QUPosium 2023

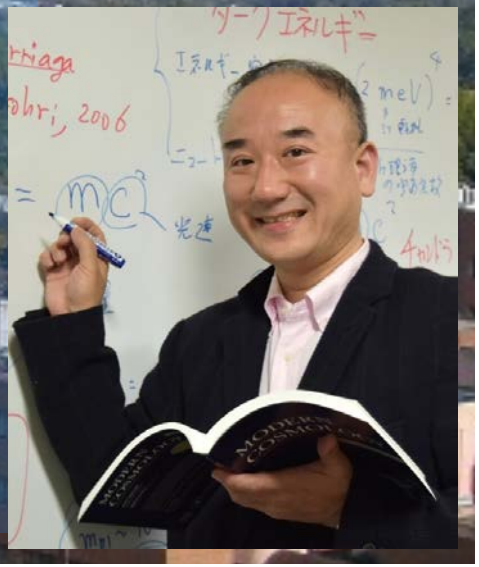
Can we observe a deviation from Starobinsky's R^2 Inflation?

Kazunori Kohri

郡　　和範

NAOJ / KEK /Sokendai/

Kavli IPMU



$$r = \frac{\text{tensor (gravitational wave)}}{\text{scalar (curvature perturbation)}} = \frac{P_h}{P_\zeta}$$

Abstract

- A future detection of tensor to scalar ratio $r \sim O(10^{-3})$ proves a quantum nature of gravity and means the (trans-)Planck-scale physics

- We will test the Starobinsky model, and discriminate it from a variety of Starobinsky-type models By LiteBIRD

- In future, we will measure tensor spectral index $n_t = dP_h/d\ln k$ within a sensitivity of $\Delta n_t \sim 0.2$

Starobinsky Inflation Model

- Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}M_{\mathrm{P}}^2 R + \boxed{\frac{M_{\mathrm{P}}^2}{12m^2}R^2}\right)$$

See also K. Maeda, Phys. Rev. D 37, 858 (1988).

$\frac{M_{\mathrm{P}}^2}{12m^2} \simeq 5 \times 10^8$ is a big number

Starobinsky Inflation in Einstein frame

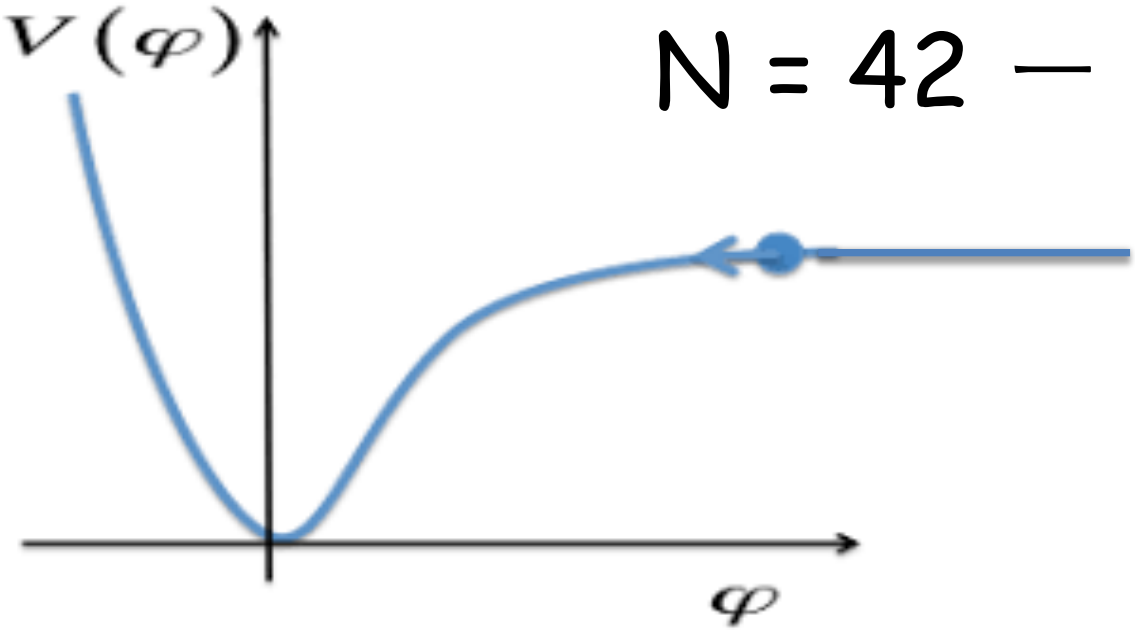
- Action

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{1}{2\kappa^2}R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- Potential of ϕ

$$V_{\text{Starobinsky}} = \frac{3}{4}m^2 M_{\text{P}}^2\left(1 - e^{-\sqrt{2/3}\phi/M_{\text{P}}}\right)^2$$

