EFT Approaches to cosmological collider program

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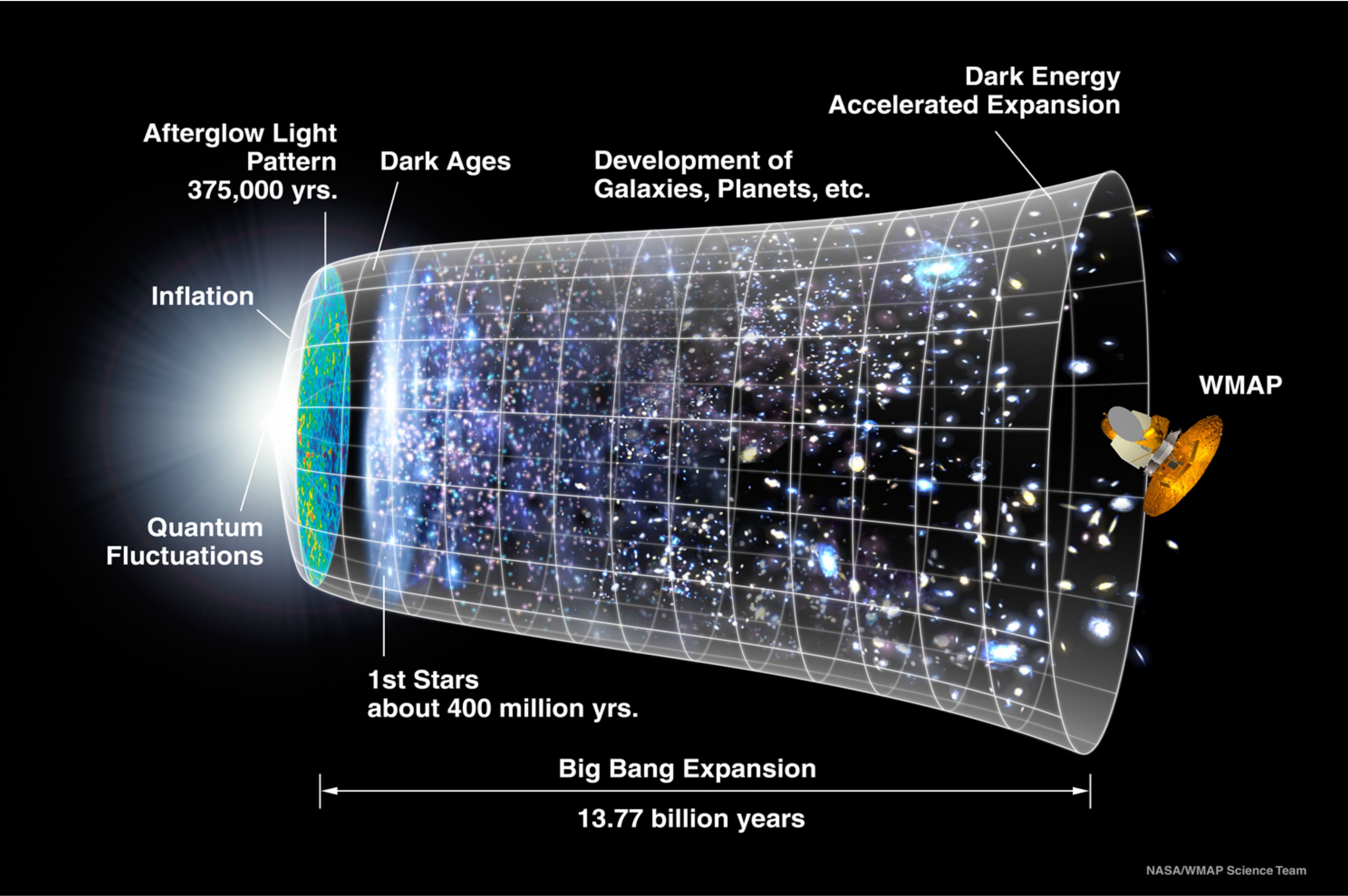
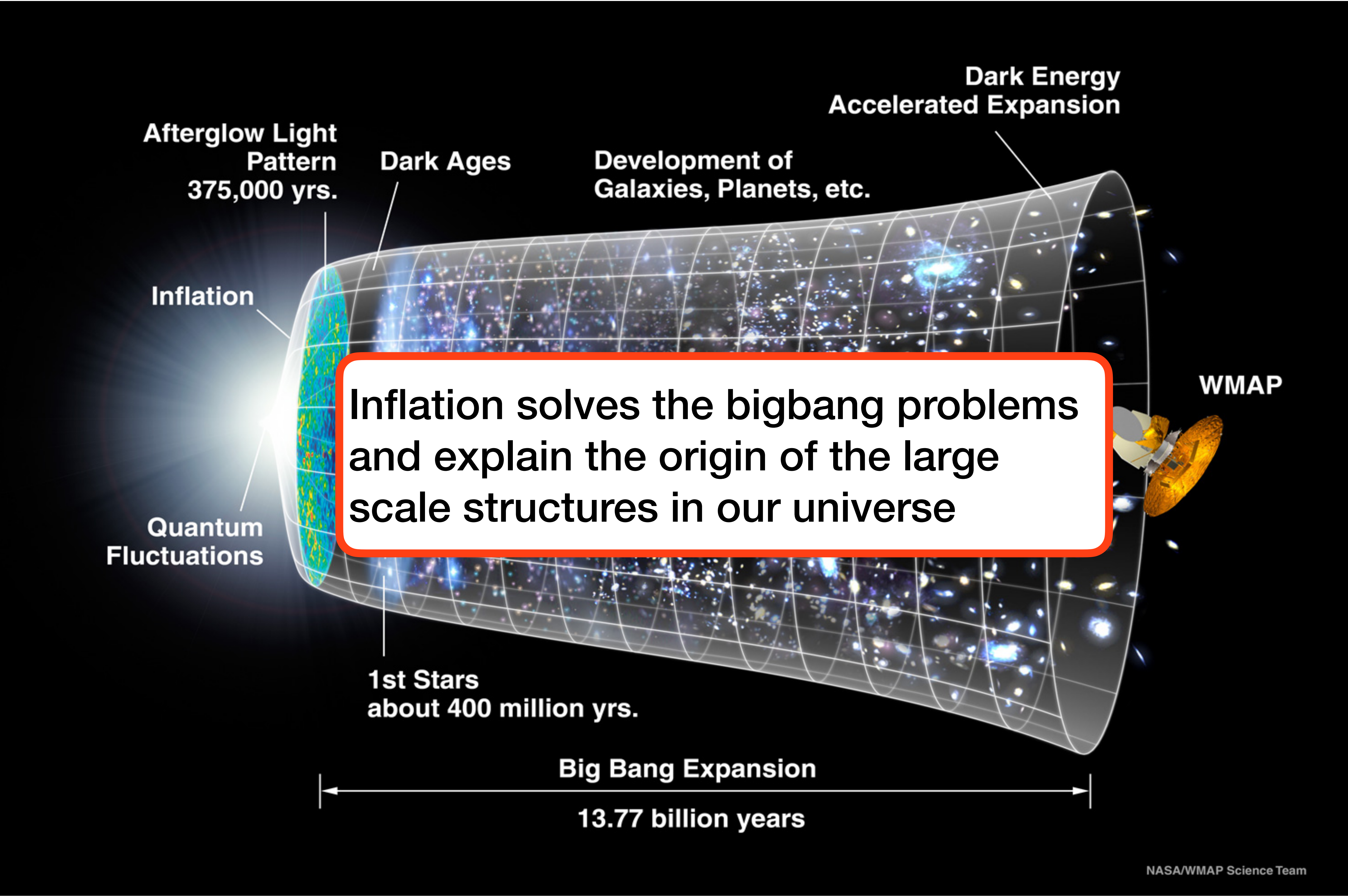
And ongoing work

Oct. 4, The 3rd “Asian-European-Institutes Workshop for BSM”

Cosmic Inflation



Cosmic Inflation



Inflation solves the bigbang problems and explain the origin of the large scale structures in our universe

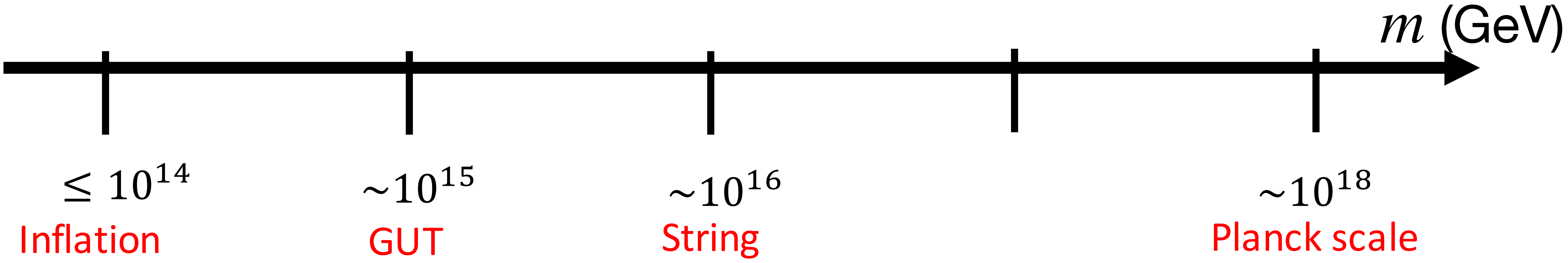
Inflation as High Energy Frontier

- Typical energy scale of inflation $H \sim 10^{14}$ GeV

c.f LHC $E \sim 10^{5}$ GeV

- It is natural to use inflation to probe high energy physics
- Primordial non-Gaussianity as a particle collider (Cosmological Collider Program)

Chen and Wang[2010]
Baumann and Green[2012]
Noumi, Yamaguchi and Yokoyama[2013]
Arkani-Hamed and Maldacena[2015]
Lee, Baumann and Pimentel[2016]



m (GeV)

$\leq 10^{14}$ Inflation

$\sim 10^{15}$ GUT

$\sim 10^{16}$ String

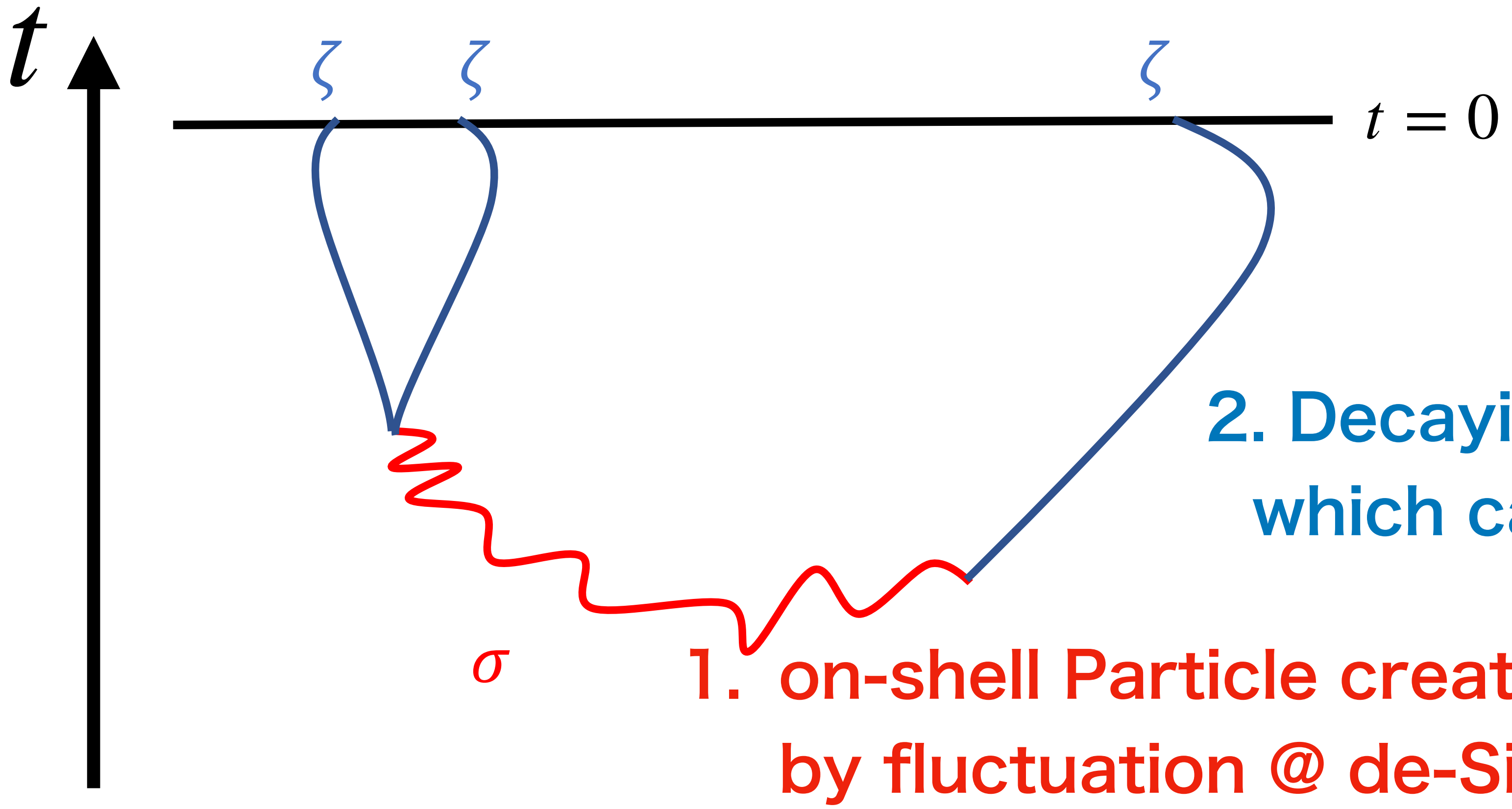
$\sim 10^{18}$ Planck scale

Cosmological Collider Physics

1. on-shell Particle creation
   by fluctuation @ de-Sitter temperature 10^{14} GeV

