Gravitational Positivity For Dark Gauge Bosons

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3rd AEI workshop for BSM, Ikaho, October 5, 2023

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arXiv: 2305.10058 [hep-ph]

How dark is “dark”?

Gravity to Rescue?

$$\text{"}\left(\begin{matrix}\text{other}\\\text{forces}\end{matrix}\right) \geq \left(\text{gravity}\right)\text{"}$$

Gravity to Rescue ?

$$\text{“}\left(\begin{matrix}\text{other}\\\text{forces}\end{matrix}\right) \gtrsim \left(\text{gravity}\right)\text{”}$$

cf. · (gauge) weak gravity conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

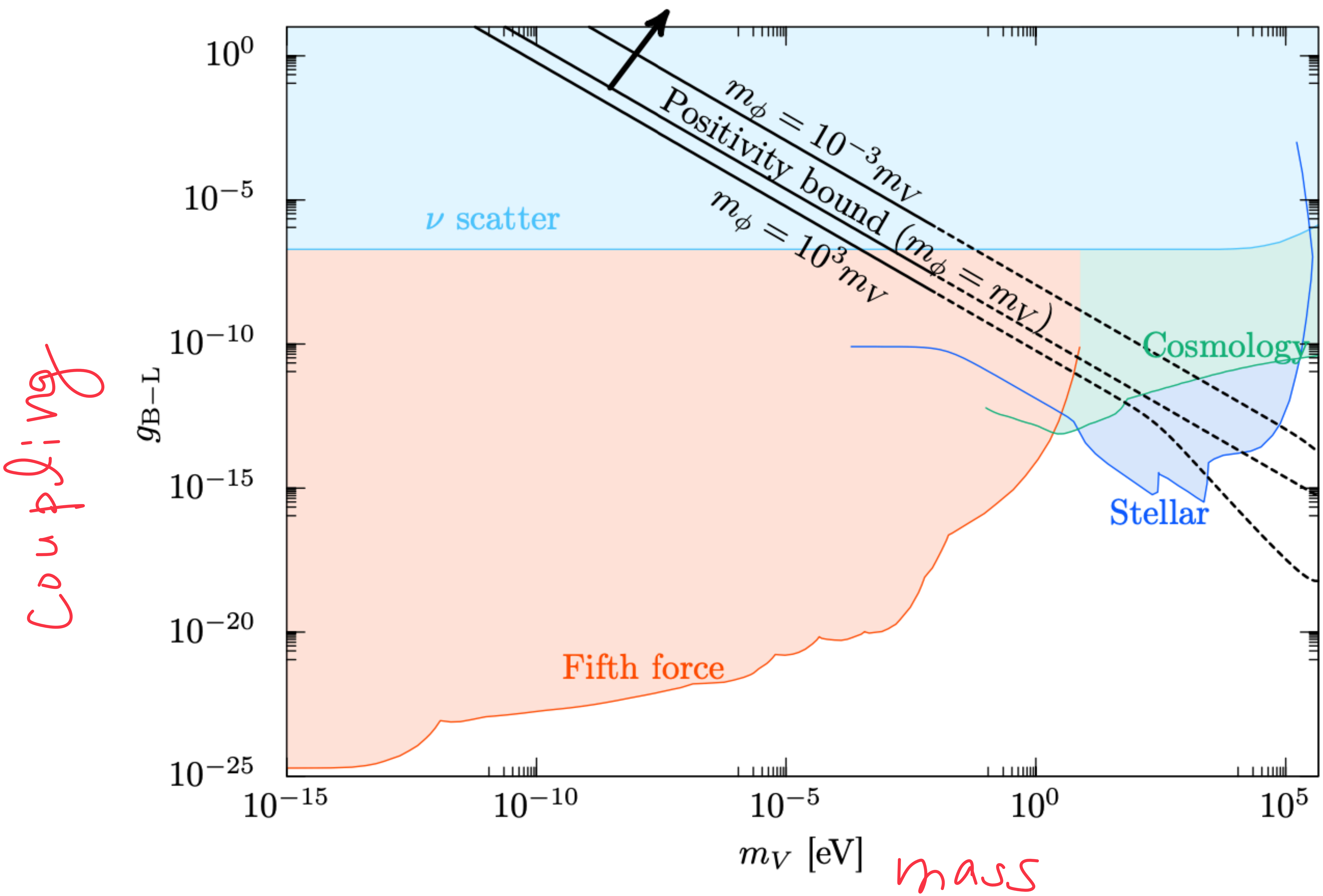
· scalar weak gravity conjecture

Palti '17, Shiraishi-MY '19, Kusenko-Takhistov-Yamada-MY '19

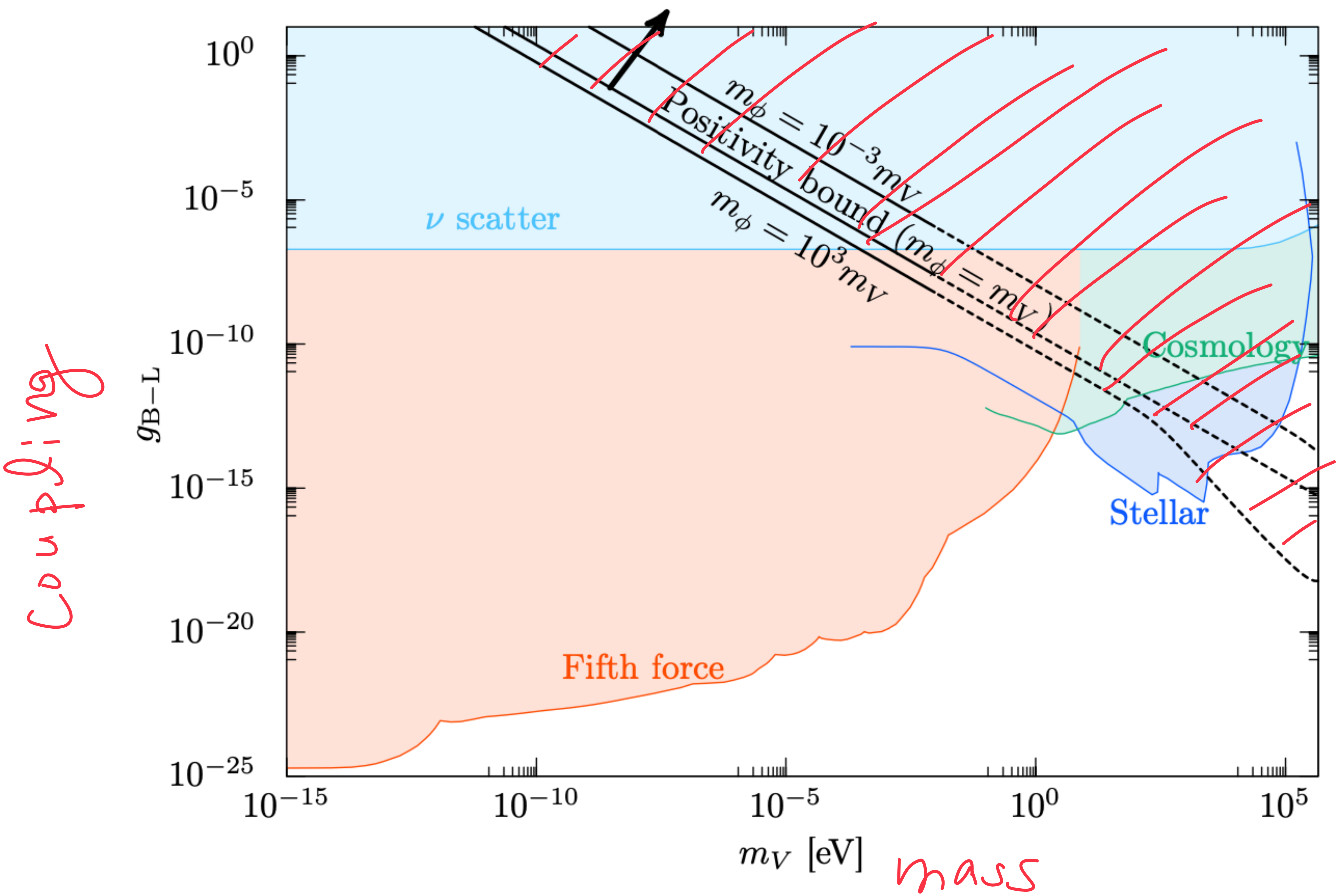
· gravitational positivity

Tokuda, Aoki, Hirano '20, ...

