Control of Casimir forces

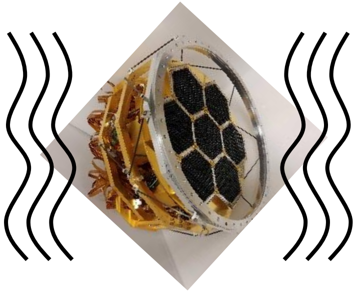
Hideo Iizuka

Principal Investigator,
International Center for Quantum-field Measurement Systems for Studies of the Universe and Particles (QUP)

Senior Fellow,
Toyota Central R&D Labs., Inc.

Research scope of QUP quantum sensor cluster

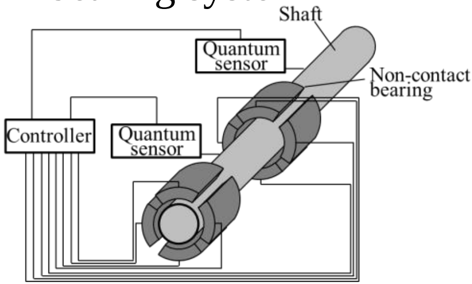
Active vibration isolation for CMB



New physics search

Circuit diagnosis

Non-contact shaft-bearing system

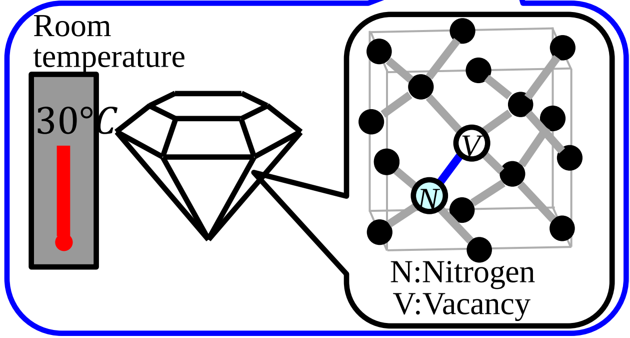


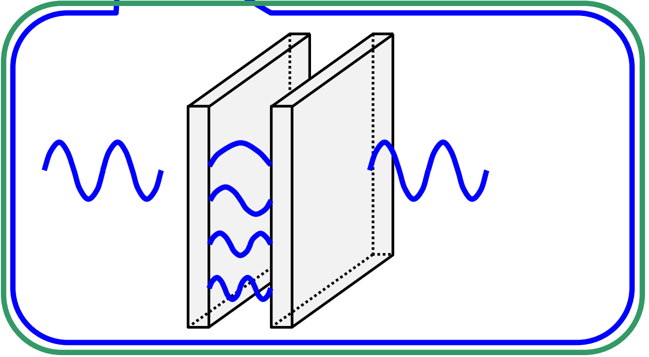
Particle physics/Universe

Industry/Social implementation

Casimir forces

Quantum sensors





Today's talk

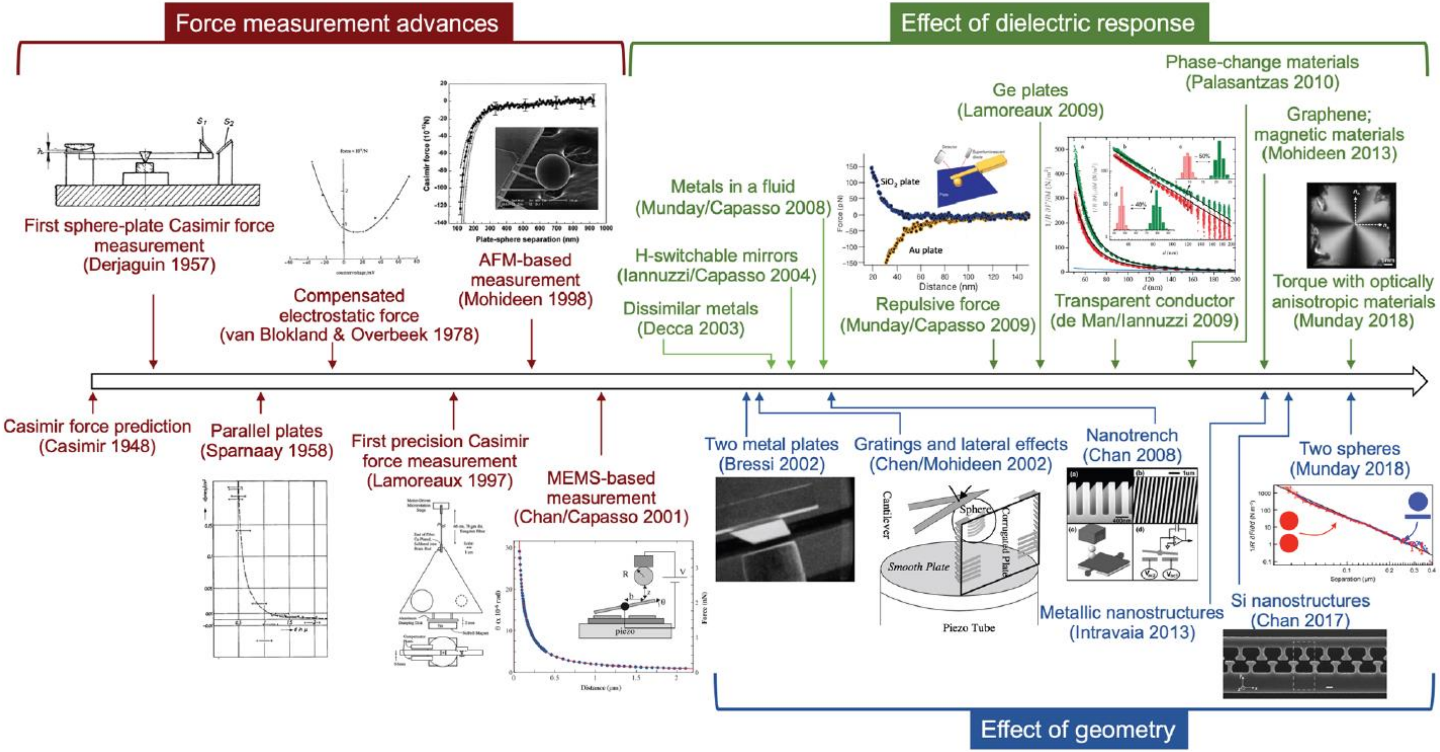
Outline

Brief overview of Casimir forces

Symmetry of Casimir forces in wavevector space

Conclusions

History of Casimir force research



Force measurement advances

Effect of dielectric response

First sphere-plate Casimir force measurement (Derjaguin 1957)

Compensated electrostatic force (van Blokland & Overbeek 1978)

AFM-based measurement (Mohideen 1998)

Dissimilar metals (Decca 2003)

H-switchable mirrors (Iannuzzi/Capasso 2004)

Metals in a fluid (Munday/Capasso 2008)

Repulsive force (Munday/Capasso 2009)

Ge plates (Lamoreaux 2009)

Transparent conductor (de Man/Iannuzzi 2009)

Phase-change materials (Palasantzas 2010)

Graphene; magnetic materials (Mohideen 2013)

Torque with optically anisotropic materials (Munday 2018)

Casimir force prediction (Casimir 1948)

Parallel plates (Sparnaay 1958)

First precision Casimir force measurement (Lamoreaux 1997)

MEMS-based measurement (Chan/Capasso 2001)

Two metal plates (Bressi 2002)

Gratings and lateral effects (Chen/Mohideen 2002)

Nanotrench (Chan 2008)

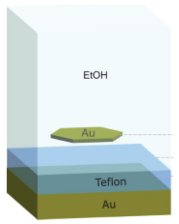
Metallic nanostructures (Intravaia 2013)

Si nanostructures (Chan 2017)

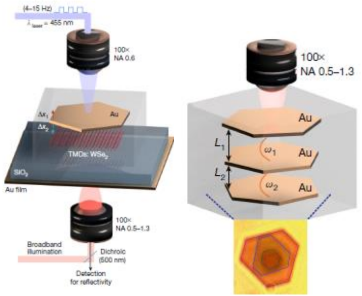
Two spheres (Munday 2018)

Effect of geometry

Past few years



Levitation in liquid
[Science 364, 984 (2019)]



Active control, many-body in liquid
[Nature, 597, 214 (2021)]

T. Gong, M. R. Corrado, A. R. Mahbub, C. Shelden, and J. N. Munday, Nanophotonics 10, 523 (2021).
J. N. Munday, KEK IPNS-IMSS-QUP joint workshop Feb. 8-10, 2022.