NG generated during inflation

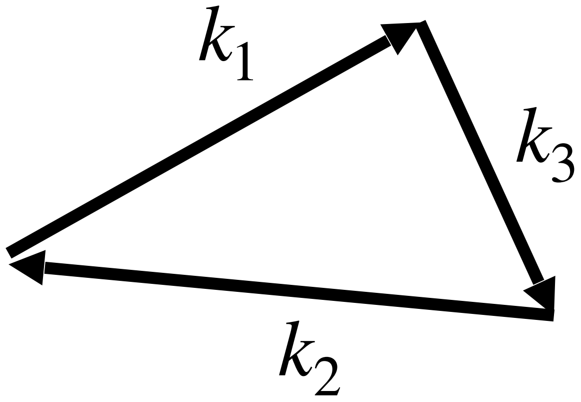
- Non-Gaussianity as particle collider



2. Decaying to scalar perturbation which can be observed by CMB

1. on-shell Particle creation by fluctuation @ de-Sitter temperature 10^{14} GeV

- $\langle \zeta_{k_1}(\tau)\zeta_{k_2}(\tau)\zeta_{k_3}(\tau)\rangle\Big|_{\tau\to0}$ includes information of σ
- Function of the triangle shape



Cosmological Collider Physics

- Non-Gaussianity as particle collider

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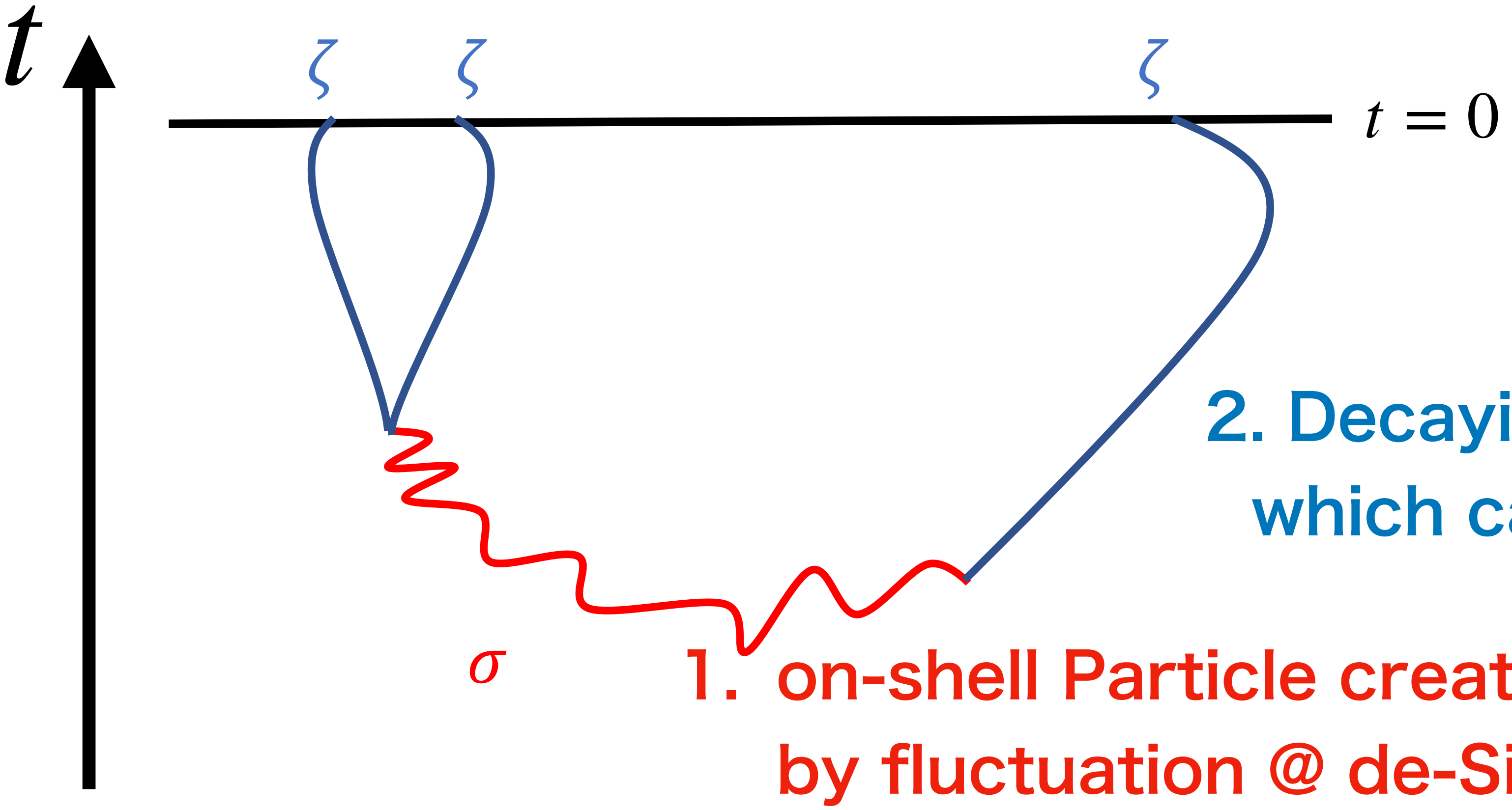


2. Decaying to scalar perturbation which can be observed by CMB

1. on-shell Particle creation by fluctuation @ de-Sitter temperature 10^{14} GeV

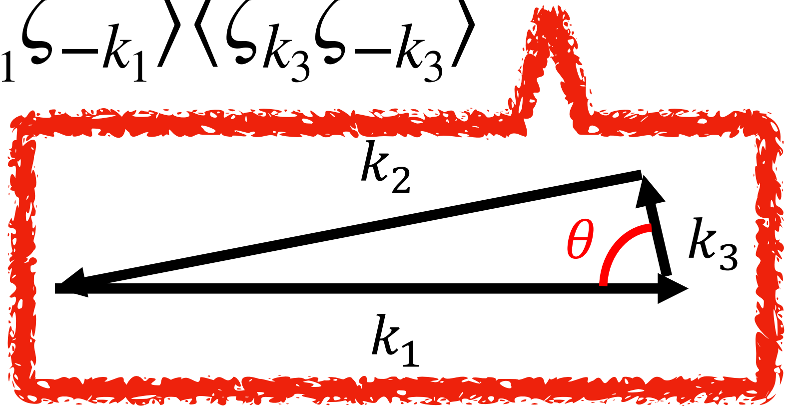
Boltzmann factor

$$\frac{\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle}{\langle\zeta_{k_1}\zeta_{-k_1}\rangle\langle\zeta_{k_3}\zeta_{-k_3}\rangle} \propto e^{-\pi\mu}(k_3/k_1)^{3/2}\sin\left[\sqrt{\frac{m^2}{H^2}-\frac{9}{4}}\log(k_3/k_1)\right]P_s(\cos\theta)$$

Oscillating

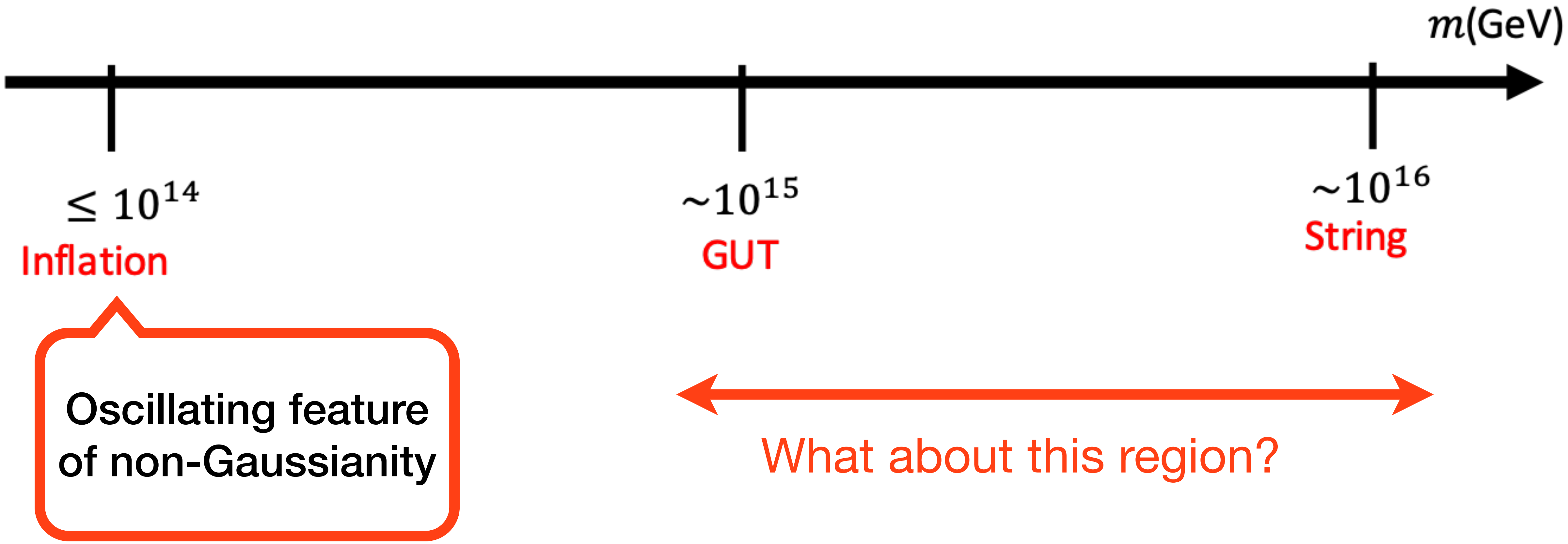
Mass of σ

Spin of σ



$$\mu = \sqrt{\frac{m^2}{H^2}-\frac{9}{4}}$$

Target energy scale



$m(\mathrm{GeV})$

$\lesssim 10^{14}$

Inflation

$\sim 10^{15}$

GUT

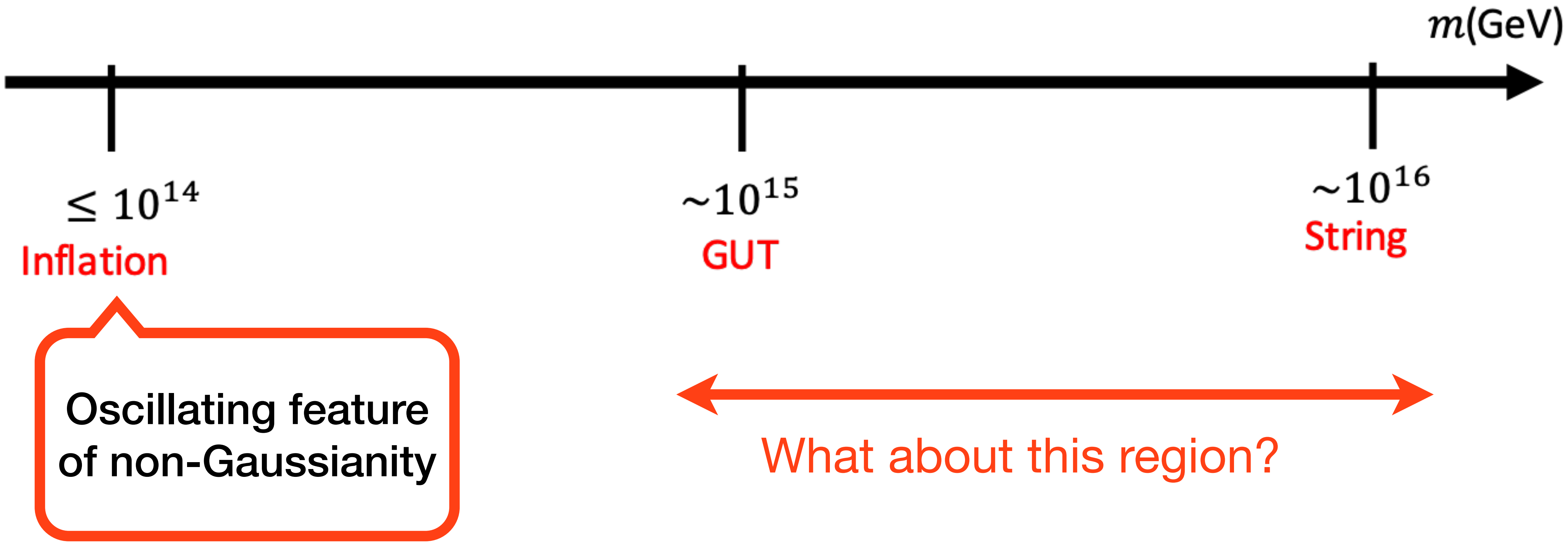
$\sim 10^{16}$

String

Oscillating feature of non-Gaussianity

What about this region?

Target energy scale



Oscillating feature of non-Gaussianity

What about this region?

Difficult with oscillating feature because of Boltzmann suppress $\propto e^{-\pi\mu}$

$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

e.g. $m \sim 3H$, the signal is suppressed by $\sim 10^{-5}$

When we want to see higher scale

@ Particle collider

1. Build a new fancy collider

LEP → LHC → ???

