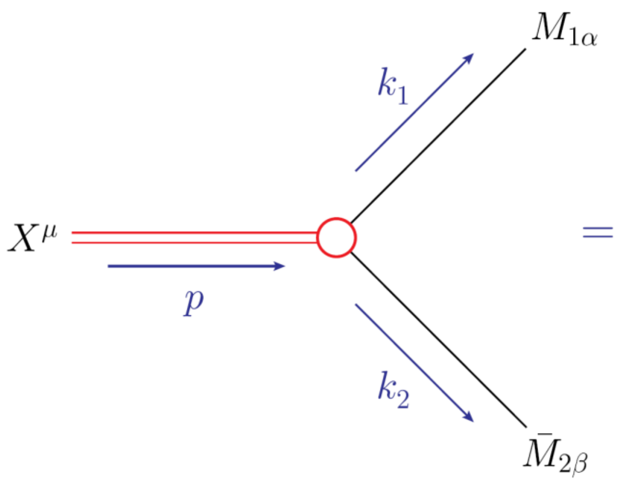
Constructing the covariant vertices systematically

Seong Youl Choi (Jeonbuk, Korea)



$= \boxed{i\,\Gamma^{\mu}_{\alpha,\beta}(p,q)}$?

SYC, Jae Hoon Jeong
PRD 103 (2021) 096013
PRD 104 (2021) 055046
PRD 105 (2022) 016016

3rd AEI workshop, Japan, October 4, 2023

Introduction

The Standard Model is established but not yet complete.

For an authoritative SM history,
Steven Weinberg,
PRL 121 (2018) 220001

Beyond the Standard Model?

⇩

Top-down route
A Lagrangian ⇒ vertices and propagators

Bottom-up effective field theory (EFT) path
All the Lorentz-invariant local vertices, …

Steven Weinberg, EPJH (2021) 46:6
All Things EFT @ YouTube

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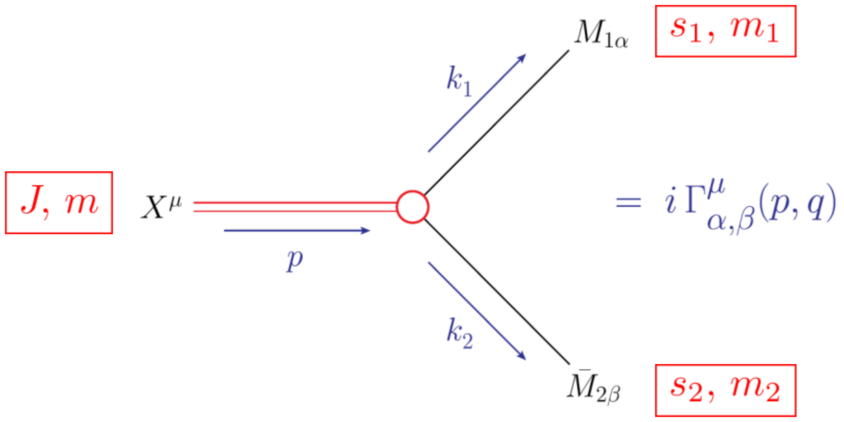
I describe an algorithm for constructing the covariant effective 3-point vertices systematically and mention a few applications.

Key Connection

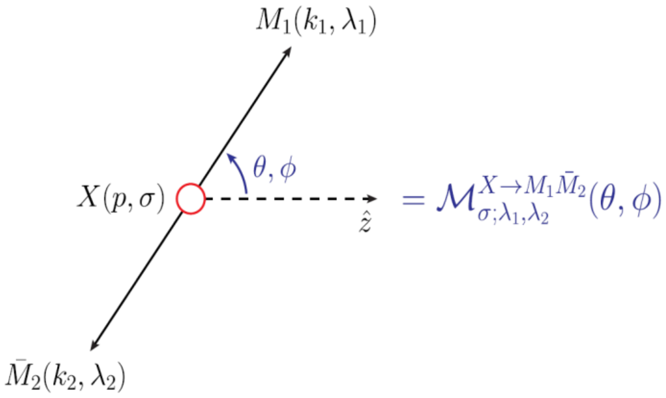
Covariant



$p = k_1 + k_2$ and $q = k_1 - k_2$

$\Longleftrightarrow$

X rest frame (XRF)



helicities: $\sigma,\ \lambda_1,\ \lambda_2$

Decay helicity amplitude

Jacob, Wick
AP7 (1959) 404

$$\mathcal{M}^{X\to M_1\bar{M}_2}_{\sigma;\lambda_1,\lambda_2}(\theta,\phi) = \mathcal{C}^{J}_{\lambda_1,\lambda_2}\, d^{J}_{\sigma,\,\lambda_1-\lambda_2}(\theta)\, e^{i(\sigma-\lambda_1+\lambda_2)\phi} \quad \text{with} \quad |\lambda_1-\lambda_2| \le J$$

XRF

$$\Leftrightarrow \quad \bar{\psi}_1^{\alpha_1\cdots\alpha_{n_1}}(k_1,\lambda_1)\;\Gamma^{\mu_1\cdots\mu_n}_{\alpha_1\cdots\alpha_{n_1},\beta_1\cdots\beta_{n_2}}(p,q)\;\psi_2^{\beta_1\cdots\beta_{n_2}}(k_2,\lambda_2)\;\psi_{\mu_1\cdots\mu_n}(p,\sigma)$$