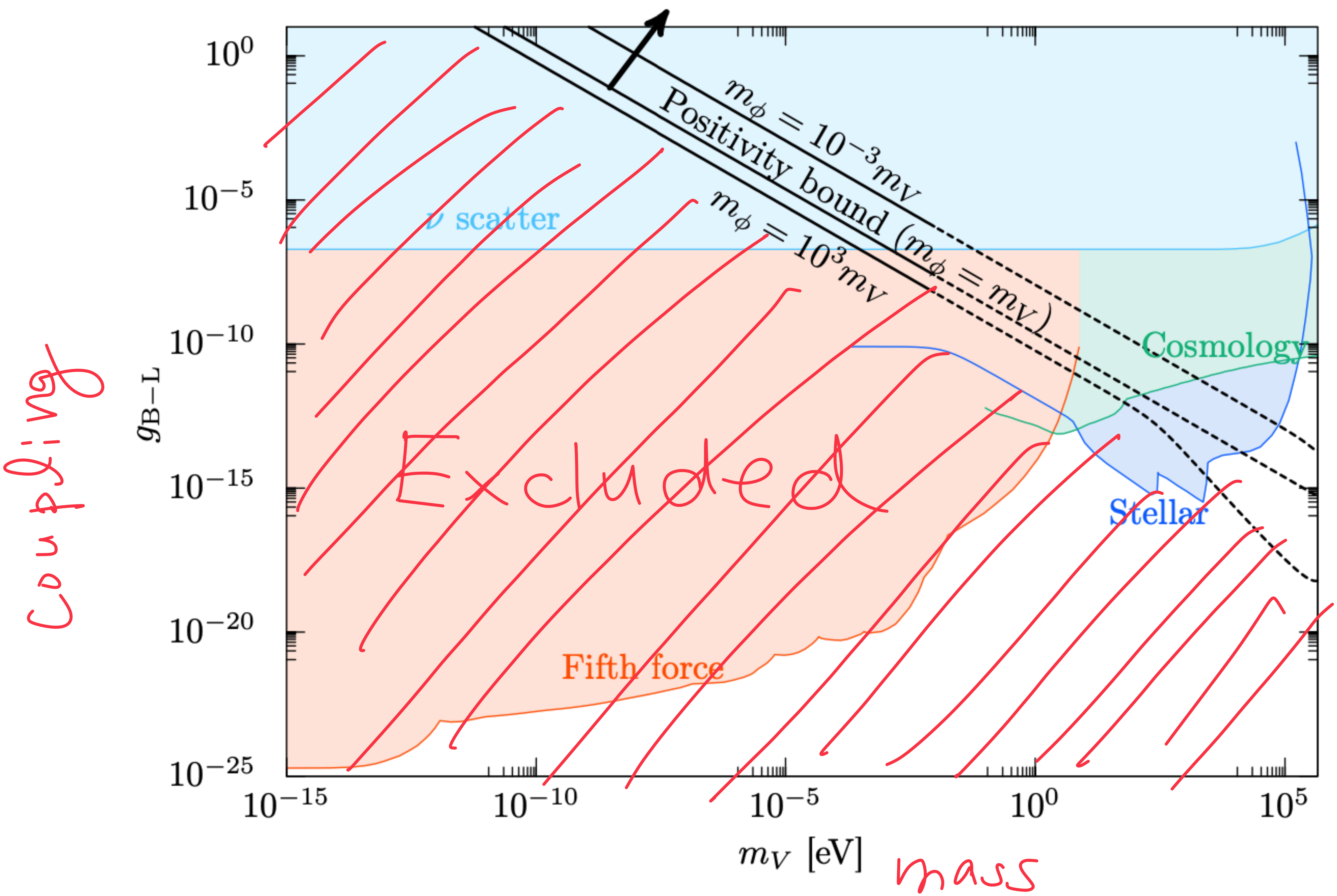
"Result" of gauged $U(1)_{B-L}$



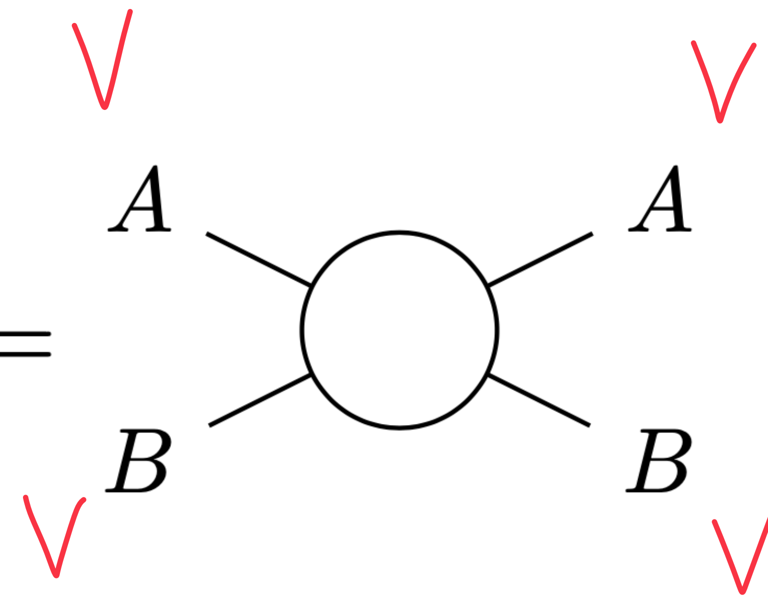
"Result" of gauged $U(1)_{B-L}$



"Result" of gauged U(1) B-L



$$i\mathcal{M}_{\text{non-grav}}(s,t) =$$



$$+\cdots$$

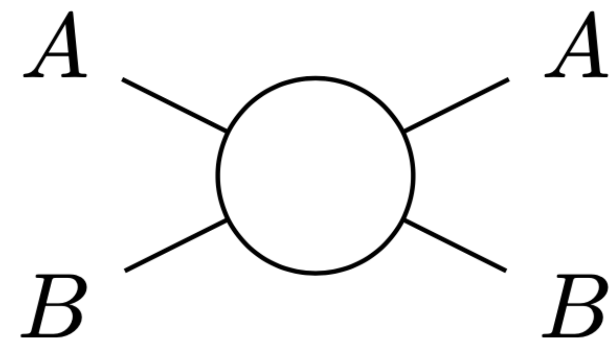
Assumptions

(i) analyticity

(ii) unitarity $(\mathrm{Im}\,\mathcal{M} \geq 0)$

(iii) s^2-boundedness $\displaystyle \lim_{|s|\to\infty} \frac{|\mathcal{M}(s,0)|}{s^2} = 0$ [Froissart ('61), Martin ('63)]