2. Study effective interactions carefully

Prediction of Weak-boson from 4-fermi interaction

When we want to see higher scale

@ Cosmological collider

1. Build a new fancy Universe

2. Study effective interactions carefully

When we want to see higher scale

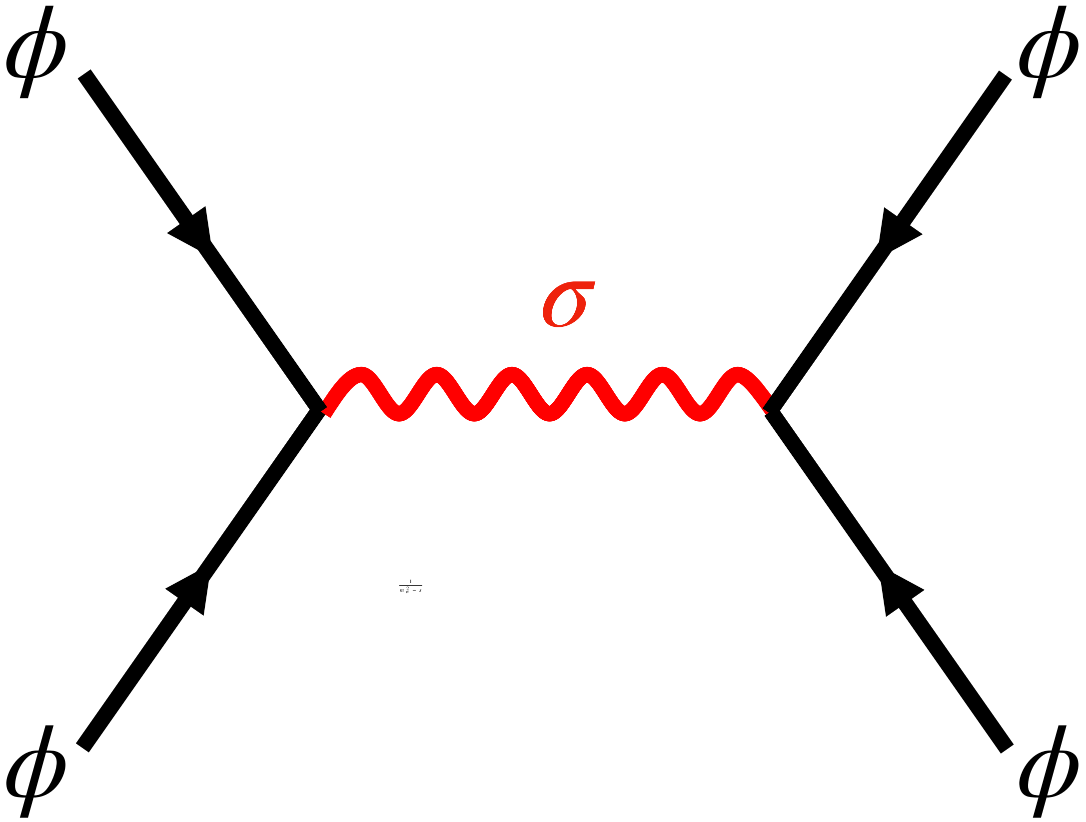
@ Cosmological collider

1. Build a new fancy Universe

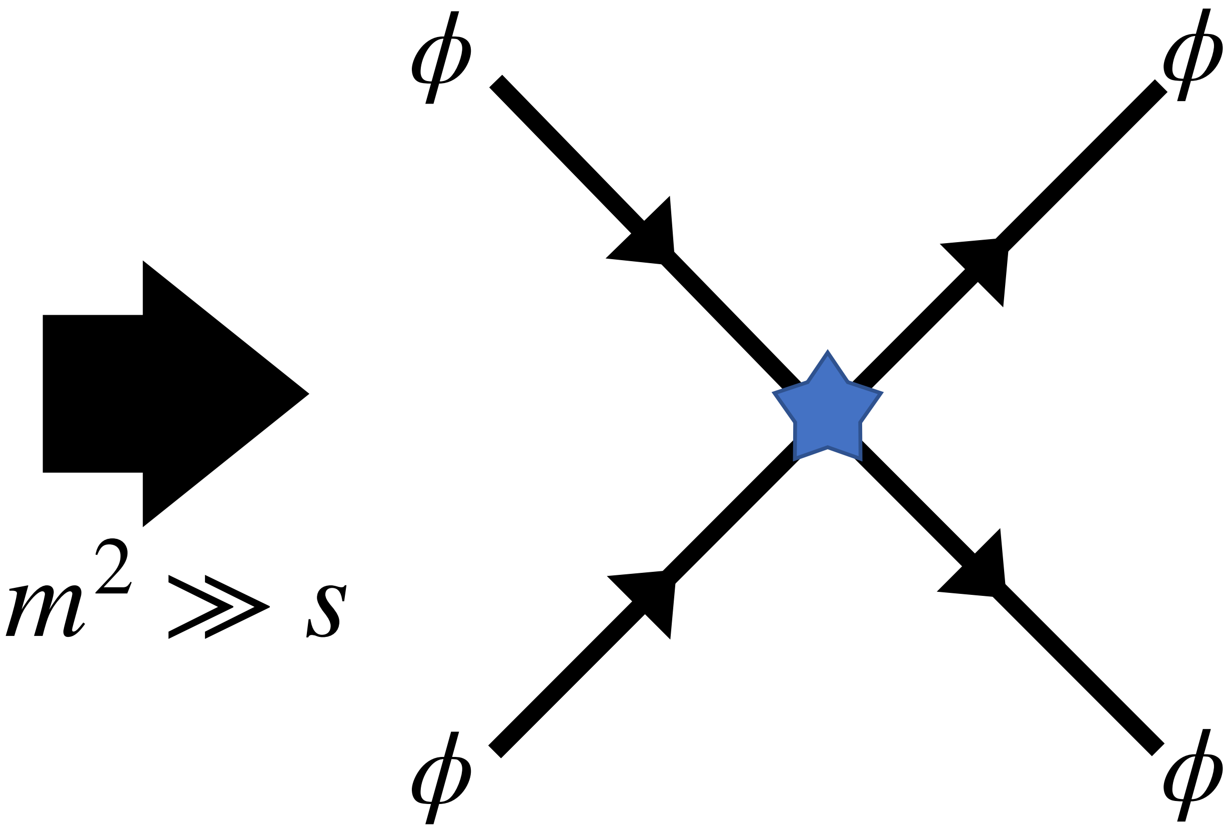
But we cannot
so we focus on

2. Study effective interactions carefully

Effective coupling @ Particle Collider



Resonance part
non-Analytic part of Propagator
(on-shell particle creation)



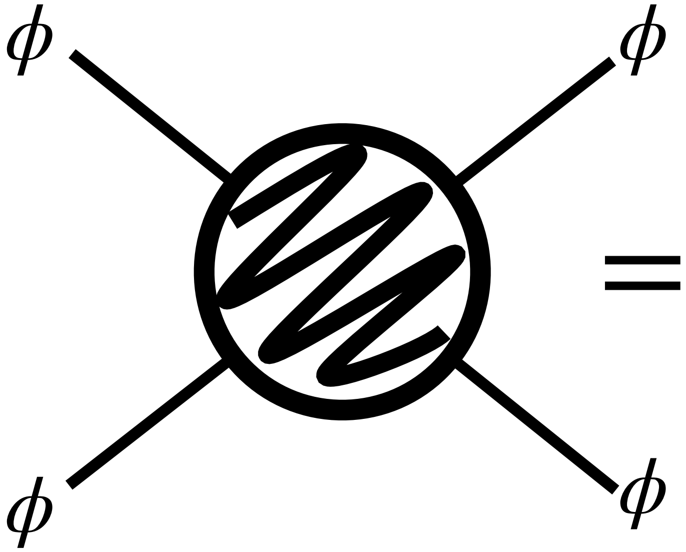
Effective coupling
Analytic part of Propagator
ex) Prediction of Weak boson
from Fermi-interaction

Q. What are important imprints
of heavy particles on
inflaton effective interactions?

A. Sign of the effective interactions

IR expansion of scattering amplitudes

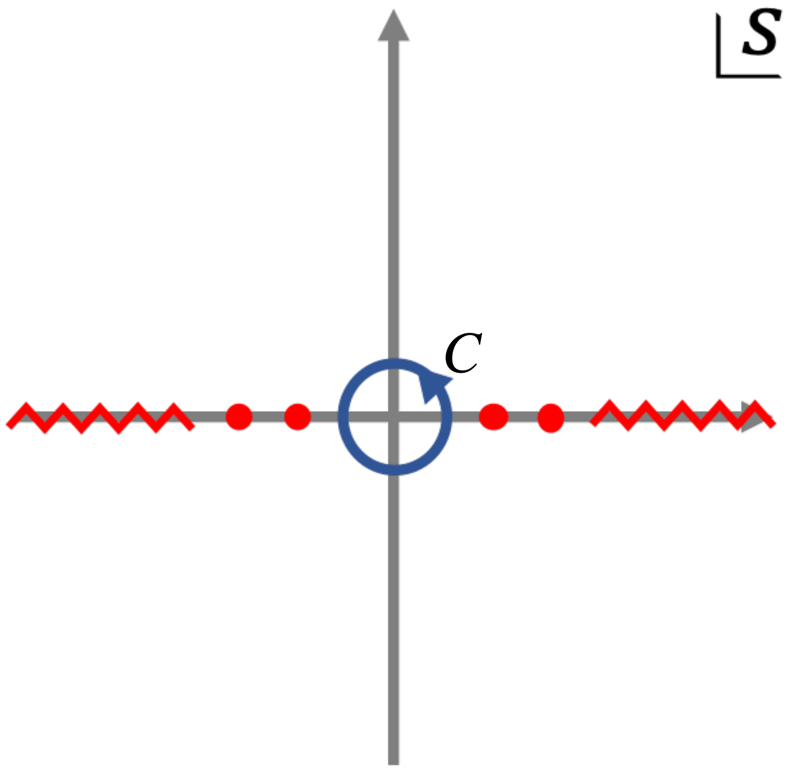
- Expand the scattering amplitude in Mandelstam variables



$$= M(s,t) = \sum_{p,q} a_{p,q} s^p t^q = \sum_p b_p(t) s^p$$

We neglected massless pole
assuming gravity is subdominant

- Coefficients of s^p



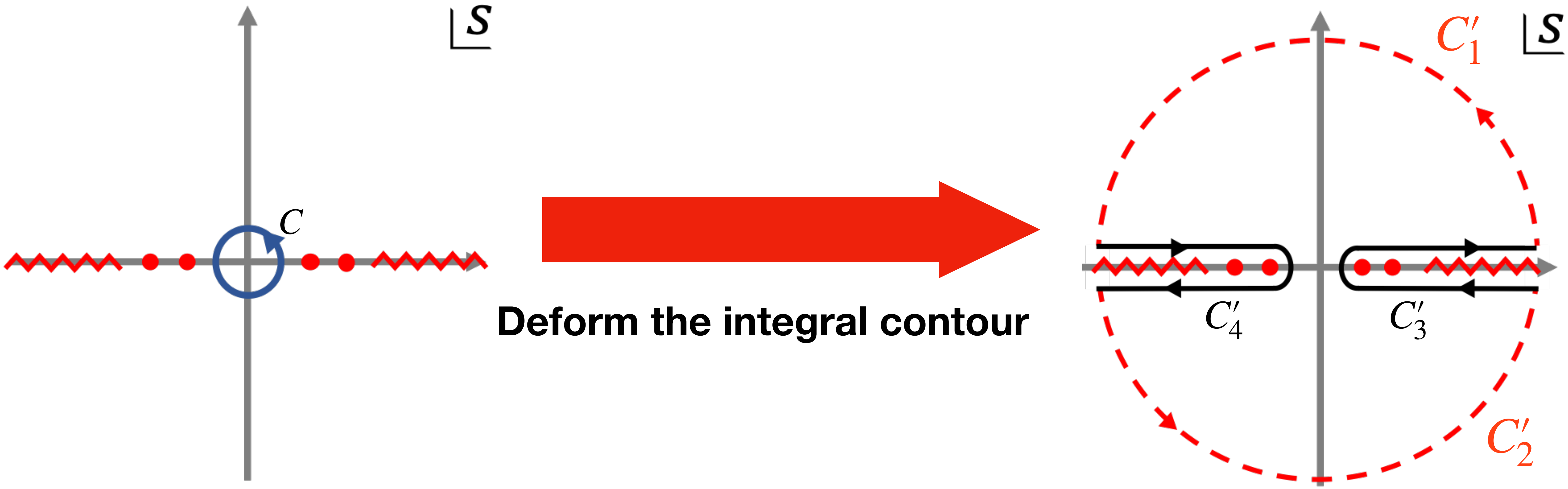
$$b_p(t) = \sum_q a_{p,q} t^q = \oint_C \frac{ds}{2\pi i} \frac{M(s,t)}{s^{p+1}}$$

Wilson-Coefficients

- Coefficients of s^p

$$b_p(t) = \sum_q a_{p,q} t^q = \oint_C \frac{ds}{2\pi i} \frac{M(s,t)}{s^{p+1}}$$

$$= \left(\int_{C_1'} + \int_{C_2'} + \int_{C_3'} + \int_{C_4'} \right) \frac{ds}{2\pi i} \frac{M(s,t)}{s^{p+1}}$$