Potential V(ϕ) and its predictions



N = 42 – 52

- Spectral index

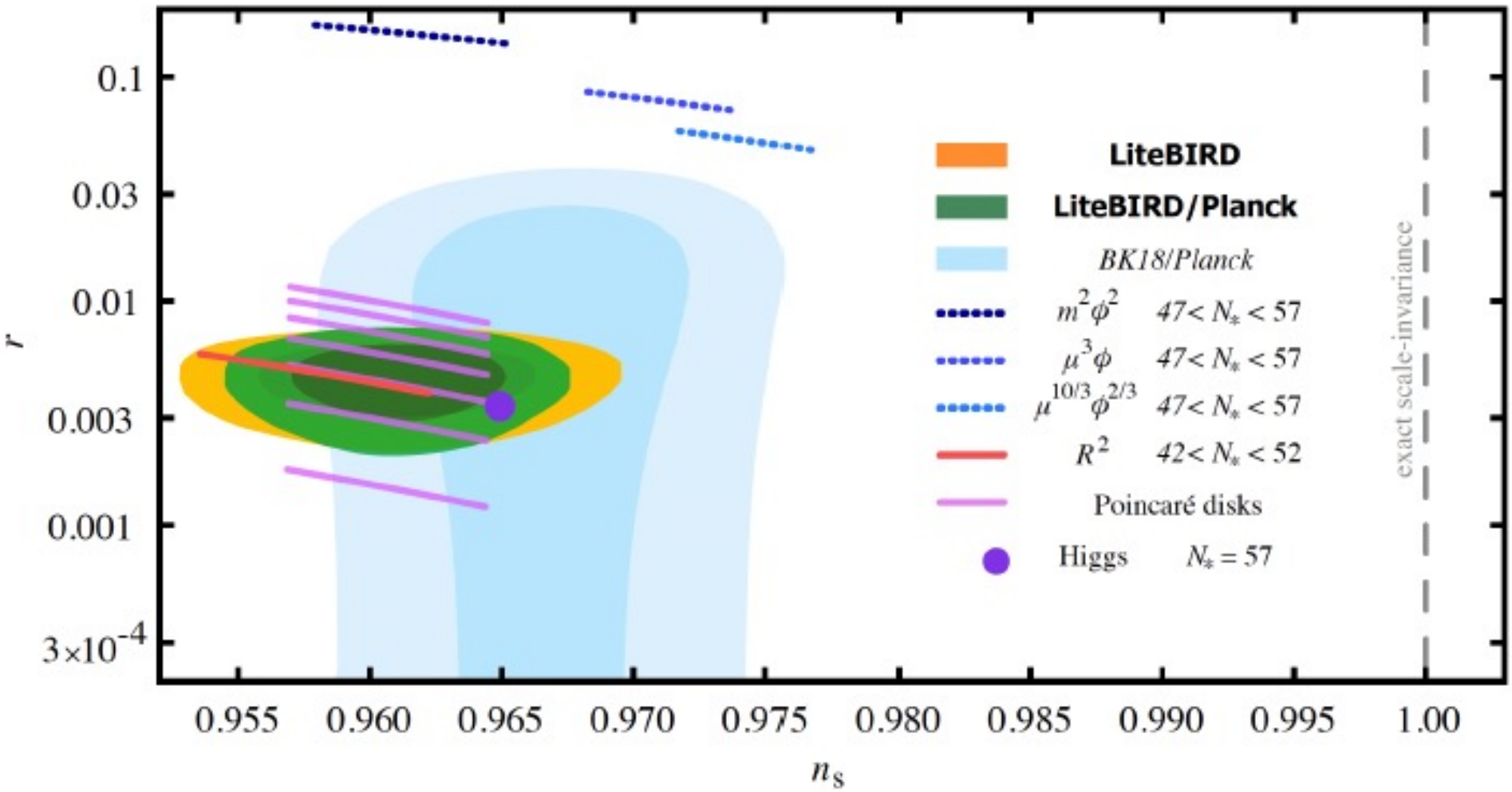
$$n_s - 1 = -\frac{2}{N} \sim -(0.038 - 0.048)$$

- Tensor to scalar ratio

$$r = \frac{12}{N^2} \sim (4.4 - 6.8) \times 10^{-3}$$

Sensitivity of LiteBIRD on *r*

E. Allys et al, PTEP 2023 (2023) 4, 042F01



We can verify the Starobinsky's R^2 model by LiteBIRD!

One day Masashi asked me

"In case the vanilla model of the R^2 inflation is rejected by LiteBIRD, what can we learn from it?"

A variety of Starobinsky-like models

- Quantum-gravity corrections on R^2

  T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, T. Terada, PTEP 2016 (2016) 12, 123E01

- Multi-field inflation models with R^2

  +Higgs inflation, $\xi R h^2$: D. Y. Cheong, K. Kohri, S.-C. Park, arXiv:2205.14813 [hep-ph]

  +Chaotic inflation,$1/2\ m^2\chi^2$: T. Mori, K. Kohri, J. White, arXiv:1705.05638 [astro-ph.CO]

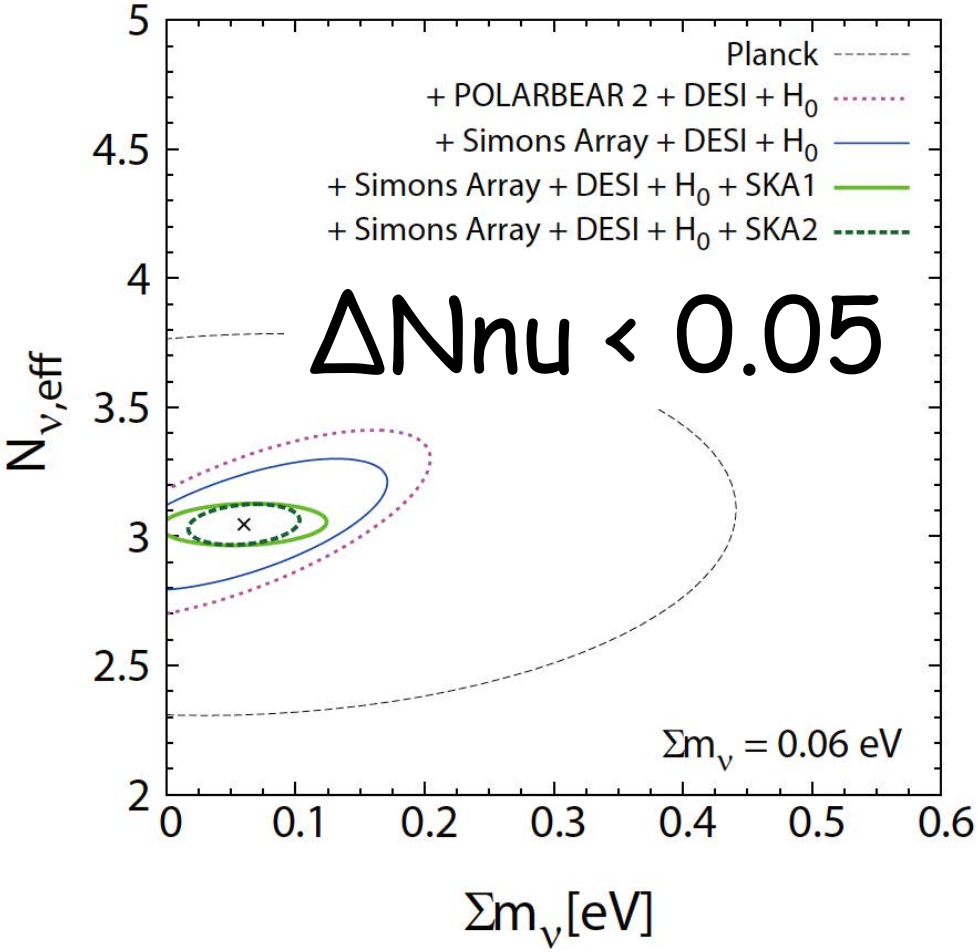
  +V0-$1/2\ m^2\chi 2+\xi R\chi^2$: S. Pi, Y.-li. Zhang, Qi.-G. Huang, M. Sasaki, arXiv:1712.09896 [astro-ph.CO]

