Natural SO(10) GUT

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1. **Introduction & summary**
2. **Natural (anomalous U(1)) GUT**
3. **Sp. SUSY breaking in natural GUT**
4. **Predictions**
5. **Summary and discussion**

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Grand Unified Theories

2 Unifications theoretically interesting

- Gauge Interactions

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$SO(10) \supset SU(5)$$

- Matter

$$\boxed{Q \quad U_R^c \quad E_R^c} \; \boxed{D_R^c \quad L} \; \boxed{N_R^c}$$

$$\mathbf{10} \quad + \quad \bar{\mathbf{5}} + \mathbf{1} \; = \; \mathbf{16}$$

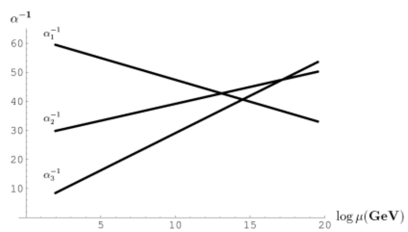
Experimental supports for both unifications

GUT is promising

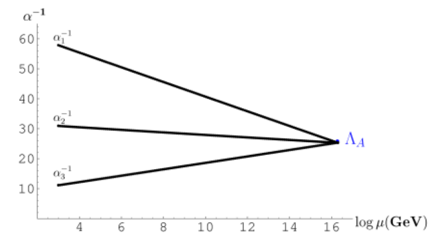
Grand Unified Theories

- Unification of gauge interactions
  quantitative evidence:

Non SUSY



SUSY GUT



- Unification of matters
  qualitative evidence:

$$(Y_u)_{ij}10_i10_j5_H + (Y_d)_{ij}10_i\bar{5}_j\bar{5}_H + (Y_\nu)_{ij}\bar{5}_i\bar{5}_j5_H5_H$$

$10_i(Q_i)$ have stronger hierarchy than $\bar{5}_i(L)$

⇒ hierarchies of masses and mixings

lepton >>quark (in hierarchies for mixings)

ups >> downs, electrons >> neutrinos (in mass hierarchies)

Two Problems of GUT

- The doublet-triplet problem (the most serious)
- Unrealistic relations for Yukawa couplings due to unification of matters.

$$Y_d = Y_e^t \qquad Y_u^{ij} 10_i 10_j 5_H + Y_d^{ij} 10_i \bar{5}_j \overline{5_H} + Y_{\nu_D}^{ij} \bar{5}_i 1_j 5_H$$

in SU(5)

$$Y_u = Y_d = Y_e = Y_{\nu_D} \qquad Y^{ij} 16_i 16_j 10_H$$

in SO(10)

$$\boxed{Q \quad U_R^c \quad E_R^c} \; \boxed{D_R^c \quad L} \;\; \boxed{N_R^c}$$

$$\mathbf{10} + \bar{\mathbf{5}} \quad \mathbf{1} = \mathbf{16}$$

Doublet—Triplet Splitting

- Minimal mulitiplet in $SU(5)$

$$\bar{\mathbf{5}}_{\bar{H}} = (\bar{H}_C, H_d) \qquad \mathbf{5}_H = (H_C, H_u)$$

$$\begin{cases} M_C = M_0 + g2v > 3 \times 10^{17}\text{GeV} \\ M_H = M_0 - g3v \sim 100\text{GeV} \end{cases}$$
: the proton

$$\Longleftrightarrow \quad \Lambda_{SGUT} \sim 2 \times 10^{16}\text{GeV} \sim v$$

$$W = \bar{\mathbf{5}}_{\bar{\mathbf{H}}}(M_0 + g\mathbf{24}_{\mathbf{A}})\mathbf{5}_{\mathbf{H}} \qquad SU(5) \to G_{SM}$$

10^{15} Finetuning $\quad g \sim 10?$ $\quad \langle \mathbf{24}_A \rangle = \begin{pmatrix} 2v & 0 \\ 0 & -3v \end{pmatrix}$

How to realize this large mass hierarchy without finetuning?

Higher dim. Int. $\bar{5}_{\bar{H}} \left(\frac{24_A}{\mathrm{M_{Planck}}} \right)^{\mathrm{n}} 24_{\mathrm{A}} 5_{\mathrm{H}}$ must be controlled.

$n \leq 7$

Several attempts

- 1, Missing partner mechanism
- 2, Sliding singlet mechanism
- 3, Missing VEV mechanism
- 4, Pseudo Nambu-Goldstone
- 5, Orbifold GUT(5 dim.)
- 6, Others, …, etc.

But it is difficult to solve this problem naturally in 4D.
In most models, int. allowed by symmetry is omitted by
hand, and/or very small couplings are assumed.

Missing VEV mechanism

Dimopoulos-Wilczek 82

- SO(10) $A : \mathbf{45}$

$$H, H' : \mathbf{10} = \mathbf{5} + \bar{\mathbf{5}} \qquad W(A) = A^2 + A^4$$

$$\langle A \rangle = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \otimes \begin{pmatrix} v & & & & \\ & v & 0 & & \\ & & v & & \\ & 0 & 0 & & \\ & & & & 0 \end{pmatrix} \propto Q_{B-L}$$

$$W = HAH' + M_{H'}H'H' \qquad \times HH, \times HH'$$

$$\begin{array}{c|cc} & \bar{\mathbf{5}}_H & \bar{\mathbf{5}}_H \\ \mathbf{5}_H & 0 & \langle A \rangle \\ \mathbf{5}_{H'} & \langle A \rangle & M_{H'} \end{array} \qquad \langle A \rangle = \begin{cases} v & (H_C) \\ 0 & (H_u, H_d) \end{cases}$$

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

An issue

Must introduce additional Higgs fields to break

$SU(2)_R \times U(1)_{B-L} \to U(1)_Y$

$C : \mathbf{16}$

$\bar{C} : \overline{\mathbf{16}}$

To obtain the VEV $\langle A \rangle$

$\mathbf{24+1}$

$(C, \bar{C})$ and A must be separated

$$W = W(C, \bar{C}) + W(A)$$

$$\langle A \rangle = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \otimes \begin{pmatrix} v & & & & \\ & v & 0 & & \\ & & v & & \\ & 0 & & 0 & \\ & & & & 0 \end{pmatrix}$$