valid in any frame

wave tensors

Number of independent terms

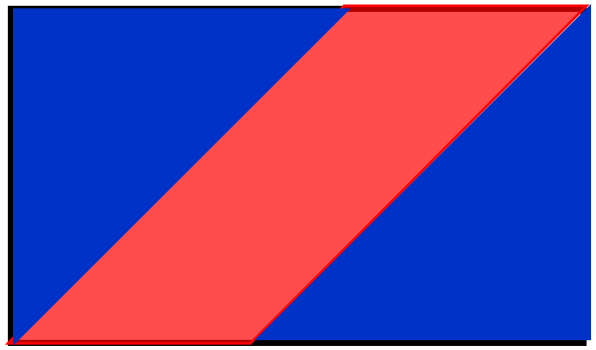
Massive

$\lambda_a = -s_a, -s_a+1, \cdots, s_a-1, s_a$ for $a = 1,2$ with $|\lambda_1-\lambda_2| \le J$

$$n^{J,s_1,s_2}_{m_1,m_2} = \begin{cases} (2s_1+1)(2s_2+1) & \text{for } J \ge s_1+s_2 \\ (2s_1+1)(2s_2+1) \\ -(s_1+s_2-J)(s_1+s_2-J+1) & \text{for } |s_1-s_2| \le J < s_1+s_2 \\ (s_1+s_2-|s_1-s_2|+1)\times(2J+1) & \text{for } J < |s_1-s_2| \end{cases}$$

SU Chung
PRD 57 (1998) 431

helicity lattice space



$s_1 > s_2$

⇩

$$n^{J,s,s}_{m_1,m_2} = \begin{cases} (2s+1)^2 & \text{for } J \ge 2s \\ 2s+1+(4s+1)J-J^2 & \text{for } J < 2s \end{cases}$$

$n^{J,0,0}_{m_1,m_2} = 1, \quad n^{0,s,s}_{m_1,m_2} = 2s+1, \quad n^{1,1/2,1/2}_{m_1,m_2} = 4, \quad n^{1,1,1}_{m_1,m_2} = 7, \quad n^{J,1,1}_{m_1,m_2} = 9$ for $J \ge 2$

Spin-s wave tensors

Behrends, Fronsdal, PR 106 (1957) 345
Scadron, PR 165 (1958) 1640

Spinless $s = 0$

$$\varphi(p) = 1 \;\; \text{independent of mass}$$

Integer spin $s = n \neq 0$

massive

$$\epsilon_{\mu_1\cdots\mu_n}(p,\sigma) = \sqrt{\frac{2^n\,(n+\sigma)!\,(n-\sigma)!}{(2n)!}} \sum_{\{\tau_i\}=\pm1,0} \delta_{\tau_1+\cdots+\tau_n,\,\sigma} \prod_{i=1}^{n} \frac{\epsilon_{\mu_i}(p,\tau_i)}{\sqrt{2}^{|\tau_i|}}$$

massless

$$\epsilon_{\mu_1\cdots\mu_n}(p,\pm s) = \epsilon_{\mu_1}(p,\pm1)\cdots\epsilon_{\mu_n}(p,\pm1)$$

$$\sigma = -s, -s+1, \ldots, s-1, s$$

$$p\cdot\epsilon(p,\tau_i) = 0 = \epsilon(p,+1)\cdot\epsilon(p,-1) + \epsilon(p,0)\cdot\epsilon(p,0)$$

Half-integer spin $s = n + 1/2$

massive

$$u_{\mu_1\cdots\mu_n}(p,\sigma) = \sum_{\tau=\pm1/2} \sqrt{\frac{J+2\tau\sigma}{2J}}\, \epsilon_{\mu_1\cdots\mu_n}(p,\sigma-\tau)\, u(p,\tau)$$

$$v_{\mu_1\cdots\mu_n}(p,\sigma) = \sum_{\tau=\pm1/2} \sqrt{\frac{J+2\tau\sigma}{2J}}\, \epsilon^*_{\mu_1\cdots\mu_n}(p,\sigma-\tau)\, v(p,\tau)$$

massless

$$u_{\mu_1\cdots\mu_n}(p,\pm s) = \epsilon_{\mu_1\cdots\mu_n}(p,\pm n)\, u(p,\pm1/2)$$

$$v_{\mu_1\cdots\mu_n}(p,\pm s) = \epsilon^*_{\mu_1\cdots\mu_n}(p,\pm n)\, v(p,\pm1/2)$$

General properties of the wave tensors

Behrends, Fronsdal,
PR 106 (1957) 345
Scadron,
PR 165 (1958) 1640

totally symmetric

$\psi = \epsilon,\ u,\ v$

$$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\psi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}(p,\sigma) = 0$$

traceless

$$g_{\mu_i\mu_j}\,\psi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}(p,\sigma) = 0$$

divergence-free

$$p_{\mu_i}\,\psi^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = 0$$

fermionic divergence-free

$$\gamma_{\mu_i}\,u^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = 0 \quad \text{and} \quad \gamma_{\mu_i}\,v^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = 0$$

Bosonic transition operators

SYC, JH Jeong
PRD 105 (2022) 016016

$$X(1,m) \to M_1(1,m_1) + \bar{M}_2(1,m_2) \quad \text{with} \quad n^{1,1,1}_{m_1,m_2} = 7$$

Each exclusive helicity combination

$$U^0_{\alpha\beta}\,\hat{k}_\mu = \hat{p}_{1\alpha}\hat{p}_{2\beta}\,\hat{k}_\mu \quad \leftrightarrow \quad \mathcal{C}^1_{0,0} = \kappa^2$$

$$U^\pm_{\alpha\mu}\,\hat{p}_{2\beta} = \frac{1}{2}\left[g_{\perp\alpha\mu} \pm i\langle\alpha\mu\hat{p}\hat{k}\rangle\right]\hat{p}_{2\beta} \quad \leftrightarrow \quad \mathcal{C}^1_{\pm1,0} = \kappa$$

