Dark Matter Detection with Qubits

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arXiv 2212.03884 [PRL 131 (2023) 211001]

arXiv 2311.10413

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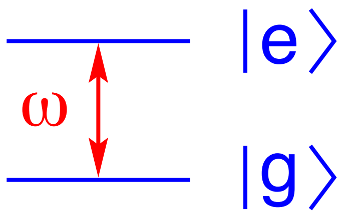
1. Introduction

What I discuss today: DM search with quantum bit (qubit)



Qubit: Two-level quantum system

- Qubit is an essential component for quantum computers
- Various types of qubits have been proposed and realized
- Qubits are excellent quantum sensors for DM detection
  [Dixit et al. ('21); Chen, Fukuda, Inada, TM, Nitta, Sichanugrist ('22, '23); Engelhardt, Bhoonah, Liu ('23); Chigusa, Hazumi, Herbschleb, Mizuochi, Nakayama ('23); Agrawal et al. ('23); Ito, Kitano, Nakano, Takai ('23)]

Outline:

1. Introduction

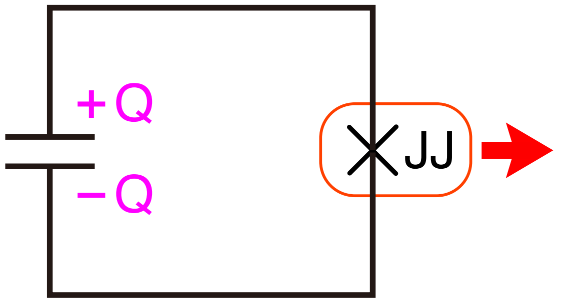
2. Hidden Photon DM Search with Transmon Qubits
   [Chen, Fukuda, Inada, TM, Nitta, Sichanugrist, 2212.03884]

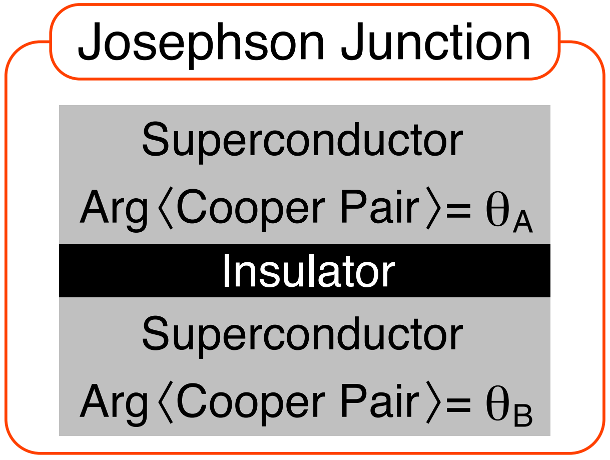
3. Quantum Enhancement of Signal Rate
   [Chen, Fukuda, Inada, TM, Nitta, Sichanugrist, 2311.10413]

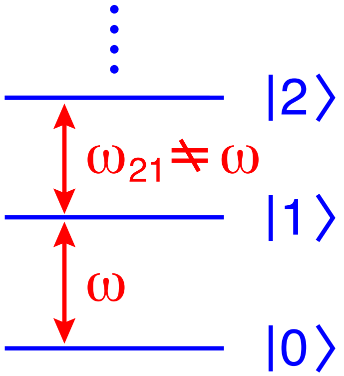
4. Summary

2. Hidden Photon DM Search with Qubits

Transmon qubit: Capacitor + Josephson junction (JJ)







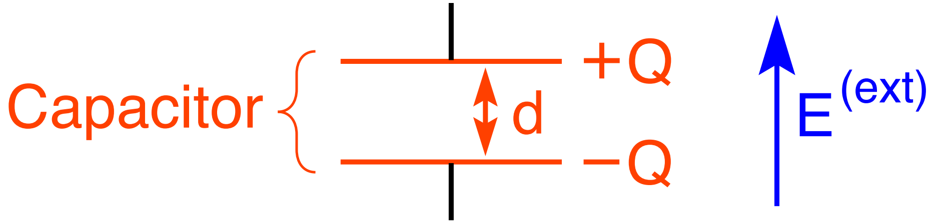
$$H_0 = \frac{1}{2C}Q^2 - J\cos\theta \simeq \frac{1}{2}\frac{C}{(2e)^2}\dot{\theta}^2 - J\cos\theta \quad \text{with } \theta = \theta_B - \theta_A$$

Transmon qubit has discrete energy levels

$\Rightarrow$ $|0\rangle$ and $|1\rangle$ can be used as $|g\rangle$ and $|e\rangle$, respectively

