\right]$: scrambler operator)

- Symmetries of the diffusion equation are inherited by the RG flow within GF-ERG
  $\Rightarrow$ S_τ is gauge-invariant at an arbitrary energy scale!

Content

- Introduction (3)
- Review of GF-ERG (5)
- GF-ERG for Scalar Quantum Electrodynamics (7)
- Summary and Discussion (3)

Our Motivation

- Perturbative analysis of QED
  - On-shell BRST invariance
  - Loop corrections to the photon kinetic term are consistent with perturbation theory in four dims.

[Y.Miyakawa, H.Sonoda and H.Suzuki 2111.15529]

- Question
  - Off-shell BRST invariance should be respected (Wilsonian effective action itself obtains quantum corrections)
  - Photon mass vanishes in general dimensions? (none-zero in the conventional ERG case)

- Scalar QED has more simple structure
  ⇒ Suitable to study these points

Diffusion Equation

- Coarse-graining along these eqs. are utilized:

$$\left.\begin{aligned} \partial_t A_\mu &= \partial_x^2 A_\mu \\ \partial_t c &= \partial_x^2 c \\ \partial_t \bar{c} &= \partial_x^2 \bar{c} \\ \partial_t B &= \partial_x^2 B \end{aligned}\right\} \text{same as WP eq.}$$

$$\partial_t \phi = D_\mu D_\mu \phi + ie_0(\partial_\mu A_\mu)\phi \quad \} \text{ differs from WP eq.}$$

- Consistent with off-shell BRST transformation
  ($\partial_t(\delta_B A_\mu) = \delta_B(\partial_t A_\mu)$ etc.)

  $\Rightarrow$ Inherited by the RG flow with GF-ERG

$$\left(\begin{aligned} \delta_B A_\mu &= \partial_\mu c \\ \delta_B c &= 0 \\ \delta_B \bar{c} &= B \\ \delta_B B &= 0 \\ \delta_B \phi &= ie_0 c\phi \end{aligned}\right)$$

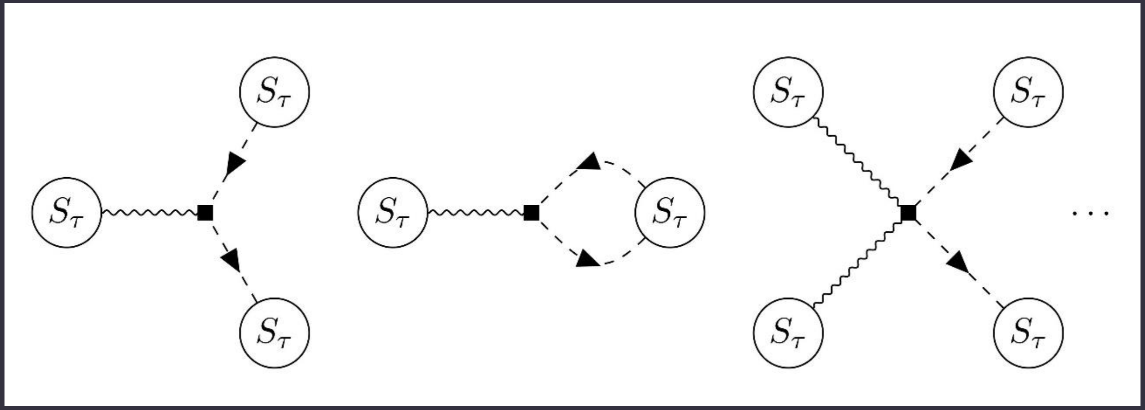
GF-ERG equation

$$\partial_\tau e^{-S_\tau} = (\text{WP part})$$
$$+\int_x \frac{\delta}{\delta\phi^*}\left[-4ie_\tau\left(A_\mu+\frac{\delta}{\delta A_\mu}\right)\partial_\mu + 2e_\tau^2\left(A_\mu+\frac{\delta}{\delta A_\mu}\right)^2\right]\left(\phi^*+\frac{\delta}{\delta\phi}\right)e^{-S_\tau} + (\text{c.c.})$$

- Gauge coupling e_τ is defined as

$$e_\tau := e_0 \exp\left(-\int_{\tau_0}^{\tau} d\tau'(D-4+2\gamma_{\tau'})/2\right)$$