Literature of Casimir forces towards new force research

PRL **94,** 240401 (2005) PHYSICAL REVIEW LETTERS week ending 24 JUNE 2005

Constraining New Forces in the Casimir Regime Using the Isoelectronic Technique

R. S. Decca,[1,*] D. López,[2] H. B. Chan,[3] E. Fischbach,[4] D. E. Krause,[5,4] and C. R. Jamell[1]
[1]*Department of Physics, Indiana University-Purdue University Indianapolis, Indianapolis, Indiana 46202, USA*
[2]*Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA*
[3]*Department of Physics, University of Florida, Gainesville, Florida 32611, USA*
[4]*Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA*
[5]*Physics Department, Wabash College, Crawfordsville, Indiana 47933, USA*

(Received 1 February 2005; published 20 June 2005)

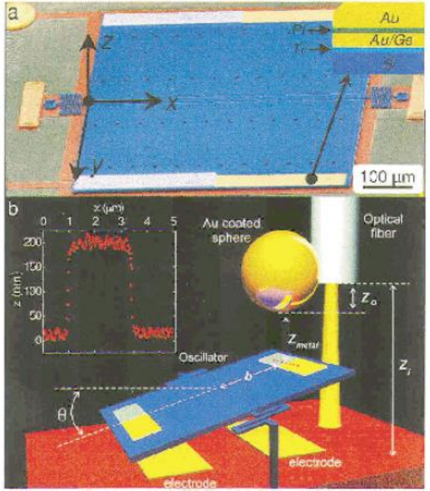


FIG. 1 (color). (a) Scanning electron microscope image of the MTO with the composite sample deposited on it. The coordinate system used in the Letter is indicated. Inset: schematic of the sample deposited on the MTO. The thickness of the different layers are (in order of deposition): $d_{\mathrm{Ti}} = 1$ nm, $d_{\mathrm{Ge}} = 200$ nm, $d_{\mathrm{Pt}} = 1$ nm, and $d^p_{\mathrm{Au}} = 150$ nm. The thickness of the layers deposited on the sphere (not shown) are: $d_{\mathrm{Cr}} = 1$ nm and $d^s_{\mathrm{Au}} = 200$ nm. (b) Experimental setup. The red dotted line indicates where AFM line cuts were taken. Inset: AFM profile of the sample interface.

Hypothetical force difference

$$\Delta F_h = -4\pi^2 G\underline{\alpha}\,\underline{\lambda}^3 e^{-z/\underline{\lambda}} R K_s K_p,$$

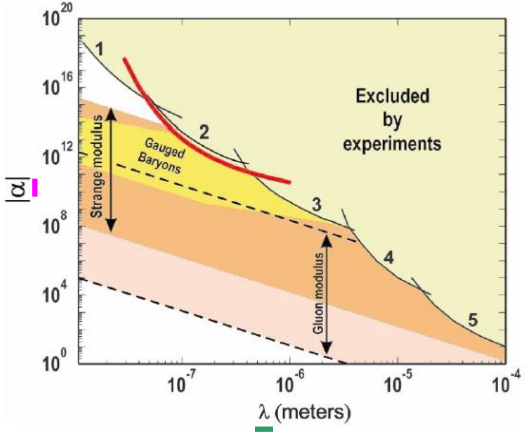
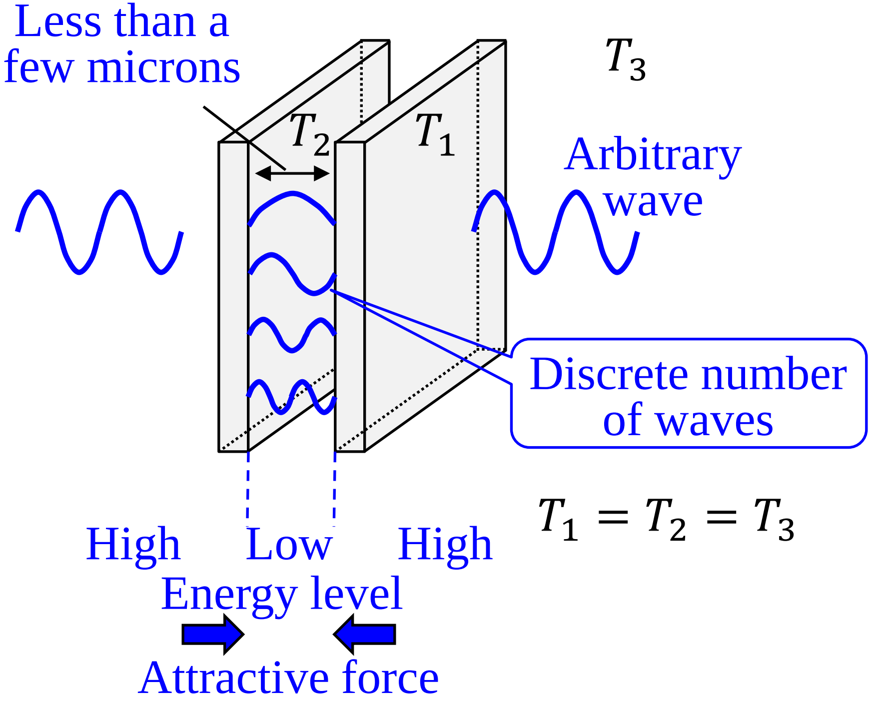


FIG. 4 (color). Values in the $\{\lambda, \alpha\}$ space excluded by experiments. The red curve represents limits obtained in this work. Curves 1 to 5 were obtained by Mohideen's group [7], our group [1], Lamoreaux [6], Kapitulnik's group [8], and Price's group [5], respectively. Also shown are theoretical predictions [21].

G: Gravitational constant
K_s, K_p: Terms relating to densities of the sphere and the plate
R: Radius of the sphere

Equilibrium and non-equilibrium Casimir forces

Equilibrium Casimir force (Attractive in vacuum)



Less than a few microns

T_3

T_2

T_1

Arbitrary wave

Discrete number of waves

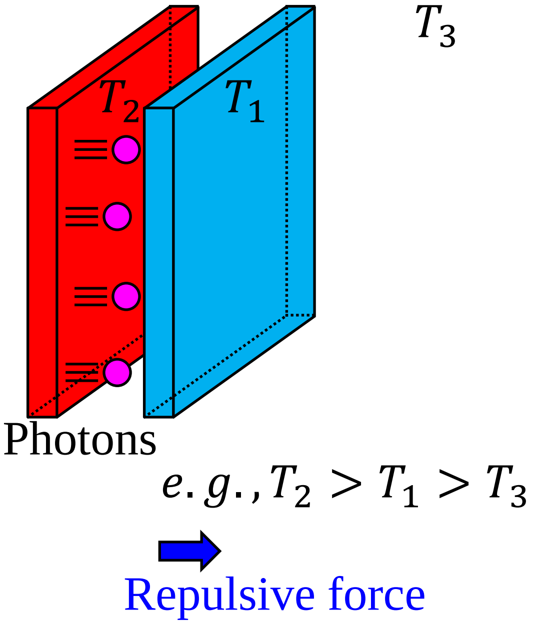
$T_1 = T_2 = T_3$

High Low High

Energy level

Attractive force

Non-equilibrium Casimir force (Can be repulsive in vacuum)



T_3

T_2

T_1

Photons

$e.g., T_2 > T_1 > T_3$

Repulsive force

Note: The plates consist of reciprocal materials.

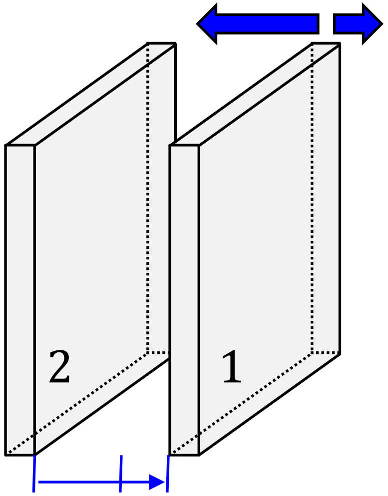
Stable and unstable Casimir force systems

When attractive and repulsive force components acting on body 1 are balanced, zero-force position can exist.