But then Pseudo Nambu-Goldstones appear

$\langle A \rangle$ $SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$(3,2)_{\frac{6}{5}} + (3,2)_{\frac{1}{6}} + (3,1)_{-\frac{2}{3}} + h.c.$$

$\langle C \rangle = \langle \bar{C} \rangle$ $SO(10) \to SU(5)$

$$(3,2)_{\frac{1}{6}} + (3,1)_{-\frac{2}{3}} + (1,1)_1 + h.c. + (1,1)_0$$

$(C, \bar{C})$ and A must be coupled each other

Natural GUT
(Anomalous U(1) GUT)

N.M.01
N.M.&Yamashita,02

Important differences (the previous models ⟺ ours)

Dvali,Hall-Raby,
Berezhiani-Tavartkiladze,…

1. **Doublet-triplet splitting problem is solved with generic interactions (incl. higher dim. Interactions) with O(1) coefficients.**

2. **Natural gauge coupling unification.**
   GUT with anomalous U(1) explains the success of gauge coupling unification in MSSM.

Natural Grand Unified Theory

- Problems of SUSY GUT are naturally solved.
- We use the word “natural”, because we

introduce all the terms which are allowed by the symmetry with O(1) coefficients

(incl. infinite higher dimensional interactions.)

- Gravity scale is not far from the GUT scale. $\frac{\Lambda_G}{\Lambda_{Pl}} \gg \frac{\Lambda_{Weak}}{\Lambda_G}$

  We have to consider higher dim. Interactions.

- We can define the model only by determining the symmetry of the model(except O(1) coefficients). The parameters for the definition are mainly about 10 $U(1)_A$ charges for the fields.
- The predictions are expected to be stable under the quantum corrections or gravity effects.

Anomalous U(1) Symmetry

- U(1) with FI-term $\xi^2 \int d^4\theta V_A, \quad (\xi^2 < \Lambda^2)$

$$D_A = \xi^2 - |\Theta|^2 \to \langle \Theta \rangle = \xi \equiv \lambda\Lambda \quad \lambda \sim 0.22$$

$\theta = -1$ small letter charges

Anomalous U(1) is broken just below the cutoff.

- Hierarchies of Yukawa couplings

Froggatt-Nielsen 79
Ibanez-Ross 94

$$W = \left(\frac{\Theta}{\Lambda}\right)^{q_i+u_j+h_u} Q_i U_j H_u \longrightarrow \lambda^{q_i+u_j+h_u} Q_i U_j H_u$$

$$(Y_u)_{ij} \sim \lambda^{q_i+u_j+h_u}, \ (Y_d)_{ij} \sim \lambda^{q_i+d_j+h_d}, \ (Y_e)_{ij} \sim \lambda^{l_i+e_j+h_d}$$

- Criticism: Any mass spectrum can be obtained.

But mixings $(V_{CKM})_{ij} \sim \lambda^{|q_i-q_j|} \to V_{13} \sim V_{12}V_{23}$

In neutrino sector $(m_\nu)_{ij} \sim \lambda^{l_i+l_j+2h_u}\frac{\langle H_u\rangle^2}{\Lambda}, \quad (V_{MNS})_{ij} \sim \lambda^{|l_i-l_j|}$

$$(V_{23})^4 \sim \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \sim \lambda^2(LMA), \lambda^{3-4}(SMA), \lambda^6(LMO), \lambda^{11}(VAC)$$

V_{23}	0.7	0.5 ◎	0.3 △	0.1 ×	0.05 ×
	Exp.			N.M.01	

Natural GUT

- Basic assumption for FN mechanism here: All interactions allowed by symmetry are introduced with O(1) coefficients.
- This basic assumption is applied to the GUT Higgs sector in addition to Yukawa sector.
- It is reckless because infinite number of terms must be considered.

How to control infinite # of generic interactions?

SUSY zero mechanism

(Holomorphic zero)

Bando, N.M.01

VEV determination

$$\langle Z \rangle = \begin{cases} 0 & z > 0 \\ \lambda^{-z} & z \le 0 \end{cases}$$

$\Lambda = 1$

An example

$Z_i^+ (i = 1, \cdots, n_+)$

$Z_i^- (i = 1, \cdots, n_-)$

$W(Z_i^+, Z_i^-)$

$$\begin{cases} \dfrac{\partial W}{\partial Z_i} = 0 \\ D_A = \xi^2 + z_i |Z_i|^2 = 0 \end{cases}$$

$n_+ + n_- - 1$ gauge inv. of W → $\langle Z_i \rangle \approx O(\Lambda)$

When $n_+ < n_-$, $\langle Z_i^+ \rangle = 0$ can be a solution.

$$\begin{cases} \dfrac{\partial W}{\partial Z_i^+} = 0 \\ D_A = \xi^2 - |z_i^-| |Z_i^-|^2 = 0 \end{cases}$$

n_+ → $\langle Z_i^- \rangle \le \xi$

VEV determination

- All the coefficients are determined by the anomalous U(1) charges. Bando-N.M.01

⇒ VEVs are determined by the charges.

$$\langle Z \rangle = \begin{cases} 0 & z > 0 \\ \lambda^{-z} & z \leq 0 \end{cases}$$

$W_S = \lambda^s S + \lambda^{s+z} SZ$

$\frac{\partial W_S}{\partial S} \propto 1 + \lambda^z Z$

$\to Z \sim \lambda^{-z}$

Z GUT gauge singlet

$\Lambda = 1$

- Holomorphic zero (SUSY zero)

The terms with negative total charges are forbidden.

$$x + y + z < 0 \longrightarrow \lambda^{x+y+z} XYZ$$

Important in solving DT splitting problem naturally.

How to control generic interactions?

