Particle dispersion in ultralight vector dark matter

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Overview

- Matter effect in neutrinos oscillations

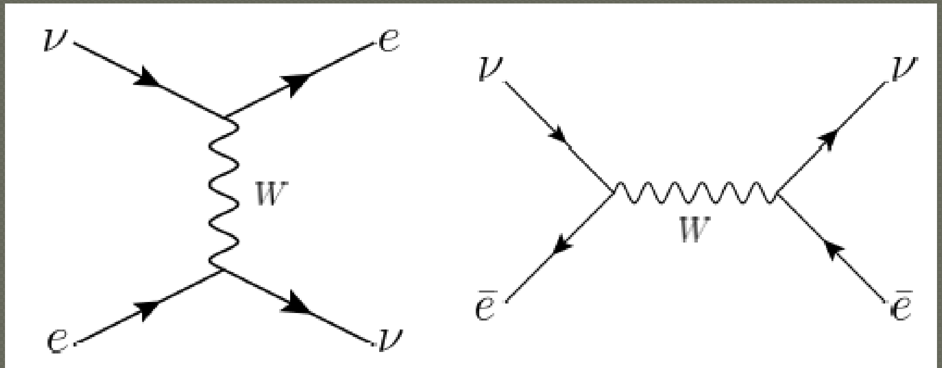
  Wolfenstein 78; Mikheyev-Smirnov 85
- Dispersion & normalization at a finite temperature/density

  Weldon 82; Nieves 89
- Ultra-Light, Fuzzy Dark Matter

  Hu-Barkana-Gruzinov 0003365; Hui-Ostriker-Tremaine-Witten 1610.08297
- Particle dispersion in vector DM EJC-Yoon, 2205.03617

Wolfenstein Effect

- Wolfenstein: "Coherent forward scattering of neutrinos leaving the medium unchanged must be taken into account."



$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F\, \overline{\nu_{eL}}\gamma^\mu e_L\, \overline{e_L}\,\gamma_\mu \nu_{eL} \Rightarrow V_W\, \overline{\nu_{eL}}\gamma^0 \nu_{eL}$$

$$V_W = \sqrt{2}\, G_F N_e$$

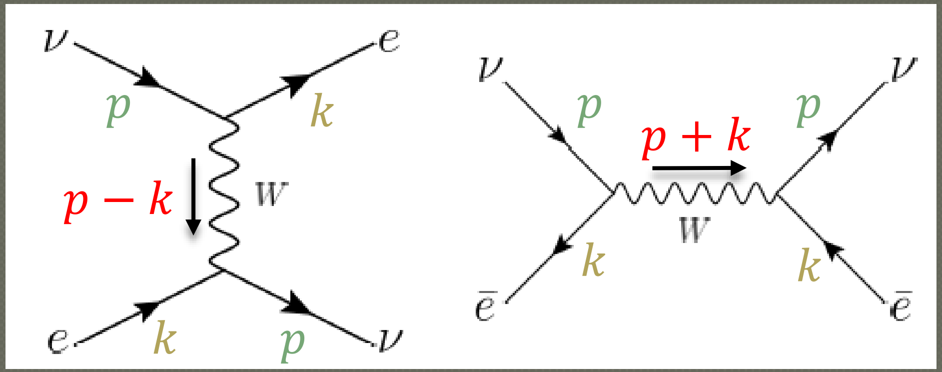
- Neutrino evolution in matter: $H_{\nu,\bar{\nu}} = \frac{M^2}{2E} \pm V_W$

MSW: $G_F N_e \sim \frac{\Delta m_\odot^2}{2E_\odot}$

Wolfenstein effect generalized

- In a medium with arbitrary N_e and $N_{\bar{e}}$

$\nu \Rightarrow$



$\Leftarrow \bar{\nu}$

$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F m_W^2 \frac{\overline{\nu_{eL}}\gamma^\mu e_L\, \overline{e_L}\gamma_\mu \nu_{eL}}{m_W^2 - q^2}$$

$$\langle \mathcal{H}_\nu \rangle = \not{k}\, \sqrt{2} G_F m_W^2 \left[+ \frac{N_e/m_e}{m_W^2 - (p-k)^2} - \frac{N_{\bar{e}}/m_e}{m_W^2 - (p+k)^2} \right]$$

$$\langle \mathcal{H}_{\bar{\nu}} \rangle = \not{k}\, \sqrt{2} G_F m_W^2 \left[+ \frac{N_{\bar{e}}/m_e}{m_W^2 - (p-k)^2} - \frac{N_e/m_e}{m_W^2 - (p+k)^2} \right]$$

$N_e \leftrightarrow N_{\bar{e}}$

$p \leftrightarrow -p$

$+ \leftrightarrow -$

$$\langle N_e, N_{\bar{e}}\, | e\bar{e} | N_e, N_{\bar{e}} \rangle = -\frac{1}{2}\sum_s u_s(k)\bar{u}_s(k)\frac{N_e}{2k^0} + \frac{1}{2}\sum_s v_s(k)\bar{v}_s(k)\frac{N_{\bar{e}}}{2k^0}$$