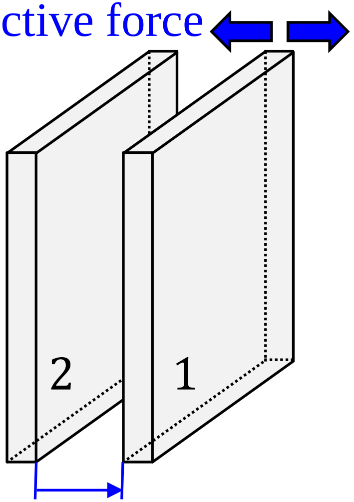
Attractive force ⇐ ⇒ Repulsive force

Stable

Attractive force is enhanced.



$z_0 + \Delta z$

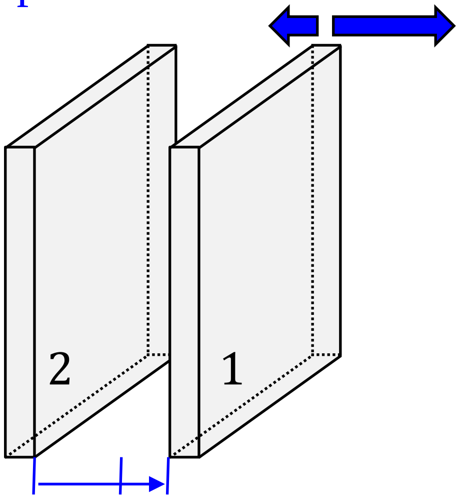


z_0

Zero-force position

Unstable

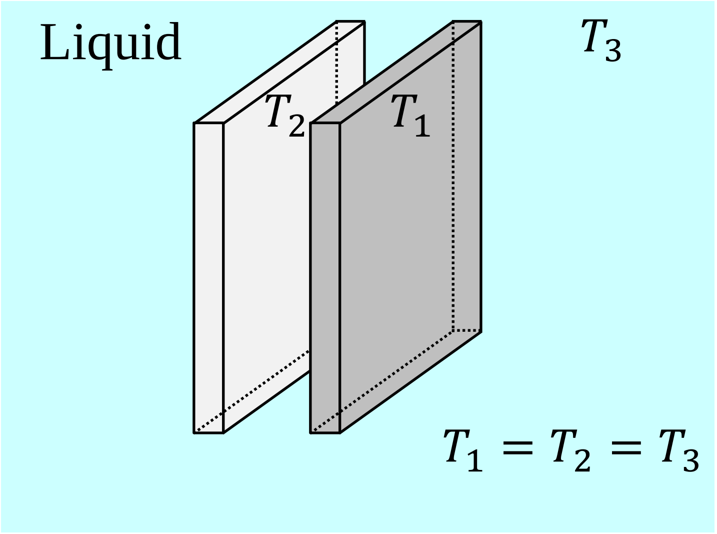
Repulsive force is enhanced.



$z_0 + \Delta z$

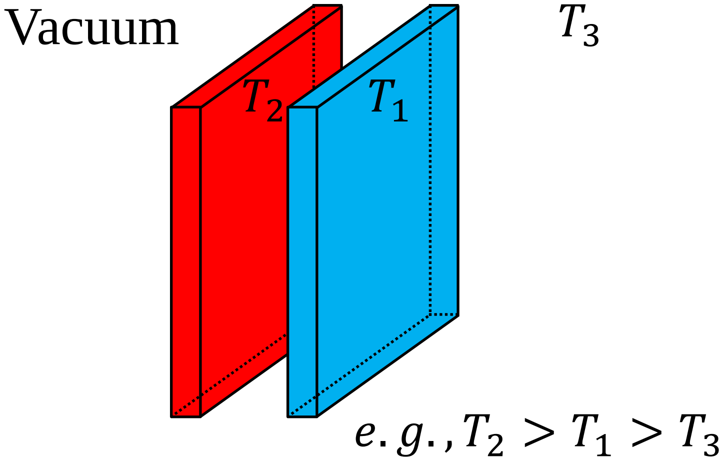
Examples of stable and unstable systems

Equilibrium Casimir force in liquid
(Can be stable)



[See *Science* 364, 984 (2019)]

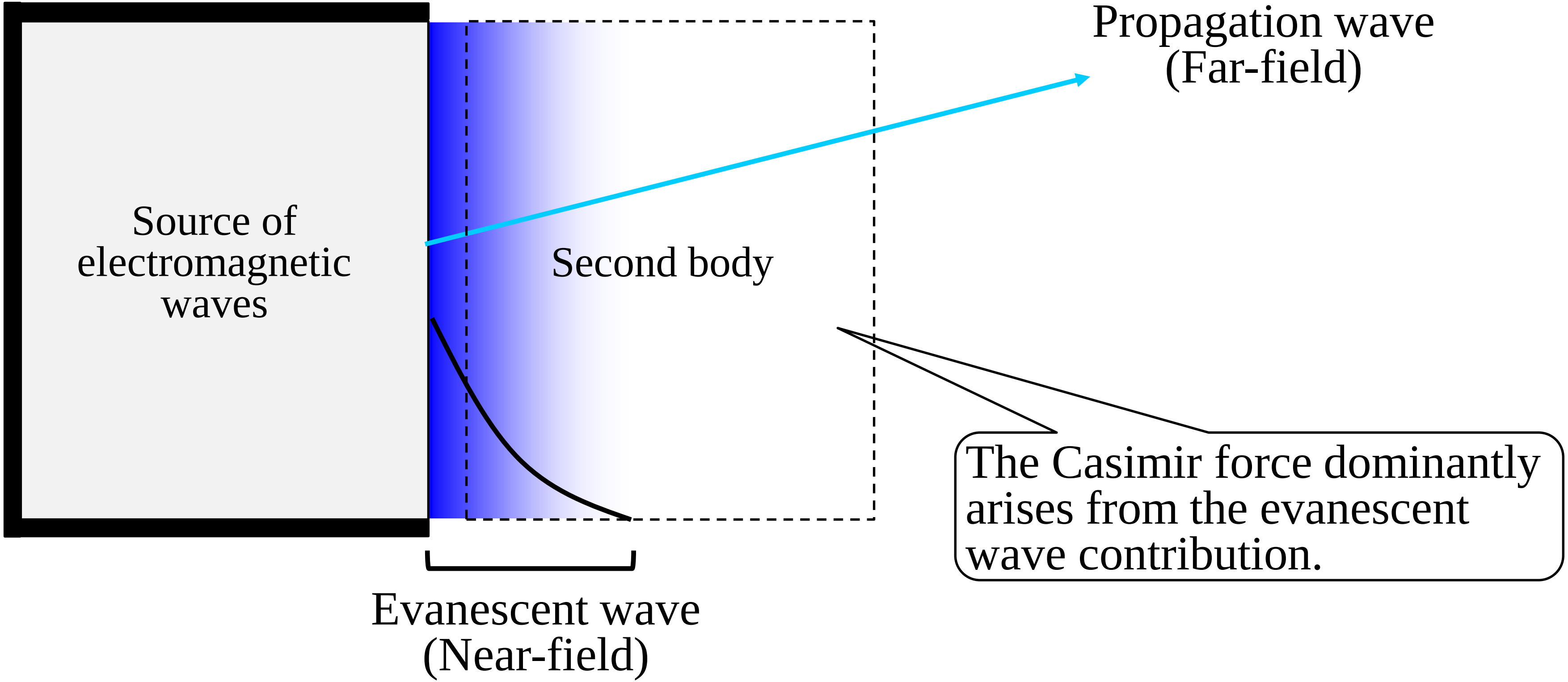
Non-equilibrium Casimir force in vacuum
(Unstable)



Note: The plates consist of reciprocal materials.

Propagation and evanescent waves

The propagation wave goes away from the electromagnetic source.
The evanescent wave stays around the electromagnetic source.