["old" S-matrix theory since '60s]

$$i\mathcal{M}_{\text{non-grav}}(s,t) =$$ 

$$+ \cdots$$

$$a_2 := \left. \frac{\partial^2 \mathcal{M}(s,t=0)}{\partial s^2} \right|_{s=2m_V^2}$$

$$B_2(\Lambda) := a_2 - \frac{2\cdot 2!}{\pi} \int_{m_{th}^2}^{\Lambda^2} ds\, \frac{\mathrm{Im}\,\mathcal{M}(s,t=0)}{(s-2m_V^2)^3}$$

$$= \frac{2\cdot 2!}{\pi} \int_{\Lambda^2}^{\infty} ds\, \frac{\mathrm{Im}\,\mathcal{M}(s,t=0)}{(s-2m_V^2)^3} \geq 0$$

dispersion rel,
analyticity + s^2-boundedness

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06
Bellazzini '16 de Rham, Melville, Tolley, Zhou '17, ...]

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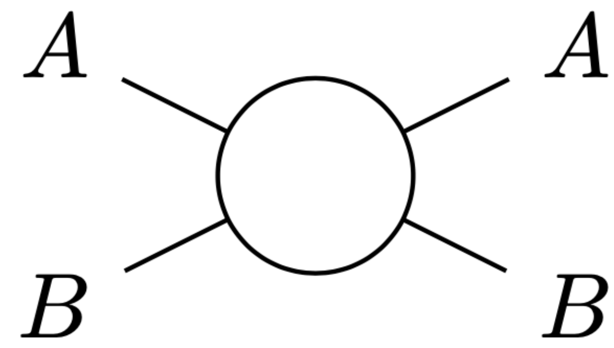


$$+\cdots$$

$$a_2 := \left.\frac{\partial^2 \mathcal{M}(s,t=0)}{\partial s^2}\right|_{s=2m_V^2}$$

IR data

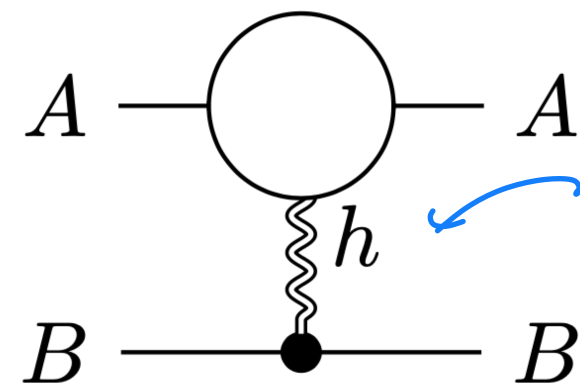
$$B_2(\underset{\text{cutoff scale}}{\Lambda}) := \boxed{a_2 - \frac{2\cdot 2!}{\pi}\int_{m_{th}^2}^{\Lambda^2} ds\, \frac{\operatorname{Im}\mathcal{M}(s,t=0)}{(s-2m_V^2)^3}}$$

$$= \boxed{\frac{2\cdot 2!}{\pi}\int_{\Lambda^2}^{\infty} ds\, \frac{\operatorname{Im}\mathcal{M}(s,t=0)}{(s-2m_V^2)^3} \geq 0}$$

UV data

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06
Bellazzini '16 de Rham, Melville, Tolley, Zhou '17, ...]

$$i\mathcal{M}_{\text{grav},t\text{-channel}}(s,t) =$$



$$+\cdots$$

graviton

$\frac{s^2}{t}$: violates s^2-boundedness ?

$$A(s,t) = \sum_J \frac{g_J^2 (-s)^J}{t - M_J^2}$$

spin J

∞ tower of states

$$A(s,t) = \sum_J \frac{g_J^2(-s)^J}{t - M_J^2}$$

$\leftarrow$ spin J

∞ tower of states

$$\left(\text{cf.} \quad \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = e^{-x} \right)$$