Wolfenstein effect generalized

- Modified propagator:

$$\mathcal{L}_{kin} \Rightarrow \overline{\nu_L}(p^\mu - k^\mu \Sigma_W)\gamma_\mu \nu_L$$

$$p^\mu = (E, \vec{\mathrm{p}})$$
$$k^\mu = (m_e, \vec{0})$$

$$\Sigma_W = \sqrt{2} G_F m_W^2 \frac{N_{e+\bar{e}}}{m_e} \frac{-2m_e E + \epsilon(m_W^2 - p^2 - m_e^2)}{(m_W^2 - p^2 - m_e^2)^2 - 4m_e^2 E^2}$$

$$\epsilon \equiv \frac{N_e - N_{\bar{e}}}{N_e + N_{\bar{e}}}$$

- Dispersion in the high-energy limit ($E \approx \mathrm{p}$):

$$(p - k\Sigma_W)^2 = (E - m_e \Sigma_W)^2 - \mathrm{p}^2 = m_\nu^2 \xrightarrow[2m_e \mathrm{p} \gg m_W^2 \; (\mathrm{p} \gtrsim 10^7 \mathrm{GeV})]{m_W^2 \gg \mathrm{p}^2} \begin{cases} E_{\nu/\bar{\nu}} \approx \sqrt{\mathrm{p}^2 + m_\nu^2} \pm \epsilon \sqrt{2} G_F N_{e+\bar{e}} + \cdots & \text{Potential} \\ E_{\nu/\bar{\nu}} \approx \sqrt{\mathrm{p}^2 + m_\nu^2 + \sqrt{2} G_F m_W^2 \frac{N_{e+\bar{e}}}{2m_e \mathrm{p}}} + \cdots & \text{Mass^2} \end{cases}$$

Why ULDM?

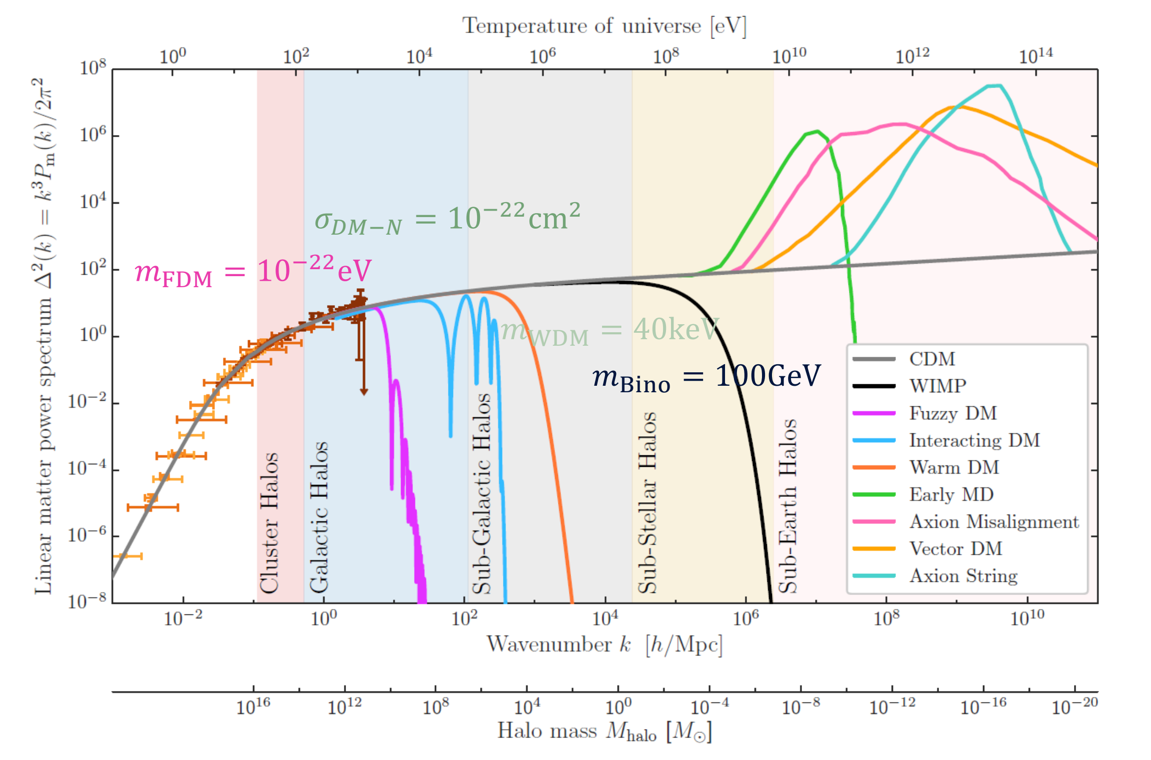
- With the WIMP paradigm approaching the limit, the territory of ideas, imagination, and experiment is expanding, particularly to the direction of arbitrarily small masses and couplings.

- Wave-like property of ULDM as light as $10^{-22}\mathrm{eV}$ shed new light on the properties of ultra-light bosonic (scalar, axion-like, vector) dark matter.