- R^2 + Curvaton models

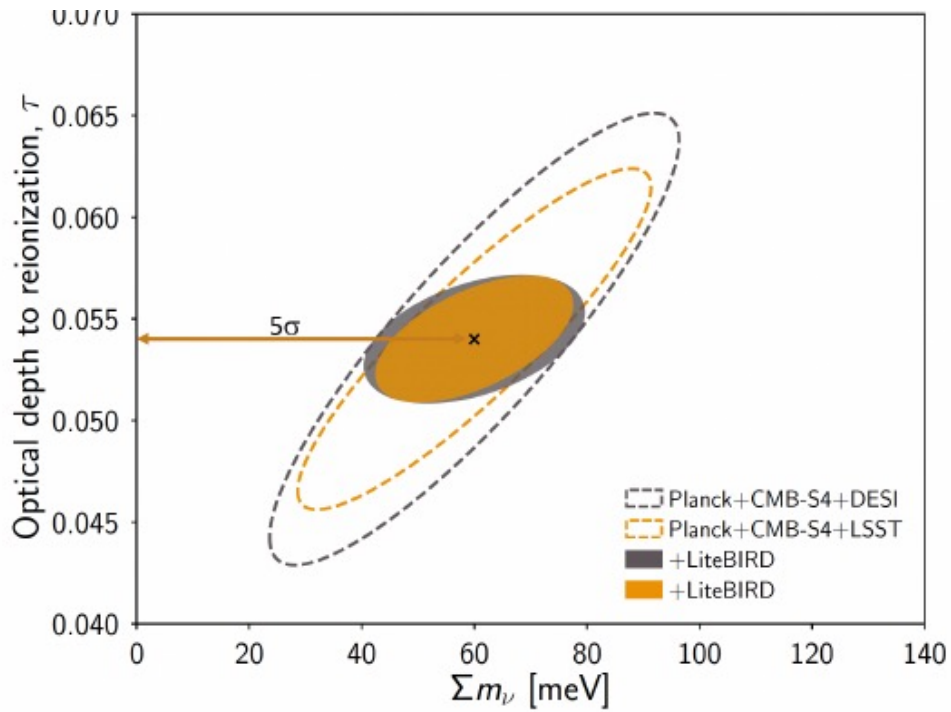
  R. Jinno, K. Kohri, T. Moroi, T. Takahashi, M. Hazumi, arXiv:2310.08158 [astro-ph.CO]

- …

Neutrino mass / hierarchy



Oyama, Kohri, Hazumi (2015)



E. Allys et al, PTEP 2023 (2023) 4, 042F01

Starobinsky-like models from extra dimensions

T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, T. Terada, PTEP 2016 (2016) 12, 123E01

- Action in D-dimension

$$S = \Lambda^D \int \mathrm{d}^D x \sqrt{-g} \sum_{n=0} b_n \left(\frac{R}{\Lambda^2}\right)^n \quad D = 10$$

- Action in 4D

Λ is a cutoff scale

$$S = c \int \mathrm{d}^4 x \sqrt{-g} \sum_{n=0} b_n \Lambda^4 \left(\frac{R}{\Lambda^2}\right)^n$$