$$A(s,t) = \sum_J \frac{g_J^2 (-s)^J}{t - M_J^2}$$

spin J

∞ tower of states

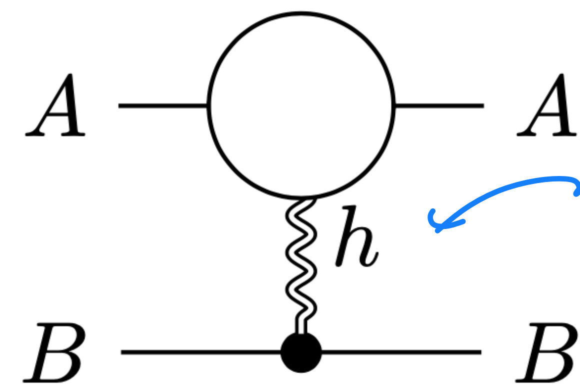
$$\left(\text{cf. } \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = e^{-x} \right)$$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = A(t,s)$$

$$\sim s^{\alpha(t)} \quad (s \to \infty)$$

$$\left(\begin{array}{l} \alpha(s) = \alpha' s + \alpha(0) \\ M_n^2 = \dfrac{n - \alpha(0)}{\alpha'} \end{array} \right)$$

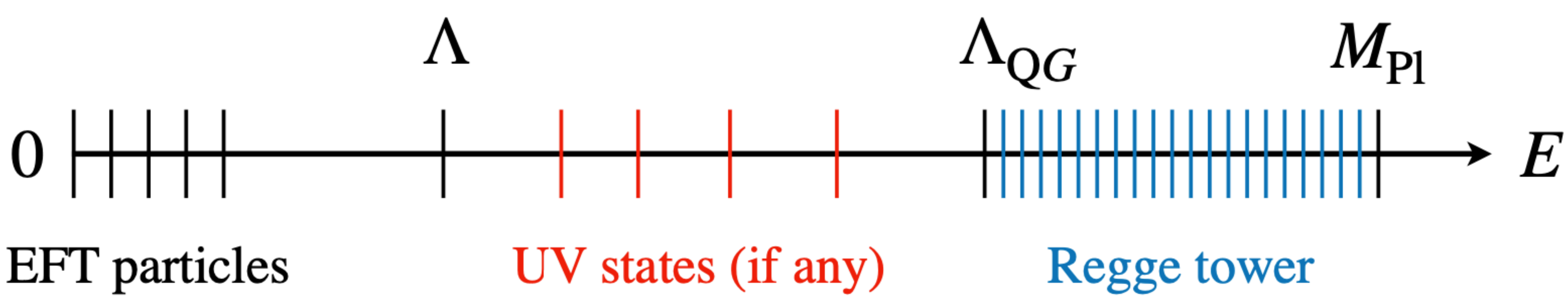
$$i\mathcal{M}_{\mathrm{grav},t\text{-channel}}(s,t) =$$



$$+ \cdots$$

graviton

$\frac{s^2}{t}$: violates s^2-boundedness ?



recovers s^2-boundedness

$$B_{\text{non-grav}}(\Lambda) \simeq \frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds\, \frac{\operatorname{Im}\mathcal{M}_{\text{non-grav}}(s,t=0)}{\left(s-2m_V^2\right)^3}$$

$$B_{\text{grav}}(\Lambda) \simeq \lim_{t\to -0}\left[\frac{\partial^2\mathcal{M}_{\text{grav},t\text{-channel}}(s,t)}{\partial s^2} + \frac{2}{M_{\text{Pl}}^2 t} - (\text{kinematic singularity})\right]_{s=2m_V^2}$$

sum

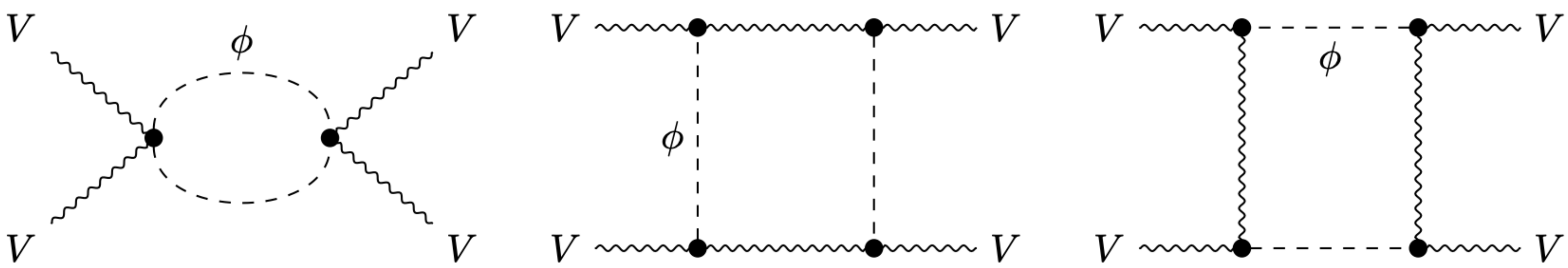
$$B(\Lambda) \geq \mathcal{O}\left(\frac{1}{M_{pl}^2 M^2}\right)$$