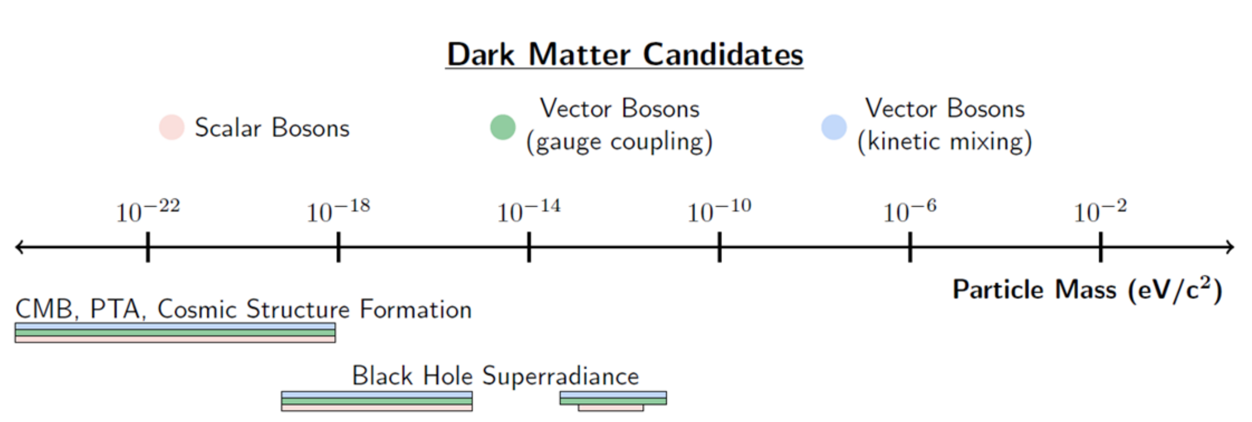
$$\lambda_{\mathrm{dB}} = \frac{2\pi}{mv} = 0.48\mathrm{kpc}\left(\frac{10^{-22}\mathrm{eV}}{m}\right)\left(\frac{250\mathrm{km/s}}{v}\right) = 1\mathrm{au}\left(\frac{10^{-14}\mathrm{eV}}{m}\right)\left(\frac{250\mathrm{km/s}}{v}\right)$$

ULDM and cosmic structures

Snowmass, 2203.07354

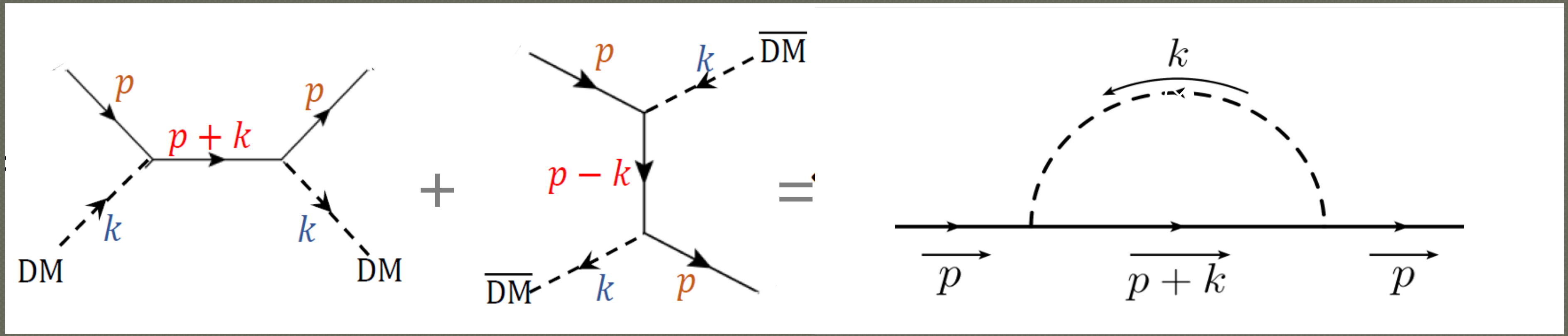


Snowmass, 2203.14915



Propagator in a finite density

- Coherent forward scattering=medium one-loop correction



- Free field solution with modified propagator

$$(\not{p} - \not{\Sigma}(p,k) - m_\psi)\psi = 0$$

Medium effect in particle propagation

- Modified propagator:

$$S_F^{-1} = p \cdot \gamma - m_\psi - \Sigma \cdot \gamma = p \cdot \gamma\left(1 - \Sigma_p\right) - k \cdot \gamma \Sigma_k - m_\psi (1 - \Sigma_m)$$

Dispersion relation:

$$\left(E\left(1 - \Sigma_p\right) - k^0 \Sigma_k\right)^2 = \left(\mathrm{p}\left(1 - \Sigma_p\right) - \mathrm{k}\Sigma_k\right)^2 + m_\psi^2 (1 - \Sigma_m)^2$$

$$\Rightarrow E = E_\mathrm{p}\left(\mathrm{p}, m_\psi; k, g\phi\right) \equiv \sqrt{\mathrm{p}^2 + m_\psi^2 + \Delta}$$

- Field normalization can also be modified:

$$Z = Z\left(\mathrm{p}, m_\psi; k, g\phi\right) \neq 1$$

Field normalization

- Propagator in vacuum:

$$S_F = \frac{1}{\not{p} - m_\psi} = \frac{\not{p} + m_\psi}{p^2 - m_\psi^2} \Longrightarrow \left[\frac{\not{p} + m_\psi}{2E}\right]_{E=\sqrt{\mathrm{p}^2 + m_\psi^2}}$$