$\Rightarrow$ Transmon qubits are used in today's quantum computers

Transmon qubit couples to external electric field



$$\Leftrightarrow \; H_{\rm int} = QdE^{(\rm ext)}$$

Hidden photon DM induces effective electric field

$$\vec{X} \simeq \bar{X}\vec{n}_X \sin(m_X t + \alpha) \quad \textbf{with} \quad \rho_{\rm DM} = \frac{1}{2} m_X^2 \bar{X}^2$$

$$\mathcal{L}_{\rm int} = \frac{1}{2}\, \epsilon\, F^{\mu\nu} X_{\mu\nu} \quad \Rightarrow$$

$$\vec{E}^{(X)} = -\epsilon \dot{\vec{X}} = -\bar{E}^{(X)} \vec{n}_X \cos(m_X t + \alpha) \quad \textbf{with} \quad \bar{E}^{(X)} = \epsilon\sqrt{2\rho_{\rm DM}}$$

Hamiltonian for transmon qubit + hidden photon system

$$H = \omega|e\rangle\langle e| - 2\eta\cos(m_X t + \alpha)\big(\,|e\rangle\langle g| + |g\rangle\langle e|\,\big)$$

$$\eta \simeq \frac{1}{2\sqrt{2}} d\bar{E}^{(X)}\sqrt{C\omega} = \frac{1}{2}\epsilon d\sqrt{C\omega\rho_{\rm DM}}$$

Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle \quad \Rightarrow \quad |\psi(t)\rangle = U_{\rm DM}(t)\,|\psi(0)\rangle$$

Resonance limit $\omega = m_X$ (for $\eta t \ll 1$)

$$\begin{pmatrix}\psi_g(t)\\ \psi_e(t)\end{pmatrix} = U_{\rm DM}(t)\begin{pmatrix}\psi_g(0)\\ \psi_e(0)\end{pmatrix} \simeq \begin{pmatrix}1 & ie^{-i\alpha}\eta t\\ ie^{i\alpha}\eta t & 1\end{pmatrix}\begin{pmatrix}\psi_g(0)\\ \psi_e(0)\end{pmatrix}$$

$$|\psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t}\,\psi_e(t)|e\rangle$$

$|g\rangle \to |e\rangle$ transition probability (assuming $|\psi(0)\rangle = |g\rangle$)

$$|\psi_e(t)|^2 \simeq \begin{cases} \eta^2 t^2 & : \omega = m_X \text{ (on-resonance)} \\ \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \text{ (off-resonance)} \end{cases}$$

Excitation probability for $\omega = m_X$:

$$P_{ge} \simeq 0.3 \times \left(\frac{\epsilon}{10^{-11}}\right)^2 \left(\frac{m_X}{10\,\mu\text{eV}}\right) \left(\frac{C}{0.1\,\text{pF}}\right) \left(\frac{d}{100\,\mu\text{m}}\right)^2 \left(\frac{\tau}{100\,\mu\text{s}}\right)^2$$