Deform the integral contour

Wilson-Coefficients

- Coefficients of s^p

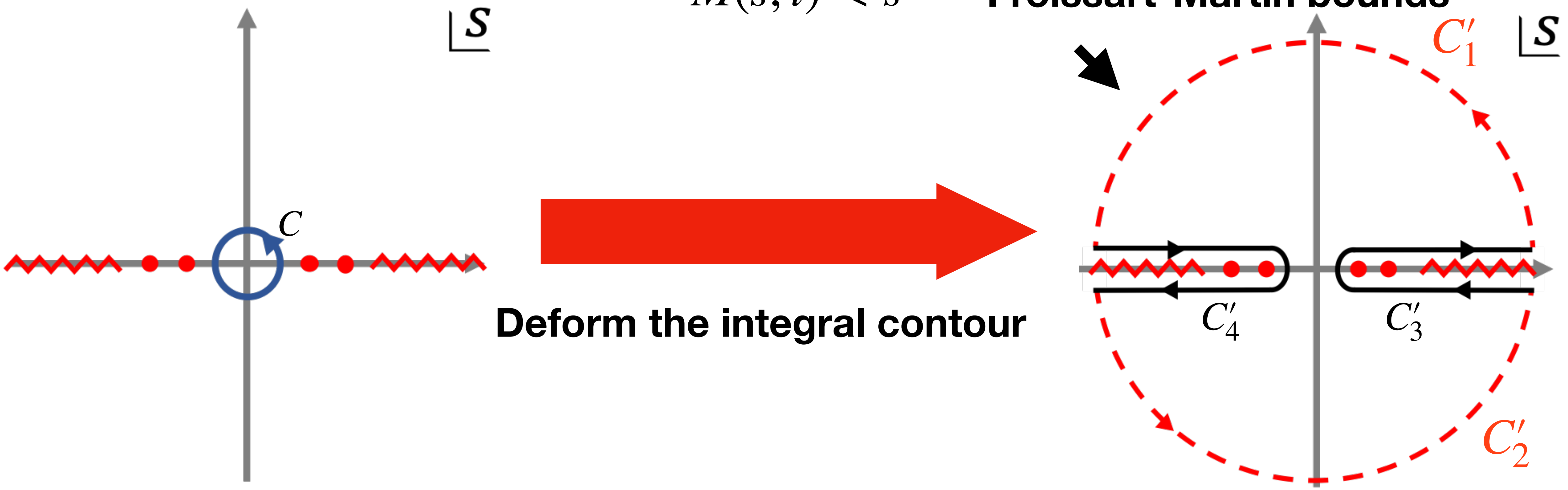
$$b_p(t) = \sum_q a_{p,q} t^q = \oint_C \frac{ds}{2\pi i}\frac{M(s,t)}{s^{p+1}}$$

0 for $p \geq 2$

$$= \left(\cancel{\int_{C_1'} + \int_{C_2'}} + \int_{C_3'} + \int_{C_4'}\right)\frac{ds}{2\pi i}\frac{M(s,t)}{s^{p+1}}$$

$M(s,t) < s^2$

Froissart-Martin bounds



Deform the integral contour

UV information in IR coefficients

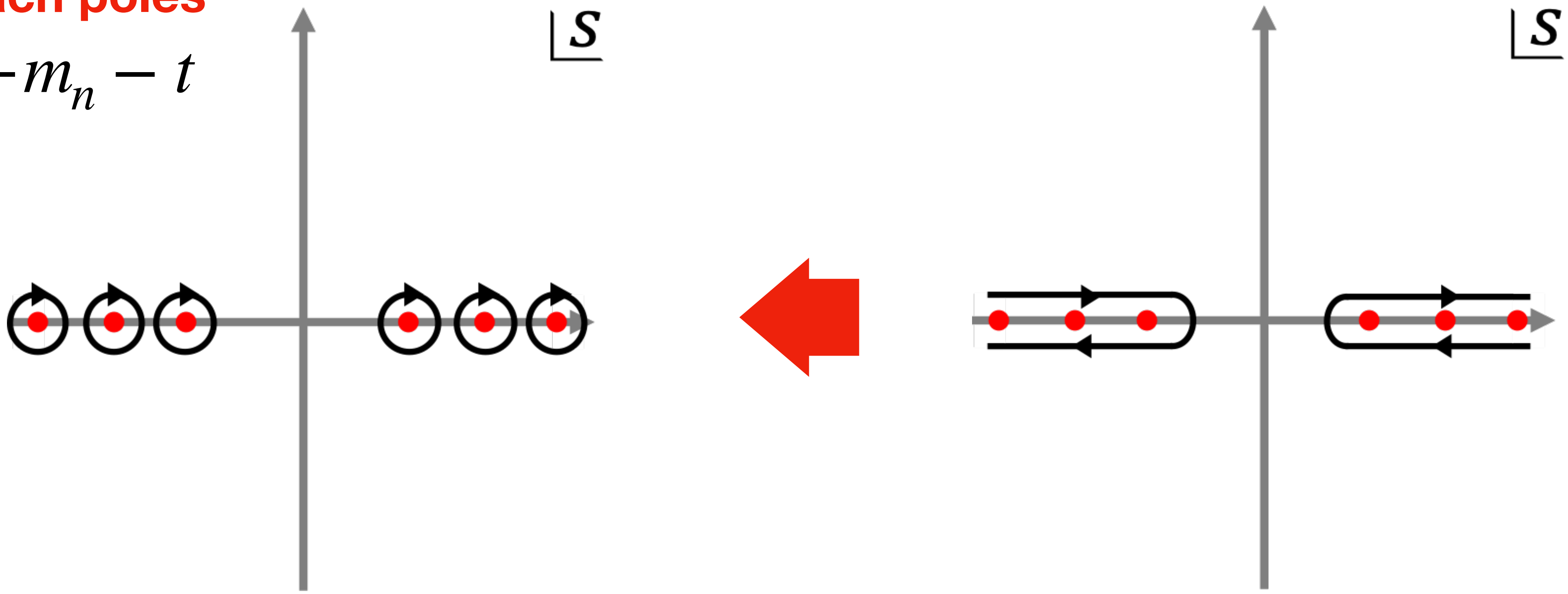
- For example, tree-level effect is

$$b_p(t) = \sum_q a_{p,q} t^q = \left(\int_{C'_3} + \int_{C'_4} \right) \frac{ds}{2\pi i} \frac{M(s,t)}{s^{p+1}}$$

$$= \sum_n \left[\frac{g_n^2 P_{\ell_n}\left(1 + \frac{2t}{m_n^2}\right)}{(m_n^2)^{p+1}} - \frac{g_n^2 P_{\ell_n}\left(1 + \frac{2t}{m_n^2}\right)}{(-m_n^2 - t)^{p+1}} \right]$$

Residue for each poles

@ m_n and $-m_n - t$



Continuous sum over n to incorporate branch cuts from loops

UV information in IR coefficients

- For example, tree-level effect is

$$b_p(t) = \sum_q a_{p,q} t^q = \left(\int_{C_3'} + \int_{C_4'} \right) \frac{ds}{2\pi i} \frac{M(s,t)}{s^{p+1}}$$

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Residue for each poles

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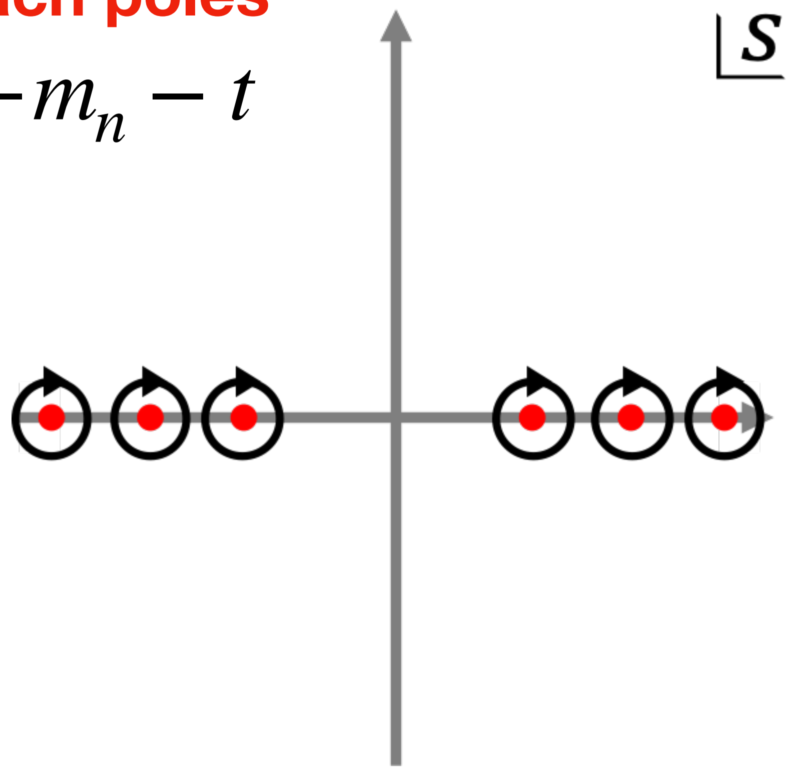
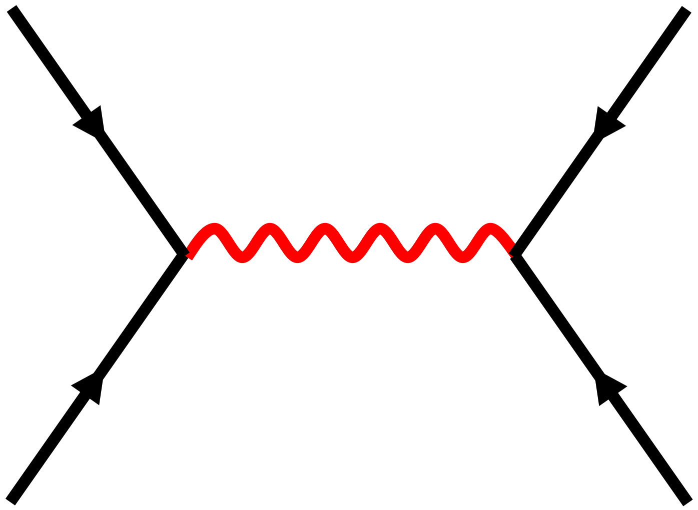


Propagation of on-shell particle

with mass m_n and spin ℓ_n



Continuous sum over n to incorporate branch cuts from loops

Positivity bound on s^{2n} coefficients

Coefficient of s^{2n}

$$a_{p,0} = \begin{cases} \sum_n \frac{g_n^2}{(m_n^2)^{p+1}} & \textbf{for even} \quad p \geq 2 \\ 0 & \textbf{for odd} \quad p \geq 2 \end{cases}$$

- $a_{2n,0}$ is always positive and a_{2n+1} is 0
- This is well-known as positivity bound

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi[2006]

- Universal and elegant, but detailed information such as spin of intermediate state is obscured at the cost

Let’s go beyond
the positivity bound!

Positivity bound on $s^p t$ coefficients

Coefficient of $s^p t$

$$a_{p,1} = \begin{cases} \sum_n \frac{g_n^2}{(m_n^2)^{p+2}} \left(2\ell_n^2 + 2\ell_n - p - 1\right) & \text{for even} \quad p \geq 2 \\ (p+1) \sum_n \frac{g_n^2}{(m_n^2)^{p+2}} & \text{for odd} \quad p \geq 2 \end{cases}$$

- For even p, the sign of $a_{p,1}$ depends on spin

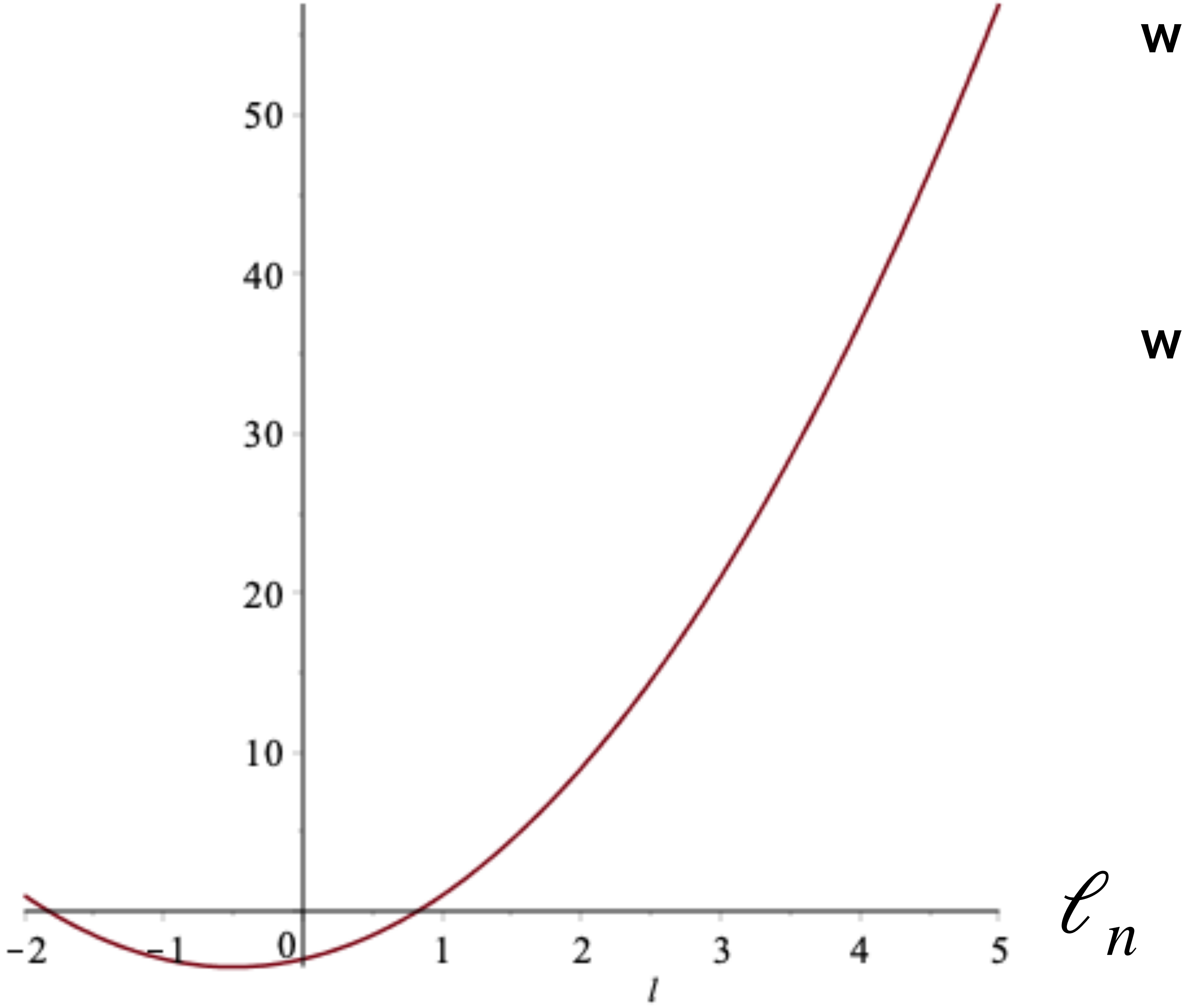
If, $l_n > l^\star = \dfrac{-1 + \sqrt{2p+5}}{2}$, $\quad a_{p,1} \geq 0$