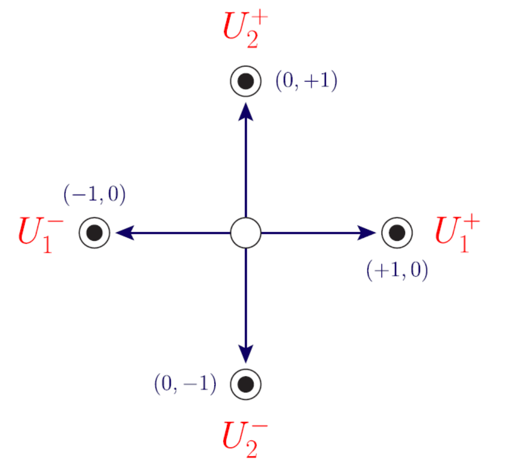
$$U^\pm_{\beta\mu}\,\hat{p}_{1\alpha} = \frac{1}{2}\left[g_{\perp\beta\mu} \pm i\langle\beta\mu\hat{p}\hat{k}\rangle\right]\hat{p}_{1\alpha} \quad \leftrightarrow \quad \mathcal{C}^1_{0,\mp1} = -\kappa$$

$$U^\pm_{\alpha\beta}\,\hat{k}_\mu = \frac{1}{2}\left[g_{\perp\alpha\beta} \pm i\langle\alpha\beta\hat{p}\hat{k}\rangle\right]\hat{k}_\mu \quad \leftrightarrow \quad \mathcal{C}^1_{\pm1,\pm1} = -1$$

$$[U_1^\pm]_{\alpha\mu} = U^\pm_{\alpha\mu}$$

$$[U_2^\pm]_{\beta\mu} = U^\mp_{\beta\mu}$$

$$[U^\pm]_{\alpha\beta} = g^{\mu_1\mu_2}U^\pm_{\alpha\mu_1}U^\mp_{\beta\mu_2}$$



$$\omega_{1,2} = m_{1,2}/m, \quad \eta^\pm = \sqrt{1-(\omega_1 \pm \omega_2)^2} \quad \text{and} \quad \kappa = \eta^+\eta^-$$

$$p = m\,\hat{p}, \quad q = m(\omega_1^2 - \omega_2^2)\,\hat{p} + m\kappa\,\hat{k} \quad \text{and} \quad \hat{p}_{1,2} = 2\omega_{1,2}\,\hat{p}$$

$$g_{\perp\mu\nu} = g_{\mu\nu} - \hat{p}_\mu\hat{p}_\nu + \hat{k}_\mu\hat{k}_\nu \quad \text{and} \quad \langle\mu\nu\hat{p}\hat{k}\rangle = \varepsilon_{\mu\nu\rho\sigma}\hat{p}^\rho\hat{k}^\sigma$$

4 fundamental bosonic operators: $\hat{k}$, $\hat{p}$ and $U^\pm$

Fermionic transition operators

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PRD 105 (2022) 016016

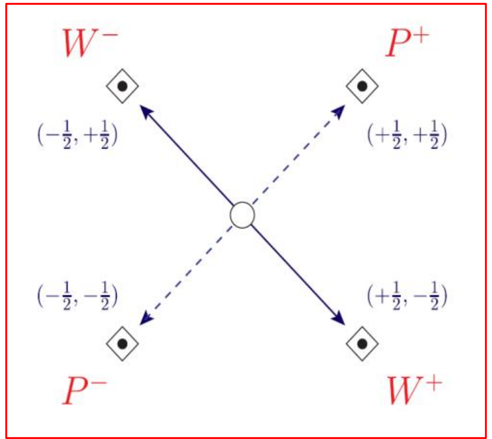
$$X(1,m) \to M_1(1/2,m_1) + \bar{M}_2(1/2,m_2) \quad \text{with} \quad n^{1,1/2,1/2}_{m_1,m_2} = 4$$

Each exclusive helicity combination

$$P^{\pm}\,\hat{k}_\mu = \frac{1}{2m}(\eta^- \mp \eta^+\gamma_5)\,\hat{k}_\mu \qquad \leftrightarrow \qquad \mathcal{C}^1_{\pm 1/2,\pm 1/2} = -\kappa$$

$$W^{\pm}_\mu = \frac{1}{2\sqrt{2}m}(\pm\eta^+\gamma^+_\mu + \eta^-\gamma^-_\mu\gamma_5) \qquad \leftrightarrow \qquad \mathcal{C}^1_{\pm 1/2,\mp 1/2} = \pm\kappa$$

$$\gamma^{\pm}_\mu = \gamma_\mu + \frac{(\omega_1 \pm \omega_2)\,\kappa}{1-(\omega_1 \pm \omega_2)^2}\,\hat{k}_\mu$$



4 fundamental fermionic operators: $P^{\pm}$ and $W^{\pm}$

Compact square-bracket notations

$$[\hat{k}]^n \rightarrow \hat{k}_{\mu_1} \cdots \hat{k}_{\mu_n}$$

$$[\hat{p}_1]^n \rightarrow \hat{p}_{1\alpha_1} \cdots \hat{p}_{1\alpha_n}$$

$$[\hat{p}_2]^n \rightarrow \hat{p}_{2\beta_1} \cdots \hat{p}_{2\beta_n}$$

$$[U^{\pm}]^n \rightarrow U^{\pm}_{\alpha_1\beta_1} \cdots U^{\pm}_{\alpha_n\beta_n}$$

$$[U_1^{\pm}]^n \rightarrow U^{\pm}_{\alpha_1\mu_1} \cdots U^{\pm}_{\alpha_n\mu_n}$$

$$[U_2^{\pm}]^n \rightarrow U^{\mp}_{\beta_1\mu_1} \cdots U^{\mp}_{\beta_n\mu_n}$$