τ = coherence time

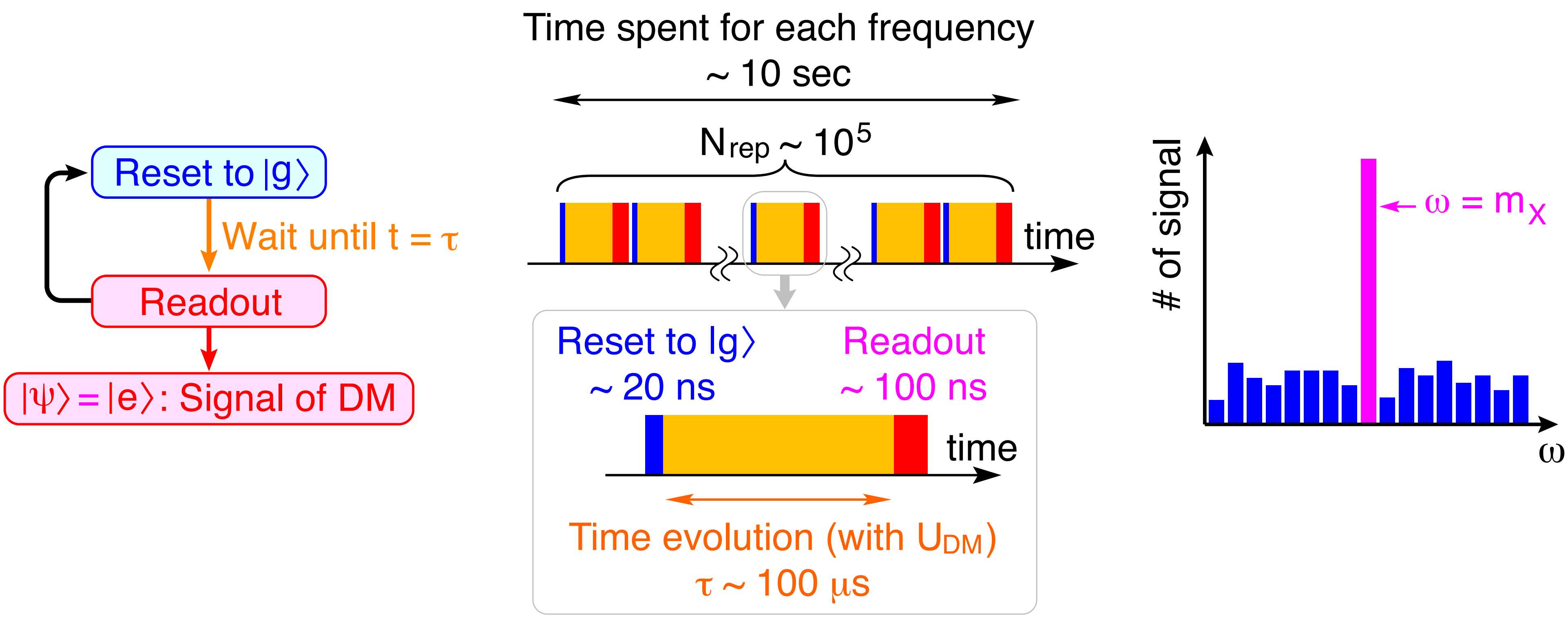
Excitation probability can be sizable

$\Rightarrow$ Transmon qubit as a DM detector

Search strategy

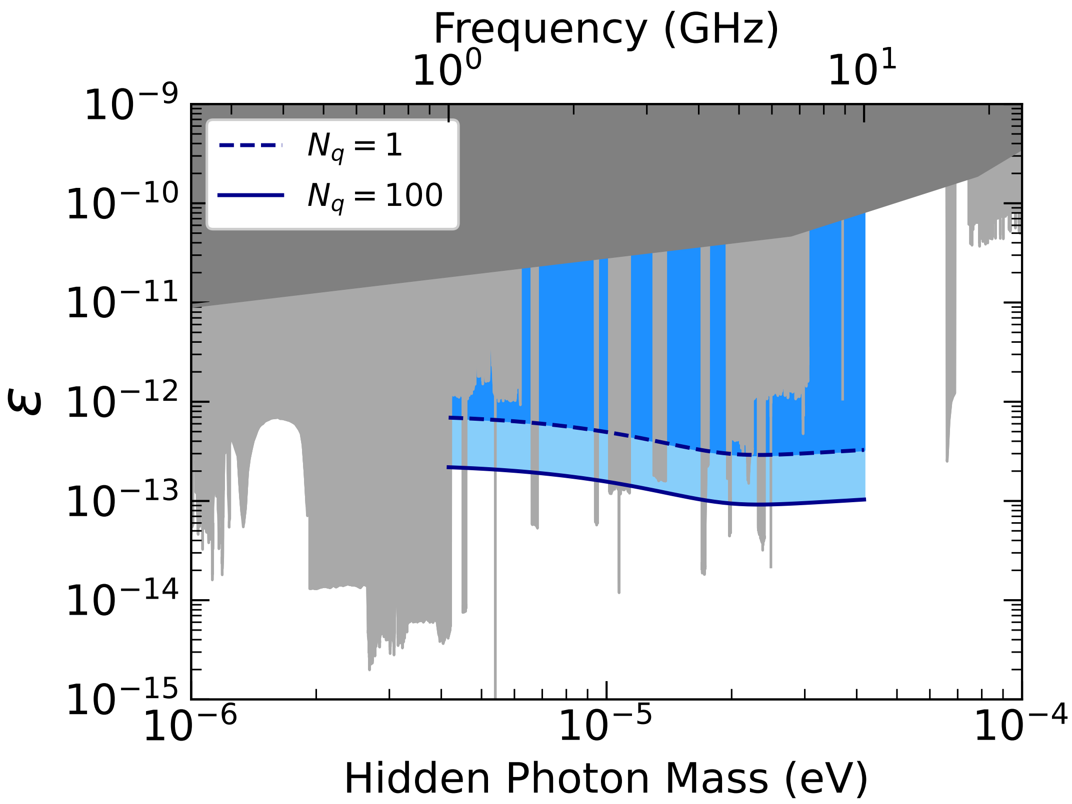
- For fixed ω, repeat the measurement cycle (reset, wait, and readout) as many time as possible

- Scan the qubit frequency ω



Discovery reach with 1 year frequency scan ($1 \leq f \leq 10$ GHz)

Bkg: thermal excitation + readout error ($0.1\ \%$)