$$c \equiv V_{D-4} \Lambda^{D-4}$$

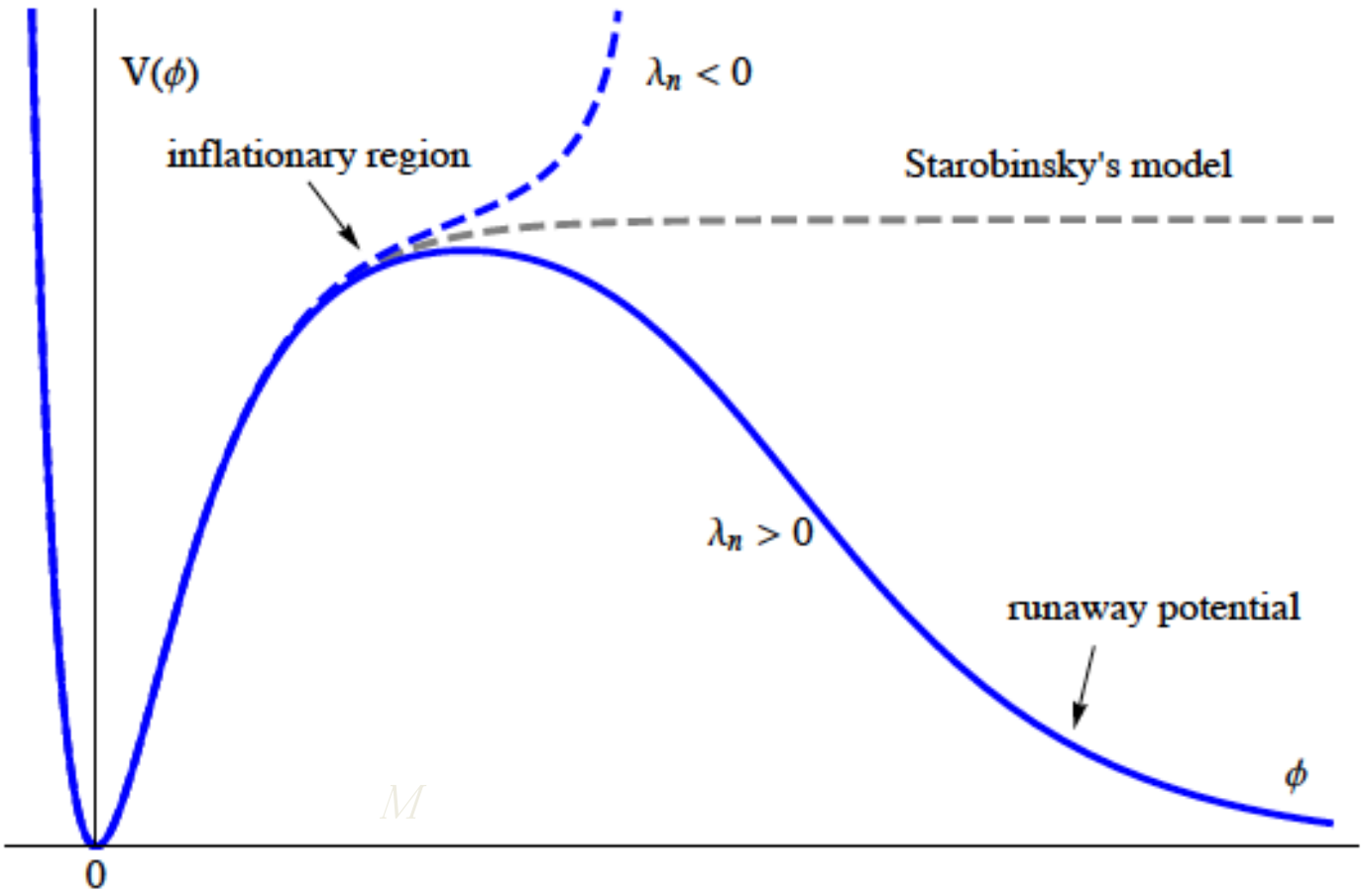
- If we take

$L = V_6^{1/6} = 30/\Lambda$

$$c = \frac{M_{\mathrm{P}}^2}{12 m^2} \simeq 5 \times 10^8$$

General potential shapes

$$f(R) = R + \frac{R^2}{6M^2} + \frac{\lambda_n}{2n}\frac{R^n}{(3M^2)^{n-1}}$$



Q.-G. Huang, 2013

Starobinsky-like Inflation models

T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, T. Terada, PTEP 2016 (2016) 12, 123E01

- Einstein term

$$cb_1\Lambda^2 = -\frac{M_{\rm P}^2}{2} \qquad b_1 \sim O(10^{-4})$$

- Including higher-order terms

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}M_{\rm P}^2 R + \frac{M_{\rm P}^2}{12m^2}\left(R^2 + \sum_{n=3}^{\infty} b_n \left(-\frac{6m^2}{b_1}\right)^{2-n} R^n\right)\right)$$

m ~ 10^{13} GeV b = $b_1 b_3$

$$V = \frac{m^2}{9b^2}e^{-2\sqrt{2/3}\phi}\left(\sqrt{1+3b\left(e^{\sqrt{2/3}\phi}-1\right)}-1\right)\left(1+6b\left(e^{\sqrt{2/3}\phi}-1\right)-\sqrt{1+3b\left(e^{\sqrt{2/3}\phi}-1\right)}\right)$$

Starobinsky-like Inflation

T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, T. Terada, PTEP 2016 (2016) 12, 123E01

- Potential

$$V = V_{\text{Starobinsky}} \times \left(1 - \frac{b}{2} e^{\sqrt{2/3}\phi} \left(1 - e^{-\sqrt{2/3}\phi}\right)\right) + \mathcal{O}(b^2),$$

- Predictions

$$b = b_1 b_3 \sim 10^{-4}$$

$$1 - n_s \simeq \frac{2}{N}\left(1 + \frac{16}{27} b N^2\right) = \frac{2}{N} + \frac{32}{27} b N,$$