- Only terms $W_{Z_i^+}$ linear in positive charged fields Z_i^+ are important to determine the VEVs of negative charged fields Z_i^-. (finite # of terms in $W_{Z_i^+}$!)

$$W \supset Z_i^+ Z_j^+ \longrightarrow \frac{\partial W}{\partial Z_i^+} \sim Z_j^+ = 0$$

- $\frac{\partial W_{Z_i^+}}{\partial Z_i^+} = 0, \ \ D_A = 0$ determine the VEVs of Z_i^-

$N_+ > N_- - 1$ Overdetermined meta-stable SUSY breaking

Kim-N.M.-Nishino-Sakurai08

$N_+ = N_- - 1$ Generically no flat direction (all massive)

$N_+ < N_- - 1$ Flat direction (massless modes appear)

SO(10) Higgs sector

- A minimal model (in addition to $\Theta(\theta=-1,+)$)

$SO(10)$	negative	positive
45	$A(a=-1,-)$	$A'(a'=3,-)$
16	$C(c=-4,+)$	$C'(c'=3,-)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$
10	$H(h=-3,+)$	$H'(h'=4,-)$
1	$Z,\bar{Z}(z=\bar{z}=-2,-)$	$Z'(z'=5,+)$

- To determine the VEVs

$$W=W_{A'}+W_{Z'}+W_{C'}+W_{\bar{C}'}+W_{H'}$$

W_X includes only terms linear in X

- It is surprising that such a minimal model includes the structure to realize DT splitting.

SO(10) Higgs sector

- $W = W_{A'} + W_{H'} + W_{Z'} + W_{C'} + W_{\bar{C}'}$
- $W_{A'}$ fixes the VEV $\langle A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \begin{pmatrix} v & 0 & 0 & 0 & 0 \\ 0 & v & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

- $W_{A'} + W_{H'}$ realizes DT splitting by Dimopoulos-Wilczek mechanism.
- $W_{Z'}$ fixes the scale of $\langle \bar{C} C \rangle \sim \lambda^{-c-\bar{c}}$.
- $W_{C'} + W_{\bar{C}'}$ realizes alighnment btw $\langle A \rangle$ and $\langle C \rangle \& \langle \bar{C} \rangle$.

$$SU(2)_R \times U(1)_{B-L} \to U(1)_Y$$

Missing VEV mechanism

Dimopoulos-Wilczek mechanism

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$$W = HAH' + M_{H'}H'H'$$

$$\times HH, \times HH'$$

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$(C, \bar{C})$ and A must be coupled each other

$$W_{A'} = \lambda^{a+a'} A'A + \lambda^{a'+3a} A'A^3 \quad \begin{matrix} a' = 3 \\ a = -1 \end{matrix}$$

$$\langle A \rangle = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \otimes \begin{pmatrix} v & & & & \\ & v & & 0 & \\ & & v & & \\ & 0 & & 0 & \\ & & & & 0 \end{pmatrix} \propto Q_{B-L}$$

$$\frac{\partial W}{\partial A'} = 0 \to \lambda^{a'+a} x_i (1 + x_i^2 \lambda^{2a}) = 0$$ for all i

$$\Rightarrow \quad x_i = 0, \lambda^{-a}$$ only two solutions

- GUT scale is determined by anomalous U(1) charges.
- SUSY zero forbids $A'A^{2L+1} \quad (L \geq 2)$

If it is allowed, $\frac{\partial W}{\partial A'} = 0$ have more solutions (less natural)

- Usual symmetry cannot forbid them.

Anomalous U(1) plays an essential role!

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$W_{H'} = \lambda^{h+a+h'} H A H' (+\lambda^{2h'} H'^2) \quad \begin{matrix} h' = 4 \\ h = -3 \\ a = -1 \end{matrix}$$

$$\frac{\partial W}{\partial H'} = 0 \rightarrow \langle H_C \rangle = 0$$

SUSY zero

$$\begin{matrix} & \bar{\mathbf{5}}_H & \bar{\mathbf{5}}_H \\ \begin{matrix} \mathbf{5}_H \\ \mathbf{5}_{H'} \end{matrix} & \begin{pmatrix} 0 & \langle A \rangle \lambda^{h+h'+a} \\ \langle A \rangle \lambda^{h+h'+a} & \lambda^{2h'} \end{pmatrix} \end{matrix}$$

$$\langle A \rangle = \begin{cases} v & (H_C) \\ 0 & (H_u, H_d) \end{cases}$$

Missing VEV mechanism
(Dimopoulos-Wilczek)

- Only one pair of doublet is massless (DT splitting).
- Proton decay is naturally suppressed. $\lambda^{2h'} < \lambda^{h+a+h'} \langle A \rangle \sim \lambda^{h'+h}$

Effective colored Higgs mass

$$M_C^{eff} \sim \lambda^{2h} \Lambda > \Lambda$$

If $h < 0$ to forbid H^2

$$W_{Z'} = \lambda^{z'} Z'(1 + \lambda^{c+\bar{c}} \bar{C} C + f(A)) \quad \begin{matrix} z' = 5 \\ c = -4 \\ \bar{c} = -1 \end{matrix}$$

$$\frac{\partial W}{\partial Z'} = 0 \rightarrow \langle \bar{C} C \rangle = \lambda^{-(c+\bar{c})}$$

D-flatness condition $\rightarrow \langle C \rangle = \langle \bar{C} \rangle = \lambda^{-\frac{1}{2}(c+\bar{c})}$

- Again VEVs are determined by the charges.
- Half integer appears
  (important in realizing bi-large neutrino mixings)
- $SU(2)_R$ breaking

$$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$$

$$W_{\bar{C}'} = \bar{C}'(\lambda^{\bar{c}'+a+c} A + \lambda^{\bar{c}'+z+c} Z) C$$

$\bar{c}' = 6$
$c = -4$
$a = -1$
$z = -2$

$$\frac{\partial W}{\partial \bar{C}'} = 0 \to (\lambda^a v Q_{B-L} + \lambda^z Z) C = 0$$

$\mathbf{16}$	$\to$	$(\mathbf{3,2,1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{2})$	$(\mathbf{1,2,1})$	$(\mathbf{1,1,2})$
Q_{B-L}		$\frac{1}{3}$	$-\frac{1}{3}$	-1	1

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$\begin{aligned} \langle \bar{C} C \rangle \neq 0 \quad &\to \quad \langle C_f \rangle \neq 0 \\ &\to \quad \lambda^a v Q^f_{B-L} + \lambda^z \langle Z \rangle = 0 \\ &\to \quad \langle C_i \rangle = 0 (i \neq f) \end{aligned}$$

- Alignment can be realized. (Barr-Raby 97)
- $f = (\mathbf{1},\mathbf{1},\mathbf{2})$ is required to obtain our world.
- These terms make PNGs massive.
- $\bar{C}A'C, \bar{C}A'AC$ are forbidden by SUSY zero.