Source of electromagnetic waves

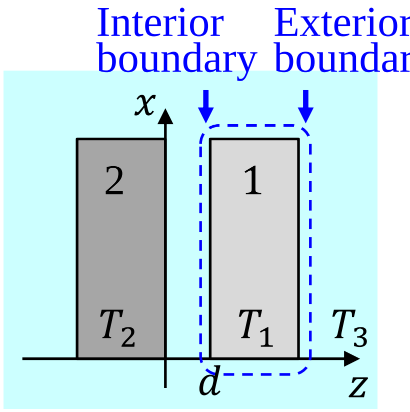
Second body

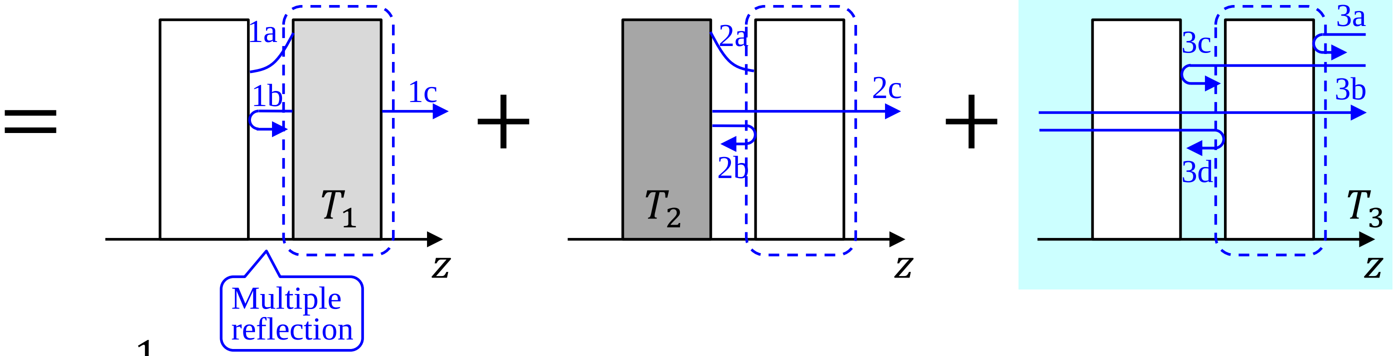
Propagation wave
(Far-field)

Evanescent wave
(Near-field)

The Casimir force dominantly arises from the evanescent wave contribution.

Casimir force calculation





Maxwell stress tensor

$$T_{ij} = \epsilon_0 E_i E_j + \mu_0 H_i H_j - \delta_{ij}\frac{1}{2}(\epsilon_0 E^2 + \mu_0 H^2), (1)$$

Casimir force acting on body 1 in the two-body system (isotropic materials)

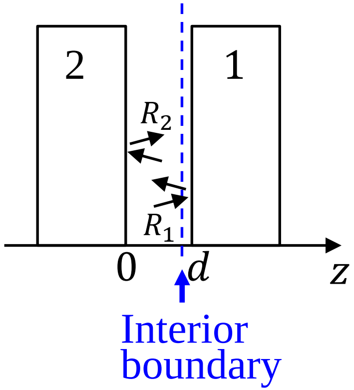
$\tilde{r}_{0j}$: reflection coefficient
$\tilde{t}_{0j}$: reflection coefficient

$$F_z = \int_S \langle T_{zz} \rangle dz\,, (2)$$

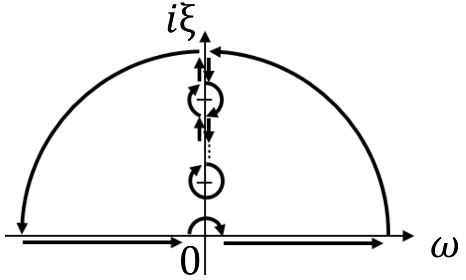
$$\begin{aligned}
&= -\sum\nolimits_{p,s} \frac{\hbar}{\pi^2}\int_0^\infty d\omega \Bigg\{\left[n(\omega,T_1)+\frac{1}{2}\right]\Bigg[\int_{k_0}^{\infty} \overbrace{\overset{\text{1a}}{k_\parallel dk_\parallel \kappa_0 \frac{Im(\tilde{r}_{01})Re(\tilde{r}_{02})e^{-2\kappa_0 d}}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{-2\kappa_0 d}|^2}}}^{\text{Evanescent waves}} + \int_0^{k_0} k_\parallel dk_\parallel k_{z0}\frac{1}{4}\Bigg(\overbrace{\overset{\text{1b}}{-\frac{(1-|\tilde{r}_{01}|^2-|\tilde{t}_{01}|^2)(1+|\tilde{r}_{02}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}} \overbrace{+1-\left|\tilde{r}_{31}+\frac{\tilde{r}_{02}\tilde{t}_{01}^2 e^{i2k_{z0}d}}{1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}}\right|^2 - \frac{|\tilde{t}_{01}|^2(1-|\tilde{r}_{02}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}}^{\text{1c}}}^{\text{Propagation waves}}\Bigg)\Bigg] \\
&\quad + \left[n(\omega,T_2)+\frac{1}{2}\right]\Bigg[\int_{k_0}^{\infty} k_\parallel dk_\parallel \kappa_0 \overset{\text{2a}}{\frac{Im(\tilde{r}_{02})Re(\tilde{r}_{01})e^{-2\kappa_0 d}}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{-2\kappa_0 d}|^2}} + + \int_0^{k_0} k_\parallel dk_\parallel k_{z0}\frac{1}{4}\Bigg(-\overset{\text{2b}}{\frac{(1-|\tilde{r}_{02}|^2-|\tilde{t}_{02}|^2)(1+|\tilde{r}_{01}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}} + \overset{\text{2c}}{\frac{|\tilde{t}_{01}|^2(1-|\tilde{r}_{02}|^2-|\tilde{t}_{02}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}}\Bigg)\Bigg] \\
&\quad + \left[n(\omega,T_3)+\frac{1}{2}\right]\int_0^{k_0} k_\parallel dk_\parallel k_{z0}\frac{1}{4}\Bigg(1+\overset{\text{3a}}{\left|\tilde{r}_{31}+\frac{\tilde{r}_{02}\tilde{t}_{01}^2 e^{i2k_{z0}d}}{1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}}\right|^2} + \overset{\text{3b}}{\frac{|\tilde{t}_{01}|^2|\tilde{t}_{02}|^2}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}} - \overset{\text{3c}}{\frac{|\tilde{t}_{01}|^2(1+|\tilde{r}_{02}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}} - \overset{\text{3d}}{\frac{|\tilde{t}_{02}|^2(1+|\tilde{r}_{01}|^2)}{|1-\tilde{r}_{01}\tilde{r}_{02}e^{i2k_{z0}d}|^2}}\Bigg)\Bigg\}(3)
\end{aligned}$$

Casimir force calculation in equilibrium

Calculations of <u>equilibrium Casimir forces</u> can be simplified since photon exchange is balanced at the exterior boundary. (The interior boundary is only considered.)



$$T \equiv T_1 = T_2 = T_3$$



Casimir force formula, integration along the real frequency axis

$$P_t = \sum_{j=p,s} \int_0^\infty \frac{k_\parallel dk_\parallel}{2\pi} \underline{\int_0^\infty \frac{d\omega}{2\pi}} 4\hbar \left[n(\omega,T) + \frac{1}{2} \right] k_z Z(\omega,\beta)\,, (11a)$$

$$Z(\omega,\beta) = \frac{R_1(\omega,\beta)R_2(\omega,\beta)e^{i2k_z d}}{1 - R_1(\omega,\beta)R_2(\omega,\beta)e^{i2k_z d}}\,, (11b)$$

- Understanding the mechanism
- Long calculation time

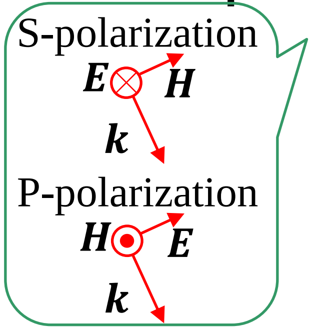
Wick rotation approach, integration along the imaginary frequency axis (E.M. Lifshitz, Sov. Phys. 1956)