Covariant 3-point helicity-specific vertices

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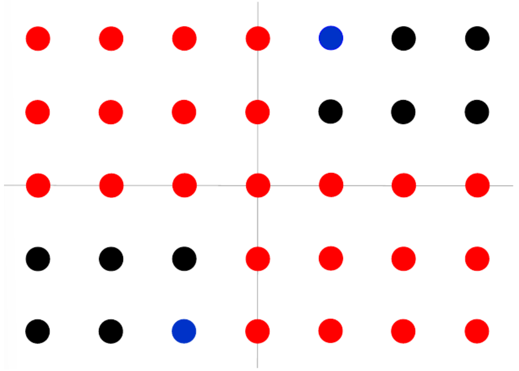
PRD 105 (2022) 016016

$$[\,\mathcal{H}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}\,] = [\,\hat{k}\,]^{J-|\lambda_1-\lambda_2|}\,[\,\hat{p}_1\,]^{s_1-|\lambda_1|}\,[\,\hat{p}_2\,]^{s_2-|\lambda_2|}\quad[\,\mathcal{T}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}\,]\quad \text{with}\quad |\lambda_1-\lambda_2|\leq J$$

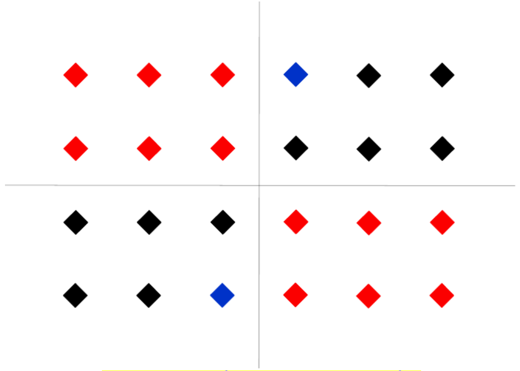
$$m_a = 0 \;\Rightarrow\; \lambda_a = \pm s_a \;\rightarrow\; [\,\hat{p}_a\,]^{s_a-|\lambda_a|} = 1 \;\text{ for }\; a = 1,2$$

$s_1 = 3,\ s_2 = 2$



$$[\,\mathcal{T}^{J,s_1,s_2}_{\mathrm{bbb}[\lambda_1,\lambda_2]}\,] = \begin{cases} [\,U^{\pm}\,]^{|\lambda_2|}\,[\,U_1^{\pm}\,]^{|\lambda_1-\lambda_2|} & \text{for}\;\; \lambda_{1,2}=\pm|\lambda_{1,2}| \;\text{ and }\; 0<|\lambda_2|\leq|\lambda_1| \\ [\,U^{\pm}\,]^{|\lambda_1|}\,[\,U_2^{\pm}\,]^{|\lambda_1-\lambda_2|} & \text{for}\;\; \lambda_{1,2}=\pm|\lambda_{1,2}| \;\text{ and }\; 0<|\lambda_1|<|\lambda_2| \\ [\,U_1^{\pm}\,]^{|\lambda_1|}\,[\,U_2^{\mp}\,]^{|\lambda_2|} & \text{for}\;\; \lambda_1=\pm|\lambda_1| \;\text{ and }\; \lambda_2=\mp|\lambda_2| \end{cases}$$

$$[\,\mathcal{T}^{J,s_1,s_2}_{\mathrm{bff}[\lambda_1,\lambda_2]}\,] = \begin{cases} [\,P^{\pm}\,]\,[\,U^{\pm}\,]^{|\lambda_2|-1/2}\,[\,U_1^{\pm}\,]^{|\lambda_1-\lambda_2|} & \text{for}\;\; \lambda_{1,2}=\pm|\lambda_{1,2}| \;\text{ and }\; |\lambda_2|\leq|\lambda_1| \\ [\,P^{\pm}\,]\,[\,U^{\pm}\,]^{|\lambda_1|-1/2}\,[\,U_2^{\pm}\,]^{|\lambda_1-\lambda_2|} & \text{for}\;\; \lambda_{1,2}=\pm|\lambda_{1,2}| \;\text{ and }\; |\lambda_1|<|\lambda_2| \\ [\,W^{\pm}\,]\,[\,U_1^{\pm}\,]^{|\lambda_1|-1/2}\,[\,U_2^{\mp}\,]^{|\lambda_2|-1/2} & \text{for}\;\; \lambda_1=\pm|\lambda_1| \;\text{ and }\; \lambda_2=\mp|\lambda_2| \end{cases}$$



$s_1 = 5/2,\ s_2 = 3/2$

Crossing relations $\Rightarrow$ $[\,\mathcal{H}^{J,s_1,s_2}_{\mathrm{ffb}[\lambda_1,\lambda_2]}\,]$ and $[\,\mathcal{H}^{J,s_1,s_2}_{\mathrm{fbf}[\lambda_1,\lambda_2]}\,]$

SYC, JH Jeong
PRD 105 (2022) 016016

General covariant 3-point vertices

$$[\Gamma_A] = \sum\nolimits'_{\lambda_1,\lambda_2} c^{J,s_1,s_2}_{\lambda_1,\lambda_2} \, [\mathcal{H}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}] \quad \text{with} \quad |\lambda_1 - \lambda_2| \leq J$$

A linear combination of all the helicity-specific 3-point vertices

Covariant 3-point vertices of two identical particles

Kayser, PRD 26 (1982) 1662
SYC, JH Jeong
PRD 104 (2021) 055046

Bose symmetry $\Gamma^{\mu}_{\beta,\alpha}(p,-q) = \Gamma^{\mu}_{\alpha,\beta}(p,q)$