- $d = 100\ \mu$m
- $C = 0.1$ pF
- $\mathcal{Q} = 10^6$
- $T = 30$ mK

$\Rightarrow$ Using qubit, we may probe parameter region unexplored

$\Rightarrow$ We hope to use qubit for the detection of other DMs

3. Quantum Enhancement of the Signal Rate

Case of N_q qubits with $N_\mathrm{q} \gg 1$

$\Rightarrow$ We may readout the qubits one-by-one

$\Rightarrow$ (# of signal) $\propto N_\mathrm{q}\,\delta^2$ with $\delta \equiv \eta\tau$

Signal rate can be $O(N_\mathrm{q}^2)$ with quantum operations

[Chen, Fukuda, Inada, TM, Nitta, Sichanugrist ('23); Ito, Kitano, Nakano, Takai ('23)]

- Quantum operations are applicable to qubits

  $\Rightarrow$ "DM search with quantum computers"

- We can design quantum circuits realizing the signal enhancement

Basic unitary operators (quantum gates)

In the following, we often use: $|\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(|g\rangle \pm |e\rangle\big)$

- Z gate

$$Z = |g\rangle\langle g| - |e\rangle\langle e| \qquad \Rightarrow \quad |+\rangle \xrightarrow{Z} |-\rangle$$

- Hadamard gate

$$H = |+\rangle\langle g| + |-\rangle\langle e| \qquad \Rightarrow \quad |g\rangle \xrightarrow{H} |+\rangle, \quad |e\rangle \xrightarrow{H} |-\rangle$$

- Controlled Z gate

$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

$U_{\rm DM}$ induces pure phase rotation of its eigenstates

E.g. for $\alpha = 0$: $U_{\rm DM} \simeq \begin{pmatrix} 1 & i\delta \\ i\delta & 1 \end{pmatrix}$ with $\delta \equiv \eta\tau$

$$\Rightarrow \; U_{\rm DM}|\pm\rangle = e^{\pm i\delta}|\pm\rangle$$

$$|\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(|g\rangle \pm |e\rangle\big)$$

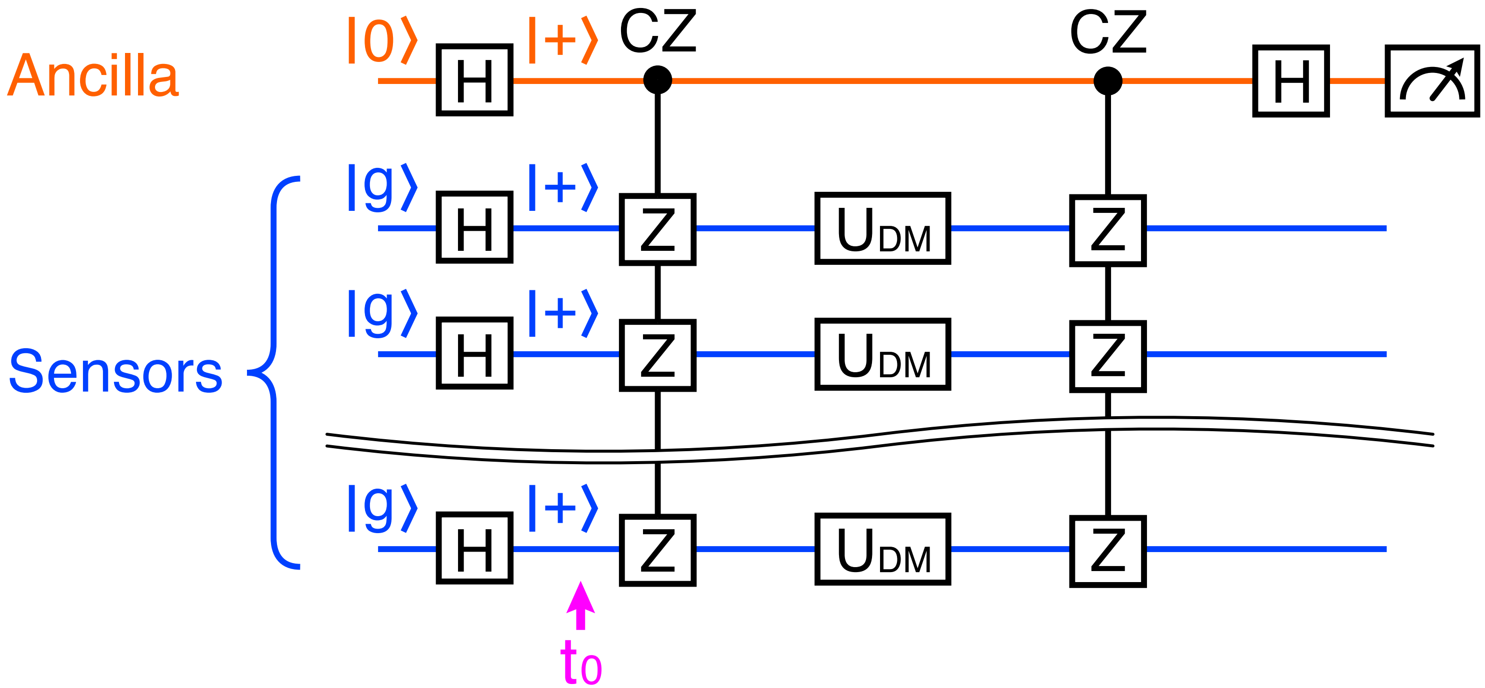
$$\Rightarrow \; U_{\rm DM}^{\otimes N_{\rm q}}|+\rangle^{\otimes N_{\rm q}} = e^{iN_{\rm q}\delta}|+\rangle^{\otimes N_{\rm q}}$$