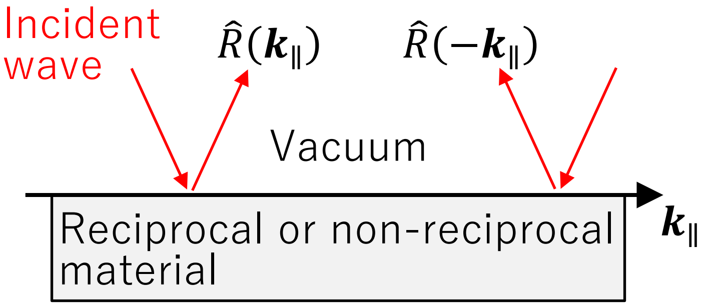
$$P_t = \sum_{j=p,s} \int_0^\infty \frac{k_\parallel dk_\parallel}{2\pi} 2k_B T \underline{\sum_{n=0}^{\infty\prime}} q_{0,n}\, Z(i\xi_n,\beta)\,, (12a)$$

$$Z(i\xi_n,\beta) = \frac{R_1(i\xi_n,\beta)R_2(i\xi_n,\beta)e^{-2q_{0,n}d}}{1 - R_1(i\xi_n,\beta)R_2(i\xi_n,\beta)e^{-2q_{0,n}d}}\,, (12b),$$

- Significantly reduced calculation time
- Little observation of the mechanism

Reciprocal and non-reciprocal materials





Reflection matrix

$$\hat{R}(\boldsymbol{k}_\parallel) = \begin{bmatrix} R^{s\to s}(\boldsymbol{k}_\parallel) & R^{p\to s}(\boldsymbol{k}_\parallel) \\ R^{s\to p}(\boldsymbol{k}_\parallel) & R^{p\to p}(\boldsymbol{k}_\parallel) \end{bmatrix}, (21)$$

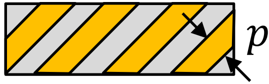
Reciprocal materials

$$\hat{R}(-\boldsymbol{k}_\parallel) = \hat{\sigma}_z \hat{R}^T(\boldsymbol{k}_\parallel) \hat{\sigma}_z, \quad (22)$$

$$\left(\hat{\sigma}_z = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} \right)$$

Bulk
(Isotropic)

Inclined nanowires
(Anisotropic)

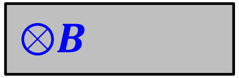


$$\hat{\epsilon}_i = \begin{bmatrix} \epsilon_p & & \\ & \epsilon_p & \\ & & \epsilon_p \end{bmatrix}, (23) \quad \hat{\epsilon}^S = \begin{bmatrix} \epsilon_d & & \epsilon_f \\ & \epsilon_p & \\ \epsilon_f & & \epsilon_d \end{bmatrix}, (24)$$

Non-reciprocal materials

Eq. (22) can be violated.

InSb



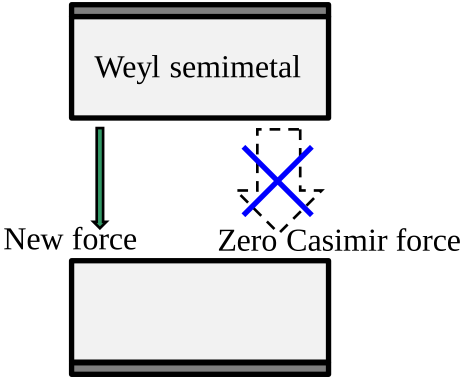
Magnetic Weyl
semimetals

$$\hat{\epsilon}^A = \begin{bmatrix} \epsilon_d & & i\epsilon_f \\ & \epsilon_p & \\ -i\epsilon_f & & \epsilon_d \end{bmatrix}, (25)$$

Our Casimir force research

Particle physics

New force search via zero Casimir force[1]


Weyl semimetal

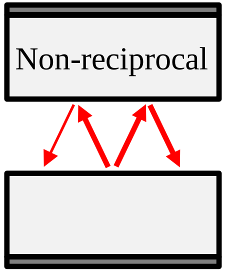
New force

Zero Casimir force

[1] Y. Ema, M. Hazumi, H. Iizuka, K. Mukaida, and K. Nakayama, Phys. Rev. D 108, 016009 (2023).

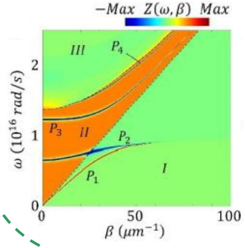
Fundamental understanding

[2] Symmetry argument in Casimir forces

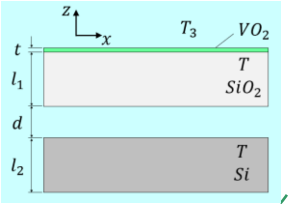

Non-reciprocal

Show detail

[3]Casimir force is insensitive to material loss



[4]Exterior control of non-equilibrium Casimir force

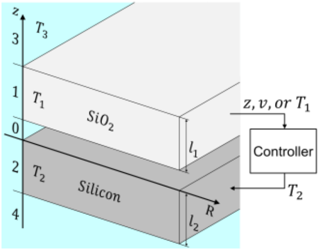


[2] H. Iizuka and S. Fan, Phys. Rev. B 108, 075429 (2023).
[3] H. Iizuka and S. Fan, J. Optical Society of America B 36, 2981 (2019).
[4] H. Iizuka and S. Fan, J. Optical Society America B 38, 151–158 (2021).

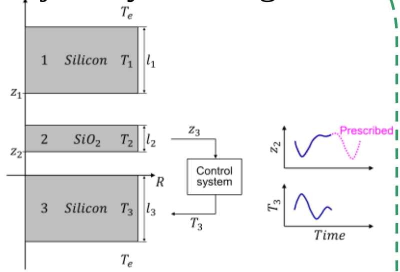
Toward industry

Dynamic control of Casimir forces in vacuum

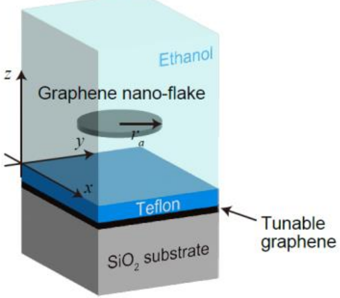
Control theory[5]



Trajectory tracking[6]



Dynamic control of Casimir forces in liquid[7]



[5] H. Iizuka and S. Fan, Applied Physics Letters 118, 144001 (2021).
[6] H. Iizuka and S. Fan, J. Quantitative Spectroscopy Radiative Transfer 289, 108281 (2022).
[7] H. Toyama, T. Ikeda, and H. Iizuka, Phys. Rev. B 108, 245402 (2023).

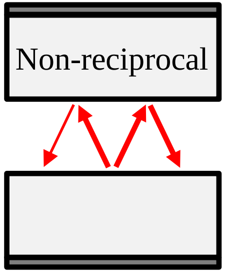
Outline

Brief overview of Casimir forces

Symmetry of Casimir forces in wavevector space

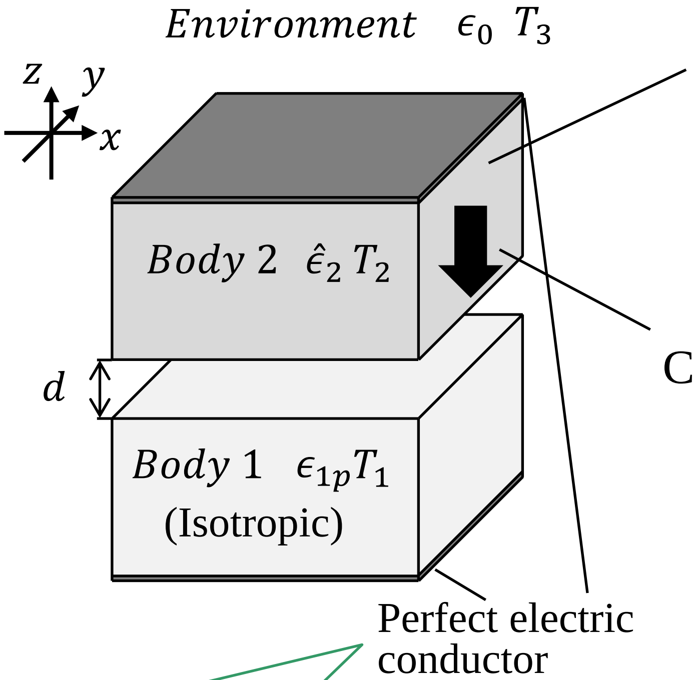
Conclusions

[2] Symmetry argument
in Casimir forces



[2] H. Iizuka and S. Fan, Phys. Rev. B 108, 075429 (2023).