Fermi symmetry $C\Gamma^{\mu}_{\beta,\alpha}(p,-q)C^{-1} = \Gamma^{\mu}_{\alpha,\beta}(p,q)$

$$\Gamma^{\mu}_{\beta,\alpha}(p,-q) \rightarrow \begin{cases} U^{\pm}_{\alpha\beta} \rightarrow U^{\pm}_{\alpha\beta} \\ U^{\pm}_{\alpha\mu} \leftrightarrow U^{\mp}_{\beta\mu} \end{cases}$$

$$C\Gamma^{\mu}_{\beta,\alpha}(p,-q)C^{-1} \rightarrow \begin{cases} P^{\pm} \rightarrow P^{\pm} \\ W^{\pm}_{\mu} \rightarrow W^{\mp}_{\mu} \end{cases}$$

$$[\mathcal{H}^{J,s,s}_{A[\lambda_1,\lambda 2]}]_{\rm IP} = [\mathcal{H}^{J,s,s}_{A[\lambda_1,\lambda_2]}] + (-1)^{J-|\lambda_1-\lambda_2|}[\mathcal{H}^{J,s,s}_{A[\lambda_2,\lambda_1]}] \quad \text{with} \quad |\lambda_1-\lambda_2| \leq J$$

$$[\mathcal{H}^{J,s,s}_{A[\lambda_1,\lambda_2]}]_{\rm IP} = (-1)^{J-|\lambda_1-\lambda_2|}[\mathcal{H}^{J,s,s}_{A[\lambda_2,\lambda_1]}]_{\rm IP}$$

$$[\mathcal{H}^{J,s,s}_{A[\lambda,\lambda]}]_{\rm IP} = [\mathcal{H}^{J,s,s}_{A[\lambda,\lambda]}] + (-1)^{J}[\mathcal{H}^{J,s,s}_{A[\lambda,\lambda]}] \Rightarrow 0 \quad \text{for odd } J$$

3-point vertices of two identical massless particles

(generalized LY theorem)

SYC, JH Jeong
PRD 104 (2021) 055046

$$m_{1,2} = 0 \quad \Rightarrow \quad \lambda_{1,2} = \pm s_{1,2} \quad \rightarrow \quad [\,\hat{p}_1\,]^{s_1 - |\lambda_1|} \; = \; [\,\hat{p}_2\,]^{s_2 - |\lambda_2|} \; = 1$$

$$[\,\mathcal{H}^{J,s,s}_{\rm bbb[\pm s,\pm s]}\,]_{\rm IP} \; = \; [\,\hat{k}\,]^J\,[1 + (-1)^J]\,[U^\pm]^s$$

$$[\,\mathcal{H}^{J,s,s}_{\rm bff[\pm s,\pm s]}\,]_{\rm IP} \; = \; [\,\hat{k}\,]^J\,[1 + (-1)^J]\,[P^\pm][U^\pm]^{s-1/2}$$

$\# \; = \; 2/0$ for even/odd J

$$\begin{aligned} [\,\mathcal{H}^{J,s,s}_{\rm bbb[+s,-s]}\,]_{\rm IP} \; &= \; (-1)^{J-2s}\,[\,\mathcal{H}^{J,s,s}_{\rm bbb[-s,+s]}\,]_{\rm IP} \\ &= \; [\,\hat{k}\,]^{J-2s}\Big([U_1^+ U_2^-]^s + (-1)^{J-2s}[U_1^- U_2^+]^s\Big) \quad \text{with} \quad J \geq 2s \end{aligned}$$

$\# \; = \; 1$

$$\begin{aligned} [\,\mathcal{H}^{J,s,s}_{\rm bff[+s,-s]}\,]_{\rm IP} \; &= \; (-)^{J-2s}\,[\,\mathcal{H}^{J,s,s}_{\rm bff[-s,+s]}\,]_{\rm IP} \\ &= \; [\,\hat{k}\,]^{J-2s}\Big([W^+][U_1^+ U_2^-]^{s-1/2} + (-1)^{J-2s}[W^-][U_1^- U_2^+]^{s-1/2}\Big) \quad \text{with} \quad J \geq 2s \end{aligned}$$

No particle with odd J < 2s can decay into two identical massless spin-s particles.

Number of independent terms (IP)

SYC, JH Jeong
PRD 104 (2021) 055046

Massive

$$n^{J,s,s}_{m,m;\mathrm{IP}} = \begin{cases} \frac{1}{2}(2s+1)[1+(-1)^J] + s(2s+1) & \text{for} \quad J \geq 2s \\ \frac{1}{2}(2s+1)[1+(-1)^J] + \frac{1}{2}[(4s+1)J - J^2] & \text{for} \quad J < 2s \end{cases}$$

Generalized Landau-Yang (LY) theorem

Landau, DANS 60 (1948) 207
Yang, PR 77 (1950) 242
SYC, JH Jeong
PRD 103 (2021) 096013

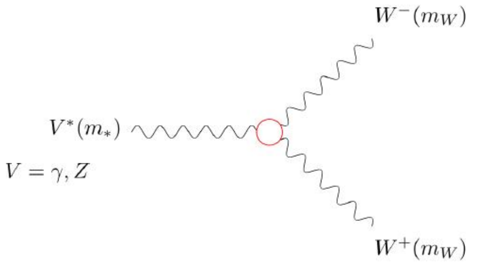
$$n^{J,s,s}_{0,0;\mathrm{IP}} = 1 + (-1)^J + \Theta(J-2s) \quad \text{for} \quad s > 0$$

$$n^{J,0,0}_{0,0;\mathrm{IP}} = \frac{1}{2}[1+(-1)^J]$$

$$n^{J,0,0}_{m,m;\mathrm{IP}} = n^{J,0,0}_{0,0;\mathrm{IP}} = \frac{1}{2}[1+(-1)^J], \quad n^{0,s,s}_{m,m;\mathrm{IP}} = 2s+1, \quad n^{1,s,s}_{m,m;\mathrm{IP}} = 2s, \quad n^{1,1,1}_{0,0;\mathrm{IP}} = 0 \ \ (\mathrm{LY}), \quad n^{2,1,1}_{0,0;\mathrm{IP}} = 3, \quad n^{3,1,1}_{0,0;\mathrm{IP}} = 1$$