Mass spectrum

$a = -1$
$a' = 3$

- Mass spectrum does not respect SU(5) symmetry.

$$W_{A'} = \lambda^{a+a'} A'A + \lambda^{a'+3a} A'A^3$$

$$A = \begin{pmatrix} G+B & X \\ \bar{X} & W+B \end{pmatrix}$$

$$M_G = M_W = M_B \sim (\lambda^{a+a'}, \lambda^{a+a'}) \sim (\lambda^2, \lambda^2)$$

- X and $\bar{X}$ are absorbed by the Higgs mechanism.

$$M_X \sim \lambda^{2a'} \sim \lambda^6 \quad \leftarrow \lambda^{2a'} A'^2$$

This may spoil the success of gauge coupling unification

⇨ No one have considered such a possibility before us.

Mass spectrum of superheavy fields

- Mass spectrum does not respect SU(5)

⇨ Naively, it spoils the success of gauge coupling unification in minimal SUSY SU(5).

- Usually, no other fields between M_W and Λ_G or the mass spectrum which respect SU(5) is required not to spoil gauge coupling unification. $\mathbf{5}+\bar{\mathbf{5}}$ or $\mathbf{10}+\overline{\mathbf{10}}$
- My expectation was

“There must be a tuning parameter because the rank of SO(10) or E6 is larger than SU(5).”

- But the fact was more exciting than my expectation.

- Mass spectrum and GUT scale are determined by the U(1) charges

⇒ We can calculate the running of gauge couplings

- The procedure is as follows.
  - From the fact that in MSSM three gauge couplings meet at a scale Λ_G , the gauge couplings $\alpha_i(\Lambda_W)(i=1,2,3)$ are obtained as functions of Λ_G, α_G .
  - Calculate all the scales by the charges and Λ
  - Calculating 1 loop RGE, rewrite the unification conditions

$$\alpha_1(\Lambda_u) = \alpha_2(\Lambda_u) = \alpha_3(\Lambda_u)$$

⇒ $\begin{cases} \Lambda \sim \Lambda_G & \text{The scale of the theory} \\ h \sim 0 & h \text{ determines the mass of triplet Higgs} \end{cases}$

- All the charges except doublet Higgs's are cancelled

We do not use any concrete values of the charges, the cutoff, and Λ_G, α_G

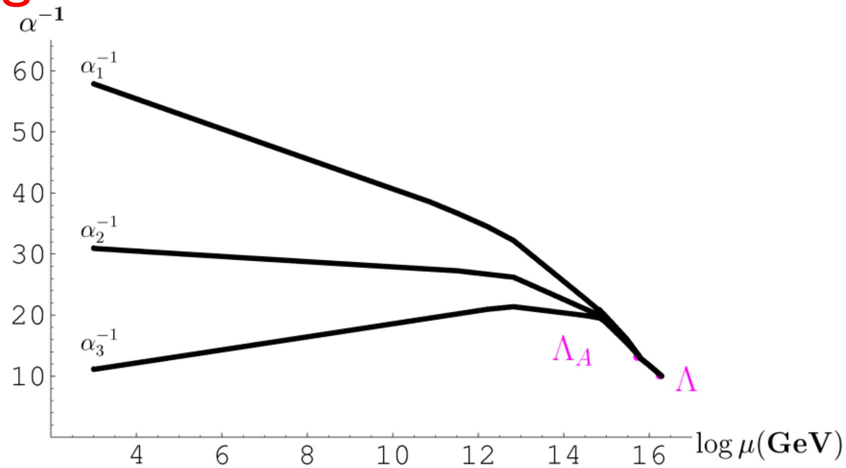
- The same freedoms as the minimal SU(5) GUT.

Natural Gauge Coupling Unification

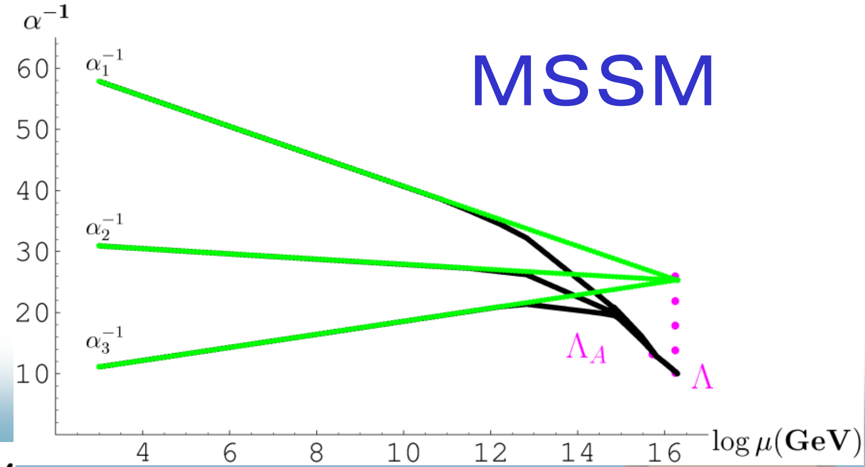
N.M. 01, N.M-Yamashita 02

New Explanation for the success

- Fix a model with Λ
- Calculate $\alpha_i(\Lambda_W)$



- Calculate $\alpha_i(\mu)$ with $\alpha_i(\Lambda_W)$
- Always meet at a scale Λ



$\Lambda_A < \Lambda \sim \Lambda_G \sim 2 \times 10^{16}$ GeV

Proton Decay

- Unification scale becomes lower.

$$\Lambda_U \sim \lambda^{-a}\Lambda_G < \Lambda_G$$

⇒ Proton decay via dimension 6 op.

$$\tau(p \to e\pi) \sim \begin{cases} 5 \times 10^{34} \text{ years } (a = -1/2) \\ 4 \times 10^{33} \text{ years } (a = -1) \end{cases}$$

$$\tau_{exp}(p \to e\pi) > 1 \times 10^{34} \text{ years}$$

$$\lambda \sim \sin\theta_C \sim 0.22$$

- Generic interactions ⇔ $\lambda^{-a} < 1$

Sufficient Conditions

N.M.-Yamashita 02

1. Simple Group
2. VEVs

$$\langle Z \rangle \sim \begin{cases} \lambda^{-z} & z \leq 0 \\ 0 & z > 0 \end{cases}$$