- Normalization in medium: $\psi \to Z^{1/2}\,\psi$

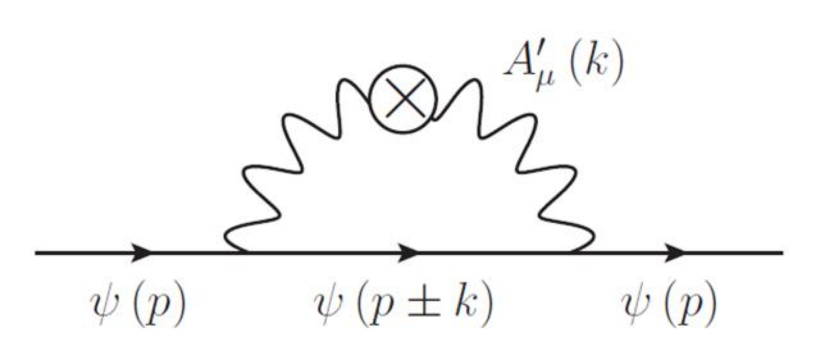
$$S_F = \frac{1}{\not{p} - \not{\Sigma} - m_\psi} \equiv \frac{1}{\not{V} - M_\psi} = \frac{\not{V} + M_\psi}{V^2 - M_\psi^2} \Longrightarrow \left[\frac{\not{V} + M_\psi}{2V_0\left(V_0 - \sqrt{\mathrm{V}_\mathrm{p}^2 + M_\psi^2}\right)}\right]_{\mathrm{pole}}$$

$$\Rightarrow Z = \left[\frac{\partial}{\partial E}\left(V_0 - \sqrt{V_\mathrm{p}^2 + M_\psi^2}\right)\right]^{-1}_{E=E_\mathrm{p}}$$

Medium effect of vector DM

$$\mathcal{L}' = g_L A'_\mu \overline{\psi_L} \gamma^\mu \psi_L + g_R A'_\mu \overline{\psi_R} \gamma^\mu \psi_R \quad \text{(vector)}$$

ψ : Dirac or Majorana ($\psi_R = \psi_L^c$)



$$\not\Sigma = g g^\dagger \int \frac{d^4k}{(2\pi)^4} \Delta_{\mu\nu}(k) \Gamma^\mu S_\psi(p+k) \Gamma^\nu$$

$$\Delta_{\mu\nu} = 2\pi\delta\left(k^2 - m_\phi^2\right) f_\phi(k) \sum_a \xi^a \, \epsilon_\mu^a \epsilon_\nu^a$$

$$f_\phi(k) = (2\pi)^3 \delta^3\left(\vec{k} - \vec{k}_\phi\right)\left(\theta(k^0) n_\phi - \theta(-k^0) n_{\bar{\phi}}\right)$$

Modified propagator

- Consider a unpolarized VDM with $\xi^a = \frac{1}{3}$ & $k \approx (m_\phi, \vec{0})$

$$S_F^{-1} = p\cdot\gamma - m_\psi - \Sigma\cdot\gamma = p\cdot\gamma\left(1-\Sigma_p\right) - k\cdot\gamma\Sigma_k - m_\psi(1-\Sigma_m)$$

$$\Sigma_p = \frac{\delta m^2}{3}\frac{\Delta + m_{\gamma'}^2}{\left(\Delta + m_{\gamma'}^2\right)^2 - 4m_{\gamma'}^2 E^2}$$

$$\Sigma_k = \frac{\delta m^2}{3}\frac{2(\Delta - 2m_{\gamma'}^2)}{\left(\Delta + m_{\gamma'}^2\right)^2 - 4m_{\gamma'}^2 E^2}\frac{E}{m_{\gamma'}}$$