Otherwise, $\quad a_{p,1} \leq 0$

- For odd p, $a_{p,1} \geq 0$ is always positive

Spin dependence of s^2t coefficients

$$a_{2,1} = \sum_n \frac{g_n^2}{(m_n^2)^4}\underline{\left(2\ell_n^2 + 2\ell_n - 3\right)}$$



where $l_n < l^\star \sim 0.82$

$$a_{2,1} < 0$$

where $l_n > l^\star \sim 0.82$

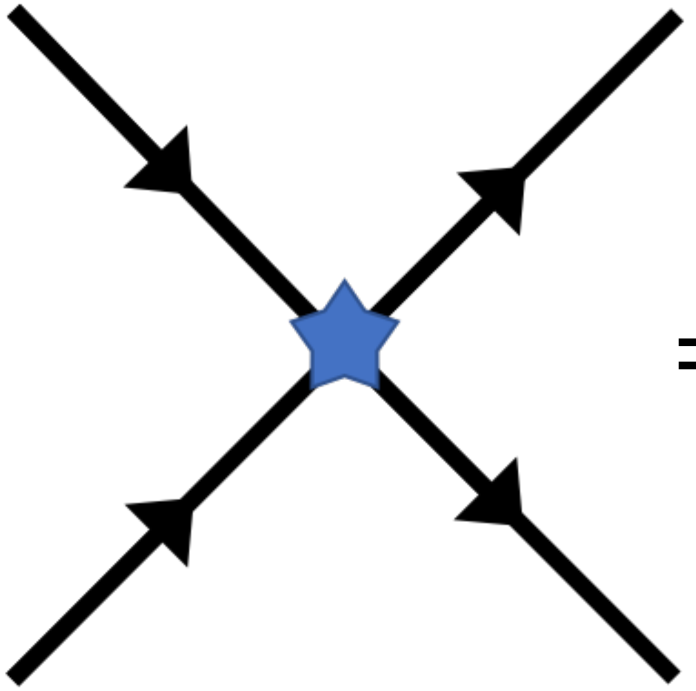
$$a_{2,1} > 0$$

Effective field theory of Inflaton

- EFT of Inflaton

$$\mathscr{L} = -\frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{\alpha}{\Lambda^4}\left(\partial_\mu \phi\right)^4 + \frac{\beta}{\Lambda^6}\left(\nabla_\mu \partial_\nu \phi\right)^2\left(\partial_\rho \phi\right)^2 + \cdots$$

- Inflaton enjoys an approximate shift symmetry when slow-roll approximation is good enough

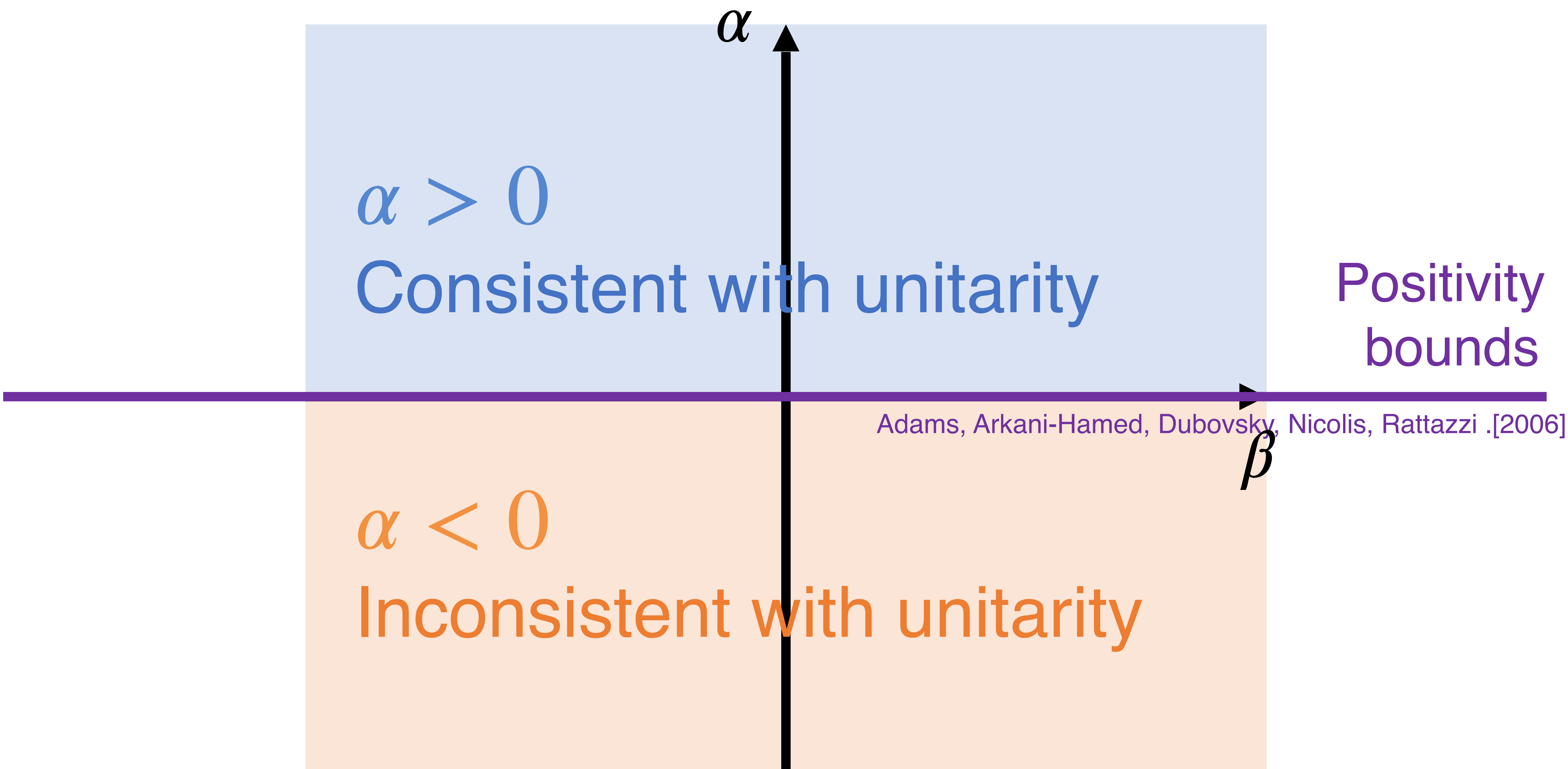


$$= \underset{a_{2,0}}{\boxed{\frac{4\alpha}{\Lambda^4}}}\left(s^2 + st + t^2\right) \underset{a_{2,1}}{\boxed{-\frac{3\beta}{\Lambda^6}}}\left(s^2 t + s t^2\right) + \cdots$$

Positivity bound

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu\phi)^4 + \frac{\beta}{\Lambda^6}(\nabla_\mu\partial_\nu\phi)^2(\partial_\rho\phi)^2 + \cdots$$

- $\alpha > 0$ follows from unitarity and analyticity.



$\alpha > 0$

Consistent with unitarity

Positivity bounds

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi .[2006]

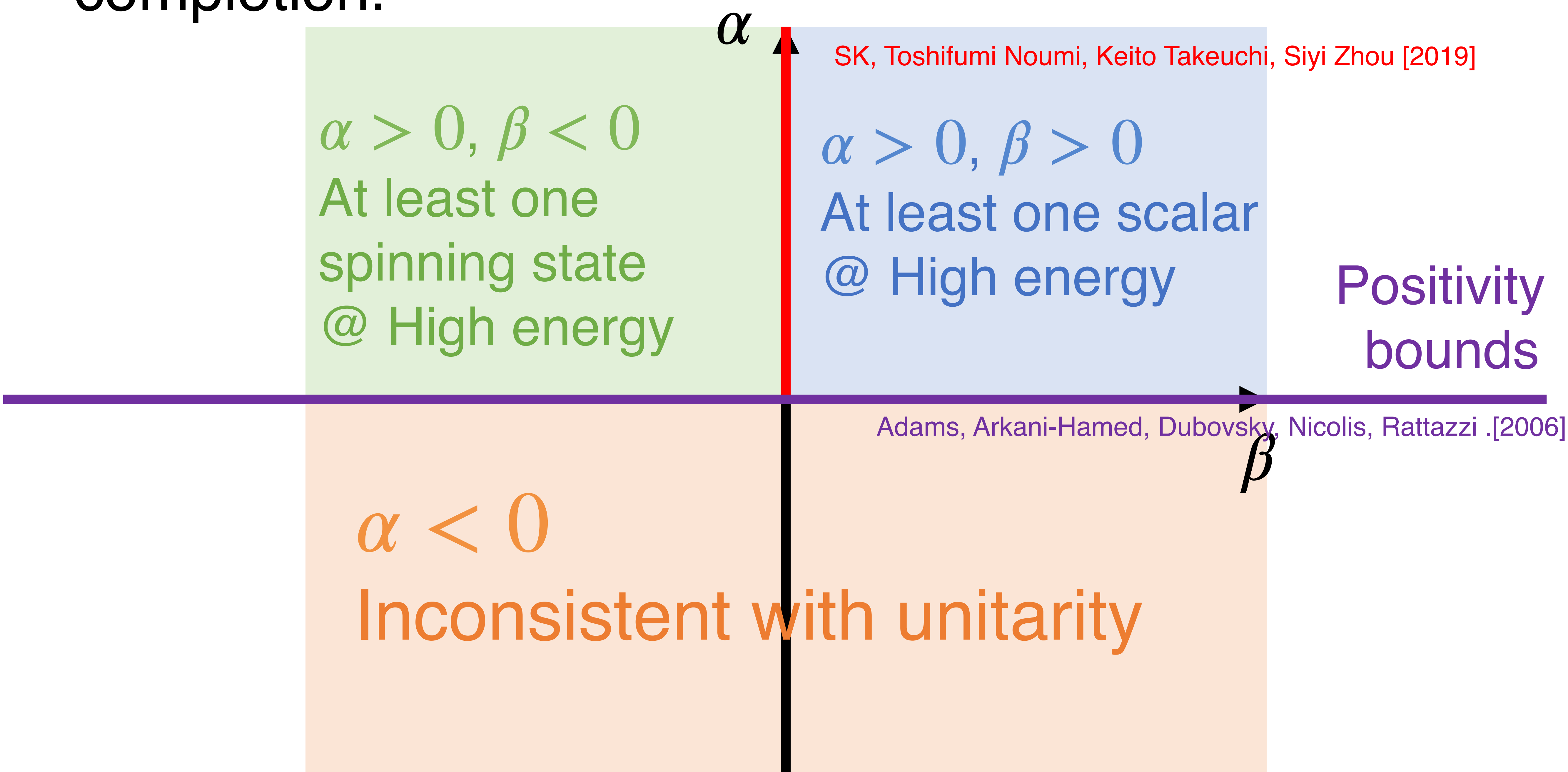
$\alpha < 0$

Inconsistent with unitarity

Beyond positivity bound

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu\phi)^4 + \frac{\beta}{\Lambda^6}(\nabla_\mu\partial_\nu\phi)^2(\partial_\rho\phi)^2 + \cdots$$

- The sign of β depends on the details of UV completion.



SK, Toshifumi Noumi, Keito Takeuchi, Siyi Zhou [2019]

$\alpha > 0,\ \beta < 0$
At least one spinning state @ High energy

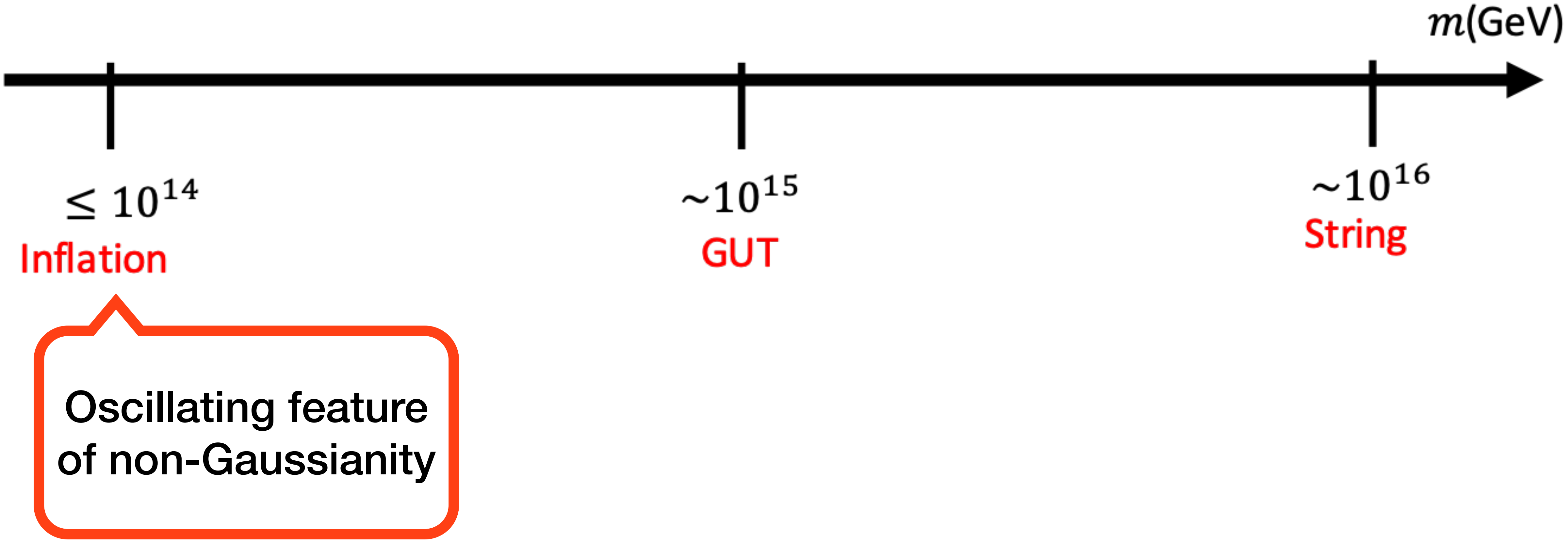
$\alpha > 0,\ \beta > 0$
At least one scalar @ High energy