3. Below a scale, MSSM(+singlets) is realized.

⇒ Independent of how to realize DT splitting

Anomalous U(1) charges

gauge group

$$SU(5) \text{ or } SO(10) \text{ or } E_6$$

Application: Generalized gauge mediation

Kawase, N.M. Sakurai

E_6 extension of Higgs sector

$E_6 \to SO(10) \to SU(5)$

N.M.-Yamashita 02,03

$$\begin{array}{lll} \mathbf{78} & A, & A' \\ \mathbf{27} & H, & C' \\ \overline{\mathbf{27}} & \bar{C}, & \bar{H}' \end{array} \qquad \begin{array}{lll} \mathbf{45} & A, & A' \\ \mathbf{16} & C, & C' \\ \overline{\mathbf{16}} & \bar{C}, & \bar{C}' \\ \mathbf{10} & H, & H' \end{array}$$

- Unification of Higgs sector

$$\begin{array}{lll} \mathbf{27} & \to & \mathbf{16}+\mathbf{10}+\mathbf{1} \\ \mathbf{78} & \to & \mathbf{45}+\mathbf{16}+\overline{\mathbf{16}}+\mathbf{1} \end{array}$$

$A \supset A + C$

$A' \supset A' + \bar{C}'$

H also breaks E_6

$C' \supset C' + H'$

What happens if adjoint Higgs is chargeless?

- Infinite number of interactions cannot be controled.

$$W_A = A^2 + A^4 + A^6 + \cdots$$

$\frac{\partial W_A}{\partial A} = 0$ has infinite number of solutions.

→ unnatural to obtain DW VEV.

- It spoils the gauge coupling unification by

$$\int d\theta^2 f(A) W_{a\alpha} W_a^\alpha \qquad \langle f(A) \rangle = \frac{1}{4 g_a^2}$$

SO(10) Matter sector

- A minimal (Higgs&Matter) sector

$SO(10)$	negative	positive
45	$A(a=-1,-)$	$A'(a'=3,-)$
16	$C(c=-4,+)$	$C'(c'=3,-)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$
10	$H(h=-3,+)$	$H'(h'=4,-)$
1	$Z,\bar{Z}(z=\bar{z}=-2,-)$	$Z'(z'=5,+)$

16 $\Psi_1\left(\psi_1=\frac{9}{2},+\right), \Psi_2\left(\psi_2=\frac{7}{2},+\right), \Psi_3\left(\psi_3=\frac{3}{2},+\right)$

10 $T(t=\frac{5}{2},+)$

- Superpotential for Matter $\langle A\rangle^{2n} \sim \lambda^{-2na} \Rightarrow Y_d \neq Y_e^t$

$$\lambda^{\psi_i+\psi_j+h}\Psi_i\Psi_j H + \lambda^{\psi_i+\psi_j+h+2na}A^{2n}\Psi_i\Psi_j H$$
$$+\lambda^{\psi_i+\psi_j+\bar{c}+\bar{c}}\Psi_i\bar{C}\Psi_j\bar{C} + \lambda^{\psi_i+t+c}\Psi_i TC + \lambda^{2t}T^2$$

- One of 4 $\bar{\mathbf{5}}$ fields $(\bar{\mathbf{5}}_T, \bar{\mathbf{5}}_{\Psi_i})$ becomes superheavy with $\mathbf{5}_T$ field. $\rightarrow Y_u \neq Y_d$

SO(10) Matter sector

- Superpotential for Matter sector

$$\lambda^{\psi_i+\psi_j+h}\Psi_i\Psi_j H + \lambda^{\psi_i+\psi_j+h+2na}A^{2n}\Psi_i\Psi_j H$$
$$+ \lambda^{\psi_i+\psi_j+\bar{c}+\bar{c}}\Psi_i\bar{C}\Psi_j\bar{C} + \lambda^{\psi_i+t+c}\Psi_i TC + \lambda^{2t}T^2$$

- Terms with A avoid $Y_d = Y_e^t$ because of $\langle A\rangle \sim \lambda^{-a}$
- One of 4 $\bar{\mathbf{5}}$ fields $(\bar{\mathbf{5}}_T, \bar{\mathbf{5}}_{\Psi_i})$ becomes superheavy with $\mathbf{5}_T$ field. $\rightarrow Y_u \neq Y_d$

16 $\Psi_1\left(\psi_1 = \frac{9}{2}, +\right), \Psi_2\left(\psi_2 = \frac{7}{2}, +\right), \Psi_3\left(\psi_3 = \frac{3}{2}, +\right)$
10 $T\left(t = \frac{5}{2}, +\right)$

$$\mathbf{5}_T(\lambda^{\psi_1+t+c}\langle C\rangle, \lambda^{\psi_2+t+c}\langle C\rangle, \lambda^{\psi_3+t+c}\langle C\rangle, \lambda^{2t})\begin{pmatrix}\bar{\mathbf{5}}_{\Psi_1}\\ \bar{\mathbf{5}}_{\Psi_2}\\ \bar{\mathbf{5}}_{\Psi_3}\\ \bar{\mathbf{5}}_T\end{pmatrix}$$

- $(\bar{\mathbf{5}}_{\Psi_1}, \bar{\mathbf{5}}_T + \lambda^{\Delta}\bar{\mathbf{5}}_{\Psi_3}, \bar{\mathbf{5}}_{\Psi_2})$ $\quad \boldsymbol{\Delta = t - \psi_3 + \frac{\bar{c}-c}{2} = 2.5}$

SO(10) Matter sector

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim Y_e^t \sim Y_\nu^t \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}$$

$$M_{\nu_R} \sim \lambda^{2\psi_3+\bar{c}-c} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$M_\nu = \frac{Y_\nu^t M_{\nu_R}^{-1} Y_\nu \langle H_u \rangle^2}{\Lambda} \sim \lambda^{4-2\psi_3+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 / \Lambda$$

$\Lambda \sim 2 \times 10^{16}$ GeV, $\langle H_u \rangle \sim 200$ GeV

$\rightarrow m_{\nu_\tau} \sim 0.04$ eV, $m_{\nu_\mu} \sim 0.008$ eV, $m_{\nu_e} \sim 0.002$ eV

$-h = 2\psi_3 = c - \bar{c} + 6$

SO(10) Matter sector

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim Y_e^t \sim Y_\nu^t \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}$$

$$M_\nu \sim \lambda^{4-2\psi_3+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 / \Lambda$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

$(10_{\Psi_1}, 10_{\Psi_2}, 10_{\Psi_3})$ and $(\bar{\mathbf{5}}_{\Psi_1}, \bar{\mathbf{5}}_T, \bar{\mathbf{5}}_{\Psi_2})$ are important.