$$\Sigma_m = -3\Sigma_p$$

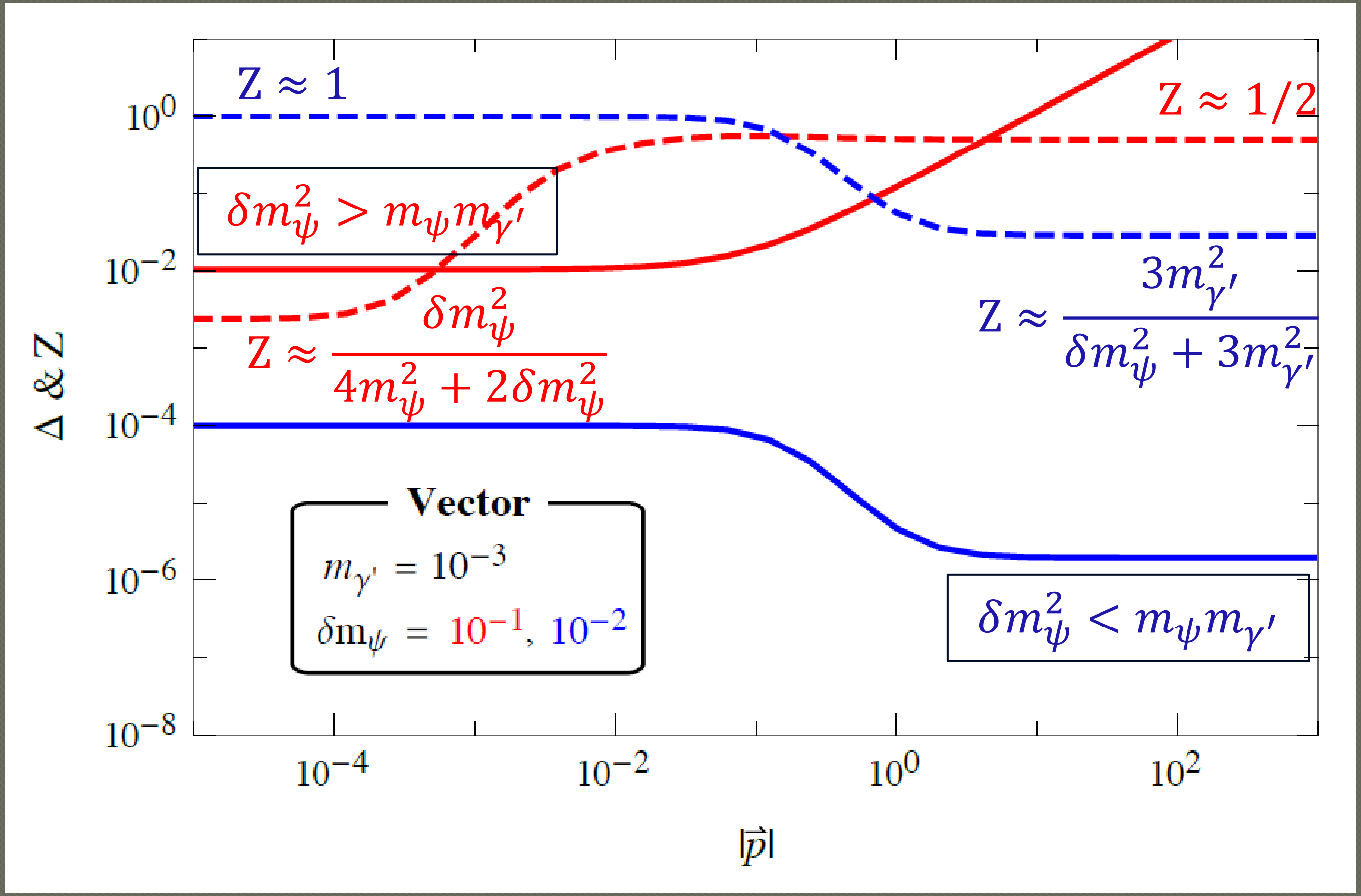
$$\delta m^2 \equiv |g|^2 \frac{n_{\gamma'}}{m_\phi}$$

$$E = \sqrt{p^2 + \mathrm{p}^2} \equiv \sqrt{\mathrm{p}^2 + m_\psi^2 + \Delta}$$