Application 1: $(e^- e^+ \to) V^* \to W^- W^+$

Hagiwara, Peccei, Zeppenfeld, Hikasa, NPB 282 (1987) 253



$m_e \approx 0$ ⇒

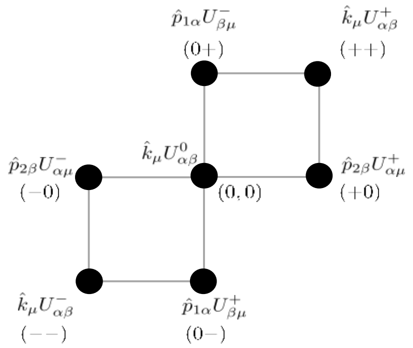
$$\Gamma_{\alpha,\beta;\mu} = f_1^V k_\mu g_{\alpha\beta} - \frac{f_2^V}{m_W^2} k_\mu p_\alpha p_\beta + f_3^V (p_\alpha g_{\beta\mu} - p_\beta g_{\alpha\mu}) + i f_4^V (p_\alpha g_{\beta\mu} + p_\beta g_{\alpha\mu})$$
$$+ i f_5^V \langle \alpha\beta\mu k\rangle - f_6^V \langle \alpha\beta\mu p\rangle - \frac{f_7^V}{m_W^2} \langle \alpha\beta p k\rangle k_\mu$$

$p = m_* \gamma\, \hat{p}_{1,2}$

$k = m_* \beta\, \hat{k}$

$U^0_{\alpha\beta} = \hat{p}_{1\alpha} \hat{p}_{2\beta}$

$$\Gamma_{\alpha,\beta;\mu} = f_{+0}^V \hat{p}_{2\beta} U^+_{\alpha\mu} + f_{0-}^V \hat{p}_{1\alpha} U^+_{\beta\mu} + f_{0+}^V \hat{p}_{1\alpha} U^-_{\beta\mu} + f_{-0}^V \hat{p}_{2\beta} U^-_{\alpha\mu}$$
$$+ f_{++}^V \hat{k}_\mu U^+_{\alpha\beta} + f_{--}^V \hat{k}_\mu U^-_{\alpha\beta} + f_{00}^V \hat{k}_\mu U^0_{\alpha\beta}$$



$(\lambda\bar{\lambda})$	$A^V_{\lambda\bar{\lambda}}$
$(+0)$	$\gamma(f_3^V - i f_4^V + \beta f_5^V + i\beta^{-1} f_6^V)$
$(0-)$	$\gamma(f_3^V + i f_4^V + \beta f_5^V - i\beta^{-1} f_6^V)$
$(0+)$	$\gamma(f_3^V + i f_4^V - \beta f_5^V + i\beta^{-1} f_6^V)$
(-0)	$\gamma(f_3^V - i f_4^V - \beta f_5^V - i\beta^{-1} f_6^V)$
$(++)$	$f_1^V + i\beta^{-1} f_6^V + 4i\gamma^2\beta f_7^V$
$(--)$	$f_1^V - i\beta^{-1} f_6^V - 4i\gamma^2\beta f_7^V$
(00)	$\gamma^2[-(1+\beta^2) f_1^V + 4\gamma^2\beta^2 f_2^V + 2 f_3^V]$

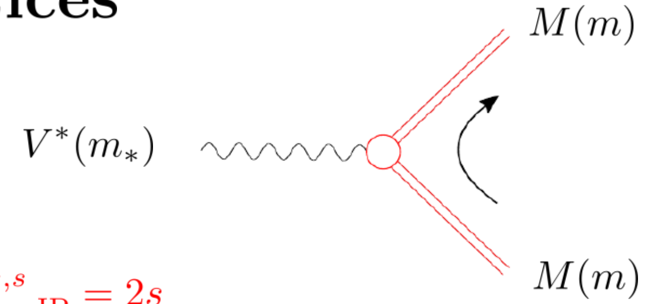
⇒

$$f_{+0}^V = -m_* \gamma \left(f_3^V - i f_4^V + \beta f_5^V + i\beta^{-1} f_6^V \right)$$
$$f_{0-}^V = m_* \gamma \left(f_3^V + i f_4^V + \beta f_5^V - i\beta^{-1} f_6^V \right)$$
$$f_{0+}^V = m_* \gamma \left(f_3^V + i f_4^V - \beta f_5^V + i\beta^{-1} f_6^V \right)$$
$$f_{-0}^V = -m_* \gamma \left(f_3^V - i f_4^V - \beta f_5^V - i\beta^{-1} f_6^V \right)$$
$$f_{++}^V = m_* \beta \left(f_1^V + i\beta^{-1} f_6^V + 4i\gamma^2 \beta f_7^V \right)$$
$$f_{--}^V = m_* \beta \left(f_1^V - i\beta^{-1} f_6^V - 4i\gamma^2 \beta f_7^V \right)$$
$$f_{00}^V = -m_* \beta^{-1} \gamma^2 \left[-(1+\beta^2) f_1^V + 4\gamma^2 \beta^2 f_2^V + 2 f_3^V \right]$$

