Possibility of macroscopic spatial superposition via double-well optomechanical potential

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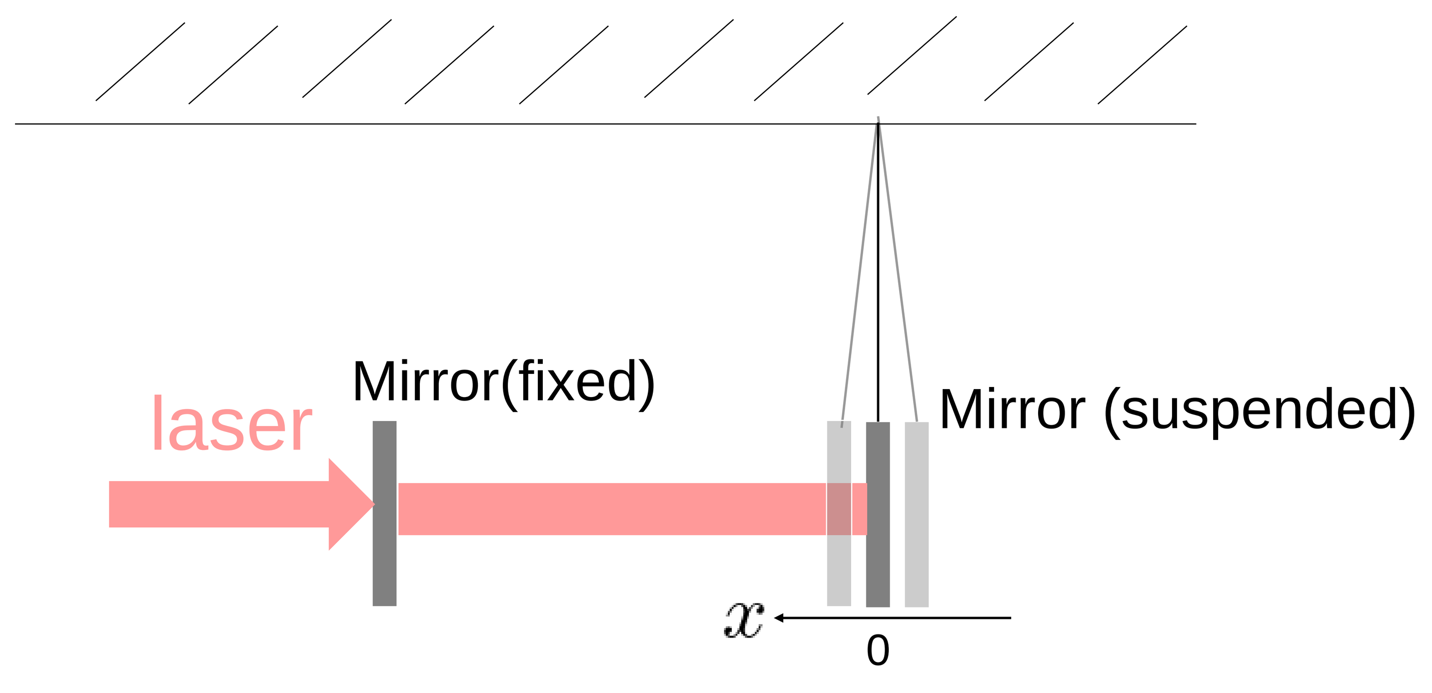
1. Optomechanics by moving mirror



・Optical cavity with a suspended mirror → Optomechanical system

Hamiltonian in rotating frame:

$$\frac{H}{\hbar} = \delta\hat{a}^\dagger\hat{a} + \frac{\Omega}{2}(\hat{p}^2+\hat{x}^2) + g\hat{x}\hat{a}^\dagger\hat{a} + iF(\hat{a}^\dagger - \hat{a})$$

Cavity mode ($\delta\hat{a}^\dagger\hat{a}$); Suspended mirror mode ($\frac{\Omega}{2}(\hat{p}^2+\hat{x}^2)$); Optomech. coupling ($g\hat{x}\hat{a}^\dagger\hat{a}$); Laser driving ($iF(\hat{a}^\dagger - \hat{a})$)

・Linearized dynamics… well studied

$$g\hat{x}\hat{a}^\dagger\hat{a} = g\hat{x}(\alpha^* + \hat{a}^\dagger)(\alpha + \hat{a}) \qquad \hat{x} = \frac{\hat{b}+\hat{b}^\dagger}{\sqrt{2}}$$

$$\approx \begin{cases} g(\alpha\hat{a}^\dagger b + \alpha^*\hat{a}\hat{b}^\dagger) & \text{For red-detuned: } \delta = \Omega \\ g(\alpha\hat{a}^\dagger\hat{b}^\dagger + \alpha^*\hat{a}\hat{b}) & \text{For blue-detuned: } \delta = -\Omega \end{cases}$$

Linearized optomech. is used *e.g.* for sideband cooling of massive mirror, quantum transducer with optomech. crystal, creation of entangled photon-phonon pair, (anti)optical spring etc.