Dispersion and normalization

$\Delta \approx \delta m_\psi^2$

$\Delta \approx \delta m_\psi^2$



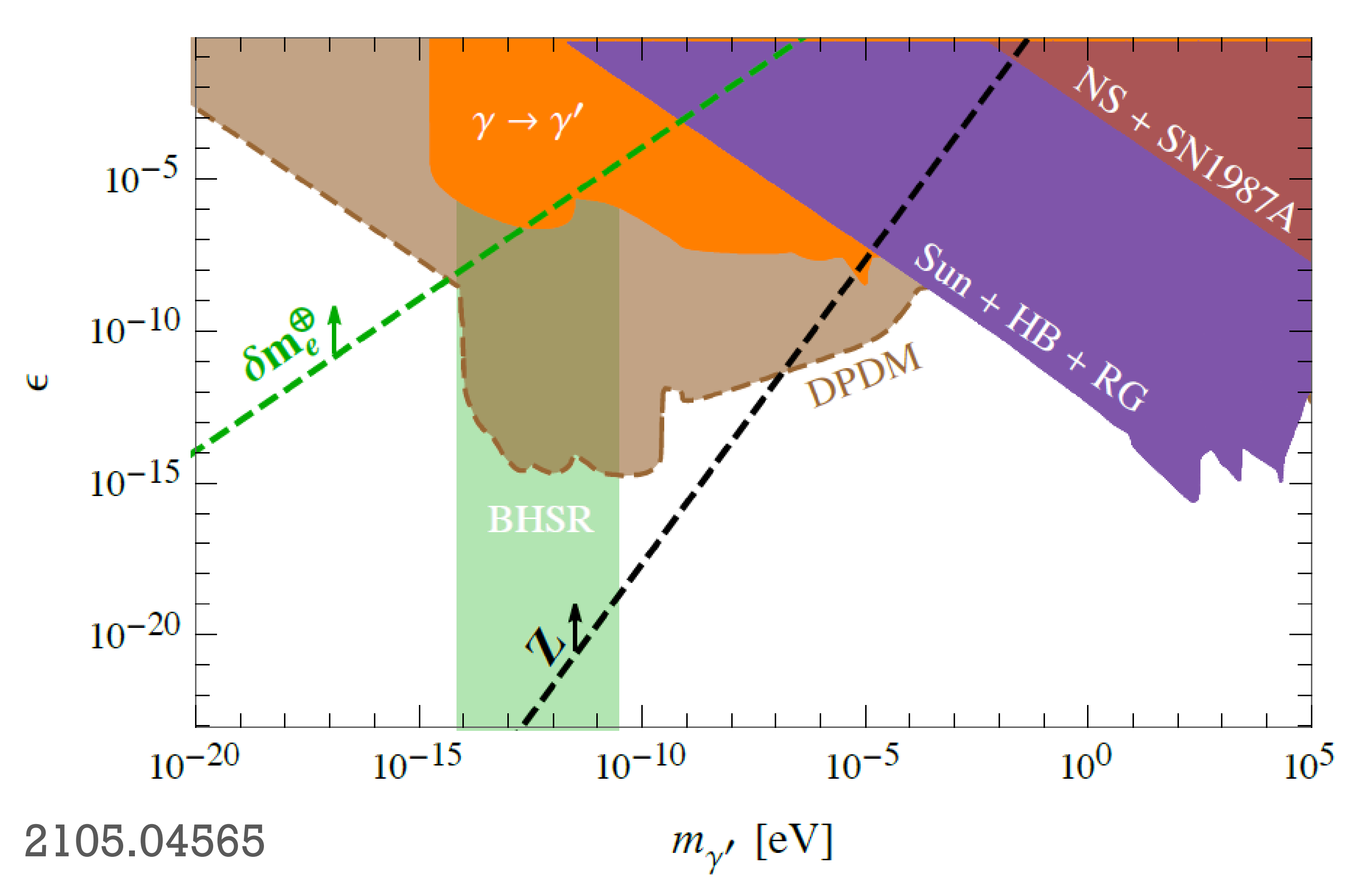
$\Delta \approx \frac{2}{\sqrt{3}} \delta m_\psi \, \mathrm{p}$