Positivity bounds

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi .[2006]

$\alpha < 0$
Inconsistent with unitarity

Summary: Cosmological collider program



m(GeV)

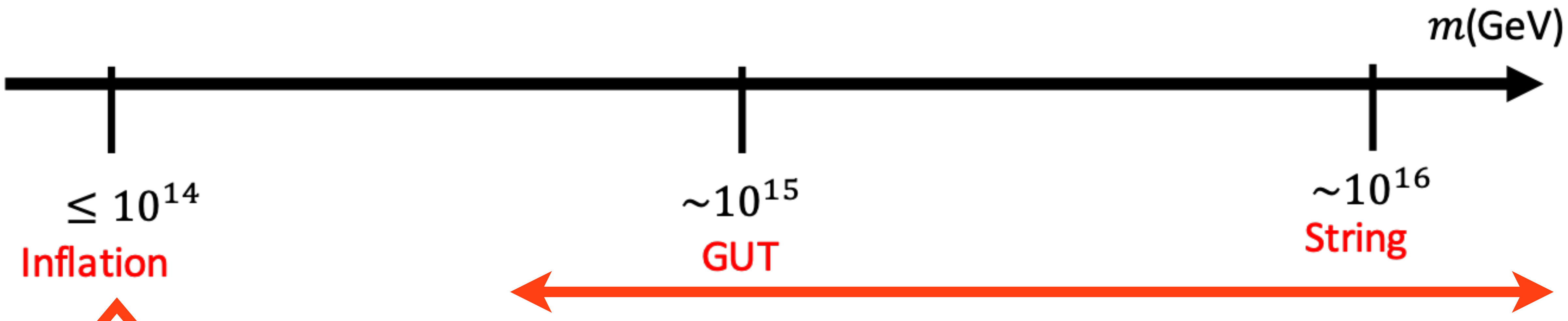
$\lesssim 10^{14}$
Inflation

$\sim 10^{15}$
GUT

$\sim 10^{16}$
String

Oscillating feature of non-Gaussianity

Summary: Cosmological collider program



m(GeV)

$\lesssim 10^{14}$
Inflation

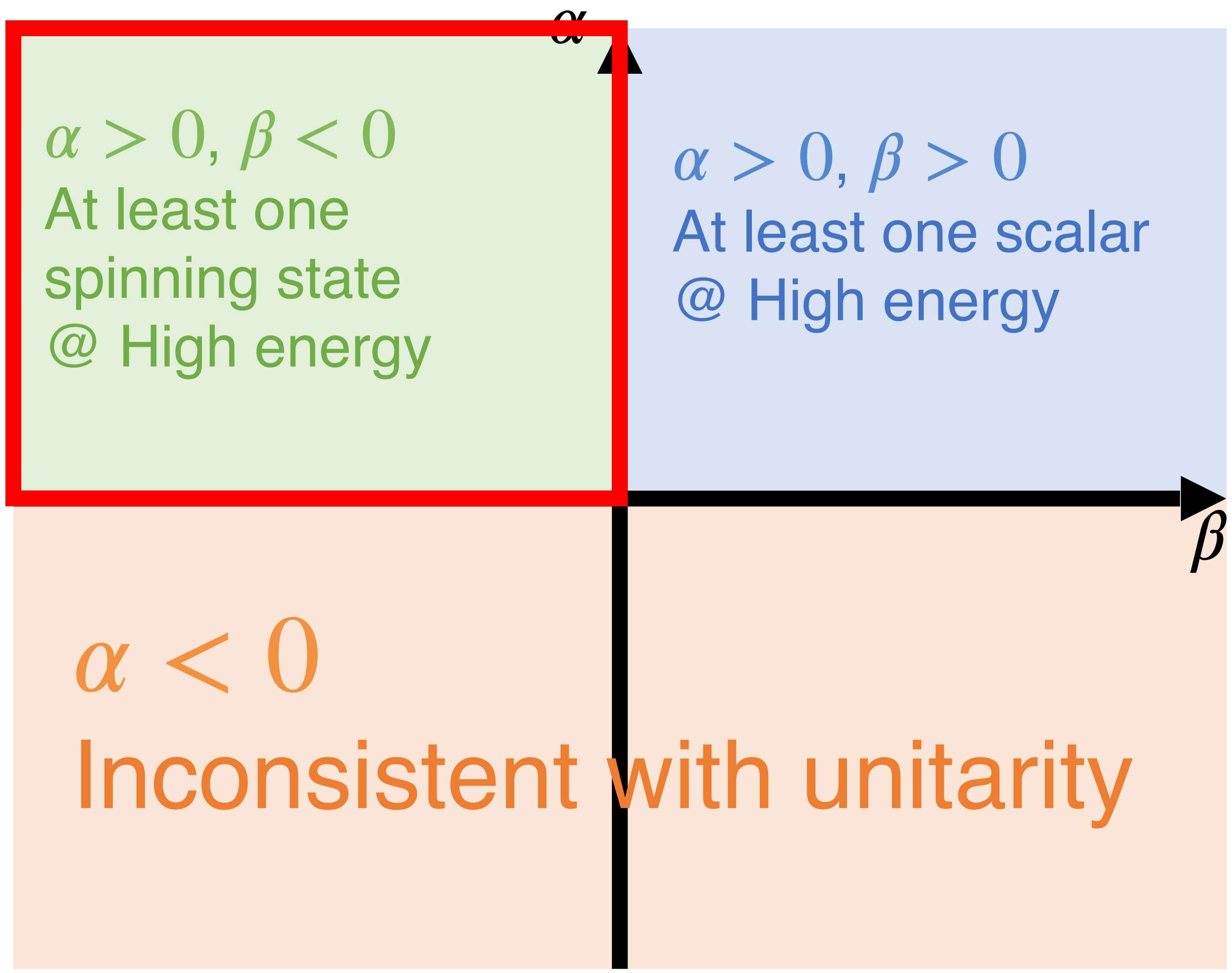
$\sim 10^{15}$
GUT

$\sim 10^{16}$
String

Oscillating feature of non-Gaussianity

Can probe with sign of effective couplings!!

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu\phi)^2+\frac{\alpha}{\Lambda^4}(\partial_\mu\phi)^4+\frac{\beta}{\Lambda^6}(\nabla_\mu\partial_\nu\phi)^2(\partial_\rho\phi)^2+\cdots$$



$\alpha > 0,\ \beta < 0$
At least one spinning state @ High energy

$\alpha > 0,\ \beta > 0$
At least one scalar @ High energy

$\alpha < 0$
Inconsistent with unitarity

Smoking gun of heavy spinning states

SK, Toshifumi Noumi, Keito Takeuchi, Siyi Zhou [2019]

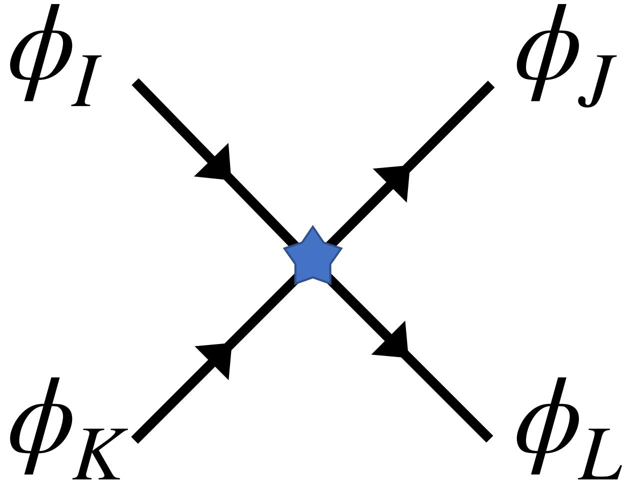
Field space geometrical approach to EFT

$$\mathscr{L} = -\frac{1}{2}G_{IJ}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J + V(\phi)$$

- $G_{IJ}(\phi)$: Field space metric
- Scattering amplitudes are invariant under field redefinitions
- Field redefinition = coordinate transformation of field space
- Scattering amplitudes can be written by geometrical invariants of the field space

Field space geometrical approach to EFT

$$\mathscr{L} = -\frac{1}{2}G_{IJ}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J + V(\phi)$$



$$\supset R_{IJKL}s + \cdots$$

Nagai, Tanabashi, Tsumura, Uchida [2019]

- 4pt scattering amplitudes proportional to s can be written by Riemann curvatures of field space

- Recently, generalizations to vector fields and fermions are discussed

Alonso, Kanshin, Saa [2017]

Finn, Karamitsos, Pilaftsis [2021]

Helset, Jenkins, Manohar [2022]

Generalization to higher derivatives

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \frac{\beta}{\Lambda^6}(\partial_\mu\partial_\nu\phi)^2(\partial_\rho\phi)^2 + \cdots$$

- Method using jet bundles (Lagrange space)

arXiv: 2305.09722 (Craig, Lee, Lu, Sutherland)
arXiv: 2307.15742 (Craig, Lee)
arXiv: 2308.00017 (Alminawi, Brivio, Davighi)

- Derivative of torsions give s^2 amplitudes
- What is the geometrical expression of s^2t amplitudes?
- What is the geometrical interpretation of the positivity bound and beyond?

Appendix

Open superstring amplitude

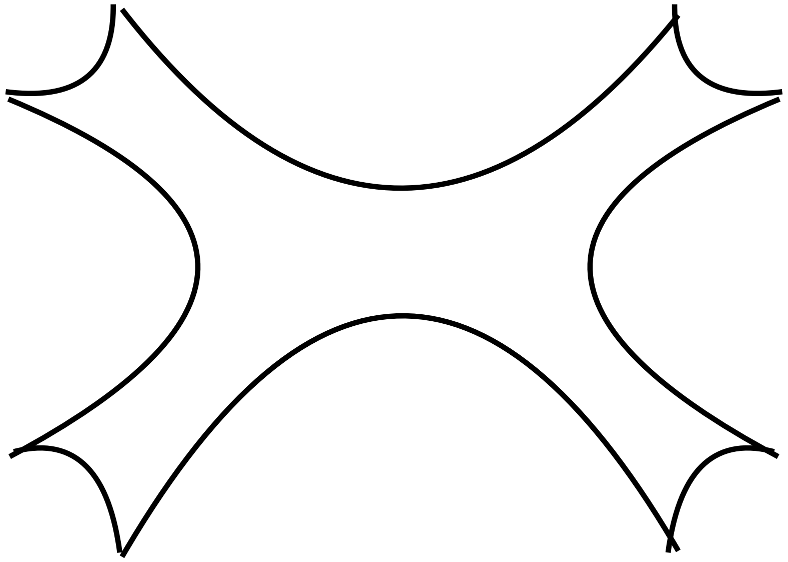
- Scalar amplitude in superstring theory

$$M(s,t) = \left(s^2 + t^2 + u^2\right)\left[\frac{B(-s,-t)}{s+t} + \frac{B(-t,-u)}{t+u} + \frac{B(-u,-s)}{u+s}\right]$$

$$\rightarrow \pi^2\left(s^2 + st + t^2\right) + \frac{\pi^4}{12}\left(s^2 + st + t^2\right)^2 + \ldots$$

low-energy limit

$$B(a,b) = \int_0^1 dx x^a (1-a)^b = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$