→Prediction $(V_{CKM})_{12} \sim (V_{MNS})_{13}$, which was confirmed in 2012.

An issue of SO(10) natural GUT

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim Y_e^t \sim Y_\nu^t \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}$$

$$M_\nu \sim \lambda^{4-2\psi_3+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 / \Lambda$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

An important term for V_{CKM} $\lambda^{\psi_1+\psi_2+h+c+\bar{c}} \bar{C}\, \Psi_1 \Psi_2 H C$

$SU(2)_R$ breaking term is needed to obtain V_{CKM}.

$$c + \bar{c} \geq -5$$

$$\boldsymbol{\Delta = t - \psi_3 + \frac{\bar{c}-c}{2} = 2.5} \qquad 2\psi_3 = c - \bar{c} + 6$$

$$t > -\psi_3 - c$$

Smallest $\psi_3 = \frac{3}{2}$ $\rightarrow h = -3 \neq 0 \rightarrow$coupling unification is not natural although superheavy particle masses with O(1) coefficients (1/2-2) can recover the gauge coupling unification.

1^{st} summary

- Natural SO(10) GUT solves various problems in SUSY GUT with natural assumption that all interactions allowed by symmetry are introduced with O(1) coefficients, and as a result we obtain a realistic GUT.
- It gives new explanation for gauge coupling unification.
- Important predictions

  Nucleon decay via dim. 6 op. is enhanced.

  $(V_{CKM})_{12} \sim (V_{MNS})_{13}$: confirmed in 2012.
- An issue : There is a tension between correct size of Neutrino mass and gauge coupling unification.

Spontaneous SUSY breaking in natural GUT

Sp. SUSY breaking with $U(1)_A$

Kim, N.M.,Nishino,Sakurai08

- $\begin{cases} N_+ > N_- - 1 & \text{meta stable SUSY breaking} \\ N_+ = N_- - 1 & \text{no flat direction} \\ N_+ < N_- - 1 & \text{flat diretion (massless mode)} \end{cases}$
- A simple model : $N_+ = N_- = 1$

A simple model with $U(1)_R$

Fayet, Iliopoulos 74

	S	Θ
$U(1)_A$	s	-1
$U(1)_R$	2	0

- Superpotential $W = S\Theta^s$
- F and D flatness conditions

$$\frac{\partial W}{\partial S} = \Theta^s = 0$$

$$\frac{\partial W}{\partial \Theta} = sS\Theta^{s-1} = 0$$

$$D_A = -g_A(\xi^2 + s|S|^2 - |\Theta|^2) = 0$$

are not satisfied.

- Global minimum of $V = |F_S|^2 + |F_\Theta|^2 + \frac{1}{2}D_A^2$ is at

$$\langle S \rangle = 0, \qquad \langle \Theta \rangle = \lambda,$$

$$\langle F_S \rangle \sim \lambda^s, \quad \langle F_\Theta \rangle = 0, \quad \langle D_A \rangle \sim \frac{s}{g_A}\lambda^{2s-2}$$

A simple model without $U(1)_R$

Kim, N.M.,Nishino,Sakurai08

- Superpotential $W = f(S\Theta^s)$
- F and D flatness conditions

	S	Θ
$U(1)_A$	s	-1

$$\frac{\partial W}{\partial S} = f'(S\Theta^s)\Theta^s = 0$$

SUSY vacua

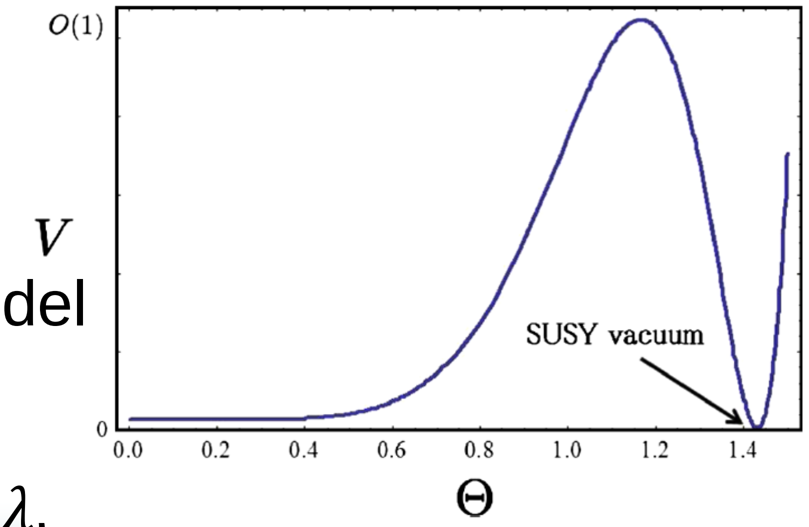
$$\frac{\partial W}{\partial \Theta} = f'(S\Theta^s)sS\Theta^{s-1} = 0$$

$\langle S \rangle \sim O(1), \ \langle \Theta \rangle \sim O(1)$

$$D_A = -g_A(\xi^2 + s|S|^2 - |\Theta|^2) = 0$$

are satisfied because $f'(S\Theta^s) = 0$

and D_A =0 are satisfied.