Two-plate system



No photon exchange between the two-body system and the environment.

Anisotropic reciprocal material

$$\epsilon_2^S = \begin{bmatrix} \epsilon_d & & \epsilon_f \\ & \epsilon_p & \\ \epsilon_f & & \epsilon_d \end{bmatrix}, (24)$$

or

non-reciprocal material

InSb/Weyl

$$\epsilon_2^A = \begin{bmatrix} \epsilon_d & & i\epsilon_f \\ & \epsilon_p & \\ -i\epsilon_f & & \epsilon_d \end{bmatrix}, (25)$$

Casimir pressure acting on body 2

$$F_2^z(T_1,T_2,T_3) = \int_0^\infty d\omega \int_{-\infty}^{\infty} d\boldsymbol{k}_\parallel \underline{\underline{F_2^z(\omega,\boldsymbol{k}_\parallel,T_1,T_2,T_3)}}, (31)$$

$$\underline{\underline{F_2^z(\omega,\boldsymbol{k}_\parallel,T_1,T_2,T_3)}} = -F_{1\to2}^z(\omega,\boldsymbol{k}_\parallel,T_1) - F_{2\to1}^z(\omega,\boldsymbol{k}_\parallel,T_2) + F_{ext,2}^z(\omega,\boldsymbol{k}_\parallel,T_3), (32)$$

$$F_{1\to2}^z(\omega,\boldsymbol{k}_\parallel,T_1) = \left[n(\omega,T_1)+\frac{1}{2}\right]\frac{\hbar|k_{z0}|}{8\pi^3}\underline{\tilde{F}_{1\to2}^z(\omega,\boldsymbol{k}_\parallel)}, (33)$$

$T_1 = T_2 = T_3 = T$ (Equilibrium)

$T_1 \neq T_2$ (Non-equilibrium)

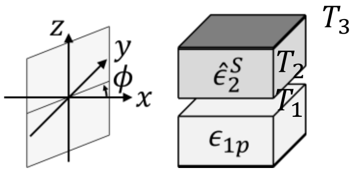
$$\underline{\tilde{F}_{l\to m}^z(\omega,\boldsymbol{k}_\parallel)} = \begin{cases} Tr\left[-\left(\hat{I}+\hat{R}_m^\dagger\hat{R}_m\right)\hat{D}_{lm}\left(\hat{I}-\hat{R}_l\hat{R}_l^\dagger\right)\hat{D}_{lm}^\dagger\right], & (k_\parallel < k_0) \\ Tr\left[-i\left(\hat{R}_m^\dagger+\hat{R}_m\right)\hat{D}_{lm}\left(\hat{R}_l-\hat{R}_l^\dagger\right)\hat{D}_{lm}^\dagger e^{-2\kappa_{z0}d}\right], & (k_\parallel > k_0) \end{cases} (34)$$

Exchange function

Casimir pressure, reciprocal

Equilibrium Casimir pressure is symmetric for $\boldsymbol{k}_\parallel$ for reciprocal systems.

$$F_2^z(\omega,\boldsymbol{k}_\parallel,T,T,T) = F_2^z(\omega,-\boldsymbol{k}_\parallel,T,T,T), (35)$$



Casimir pressure in $(\omega,\boldsymbol{k}_\parallel)$ space

$F_2^z(\omega,\boldsymbol{k}_\parallel,T_1,T_2,T_3)$

Exchange function

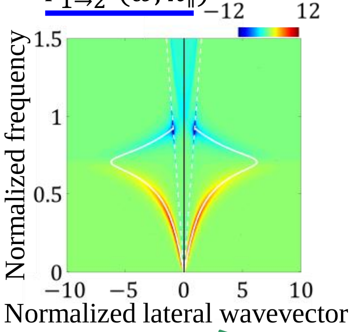
$\tilde{F}^z_{l\to m}(\omega,\boldsymbol{k}_\parallel)$

White lines:

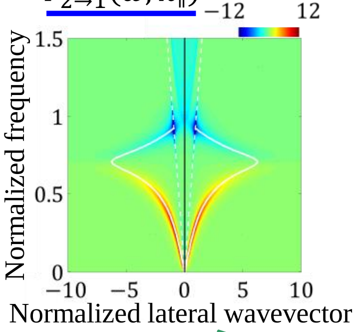
Dispersion curves

Asymmetric for $\boldsymbol{k}_\parallel$

$\tilde{F}^z_{1\to2}(\omega,\boldsymbol{k}_\parallel)$

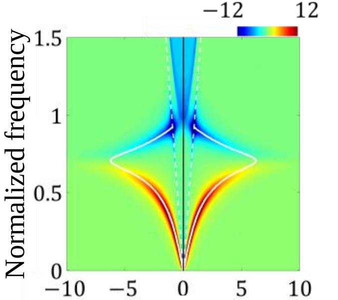


$\tilde{F}^z_{2\to1}(\omega,\boldsymbol{k}_\parallel)$



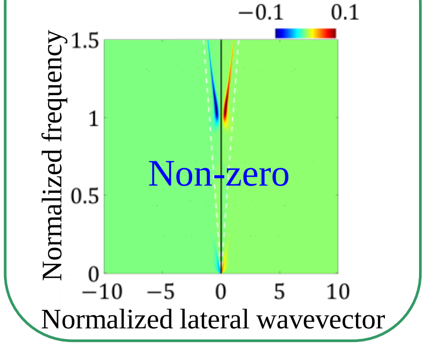
Symmetric for $\boldsymbol{k}_\parallel$

$\tilde{F}^z_{1\to2}(\omega,\boldsymbol{k}_\parallel)+\tilde{F}^z_{2\to1}(\omega,\boldsymbol{k}_\parallel)$



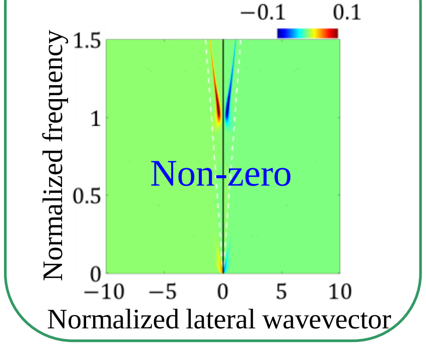
Normalized lateral wavevector

$\tilde{F}^z_{1\to2}(\omega,\boldsymbol{k}_\parallel)-\tilde{F}^z_{1\to2}(\omega,-\boldsymbol{k}_\parallel)$



Non-zero

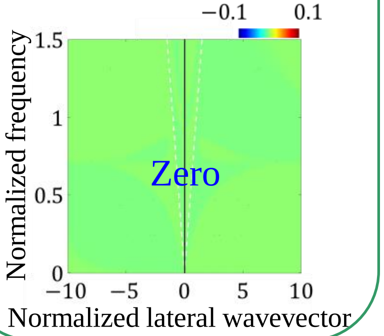
$\tilde{F}^z_{2\to1}(\omega,\boldsymbol{k}_\parallel)-\tilde{F}^z_{2\to1}(\omega,-\boldsymbol{k}_\parallel)$



Non-zero

$\tilde{F}^z_{1\to2}(\omega,\boldsymbol{k}_\parallel)-\tilde{F}^z_{1\to2}(\omega,-\boldsymbol{k}_\parallel)$

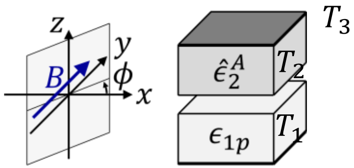
$+\tilde{F}^z_{2\to1}(\omega,\boldsymbol{k}_\parallel)-\tilde{F}^z_{2\to1}(\omega,-\boldsymbol{k}_\parallel)$



Zero

Eq. (35) is not true for non-equilibrium Casimir forces. $a\tilde{F}^z_{1\to2}(\omega,\boldsymbol{k}_\parallel)+b\tilde{F}^z_{2\to1}(\omega,\boldsymbol{k}_\parallel)$

Casimir pressure, non-reciprocal



Casimir pressure
in $(\omega, \boldsymbol{k}_\parallel)$ space
$F_2^z(\omega, \boldsymbol{k}_\parallel, T_1, T_2, T_3)$

Exchange function
$\tilde{F}^z_{l\to m}(\omega, \boldsymbol{k}_\parallel)$

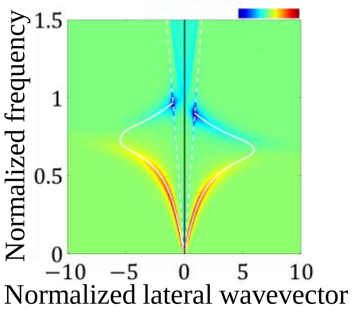
White lines:
Dispersion curves

Symmetry of the Casimir pressure for $\boldsymbol{k}_\parallel$ is broken for non-reciprocal systems in equilibrium and non-equilibrium.