related to Λ_{QG}

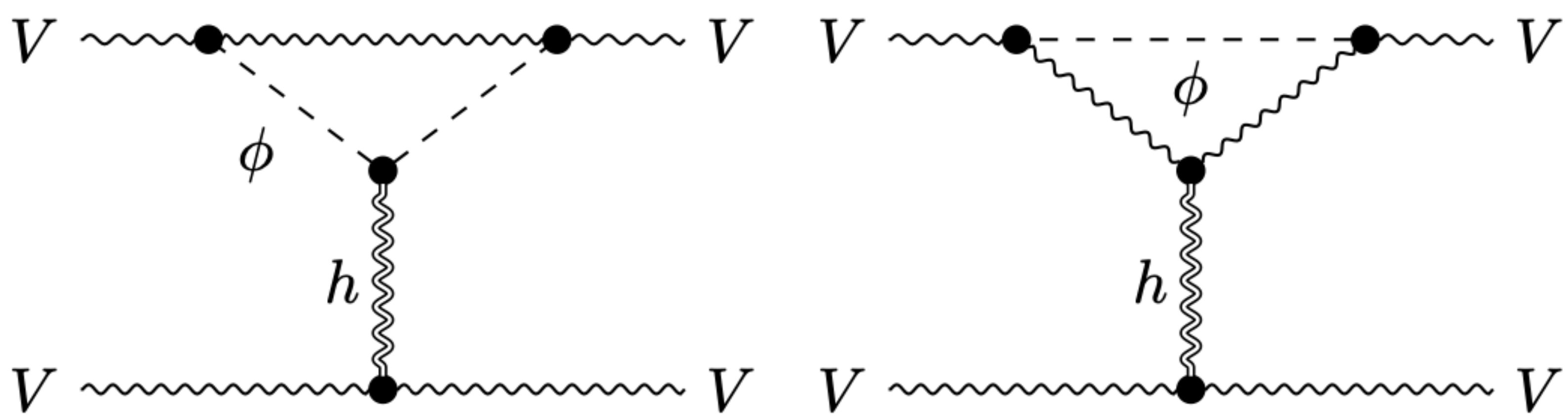
[Tokuda, Aoki, Hirano '20

cf. Alberte, de Rham, Jaitly, Tolley '20]