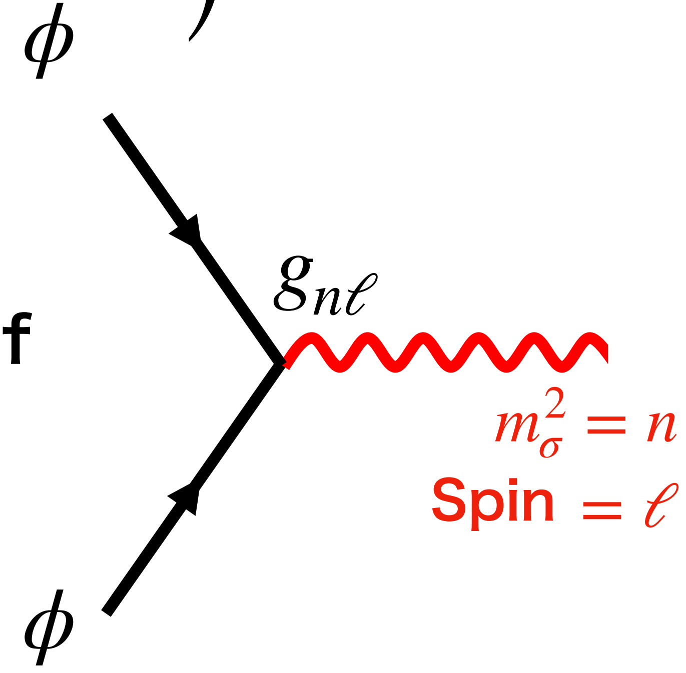
- Coefficients of $s^2 t$ is 0

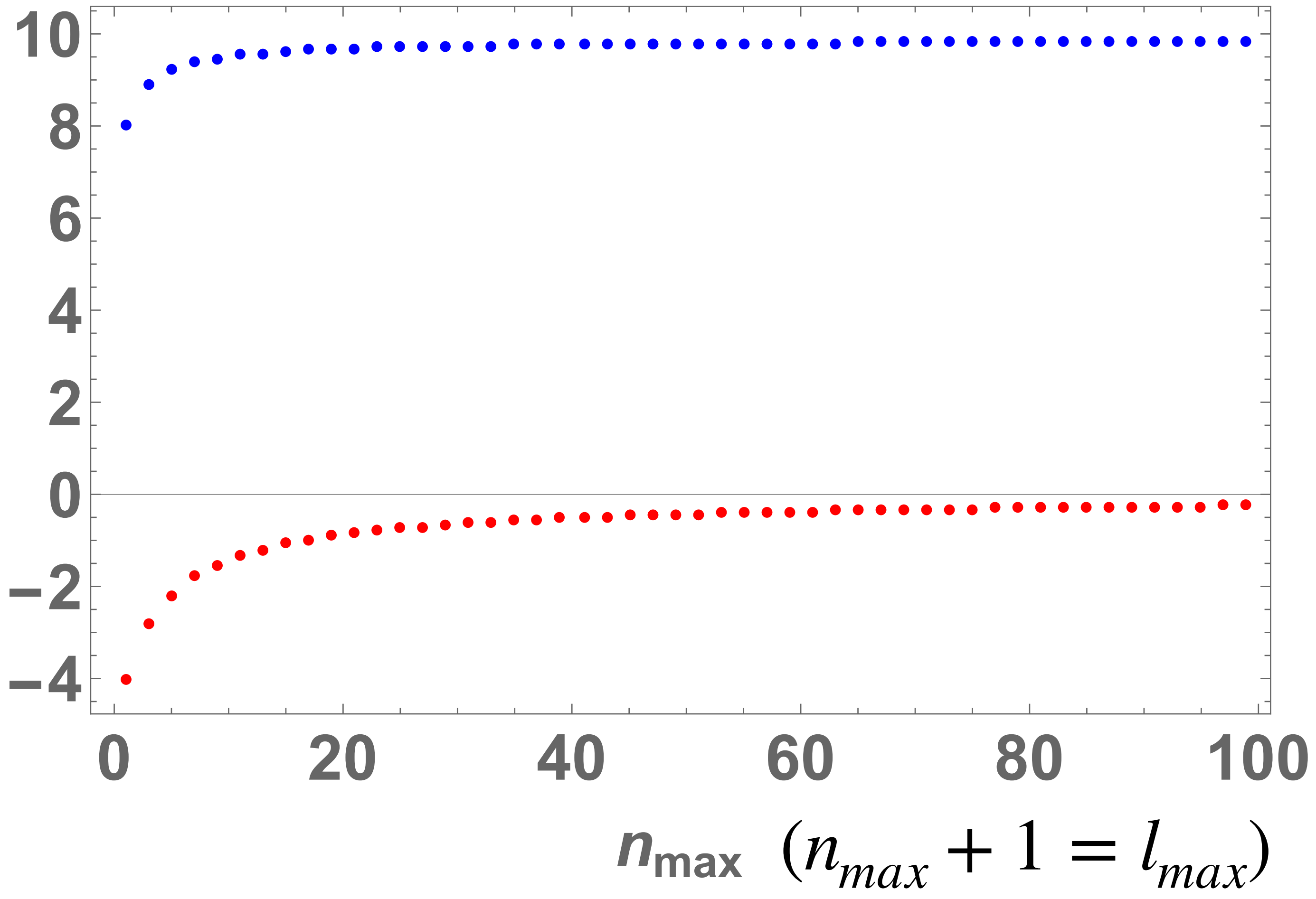
- Exact cancellation between contributions of scalar and higher spins

Open superstring amplitude

$$a_{2,1} = \sum_{n}^{m_{max}} \sum_{\ell=0}^{n+1} \frac{g_{n\ell}^2}{n^4}\left(2\ell_n^2 + 2\ell_n - 3\right)$$

$g_{n\ell}$ Is the coupling constant of



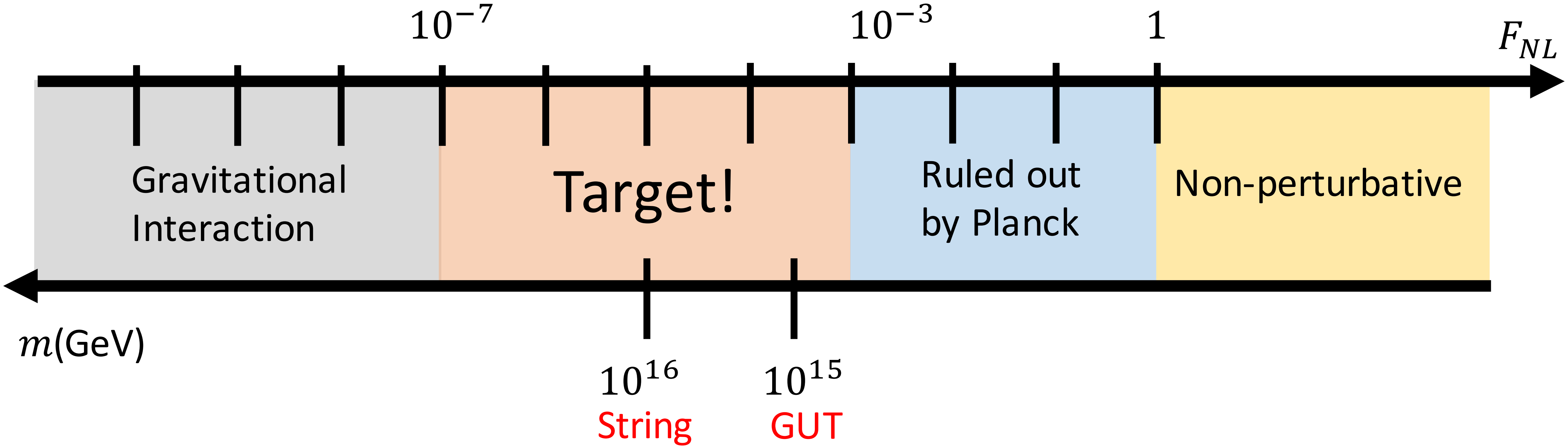


- Coefficient of s^2
- Coefficient of $s^2 t$

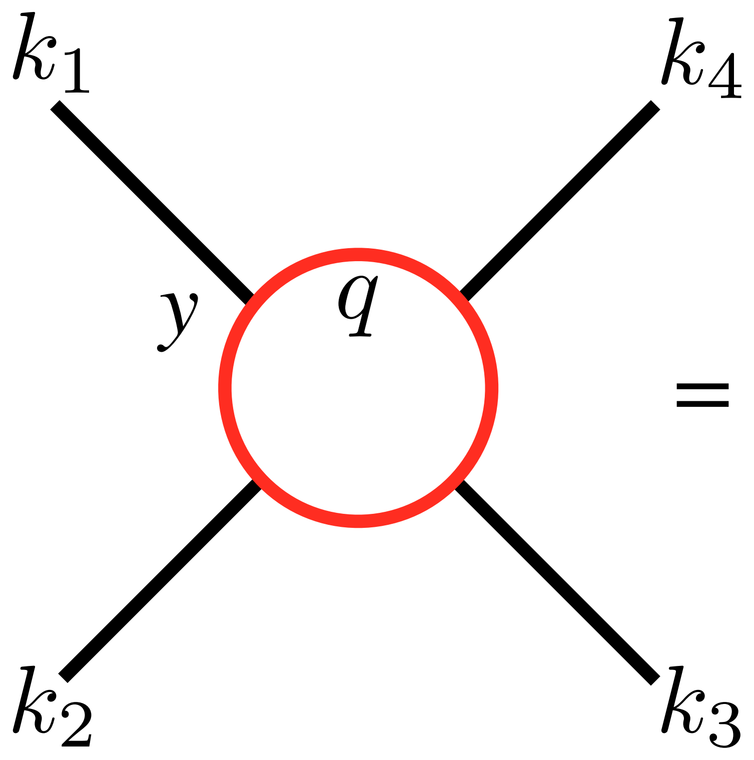
Energy scale of target

- Typically $F_{NL} = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} \sim \frac{H^2}{m^2}$

- For example, if $H \sim 3 \times 10^{13}$



Fermion Loop



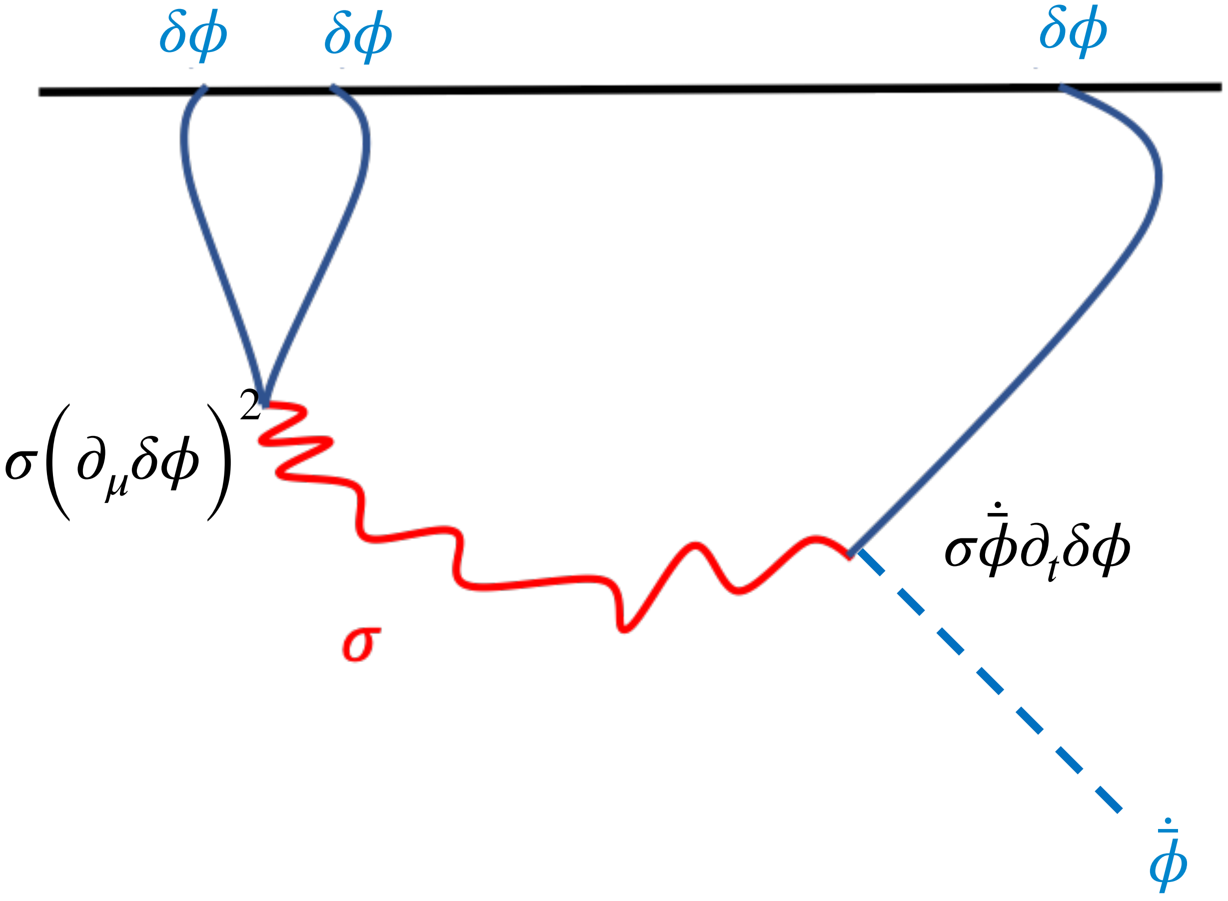
$$= \frac{11y^4}{720\pi^2 m^4}(s^2 + st + t^2) \boxed{- \frac{13y^4}{10080\pi^2 m^6}} st(s+t)$$

- Scalar intermediate states dominate over spinning ones.