Phases from different qubits may coherently accumulate

$\Rightarrow$ Quantum enhanced parameter estimation

[Giovannetti, Lloyd, Maccone ('04)]

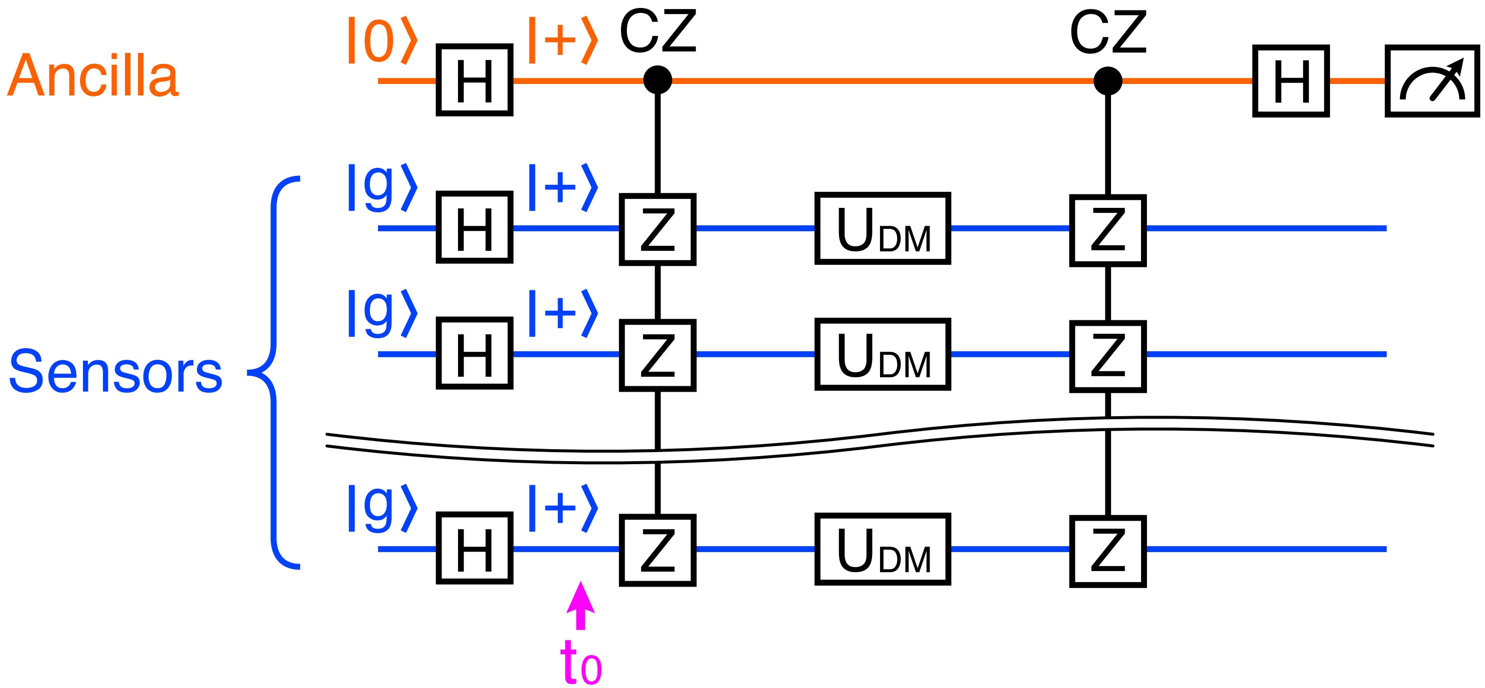
One measurement cycle for the signal enhancement