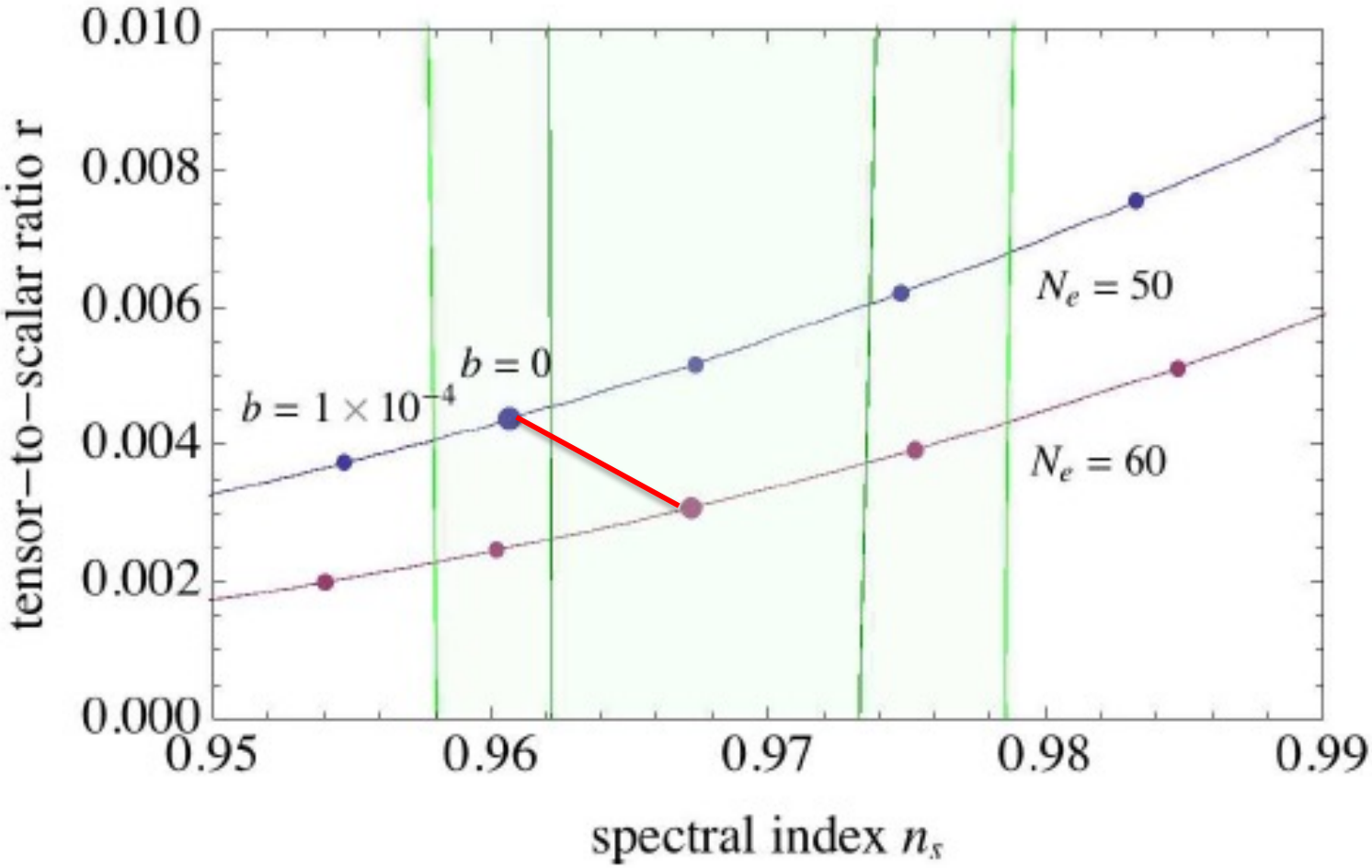
$$r \simeq \frac{12}{N^2}\left(1 - \frac{16}{27} b N^2\right) = \frac{12}{N^2} - \frac{64}{9} b,$$

$$\alpha_s = dn_s / d\ln k \qquad \alpha_s \simeq -\frac{2}{N^2}\left(1 - \frac{16}{27} b N^2\right) = -\frac{2}{N^2} + \frac{32}{27} b$$

$$\beta_s = d\alpha_s / d\ln k \qquad \beta_s \simeq -\frac{4}{N^3}\left(1 + \frac{4}{9} b N\right) = -\frac{4}{N^3} - \frac{16}{9N^2} b$$

Tensor to scalar ratio

T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, T. Terada, PTEP 2016 (2016) 12, 123E01



$$b = b_1 b_3 \sim O(10^{-4})$$

Inflaton/scalaron ϕ + curvaton σ

R. Jinno, K. Kohri, T. Moroi, T. Takahashi, M. Hazumi, arXiv:2310.08158 [astro-ph.CO]

- During inflation, one more light field existed?

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta^{(\mathrm{inf})}(k) + \mathcal{P}_\zeta^{(\mathrm{cur})}(k) = (1+R_\sigma)\mathcal{P}_\zeta^{(\mathrm{inf})}(k)$$

$$R_\sigma \equiv \frac{\mathcal{P}_\zeta^{(\mathrm{cur})}}{\mathcal{P}_\zeta^{(\mathrm{inf})}} > 1$$

- Modifications on observables (with $f_{NL} \sim O(1)$)

$$r = \frac{16\epsilon}{1+R_\sigma}$$ << 3.E-3 ?

$$n_s - 1 = -2\epsilon + 2\eta_\sigma + \frac{-4\epsilon + 2\eta_\phi - 2\eta_\sigma}{1+R_\sigma}$$

$n_t \equiv dP_h/d\ln k$

$$n_T = -(1+R_\sigma)\frac{r}{8}$$ < ~ -0.2?