- The global min. in the previous model becomes local min. at

$$\langle |S| \rangle = 0 \to \frac{1}{s^2}\lambda^{s+2}, \quad \langle \Theta \rangle \sim \lambda,$$

$$\langle F_S \rangle \sim \lambda^s, \quad \langle F_\Theta \rangle = 0 \to s\lambda^{s-1}\langle |S| \rangle, \quad \langle D_A \rangle \sim \frac{s}{g_A}\lambda^{2s-2}$$

No sizable gaugino mass

- Sfermion masses $m_0 \sim \frac{F_S}{\Lambda} \sim \lambda^s \to s \sim 19 (\Lambda \sim \Lambda_{\mathrm{GUT}})$
- Gaugino masses $m_{1/2} \sim \frac{\lambda^s F_S}{\Lambda} \sim \lambda^{2s}$ too small

$$\int \theta^2 \lambda^s \frac{S}{\Lambda} W_\alpha W^\alpha \Rightarrow \lambda^s \frac{F_S}{\Lambda} \lambda_\alpha \lambda^\alpha$$

gauge mediation too small

anomaly med. $m_{1/2} \sim \frac{m_{3/2}}{16\pi^2} \sim \frac{F_S}{16\pi^2 M_{pl}} \sim 10^{-4} m_0$

SUGRA $m_{1/2} \sim m_{3/2} \sim \frac{F_S}{M_{pl}} \sim 10^{-2} m_0$

- High scale SUSY unnatural

$m_{1/2} \sim 1\,\mathrm{TeV}, m_0 \sim 10^2\,\mathrm{TeV} \to s \sim 17$

Sp. SUSY breaking in natural GUT

- Higgs sector of SO(10)

$SO(10)$	$negative$	$positive$
45	$A(a=-1,-)$	$A'(a'=3,-)$
16	$C(c=-4,+)$	$C'(c'=3,-)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$
10	$H(h=-3,+)$	$H'(h'=4,-)$
1	$Z,\bar{Z}(Z=\bar{z}=-2,-),$	$Z'(z'=5,+)$

- Let us decrease one negatively charged field.
- One F of $W_{C'}=\bar{C}(A+Z)C',\ \ W_{\bar{C}'}=\bar{C}'(A+Z)C$ is not vanishing.

$$\frac{\partial W_{\bar{C}'}}{\partial \bar{C}'}=(A+Z)C\sim\lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})}=\lambda^{\frac{9}{2}}\sim(2\times 10^{13}\mathrm{GeV})$$ too large

- SUSY breaking scale can be smaller by choosing larger $\bar{c}'$.

Gauge messenger gives sizable gaugino mass?

- Massive vector multiplets do not respect SUSY$(F_{\bar{C}'} \neq 0)$

  Generically they induce $m_{1/2} \sim c_i \frac{\alpha_i}{4\pi}\frac{F_{\bar{C}'}}{\Lambda} \sim 10^{-2} m_0$(gauge messenger)

- Unfortunately, induced gaugino masses are quite small.

because of approximate $U(1)_R$ symmetry.

$SO(10)$	$negative$	$positive$	
45	$A(a=-1,-)$	$A'(a'=3,-)$	$\langle A\rangle \neq 0$
16	$C(c=-4,+)$	$C'(c'=3,-)$	$\langle F_{C'}\rangle \neq 0$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$	
10	$H(h=-3,+)$	$H'(h'=4,-)$	massive chiral multiplets do not
1	$Z(Z=-2,-)$,	$Z'(z'=5,+)$	respect SUSY but $m_{1/2} \sim \mathrm{m}_0^2/\Lambda$
$U(1)_R$	0	$2(F_{C'}:0)$	Very small $\langle Z'\rangle, \langle F_Z\rangle$

Very small $U(1)_R$ breaking like $\lambda^{c'+\bar{c}'}\bar{C}'C'$ must be picked up for gaugino mass.

Anomaly mediation gives $m_{1/2} \sim 10^{-5} m_0$.

15 years ago, we gave up to build model because of too small gaugino mass.

SUGRA effects induce $m_{1/2} \sim m_{3/2}$

N.M.-Omura-Shigekami-Yoshida17

- R symmetry breaking $\langle W \rangle$ gives larger contribution to gaugino masses
- SUSY breaking spectrum becomes

$\bar{c}'$	$F_{\bar{c}'}/\Lambda$	$m_{1/2} \sim m_{3/2}$	$m_0 \sim \sqrt{D_A}$	
18	200 TeV	2TeV	2000TeV	
19	40 TeV	400 GeV	400TeV	

- High scale SUSY is predicted.

  (D_A dominates in a simple model.)

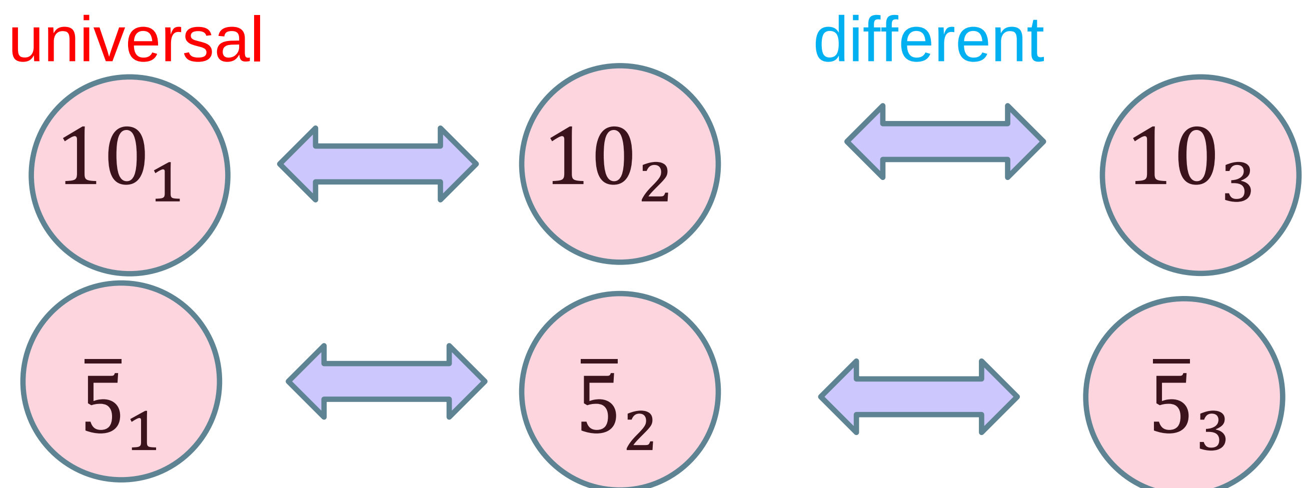
  Roughly $m_{1/2} \sim 1$ TeV, $m_0 \sim (100-)1000$ TeV
- No SUSY flavor and CP problem.