The above is an example of the quantum circuit

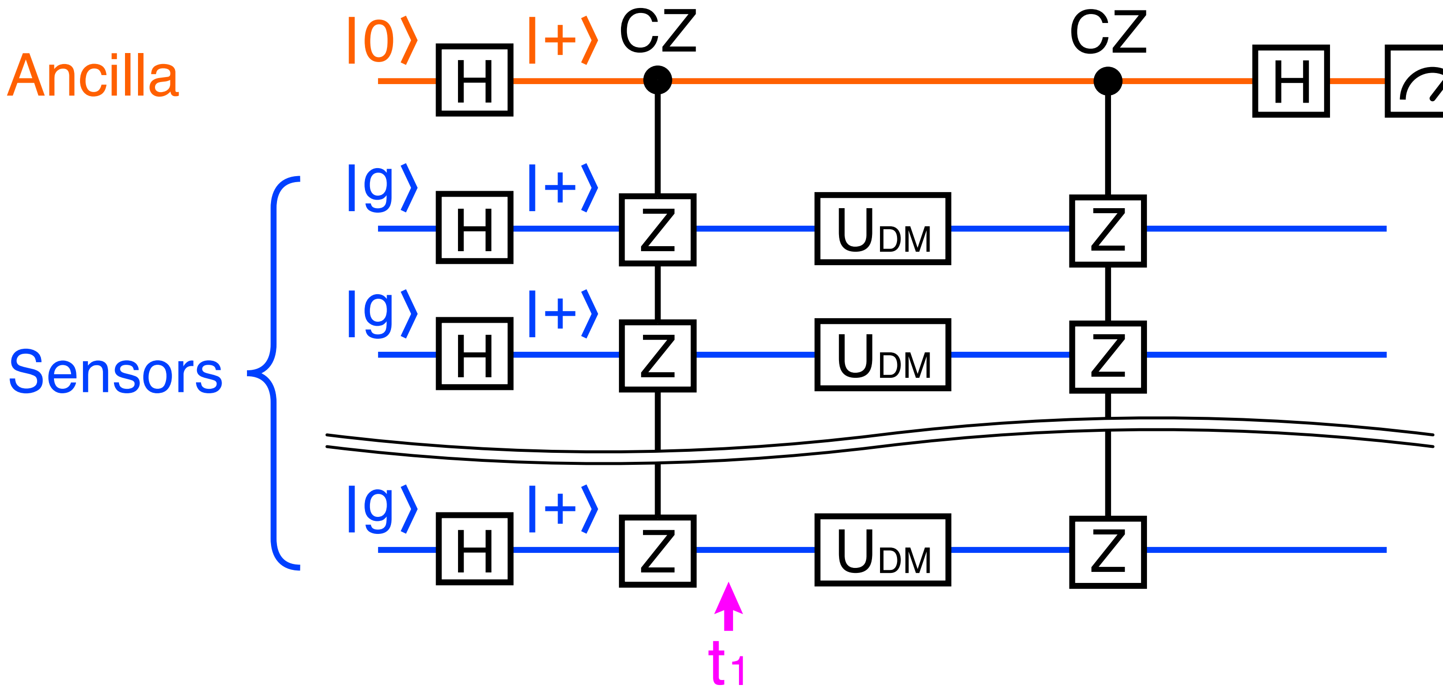
$\Rightarrow$ Let us first see how it works when $\alpha = 0$