Application 2: U(1) anapole VMM vertices

Boudjema, Hamzaoui, PRD 43 (1991) 3748

SYC, JH Jeong, IG Jeong, DW Kang, SD Shin, in progress



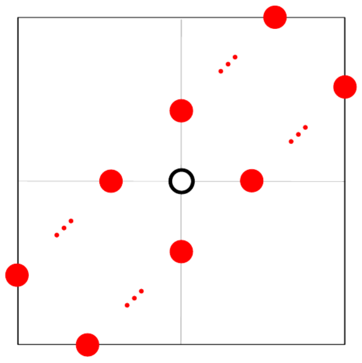
$$[\mathcal{H}^{1,s,s}_{A[\lambda,\lambda]}]_{\rm IP} = 0 \quad \text{and} \quad [\mathcal{H}^{1,s,s}_{A[\lambda,\lambda\pm 1]}]_{\rm IP} = [\mathcal{H}^{1,s,s}_{A[\lambda\pm 1,\lambda]}]_{\rm IP} \neq 0 \quad \text{with} \quad n^{1,s,s}_{m,m;\rm IP} = 2s$$

$$\mathcal{L}_{\rm anapole} = \mathcal{X}_\nu \, \partial_\mu B^{\mu\nu}$$

$$\Rightarrow p^2 \left(g_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu\right) \mathcal{X}^\nu \epsilon^\mu(p)$$

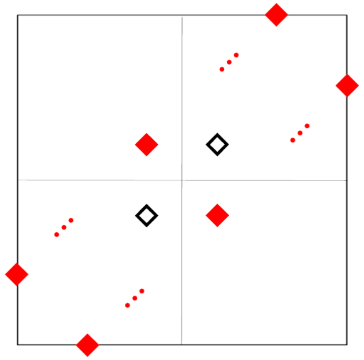
$\mathcal{X}_\nu$: anapole vector current of two identical Majorana particles of any spin

$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$: field strength of the U(1) gauge boson B^μ



$$[\Gamma_{\rm bbb}] = \sum_{\tau=1}^{n} \left(b^+_\tau [V^+][U^+]^{\tau-1} + b^-_\tau [V^-][U^-]^{\tau-1}\right)[U^0]^{n-\tau} \quad \text{with} \quad s = n \neq 0$$

$$[\Gamma_{\rm bff}] = [A]\left\{f^0[U^0]^n + \sum_{\tau=1}^{n}\left(f^+_\tau\,[U^+]^\tau + f^-_\tau\,[U^-]^\tau\right)[U^0]^{n-\tau}\right\} \quad \text{with} \quad s = n + 1/2$$

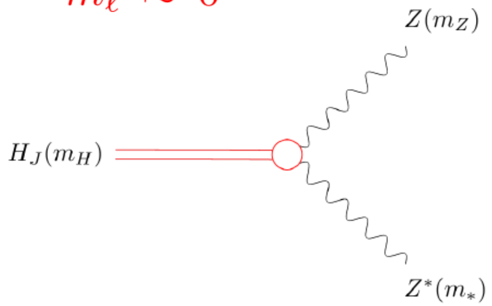


$$V^\pm_{\alpha\beta;\mu} = \hat{p}_\beta U^\pm_{\alpha\mu} + \hat{p}_\alpha U^\mp_{\beta\mu} \quad \text{and} \quad A_\mu = \gamma_{\perp\mu}\gamma_5 \quad \text{with} \quad \gamma_{\perp\mu} = g_{\perp\mu\nu}\gamma^\nu$$

Application 3: $H_J \to ZZ^* (\to \ell_1^- \ell_1^+ \ell_2^- \ell_2^+)$ $m_\ell \approx 0$

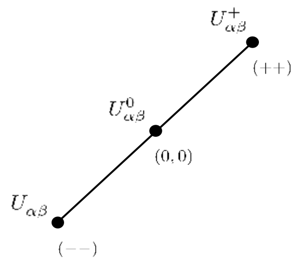
SYC, Miller, Muhlleitner, Zerwas, PLB 553 (2003) 61

SYC ea, in progress (including 4-point *HZll* vertices)



on-shell

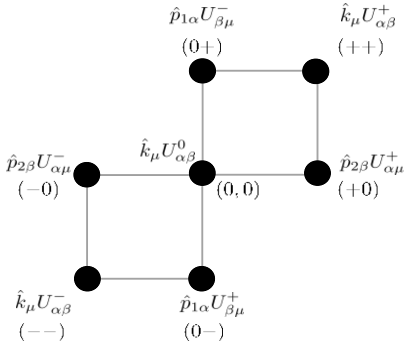
$J = 0$



$$\Gamma^{[0,1,1]}_{\alpha,\beta} = a_1 g_{\alpha\beta} + a_2 p_\alpha p_\beta + a_3 \langle \alpha\beta p q \rangle$$

$$\Gamma^{[0,1,1]}_{\alpha,\beta} = a_{00}\, U^0_{\alpha\beta} + a_{++}\, U^+_{\alpha\beta} + a_{--}\, U^-_{\alpha\beta}$$

$J = 1$



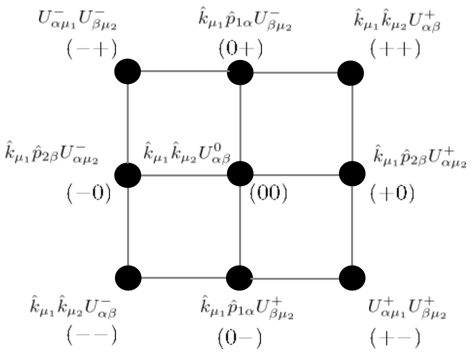
$$\Gamma^{[1,1,1]}_{\alpha,\beta;\mu} = b_1 q_\mu g_{\alpha\beta} + b_2 p_\beta g_{\alpha\mu} + b_3 p_\alpha g_{\beta\mu} + b_4 q_\mu p_\alpha p_\beta + b_5 \langle \mu\alpha\beta p \rangle + b_6 \langle \mu\alpha\beta q \rangle + b_7 \big(p_\beta \langle \alpha\mu p q \rangle + p_\alpha \langle \beta\mu p q \rangle \big)$$