$\Delta \approx \frac{2\delta m_\psi^2 m_{\gamma'}^2}{\delta m_\psi^2 + 3m_{\gamma'}^2}$

Constraining VDM couplings

- The rest mass correction, $E \sim \sqrt{m_\psi^2 + \delta m_\psi^2}$, may be in conflict with the observations $\left(m_\psi\right)_{\rm obs.}$: m_e and $m_\nu^{\rm eq}$.
- In high-momentum limit, $\Delta \propto \mathrm{p}\, \delta m_\nu$ amounts to add an constant potential $\delta E_\nu \propto \delta m_\nu$ spoiling the MSW effect if VDM is flavor-dependent.
- The normalization $Z = 1/2$ in the relativistic limit contradicts to various SM precision measurements such as lepton-flavor universality and so on.

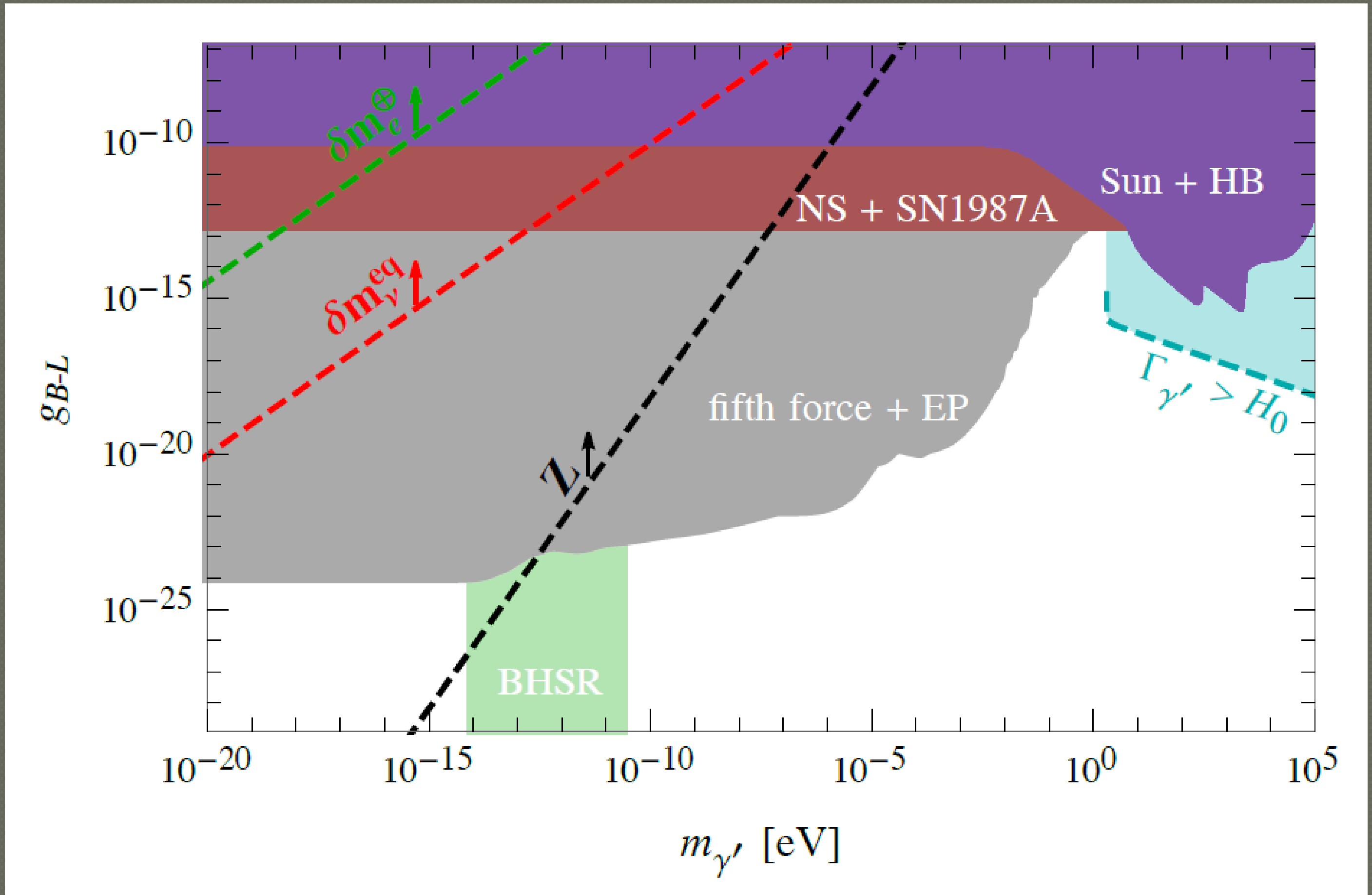
Constraints on the kinetic mixing



$Z_l \neq Z_\nu = 1$