Inflaton/scalaron ϕ + curvaton σ

R. Jinno, K. Kohri, T. Moroi, T. Takahashi, M. Hazumi, arXiv:2310.08158 [astro-ph.CO]

- Fraction of perturbation

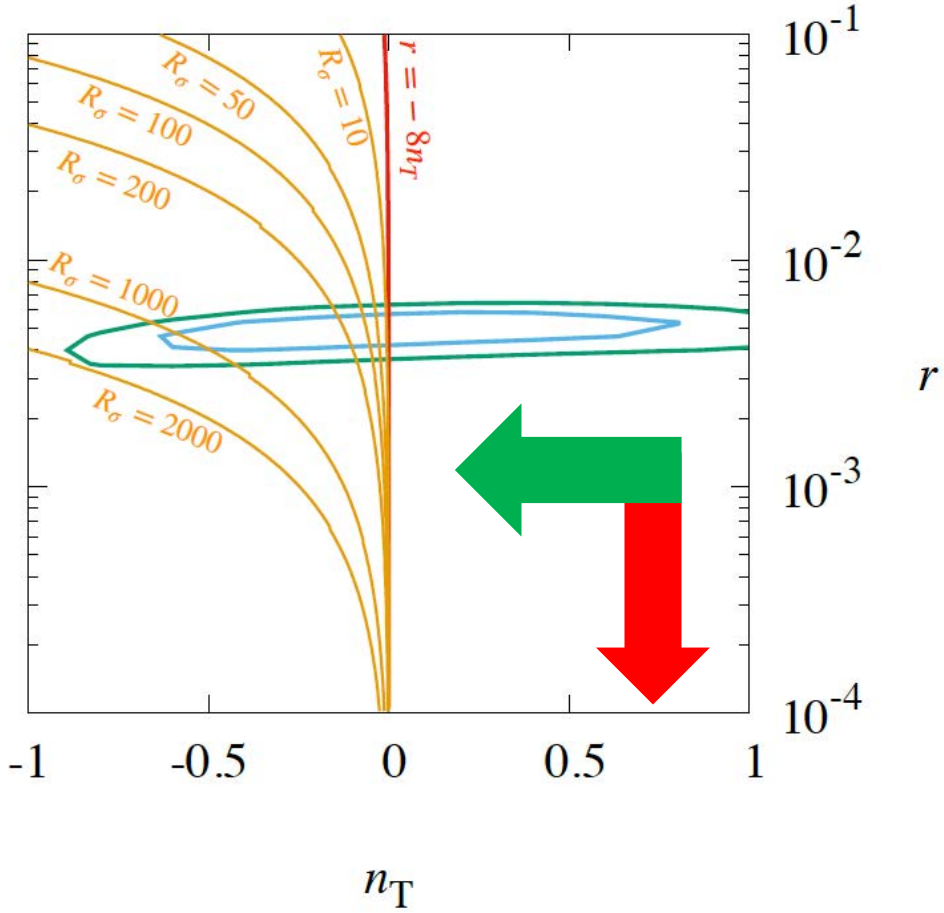
$$R_\sigma \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

- tensor to scalar ratio

$$r = \frac{16\epsilon}{1 + R_\sigma}$$

- tensor spectral index

$$n_T = -(1 + R_\sigma)\frac{r}{8}$$



R^2 + Higgs Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

- Action of Higgs and R^2

$$S_J = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(R_J + \frac{\xi h^2}{M_P^2}R_J + \frac{R_J^2}{6M^2}\right) - \frac{1}{2}g^{\mu\nu}\nabla_\mu h\nabla_\nu h - \frac{\lambda(\mu)}{4}h^4\right]$$

- Conformal transformation

$$\alpha = M_P^2/12M^2$$

$$\sqrt{\frac{2}{3}}\frac{s}{M_P} = \ln\left(1 + \frac{\xi h^2}{M_P^2} + \frac{R_J}{3M^2}\right) \equiv \Omega(s).$$

- Action of scalaron (s) and Higgs (h)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}G_{ab}g^{\mu\nu}\nabla_\mu\phi^a\nabla_\nu\phi^b - U(\phi^a)\right]$$

$$U(\phi^a) \equiv e^{-2\Omega(s)}\left\{\frac{3}{4}M_P^2M^2\left(e^{\Omega(s)} - 1 - \frac{\xi h^2}{M_P^2}\right)^2 + \frac{\lambda(\mu)}{4}h^4\right\}$$

$$g_{\mu\nu} = e^{\Omega(s)}g^J_{\mu\nu} \qquad G_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Omega(s)} \end{pmatrix}$$

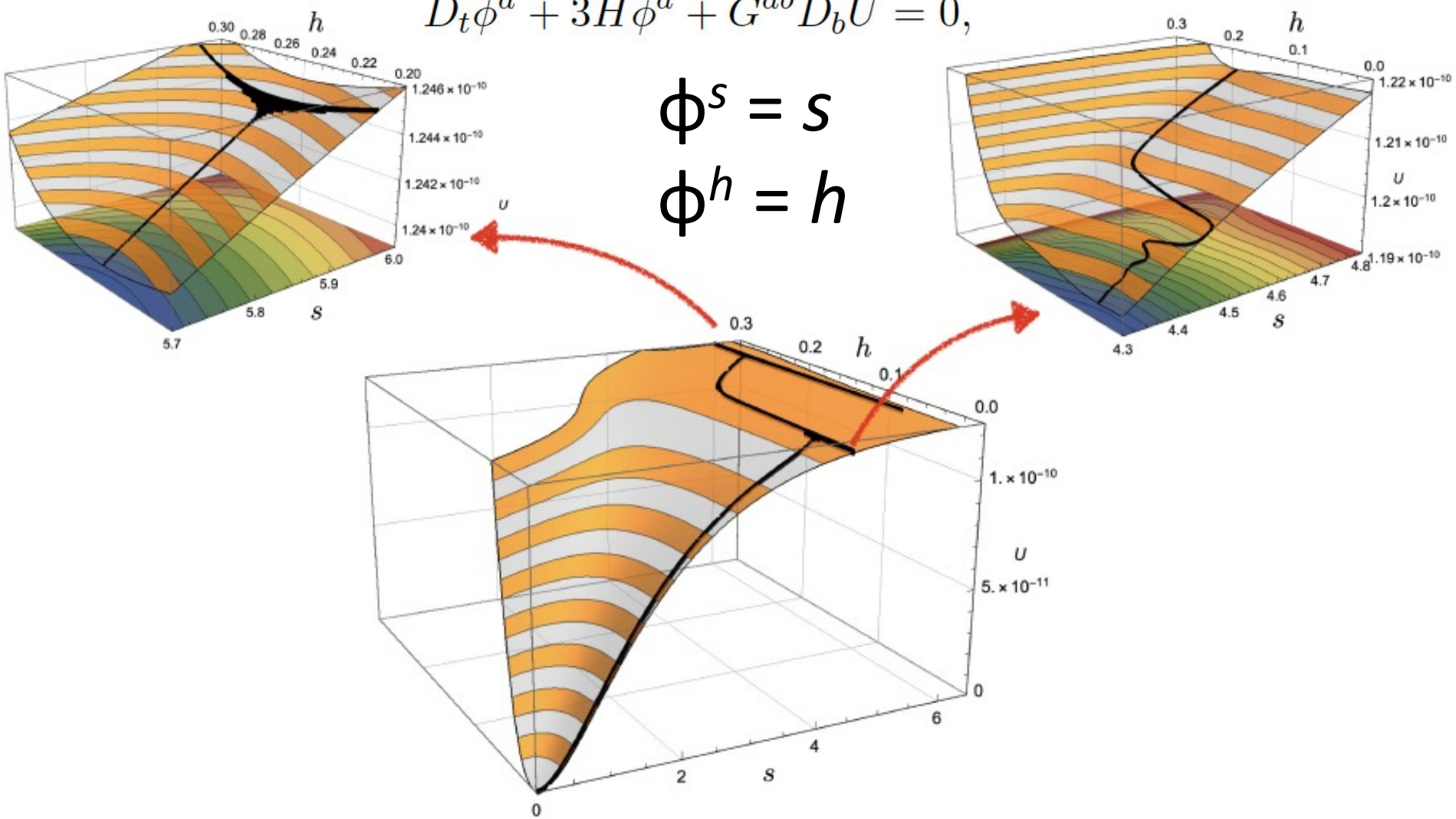
Motions on the potential of the Higgs-scalaron (s) system