Example: U(1) gauge boson with Higgs mass

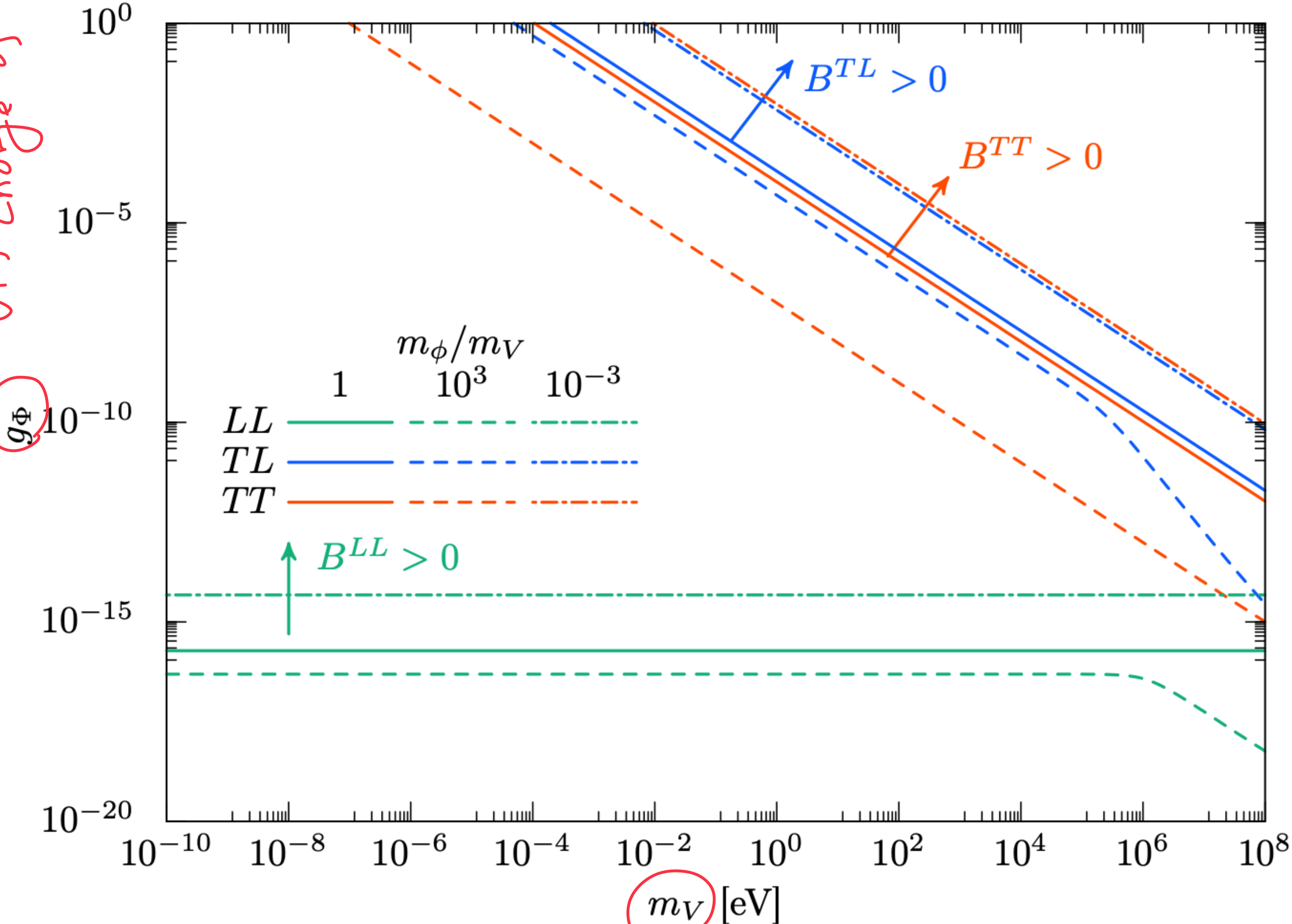


non-gravitational $\qquad B_{\text{non-grav}} \overset{\text{e.g.}}{\sim} + \frac{g_\Phi^4}{\Lambda^4} + \cdots$

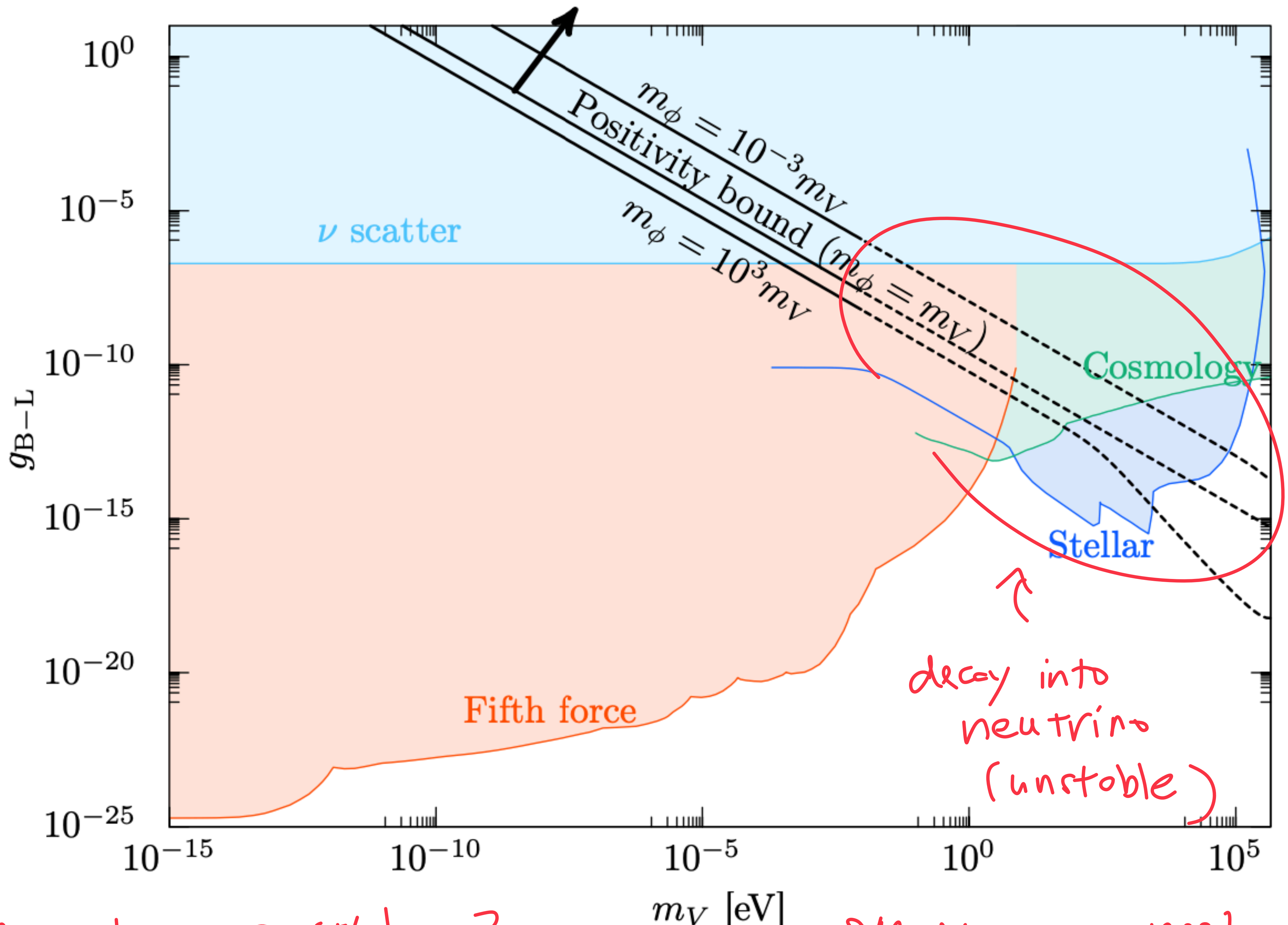


gravitational $\qquad B_{\text{grav}} \overset{\text{e.g.}}{\sim} - \frac{g_\Phi^2}{m_\Phi^2 M_{pl}^2}$