$Z_V = Z_H = 1$

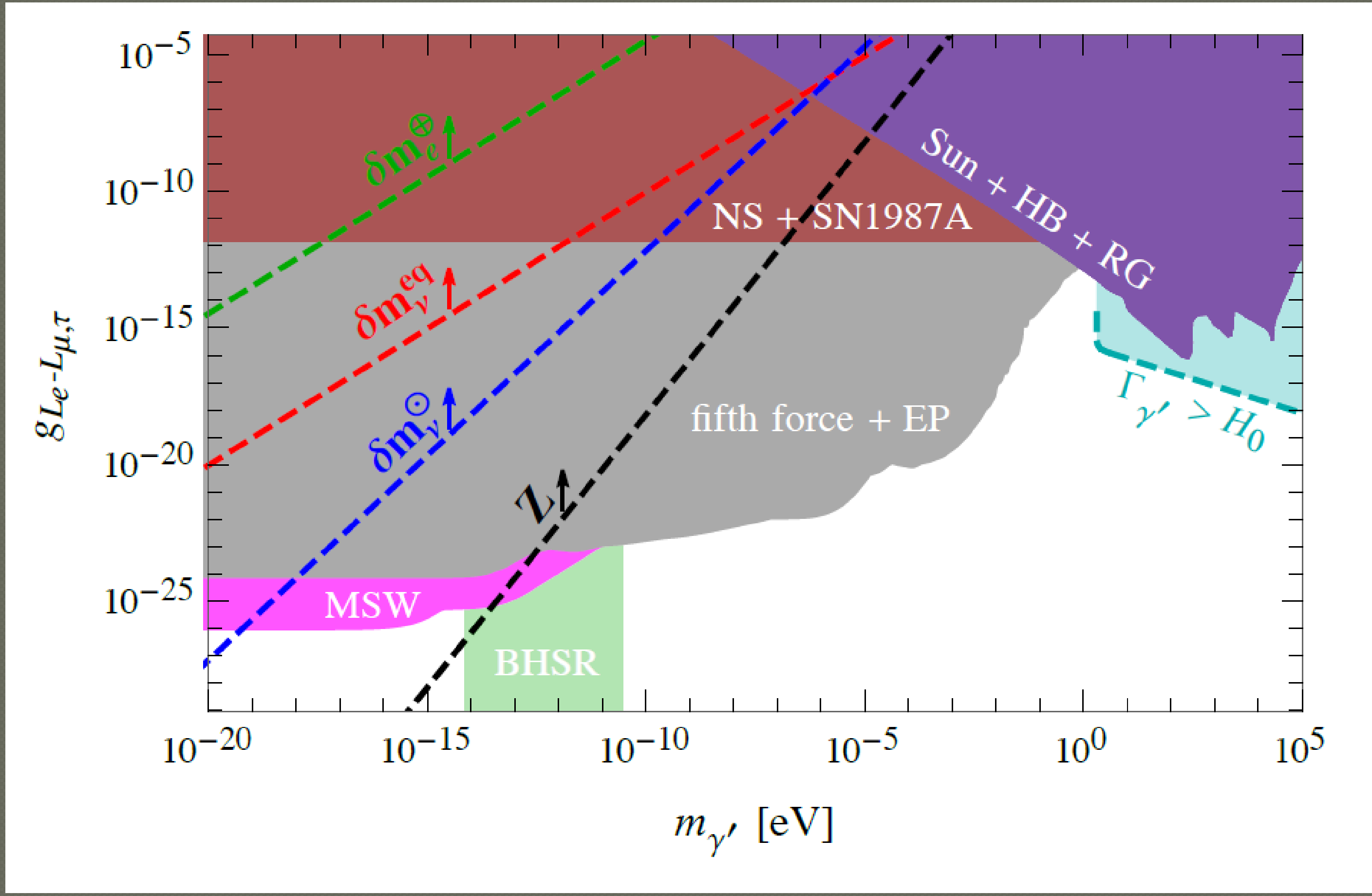
Constraints on $B-L$



$Z_{q,l} \neq 1$

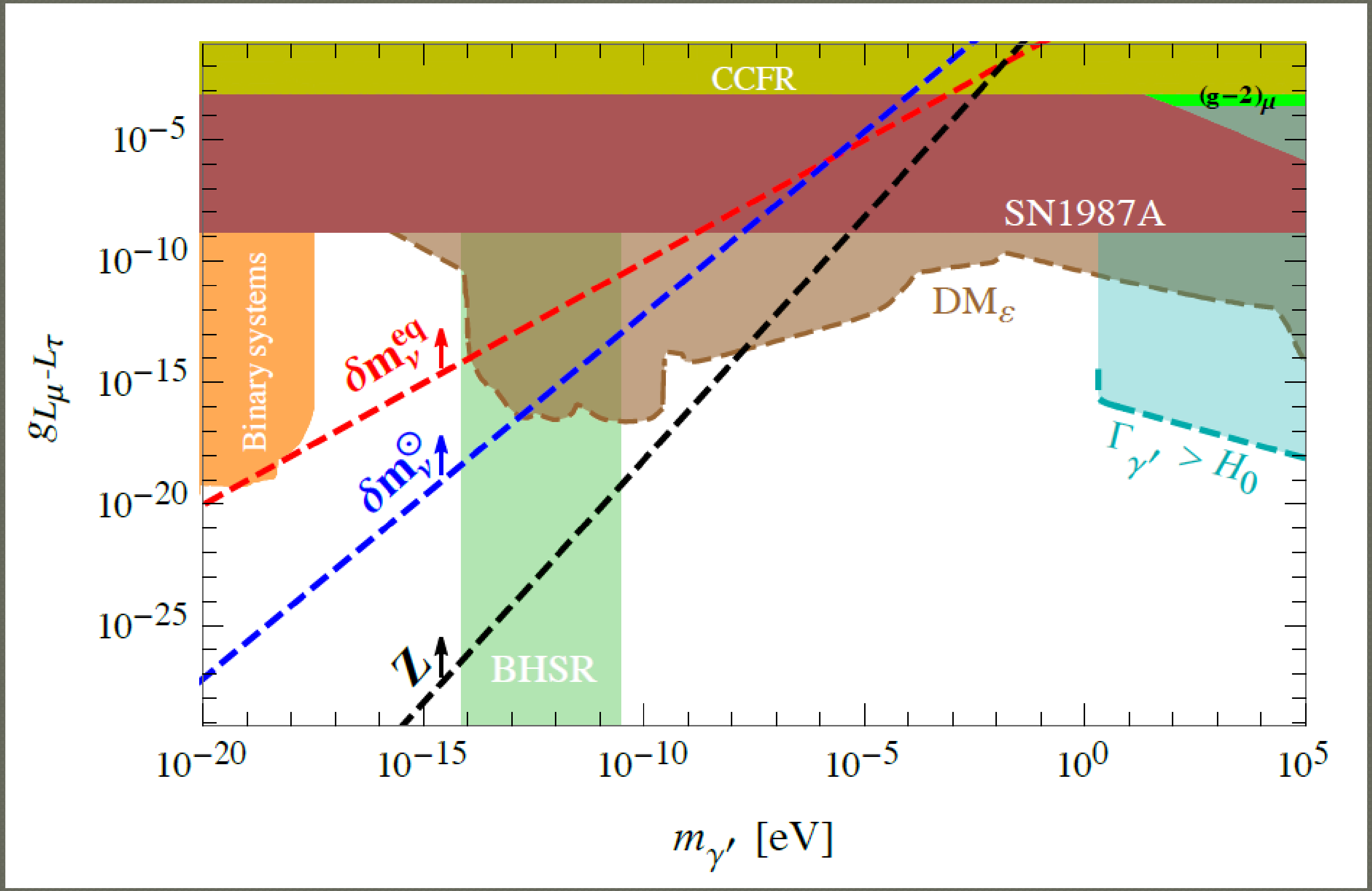
$Z_V = Z_H = 1$

Constraints on $L_e - L_{\mu,\tau}$



$Z_{l_e,l_{\mu,\tau}} \neq 1$

Constraints on $L_\mu - L_\tau$



$\delta m_\nu^{\rm eq} \sim T_{\rm eq}^{3/2}$

$\delta E_\nu \sim \delta m_\nu$

$Z_{\mu,\tau} \neq 1$

Summary

- When SM fermions couple to ULDM, the medium effect in their dispersion and normalization has to considered.

- The medium-induced rest mass, or potential can be sizable and thus constrained by the observations.

- For VDM, a peculiar field normalization $Z = 1/2$ appears in the relativistic regime which highly constrains the parameter space.