- GF-ERG eq. includes third and forth functional derivatives



Modified BRST Invariance

- GF-ERG eq. is invariant under modified BRST trf. due to the scrambler operator:

$$0 = \widetilde{\hat{\delta}_B} e^{-S_\tau} = \hat{s}^{-1} \hat{\delta}_B \hat{s} e^{-S_\tau}$$

$$= \int_x \left[\partial_\mu \left(c + \frac{\delta}{\delta \bar{c}} \right) \cdot \frac{\delta}{\delta A_\mu} + \left(B - \frac{\delta}{\delta B} \right) \frac{\delta}{\delta \bar{c}} + i e_\tau \left(c + \frac{\delta}{\delta \bar{c}} \right) \left(\phi \frac{\delta}{\delta \phi} - \phi^* \frac{\delta}{\delta \phi^*} \right) \right] e^{-S_\tau}$$

- Result I: Assuming

$$S_\tau = S_0[A_\mu, B, c, \bar{c}, \phi, \phi^*] + S_I[\mathcal{A}_\mu, \phi, \phi^*],$$

it reduces to the ordinary WT identity with $\mathcal{A}_\mu$

$$0 = e^{k^2} k_\mu \frac{\delta S_I}{\delta \mathcal{A}_\mu(k)} + i e_\tau \int_p \left[\phi(p+k) \frac{\delta S_I}{\delta \phi(p)} - \phi^*(p+k) \frac{\delta S_I}{\delta \phi^*(p)} \right]$$

$$\mathcal{A}_\mu(k) \coloneqq e^{-k^2} \left(A_\mu(k) + \frac{\delta S^{(0)}}{\delta A_\mu(-k)} \right)$$

$S^{(0)}$: Gaussian fixed-point action

Perturbative Analysis

- Expand S_τ with respect to e_τ:

$$S_\tau = S^{(0)} + e_\tau S^{(1)} + e_\tau^2 S^{(2)} + \cdots$$

- Assume $S^{(i)}$ has no explicit τ-dependence:

$$\partial_\tau S_\tau = \partial_\tau e_\tau \cdot \frac{\partial S}{\partial e_\tau} = -\left(\frac{D-4}{2} + \gamma_\tau\right) e_\tau \frac{\partial S}{\partial e_\tau}$$

- To do

1. Substitute the ansatz into GF-ERG eq. and focus on the terms of each order of e_τ
2. Solve the RG equation order by order
3. Determine integral constants by WT identity

Two Point Function at $O(e_\tau^2)$

- Perturbative solution at second order in four dims.

$$S_{AA}^{(2)} = \frac{1}{2}\int_k V_{\mu\nu}^A(k)\mathcal{A}_\mu(k)\mathcal{A}_\nu(-k)$$

$$V_{\mu\nu}^A(k) = m_A^2\delta_{\mu\nu} + g_1\left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right) + g_2 k_\mu k_\nu + O\left(k^4\right)$$

$$m_A^2 = 0, \quad g_1 = 1/\left(48\pi^2\right), \quad g_2 = 0$$

$\Rightarrow$ consistent with the perturbation theory

- Result 2:

m_A^2 becomes exactly zero in general dims.
$\Rightarrow$ GF-ERG gives a gauge-invariant RG flow?

(c.f. $m_A^2 \neq 0$ in the conventional ERG (WP eq.))

Content

Summary

- Applied Gradient Flow Exact Renormalization Group to Scalar Quantum Electrodynamics
- Gave an RG flow based on the diffusion equation consistent with the off-shell BRST symmetry
- Showed the modified BRST invariance reduces to the ordinary Ward-Takahashi identity under some assumption
- Solved the GF-ERG equation perturbatively
  - The consistent results with the perturbation theory
  - Showed the 1-loop contribution to the photon mass vanishes in general dimensions
- Implication: GF-ERG gives a gauge-invariant RG flow

Discussion

- Consistency with the perturbation theory in the matter sector
  (NOT in QED case, but consistent with those of the gradient-flowed field)

- Application of GF-ERG to quantum gravity
  - Can we define an RG flow in a manifestly diffeomorphism-invariant way?

Thank you!