“Direct” signature for GUT

- D term dominates sfermion masses.

$$\tilde{m}^2_{10} \sim \left(\frac{9}{2},\frac{7}{2},\frac{3}{2}\right) D_A + D_V \qquad SO(10) \supset SU(5) \times U(1)_V$$

$$\tilde{m}^2_{\bar{5}} \sim \left(\frac{9}{2},\frac{5}{2},\frac{7}{2}\right) D_A + (-3,2,-3) D_V$$



- Sfermion masses respect SU(5) unification!

They can be a direct signature of unification of matter in SU(5) GUT. (RGE effects can be negligible.)

- Sfermion masses are fixed by D_A and D_V.

Predictions from E_6 GUT

- $E_6 \supset SO(10) \times U(1)_{V'}$, has 3 D terms

Bando-N.M.01
N.M.-Yamashita 02

$$\tilde{m}^2_{10} \sim \left(\frac{9}{2}, \frac{7}{2}, \frac{3}{2}\right) D_A + D_{V'} + D_V$$

$$\tilde{m}^2_{\bar{5}} \sim \left(\frac{9}{2}, \frac{9}{2}, \frac{7}{2}\right) D_A + (1, -2, 1) D_{V'} + (-3, 2, -3) D_V$$

- $E_6 \times SU(2)_F$ has 4 D terms

N.M.02
Ishiduki-Kim-N.M.-Sakurai 09
N.M.-Muramatsu-Shigekami 14

$$\tilde{m}^2_{10} \sim \left(4, 4, \frac{3}{2}\right) D_A + D_{V'} + D_V + \frac{1}{2}(1, -1, 0) D_F$$

$$\tilde{m}^2_{\bar{5}} \sim (4, 4, 4) D_A + (1, -2, 1) D_{V'} + (-3, 2, -3) D_V + \frac{1}{2}(1, 1, -1) D_F$$

- The sfermion mass scale is much smaller than the GUT scale, although it is too large to reach by experiments in near future.
- Various GUT can be tested.

More rigid prediction is possible

- D term dominates F term.

→Various D can be written by D_A.

For example in natural SO(10) GUT,

$$D_V = -\frac{\widetilde{m}_C^2 - \widetilde{m}_{\bar{C}}^2}{2} = \frac{\bar{c} - c}{2} D_A$$

More rigid prediction is possible.

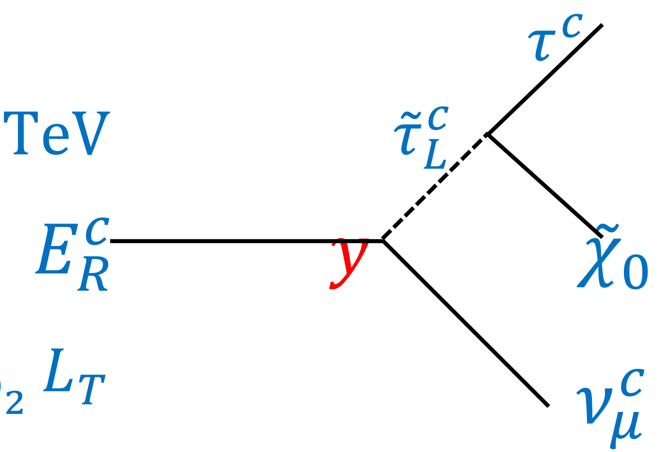
An interesting prediction

- Long-lived heavy electron(R-parity odd)

Lightest particle in Higgs sector is $E_R^c + \overline{E_R^c}$ in $16_{C'} + \overline{16}_{\bar{C}'}$

Roughly we take

$m_0 \sim 1000\mathrm{TeV}, m_{1/2} \sim 1\mathrm{TeV},\ m_{E_R^c} \sim 1\mathrm{TeV}$



- Decay mode is $\tau^c \nu_\mu^c \tilde{\chi}_0$ or $\mu^c \nu_\tau^c \tilde{\chi}_0$

$y = \lambda^{c'+\psi_2+t}$, $\quad 16_{C'} 16_{\psi_2} 10_T \to E_R^c L_{\psi_2} L_T$

- $\tau_{E_R^c} \sim O(1)\mathrm{sec}\left(\frac{10^{-6}}{y}\right)^2 \left(\frac{m_0}{1000\mathrm{TeV}}\right)^4 \left(\frac{1\mathrm{TeV}}{m_{E_R^c}}\right)^5$ sec

$\tau < 1$ is needed for BBN.

- LHC gives a constraint for long-lived charged particle.

$m_{E_R^c} > 574$ GeV (CMS 1305)

- LHC may find this particle.

Summary and discussions

Good points

- SUSY and GUT breaking in a model (just by decreasing a singlet).
- No R-axion. Constant superpotential is allowed by symmetry. (No $U(1)_R$.)
- It produces gaugino mass by gravity mediation. High scale SUSY!

  $m_{1/2} \sim m_{3/2} \sim 1\text{TeV}$, $m_0 \sim 1000\text{TeV}$.

  No SUSY flavor and CP problems
- Interesting phenomenology.

  Long lived charged lepton appears.

$$\tau_{E_R^c} \sim O(1)\ \sec\left(\frac{m_0}{1000\text{TeV}}\right)^4\left(\frac{1\text{TeV}}{m_{E_R^c}}\right)^5$$

  LHC may discover it.
- "Direct" signatures of GUT in sfermion mass spectrum

Bad points

- High scale SUSY needs finetuning. It is caused by $\Lambda \sim \Lambda_G \ll M_{Pl}$, which is required to explain the success of RGE gauge couplings.
- Artificial discrete symmetry and singlets.Otherwise, SUSY vacua appear by fixing the SM Higgs VEV. (E6 GUT may avoid this issue.) Artificially large $U(1)_A$ charge.
- Gravitino problem
- Bino LSP (overproduction?)

The upper 2 bad points are based on the assumption of O(1) coefficients. Suppression factors from extra dimension may avoid these.