・**Can we make use of nonlinear dynamics, in particular to realize a quantum superposition of a massive mirror?** (typically, 1mg or heavier!)

2. Optomechanical potential force and friction for mirror

・Stationary solution for the Heisenberg eqs. with relaxations:

$$\langle\hat{a}\rangle = \frac{F}{i(\delta + gx_s) + \kappa/2} \equiv \sqrt{n}\,e^{i\phi} \qquad \langle\hat{x}\rangle = -\frac{gn}{\Omega^2} \equiv x_s$$

・By solving the eq. for the fluctuation of the cavity mode, and by substituting it to the eq. for the mirror mode, we can read off the effective potential force and friction affecting the mirror.

$$\frac{d}{dt}\hat{p} = -\Omega\hat{x} - \frac{\Gamma}{2}\hat{p} - gn\left(\frac{(\delta+gx_s)^2+\kappa^2/4}{1+2g(\delta+gx_s)\hat{x}+g^2\hat{x}^2} - 1\right)$$
$$-gn\Omega\,\mathrm{Im}\left(\frac{1}{(\delta+gx_s-i\kappa/2+g\hat{x})^2}\right)\hat{p} + (\text{ higher order terms in } \Omega/|\delta - i\kappa/2| \,)$$

Although it is an adiabatic expansion, each term is of all-order in *g*!

Nonlinear extensions of (anti)optical spring & (anti)friction.

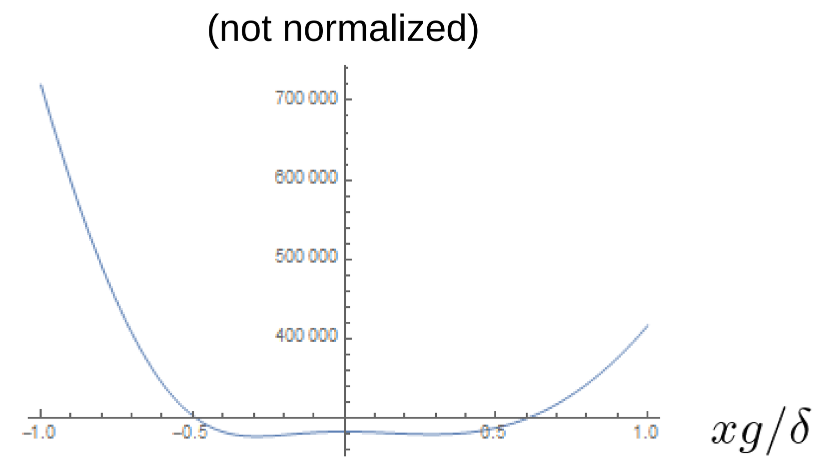
3. Effective potential

・The origin is always stable/unstable since x is the fluctuation from stationary pt. x_s → Easy to evaluate an existence of multiple minimal pt.s.

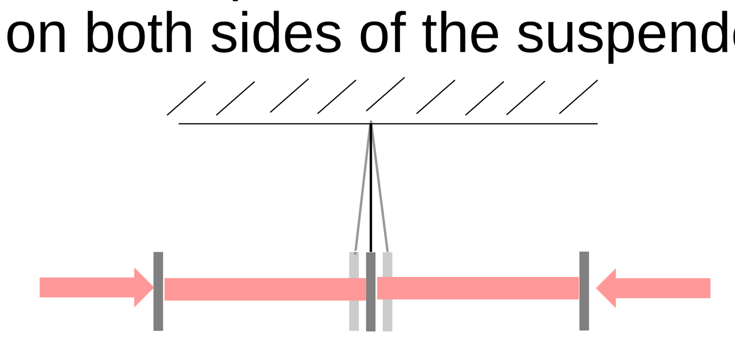
・The following conditions for laser detuning, loss rate and cavity photon number realize a double-well like potential