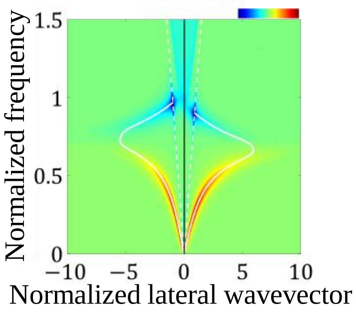
$\tilde{F}^z_{1\to2} + \tilde{F}^z_{2\to1}$ $a\tilde{F}^z_{1\to2} + b\tilde{F}^z_{2\to1}$

Asymmetric for $\boldsymbol{k}_\parallel$

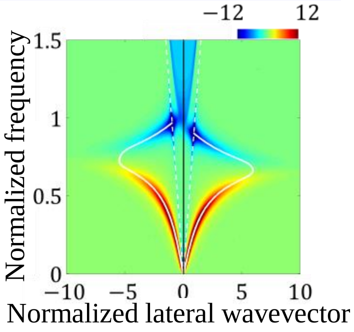
$\tilde{F}^z_{1\to2}\ (\omega, \boldsymbol{k}_\parallel)$



$\tilde{F}^z_{2\to1}(\omega, \boldsymbol{k}_\parallel)$

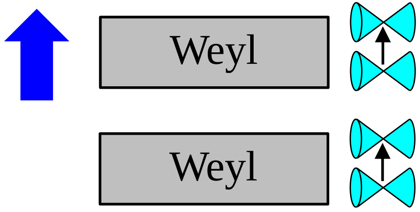


$\tilde{F}^z_{1\to2}(\omega, \boldsymbol{k}_\parallel) + \tilde{F}^z_{2\to1}(\omega, \boldsymbol{k}_\parallel)$

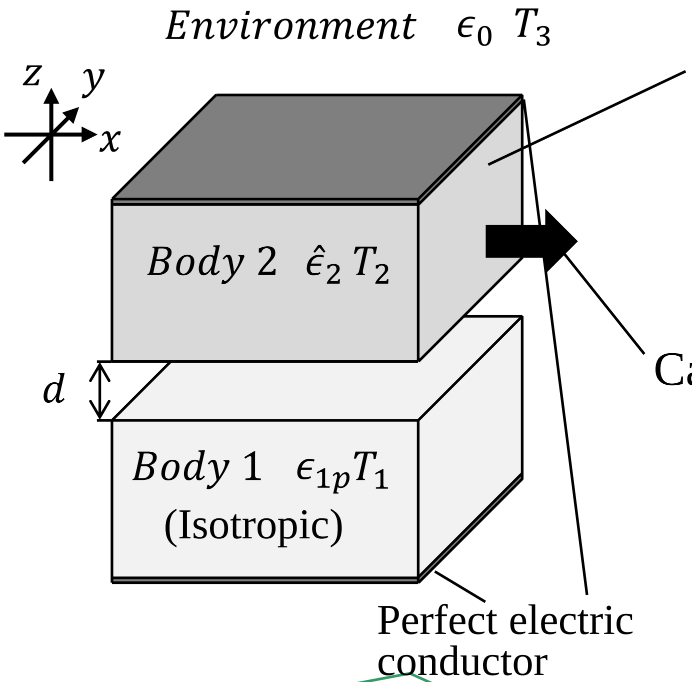


The analysis of real frequency spectra will be helpful for understanding the mechanism of repulsive Casimir force in equilibrium.

Why repulsive?



Casimir lateral force

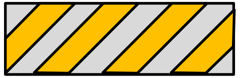


No photon exchange between the two-body system and the environment.

$T_1 = T_2 = T_3 = T$ (Equilibrium)

$T_1 \neq T_2$ (Non-equilibrium)

Anisotropic reciprocal material



non-reciprocal material

InSb/Weyl

$$\epsilon_2^S = \begin{bmatrix} \epsilon_d & & \epsilon_f \\ & \epsilon_p & \\ \epsilon_f & & \epsilon_d \end{bmatrix}, (24) \quad \text{or} \quad \epsilon_2^A = \begin{bmatrix} \epsilon_d & & i\epsilon_f \\ & \epsilon_p & \\ -i\epsilon_f & & \epsilon_d \end{bmatrix}, (25)$$

Casimir lateral force acting on body 2

$$\boldsymbol{F}_2^{\parallel}(T_1, T_2) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d\boldsymbol{k}_{\parallel} \underline{\underline{\boldsymbol{F}_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2)}}, (41)$$

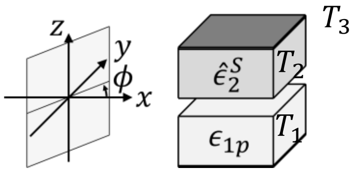
$$\underline{\underline{\boldsymbol{F}_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2)}} = -\boldsymbol{F}_{1\to2}^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1) - \boldsymbol{F}_{2\to1}^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_2), (42)$$

$$\boldsymbol{F}_{1\to2}^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1) = \left[n(\omega, T_1) + \frac{1}{2}\right] \frac{\hbar \boldsymbol{k}_{\parallel}}{8\pi^3} \underline{\tilde{F}_{1\to2}^{\parallel}(\omega, \boldsymbol{k}_{\parallel})}, (43)$$

$$\underline{\tilde{F}_{l\to m}^{\parallel}(\omega, \boldsymbol{k}_{\parallel})} = \begin{cases} Tr\left[(-1)^l \left(\hat{I} - \hat{R}_m^{\dagger}\hat{R}_m\right) \widehat{D}_{lm} \left(\hat{I} - \hat{R}_l \hat{R}_l^{\dagger}\right) \widehat{D}_{lm}^{\dagger}\right], & (k_{\parallel} < k_0) \\ Tr\left[(-1)^l \left(\hat{R}_m^{\dagger} - \hat{R}_m\right) \widehat{D}_{lm} \left(\hat{R}_l - \hat{R}_l^{\dagger}\right) \widehat{D}_{lm}^{\dagger} e^{-2\kappa_{z0} d}\right], & (k_{\parallel} > k_0) \end{cases}, (44)$$

Exchange function

Casimir lateral force, reciprocal



Casimir lateral force is symmetric for reciprocal systems in equilibrium and non-equilibrium.

$$F_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2) = -F_2^{\parallel}(\omega, -\boldsymbol{k}_{\parallel}, T_1, T_2), (45)$$