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

$$D_t\dot{\phi}^a + 3H\dot{\phi}^a + G^{ab}D_bU = 0,$$

$\phi^s = s$

$\phi^h = h$



Tachyonic Instability induced in R^2-Higgs Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}_0^2}{2H^2}\,, \quad \eta^a \equiv -\frac{1}{H\dot{\phi}_0} D_t \dot{\phi}^a.$$

$$\mathcal{S} = \frac{H}{a\dot{\phi}_0} v_N \equiv \frac{H}{\dot{\phi}_0} Q_N.$$

$$\eta^a = \eta_\parallel T^a + \eta_\perp N^a$$

$$\eta_\parallel \equiv -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}\,, \quad \eta_\perp \equiv \frac{U_N}{\dot{\phi}_0 H}$$

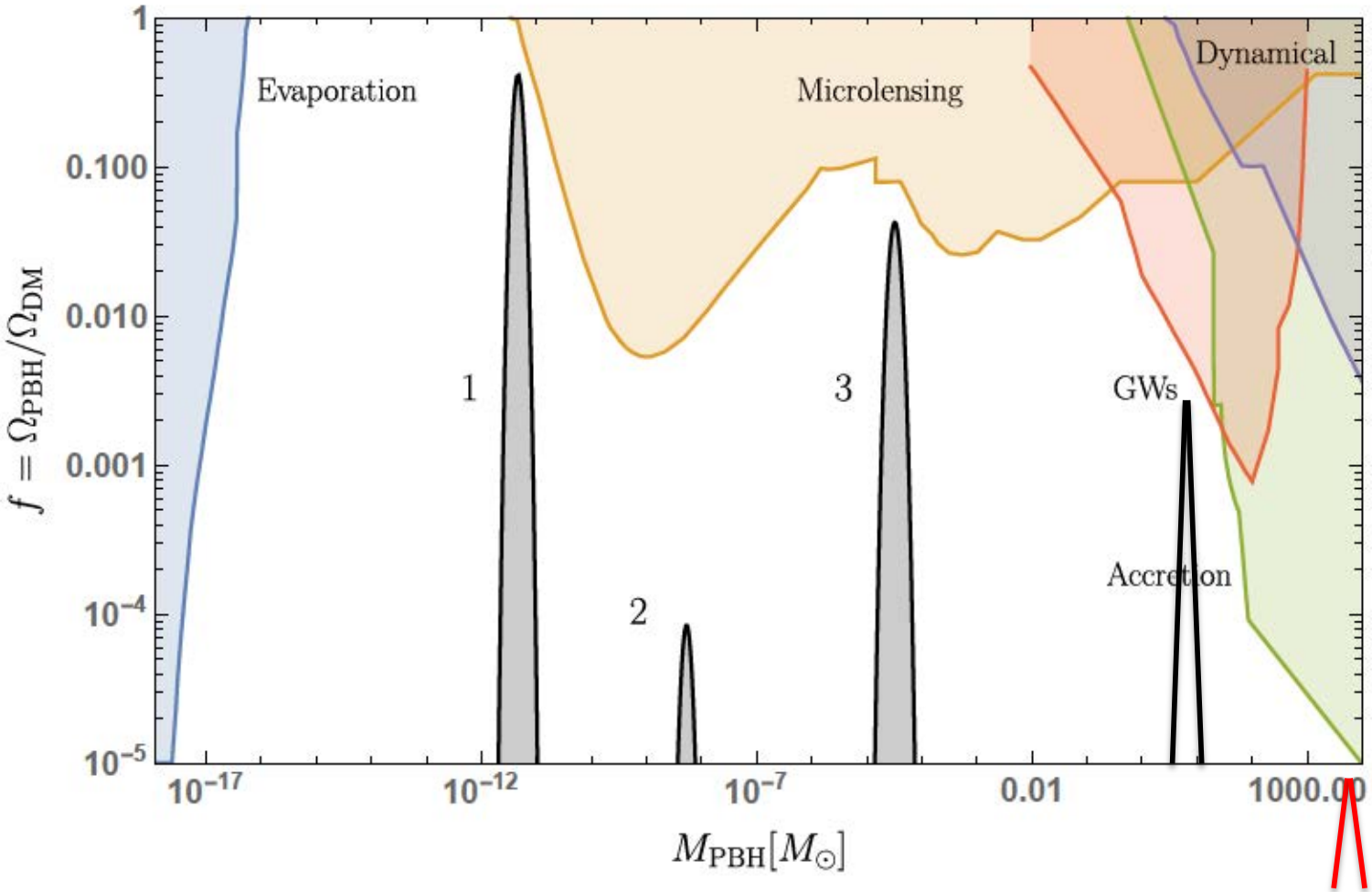
$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + M_{\rm eff}^2\right) Q_N = 2\dot{\phi}_0 \eta_\perp \dot{\mathcal{R}}$$