U(1) charge of Φ



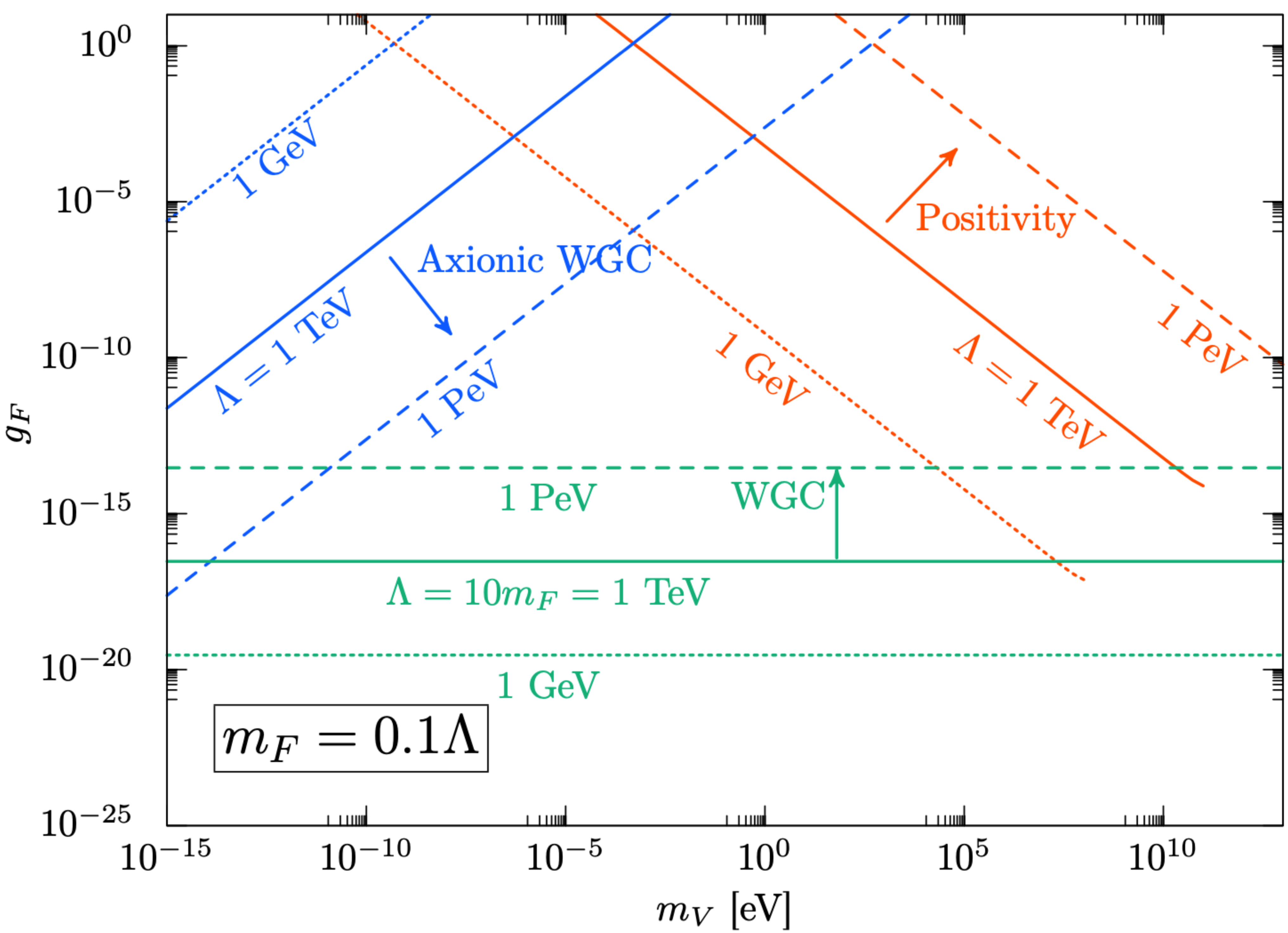
Realistic Cases: Theoretically Subtle



massless particles?

SM diagrams neglected

Example: Stückelberg fermion m_F, g_F, m_V



Summary

(Gravity
UV completion) $\xrightarrow[\text{recipes}]{\text{practical}}$ (lower bound
on dark sector
couplings)

Implications of "Quantum Gravity"
for Dark Sector