$\tilde{F}^{\parallel}_{1\to2} + \tilde{F}^{\parallel}_{2\to1}$

$a\tilde{F}^{\parallel}_{1\to2} + b\tilde{F}^{\parallel}_{2\to1}$

Casimir lateral force in $(\omega, \boldsymbol{k}_{\parallel})$ space

$F_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2, T_3)$

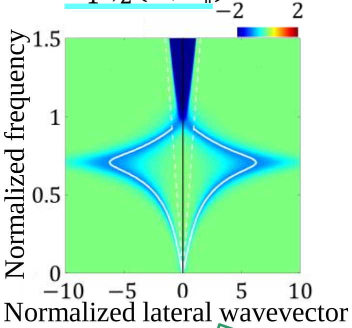
Exchange function

$\tilde{F}^{\parallel}_{l\to m}(\omega, \boldsymbol{k}_{\parallel})$

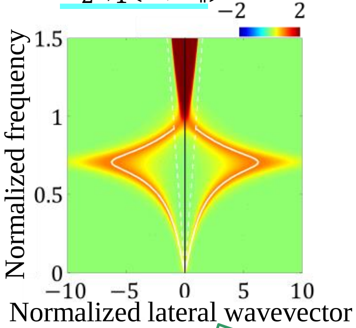
White lines:
Dispersion curves

Symmetric for $\boldsymbol{k}_{\parallel}$

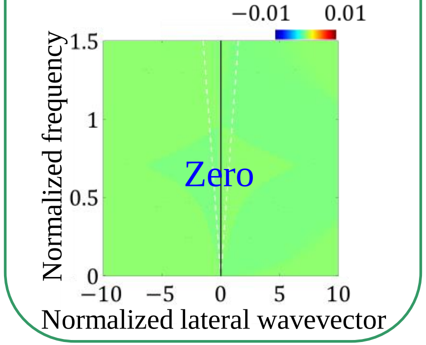
$\tilde{F}^{\parallel}_{1\to2}(\omega, \boldsymbol{k}_{\parallel})$



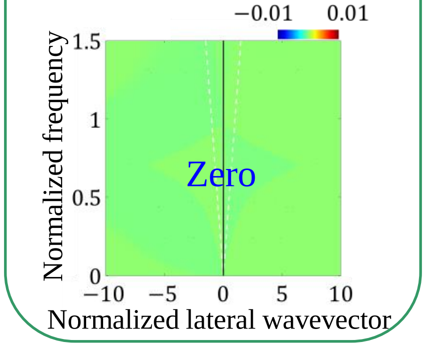
$\tilde{F}^{\parallel}_{2\to1}(\omega, \boldsymbol{k}_{\parallel})$



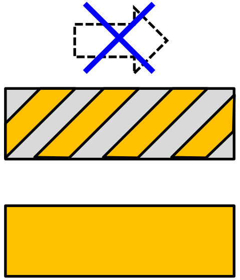
$\tilde{F}^{\parallel}_{1\to2}(\omega, \boldsymbol{k}_{\parallel}) - \tilde{F}^{\parallel}_{1\to2}(\omega, -\boldsymbol{k}_{\parallel})$



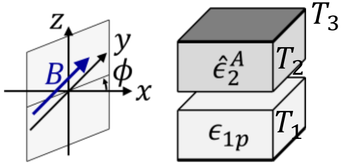
$\tilde{F}^{\parallel}_{\boldsymbol{2}\to\boldsymbol{1}}(\omega, \boldsymbol{k}_{\parallel}) - \tilde{F}^{\parallel}_{\boldsymbol{2}\to\boldsymbol{1}}(\omega, -\boldsymbol{k}_{\parallel})$



Lateral force does not occur in equilibrium and nonequilibrium for reciprocal systems.



Casimir lateral force, non-reciprocal



Casimir lateral force
in $(\omega, \boldsymbol{k}_\parallel)$ space

$F_2^{\parallel}(\omega, \boldsymbol{k}_\parallel, T_1, T_2, T_3)$

Exchange function
$\tilde{F}^{\parallel}_{l\to m}(\omega, \boldsymbol{k}_\parallel)$

White lines:
Dispersion curves

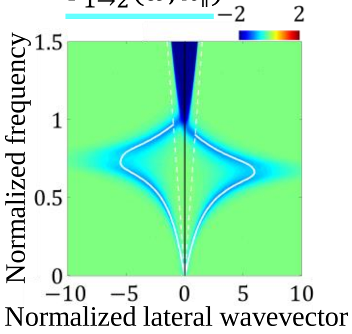
Symmetry of the Casimir lateral force is broken for non-reciprocal systems in equilibrium and non-equilibrium.

$\tilde{F}^{\parallel}_{1\to 2} + \tilde{F}^{\parallel}_{2\to 1}$ $\quad$ $a\tilde{F}^{\parallel}_{1\to 2} + b\tilde{F}^{\parallel}_{2\to 1}$

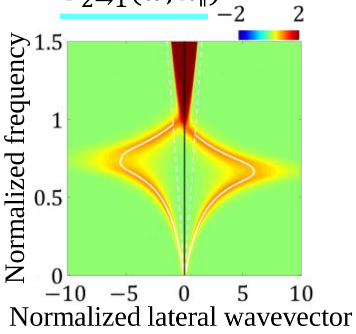
Cancelled out

Asymmetric for $\boldsymbol{k}_\parallel$

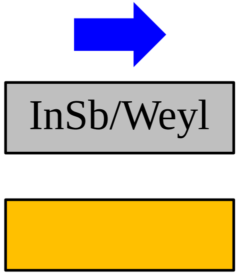
$\tilde{F}^{\parallel}_{1\to 2}(\omega, \boldsymbol{k}_\parallel)$



$\tilde{F}^{\parallel}_{2\to 1}(\omega, \boldsymbol{k}_\parallel)$



Lateral force occurs in non-equilibrium
for non-reciprocal systems.



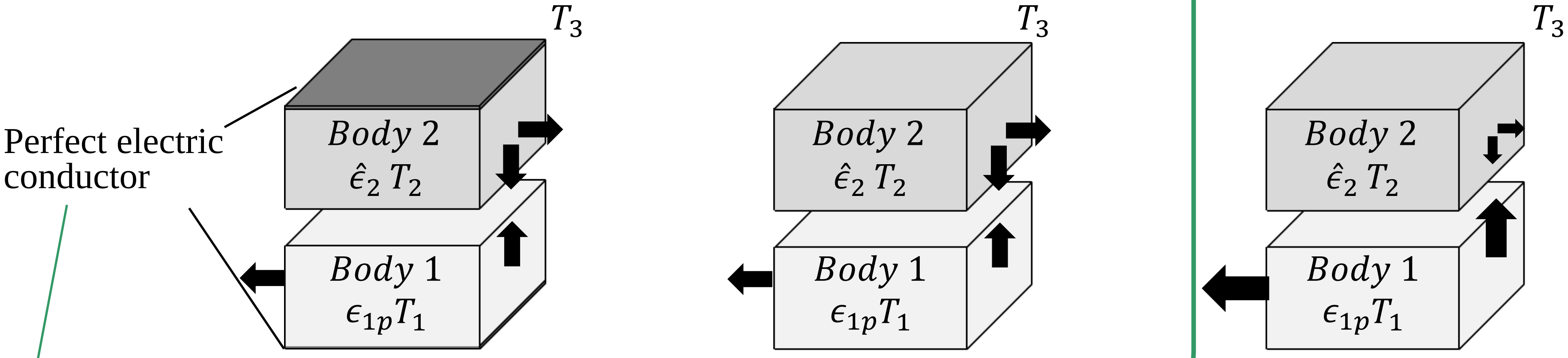
Newton’s third law

Newton’s third law holds for every frequency and wavevector, as long as no exchange of photons occurs between the two-body system and the environment.

Casimir pressure: $\underline{\underline{F_1^z(\omega, \boldsymbol{k}_\parallel, T_1, T_2, T_3) = -F_2^z(\omega, \boldsymbol{k}_\parallel, T_1, T_2, T_3)}}$

Casimir lateral force: $\underline{\underline{F_1^\parallel(\omega, \boldsymbol{k}_\parallel, T_1, T_2) = -F_2^\parallel(\omega, \boldsymbol{k}_\parallel, T_1, T_2)}}$

Newton’s third law does not hold.



Perfect electric conductor

Equilibrium ($T_1 = T_2 = T_3$) and non-equilibrium ($T_1 \neq T_2$)

Equilibrium ($T_1 = T_2 = T_3$)

Non-equilibrium ($T_1 \neq T_3$ or $T_2 \neq T_3$)

No exchange of photons

Exchange of photons

The above is true for both reciprocal $\hat{\epsilon}_2$=$\hat{\epsilon}_2^S$ and non-reciprocal $\hat{\epsilon}_2$=$\hat{\epsilon}_2^A$ materials.

Conclusions

A brief overview of Casimir forces was presented.

Symmetry of Casimir forces in wavevector space was discussed. This understanding is helpful for investigating Casimir forces using Weyl semimetals.

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