$$\delta > 0 \quad \kappa < \frac{2}{\sqrt{3}}\delta,$$

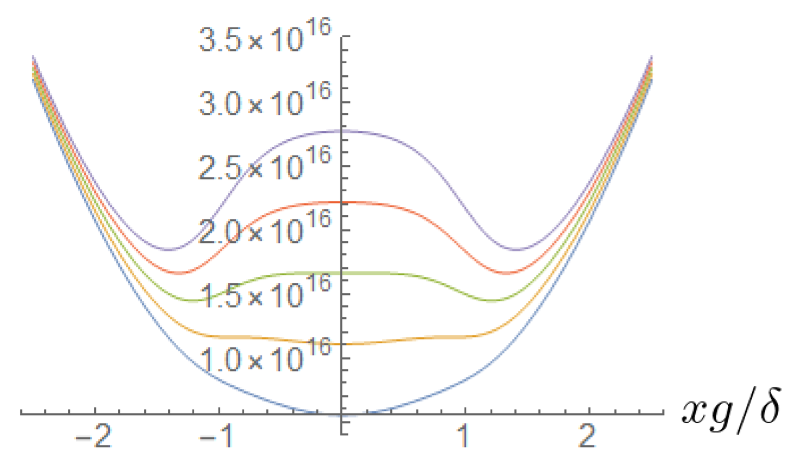
$$\frac{\delta\Omega}{3g^2}\left(2-\sqrt{1-\frac{3^2}{4\delta}}\right) < n < \frac{\delta\Omega}{3g^2}\left(2-\sqrt{1+\frac{3^2}{4\delta}}\right)$$



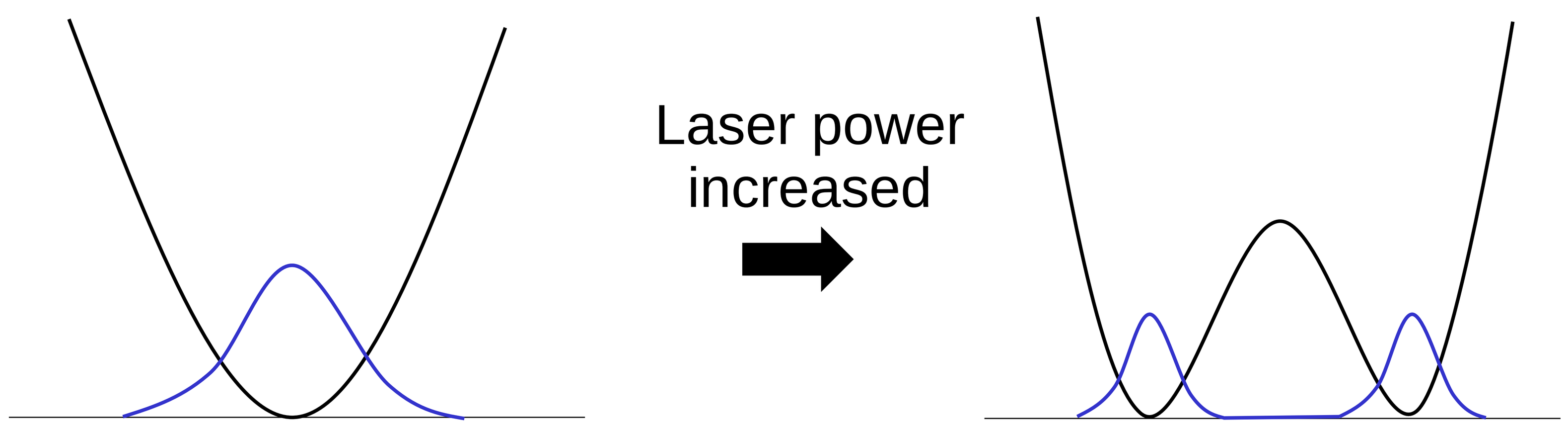
・If we input double laser beams with the same detuning and power on both sides of the suspended mirror, we have symmetric potential



$$\delta > 0, \quad n > \frac{\delta\Omega}{4g^2}\left(1+\frac{\kappa^2}{4\delta^2}\right)$$



・Consider the following adiabatic tuning:



Mirror at the ground or squeezed state of gravitational potential

Mirror at the ground state of double-well potential

Spatial superposition!

Such a superposition of a massive object is expected to be useful in detection of the dark matter or quantum graviton by decoherence

4. Parameters for experiments

Suspended mirror freq.: $\Omega/2\pi = O(1)\mathrm{Hz}$ (1mg mirror, 10cm suspension)

Optomech. Coupling: $g/2\pi = O(1)\mathrm{Hz}$ (normalized for quantum fluctuation length of mirror)

Laser detuning: $\delta/2\pi \gtrsim O(10)\mathrm{kHz}$

Laser loss rate: $\kappa/2\pi \gtrsim O(1)\mathrm{MHz}$

For the symmetric potential case, the requirement on the photon num. for the double-well is $n \gtrsim O(10^8)$.
→ O(1) μW power is sufficient(!?) for O(100THz) laser.

5. Outlook

・Feasibility of the adiabatic change in the potential and the wavefunction (in a case $\delta \simeq 10g$, we confirmed it is easily possible)

・Detection scheme of the dark matter and graviton by decoherence of the mirror (cf. poster by Jinyang Li)