Inflationary fluctuation

- Inflationary background $\langle \phi \rangle = \bar{\phi}(t)$ and fluctuation $\delta\phi$

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$$



Inflaton sector

$$(\partial_\mu\phi)^2 = \left(-\dot{\bar{\phi}}(t)^2 - 2\dot{\bar{\phi}}\delta\dot{\phi} + \left(\partial_\mu\delta\phi\right)^2\right)$$

Mixing

$$\sigma\left(\partial_\mu\phi\right)^2 \ni \underline{\sigma\left(-2\dot{\bar{\phi}}\delta\dot{\phi} + \left(\partial_\mu\delta\phi\right)^2\right)}$$

Time dependent background

New massive particle sector

$$\mathscr{L}_\sigma = -\frac{1}{2}\left(\partial_\mu\sigma\right)^2 - \frac{1}{2}m^2\sigma^2 - V(\sigma)$$

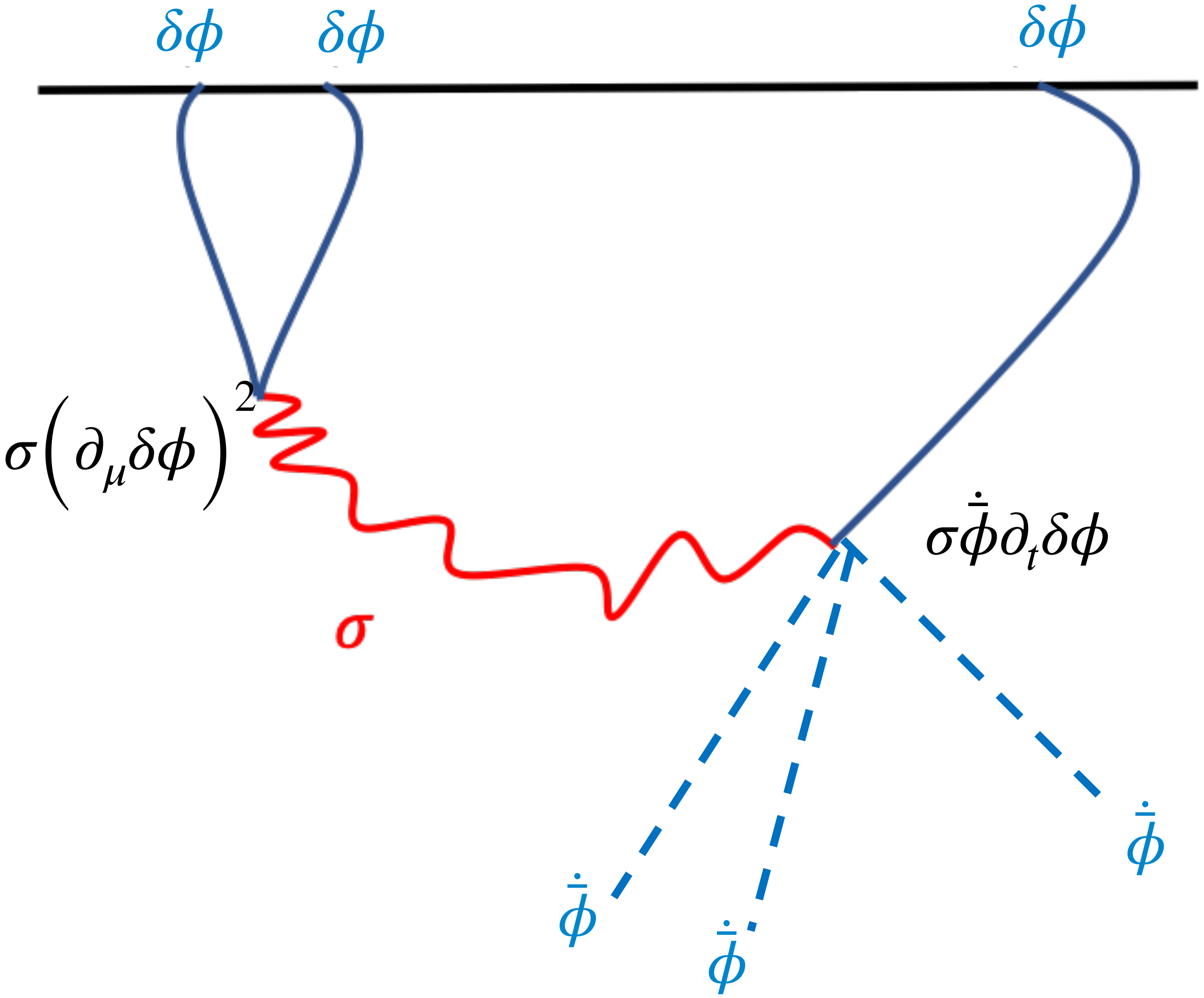
Inflationary fluctuation

- Inflationary background $\langle \phi \rangle = \bar{\phi}(t)$ and fluctuation $\delta\phi$

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$$

Inflaton sector

$$(\partial_\mu \phi)^2 = \left(-\dot{\bar{\phi}}(t)^2 - 2\dot{\bar{\phi}}\dot{\delta\phi} + (\partial_\mu \delta\phi)^2 \right)$$



Mixing

$$\sigma(\partial_\mu \phi)^2 \ni$$

$$\sigma\underline{\left(-2\dot{\bar{\phi}}\dot{\delta\phi} + (\partial_\mu \delta\phi)^2 \right)}$$

Time dependent background

New massive particle sector

$$\mathcal{L}_\sigma = -\frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{1}{2}m^2\sigma^2 - V(\sigma)$$

KK graviton

- Scattering amplitude which is generated by the exchange of the KK graviton

$$M(s,t) = \frac{g^2}{6}\left[\frac{-2s^2+3\left(t^2+u^2\right)}{m^2-s} + \frac{-2t^2+3\left(u^2+s^2\right)}{m^2-t} + \frac{-2u^2+3\left(s^2+t^2\right)}{m^2-u}\right]$$

$$\to \frac{4g^2}{3m^2}\left(s^2+st+t^2\right) + \frac{5g^2}{2m^4}\left(s^2t+st^2\right)$$

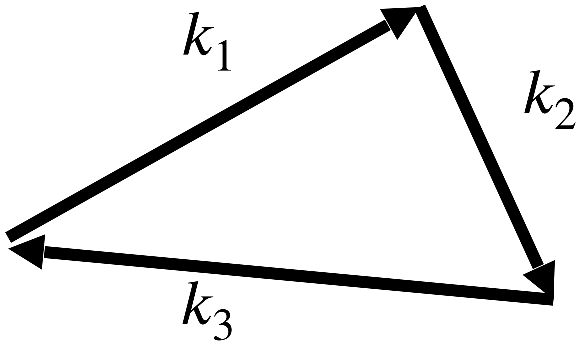
Q. How to read off the sign of the α and β

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu \phi)^4 + \frac{\beta}{\Lambda^6}(\nabla_\mu \partial_\nu \phi)^2(\partial_\rho \phi)^2 + \cdots$$

A. See the shape of the non-Gaussainities

3-point function

- $\left\langle \delta\phi_{k_1}(\tau)\delta\phi_{k_2}(\tau)\delta\phi_{k_3}(\tau) \right\rangle \Big|_{\tau\to 0}$ is the function of the triangle shape



- If α term and β term make the different shape dependence, we can distinguish and fix the sign of the α and β

- Schematically, we write

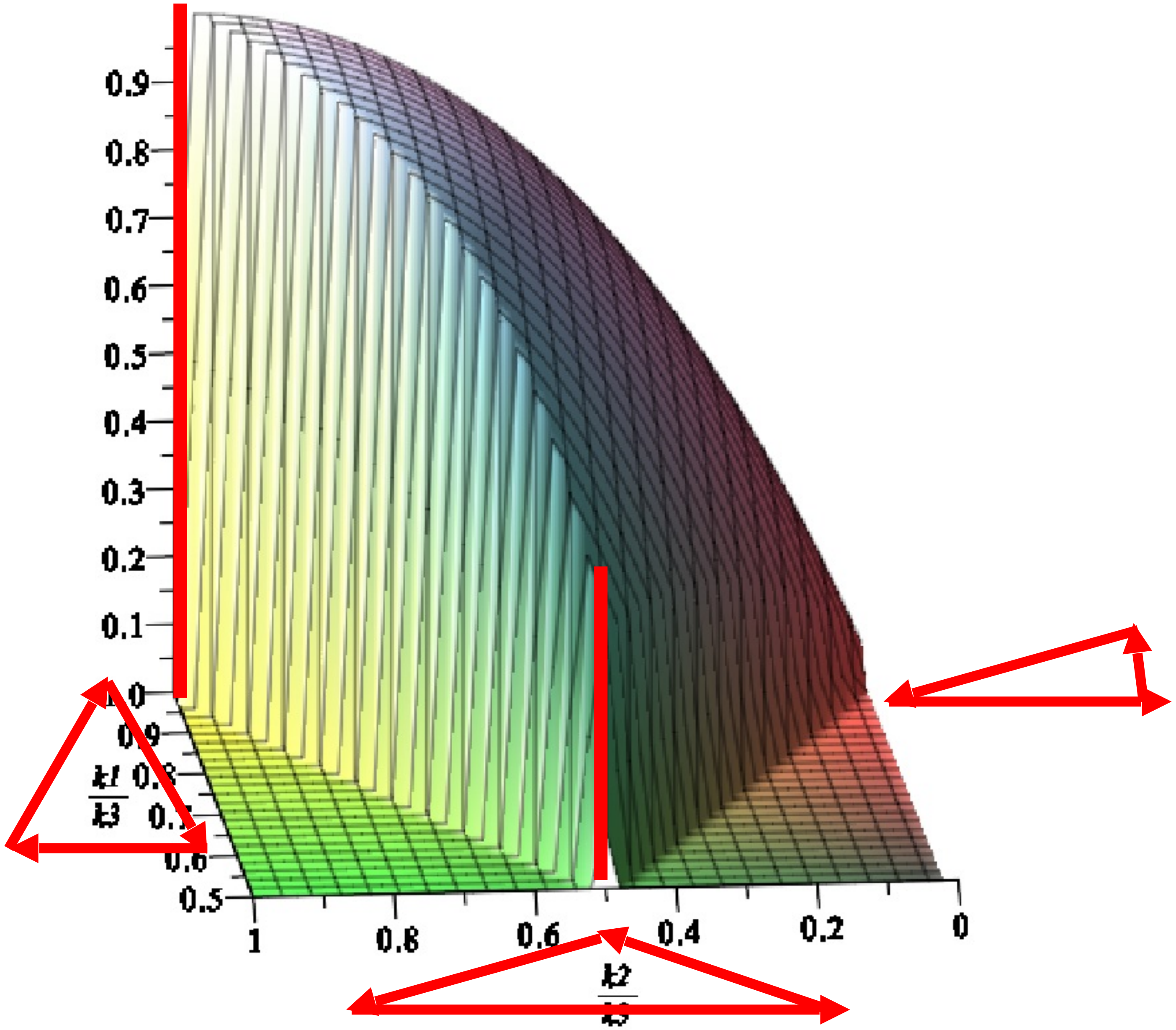
Shape function

$$\left\langle \delta\phi_{k_1}\delta\phi_{k_2}\delta\phi_{k_3} \right\rangle \propto \frac{1}{k_1^2 k_2^2 k_3^2} \boxed{\delta^3\left(\sum_i k_i\right)} \left[\alpha \boxed{\mathscr{A}_{3pt}\left(k_i\right)} + \frac{\beta m^2}{H^2} \boxed{\mathscr{B}_{3pt}\left(k_i\right)} \right]$$

Momentum conservation

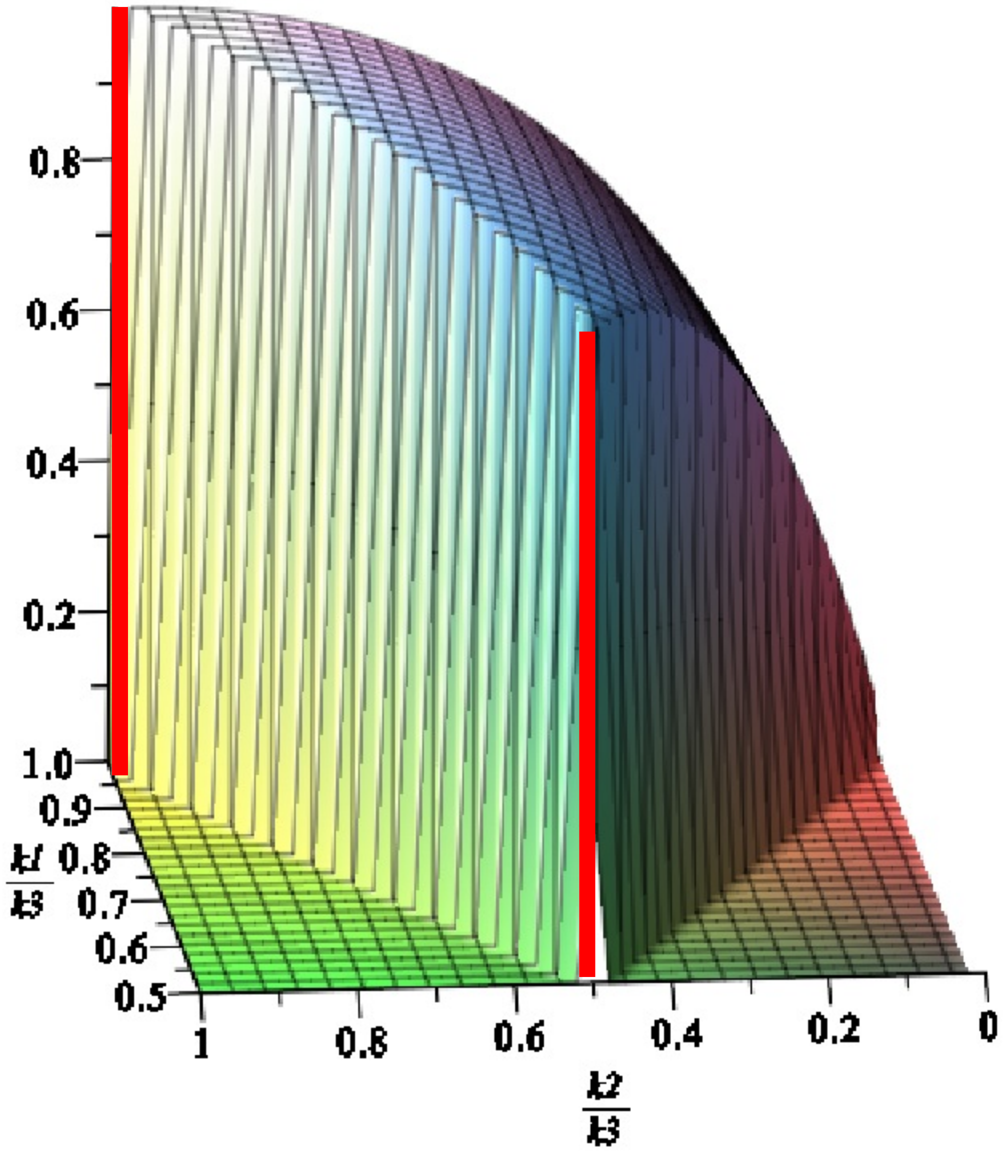
Shape function of α term and β

$\mathscr{A}_{3pt}(k_i)$



$$\frac{\mathscr{A}_{3pt}(0.5,\ 0,5,\ 1)}{\mathscr{A}_{3pt}(1,1,1)} = 0.32$$

$\mathscr{B}_{3pt}(k_i)$



$$\frac{\mathscr{B}_{3pt}(0.5,\ 0,5,\ 1)}{\mathscr{B}_{3pt}(1,1,1)} = 0.84$$