One measurement cycle for the signal enhancement



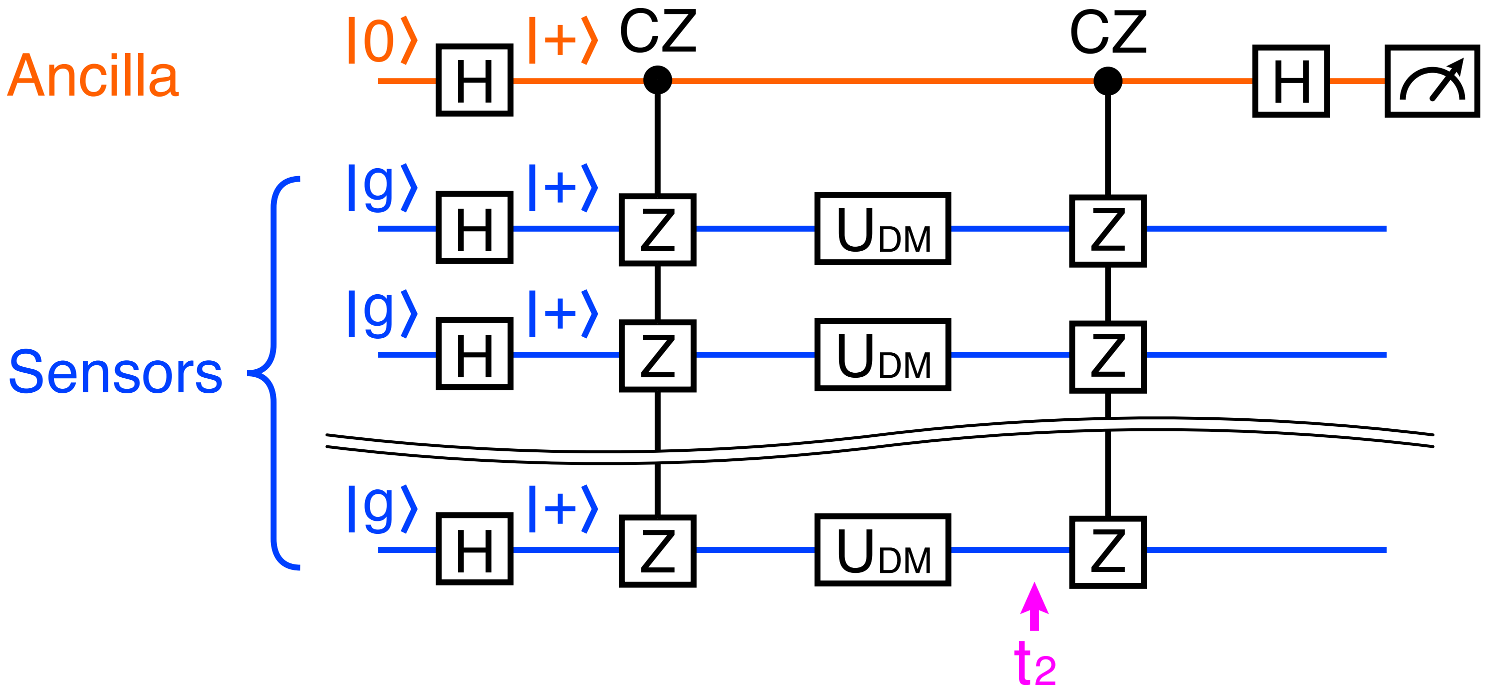
$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_q} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



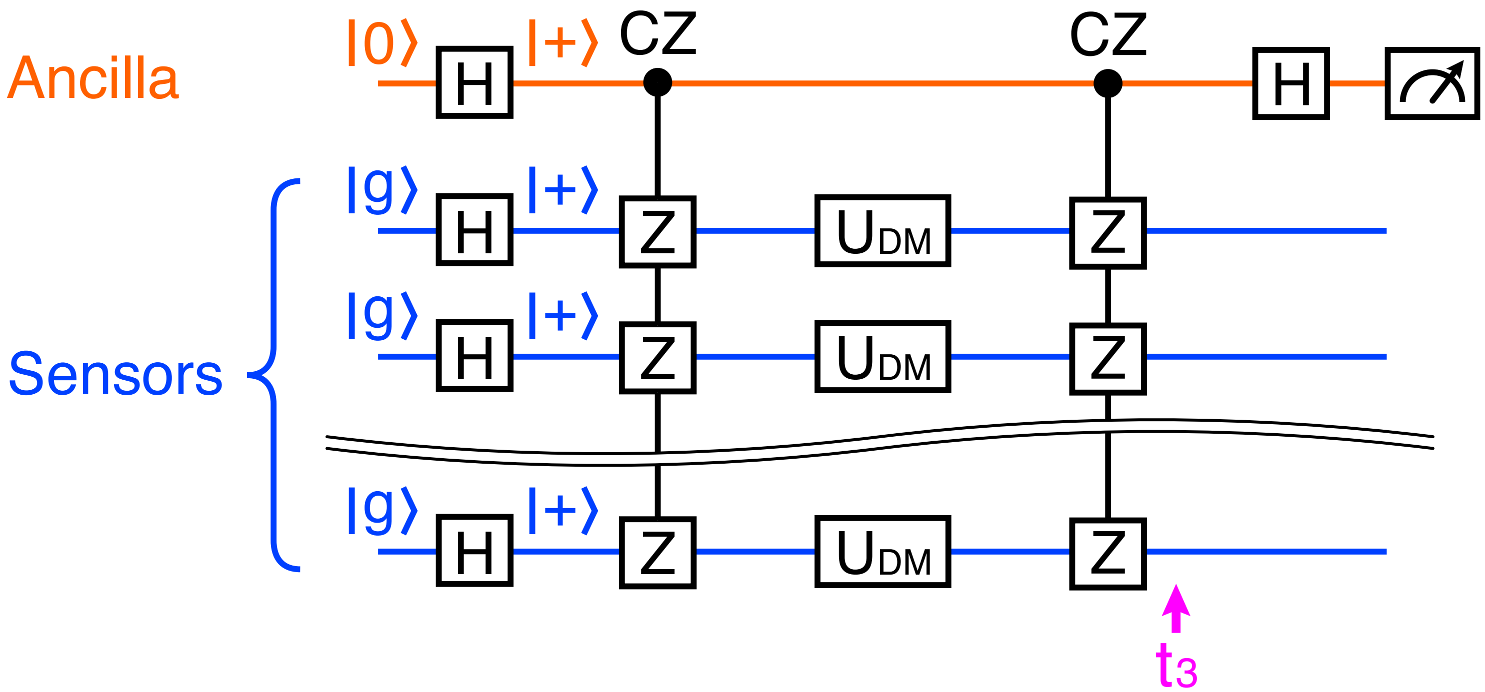
$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_{\mathrm{q}}} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_{\mathrm{q}}}$$