$$M_{\rm eff}^2 = U_{NN} + H^2 \epsilon \mathbb{R} - \dot{\theta}^2 \qquad \dot{\theta} \equiv H\eta_\perp$$

$$\boxed{U_{NN} < 0,}$$ during the tachyonic phase

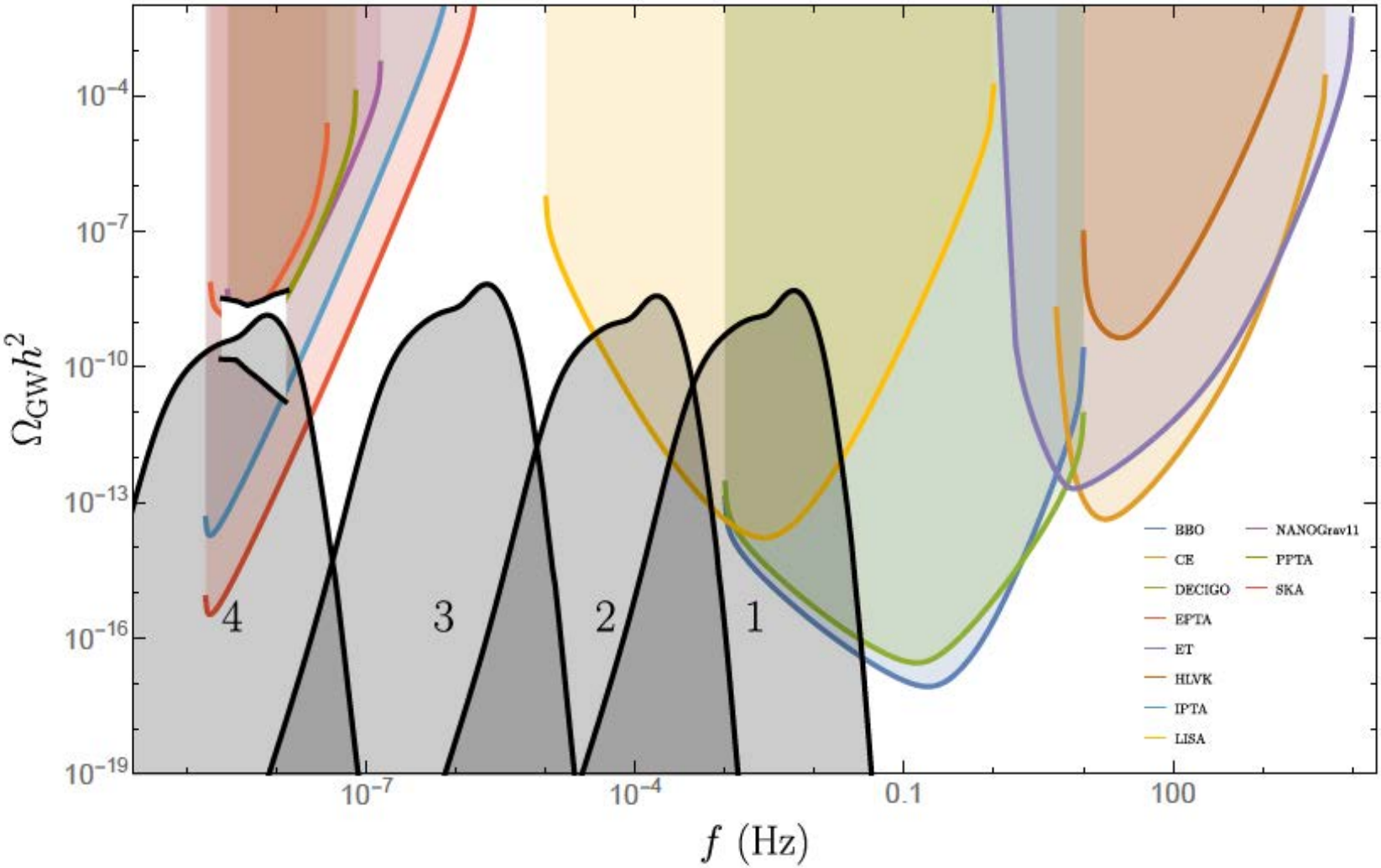
Primordial Black Holes and Second Order Gravitational Waves from Tachyonic Instability induced in Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]



Primordial Black Holes and Second Order Gravitational Waves from Tachyonic Instability induced in Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]



Summary

- A future detection of tensor to scalar ratio $r \sim O(10^{-3})$ proves a quantum nature of gravity and means the (trans-)Planck-scale physics

- We will test the Starobinsky model, and discriminate it from a variety of Starobinsky-type models By LiteBIRD

- In future, we will measure tensor spectral index $n_t = dP_h/d\ln k$ within a sensitivity of $\Delta n_t \sim 0.2$

Various scales with *r*

$$r = \frac{\text{tensor (gravitational wave)}}{\text{scalar (curvature perturbation)}} = \frac{P_h}{P_\zeta}$$

- Potential energy of infaton field

$$V^{1/4} \sim 10^{16}\,\mathrm{GeV} \left(\frac{r}{0.1}\right)^{1/4}$$

- Hubble expansion rate at inflation

$$H \sim 10^{14}\,\mathrm{GeV} \left(\frac{r}{0.1}\right)^{1/2}$$

- Cosmic time at inflation

$$t \sim 10^{-38}\,\mathrm{sec} \left(\frac{r}{0.1}\right)^{-1/2}$$

Lyth bound on tensor to scalar ratio in slowroll inflation

- Lyth bound

$$r \leq 0.1\left(\frac{N}{60}\right)^{-2}\left(\frac{\Delta\phi}{7m_{\mathrm{pl}}}\right)^{2}$$

$$\leq 0.002\left(\frac{N}{60}\right)^{-2}\left(\frac{\Delta\phi}{m_{\mathrm{pl}}}\right)^{2}$$

Lyth (1998)

Boubekeur and Lyth (2005)