$$\Gamma^{[1,1,1]}_{\alpha,\beta;\mu} = \hat{k}_\mu \big(b_{00}\, U^0_{\alpha\beta} + b_{++}\, U^+_{\alpha\beta} + b_{--}\, U^-_{\alpha\beta} \big) + b_{+0}\, \hat{p}_{2\beta} U^+_{\alpha\mu} + b_{-0}\, \hat{p}_{2\beta} U^-_{\alpha\mu} + b_{0+}\, \hat{p}_{1\alpha} U^-_{\beta\mu} + b_{0-}\, \hat{p}_{1\alpha} U^+_{\beta\mu}$$

$$\omega_1 = \frac{m_Z}{m_H}, \quad \omega_2 = \frac{m_*}{m_H}$$

$$\hat{p}_{1,2} = 2\omega_{1,2}\, \hat{p}$$

$J \geq 2$



$$\Gamma^{[2,1,1]}_{\alpha,\beta;\mu_1,\mu_2} = c_1 g_{\alpha\mu_1} g_{\beta\mu_2} + c_2 q_{\mu_1} q_{\mu_2} g_{\alpha\beta} + c_3 q_{\mu_1} p_\beta g_{\alpha\mu_2} + b_4 q_{\mu_1} p_\alpha g_{\beta\mu_2} + b_5 q_{\mu_1} q_{\mu_2} p_\alpha p_\beta + c_6 q_{\mu_1} \langle \mu_2 \alpha\beta p \rangle + c_7 q_{\mu_1} \langle \mu_2 \alpha\beta q \rangle + c_8 q_{\mu_1} \big(p_\beta \langle \alpha\mu_2 p q \rangle + p_\alpha \langle \beta\mu_2 p q \rangle \big) + c_9 q_{\mu_1} q_{\mu_2} \langle \alpha\beta p q \rangle$$

$$\Gamma^{[2,1,1]}_{\alpha,\beta;\mu_1,\mu_2} = \hat{k}_{\mu_1} \hat{k}_{\mu_2} \big(c_{00}\, U^0_{\alpha\beta} + c_{++}\, U^+_{\alpha\beta} + c_{--}\, U^-_{\alpha\beta} \big) + \hat{k}_{\mu_1} \big(c_{+0}\, \hat{p}_{2\beta} U^+_{\alpha\mu_2} + c_{-0}\, \hat{p}_{2\beta} U^-_{\alpha\mu_2} + c_{0+}\, \hat{p}_{1\alpha} U^-_{\beta\mu_2} + c_{0-}\, \hat{p}_{1\alpha} U^+_{\beta\mu_2} \big) + c_{+-} U^+_{\alpha\mu_1} U^+_{\beta\mu_2} + c_{-+} U^-_{\alpha\mu_1} U^-_{\beta\mu_2}$$

$$\Gamma^{[3,1,1]}_{\alpha,\beta;\mu_1,\mu_2,\mu_3} = \hat{k}_{\mu_3} \Gamma^{[2,1,1]}_{\alpha,\beta;\mu_1,\mu_2}, \quad \Gamma^{[4,1,1]}_{\alpha,\beta;\mu_1,\mu_2,\mu_3,\mu_4} = \hat{k}_{\mu_3} \hat{k}_{\mu_4} \Gamma^{[2,1,1]}_{\alpha,\beta;\mu_1,\mu_2} \quad \cdots$$

Further investigations

- Gauge and discrete symmetries
- Link to spinor helicity formalism

  (Dirac, PRSLA 155 (1936) 447, … ,
  Arkani-Hamed, TC Huang, Yt Huang,
  JHEP 11 (2021) 070)
- Hypercharge anapole DM

  (SYC, JH Jeong, IG Jeong, DW Kang, SD Shin, in progress)
- High-spin DM particles

  (Babichev ea, PRD 94 (2016) 084055, …, Gondolo,
  S Kang, Scopel, Tomar, PRD 104 (2021) 063017)
- High-spin targets for DM detection

  (SYC, Drees, JH Jeong, in progress)

- Off-shell vertices and propagators
- 4-point covariant vertices, …
- Application to various processes

  (SYC ea in progress)
- Program for automatic evaluation

  (JH Jeong ea, in progress)
- ⋮

Any connection? to Suro Kim,
Rodrigo Alonso & Dong Woo
Kang's talks … at this 3rd AEI

$$
\begin{aligned}
B^{\pm}(0) &\to K^*(1)^{\pm} + \gamma(1)\\
H(0) &\to \gamma(1) + \gamma(1)\\
H(0) &\to g(1) + g(1)\\
H(0) &\to Z(1) + \gamma(1)\\
H(0) &\to Z^*(1) + Z(1)\\
t(1/2) &\to b(1/2) + W^+(1)\\
\tau(1/2) &\to \pi(0) + \nu_\tau(1/2)\\
\tau(1/2) &\to \rho(1) + \nu_\tau(1/2)\\
Z(1) &\to \tau(1/2) + \bar{\tau}(1/2)\\
V^*(1) &\to W^-(1) + W^+(1)\\
J/\psi(1) &\to a_2(1320)(2) + \rho(1)\\
J/\psi(1) &\to f_4(2050)(4) + \gamma(1)\\
&\vdots
\end{aligned}
$$

Workman ea, PTEP (2022) 038C01 [PDG]

Summary

Exploiting the equivalence between the helicity formalism and the covariant formulation, we identified all the basic operators for constructing any Lorentz covariant 3-point vertices.

We found all the helicity-specific covariant 3-point vertices to be combined into a general covariant three-point vertex.

We worked out the case with two identical particles fully.

We expect the general algorithm to enable us to work out various theoretical and phenomenological aspects effectively.

A valuable work is to synthesize the bosonic and fermionic cases in a compact and unified way.

⋮