One measurement cycle for the signal enhancement



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |-\rangle^{\otimes N_q}$$

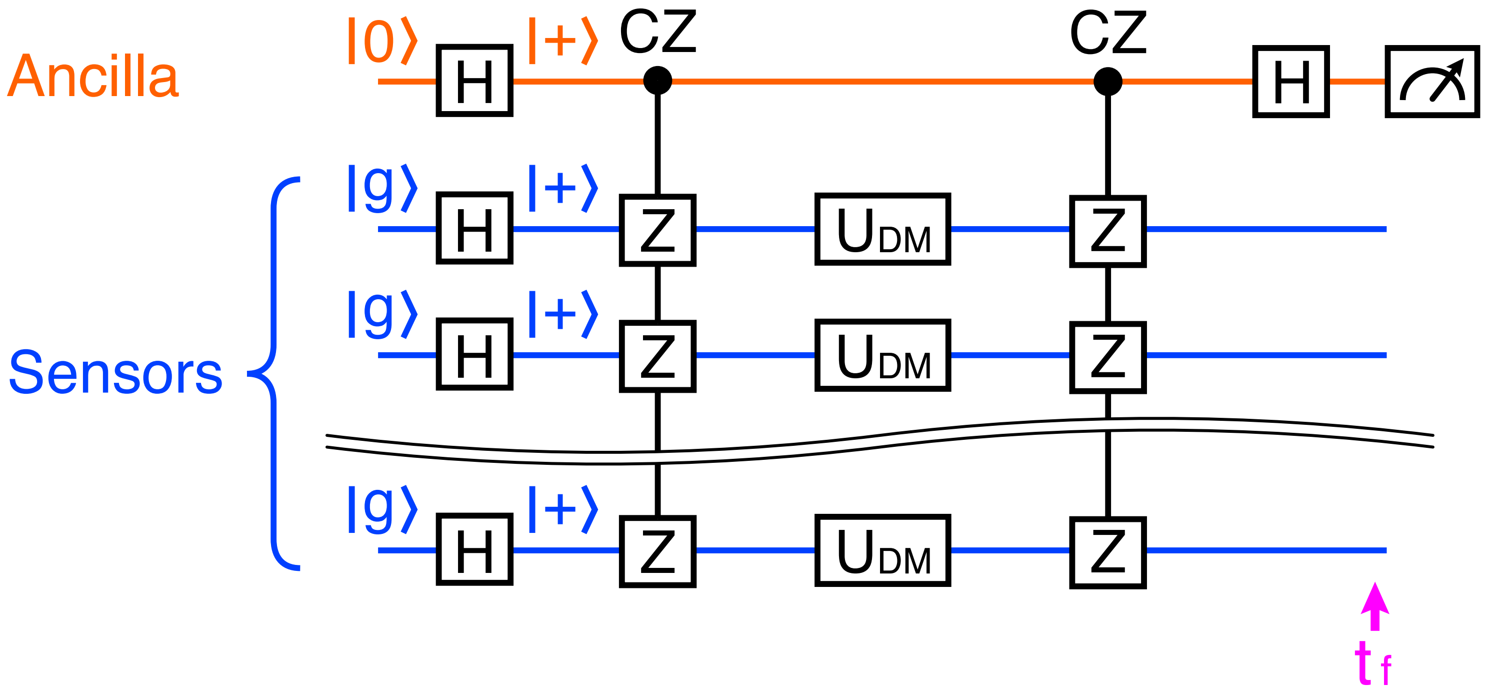
One measurement cycle for the signal enhancement



$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}} e^{iN_q\delta}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta}|1\rangle \otimes |+\rangle^{\otimes N_q}$$

$$= \left(\cos\, N_q\delta\, |+\rangle + i \sin\, N_q\delta\, |-\rangle \right) \otimes |+\rangle^{\otimes N_q}$$

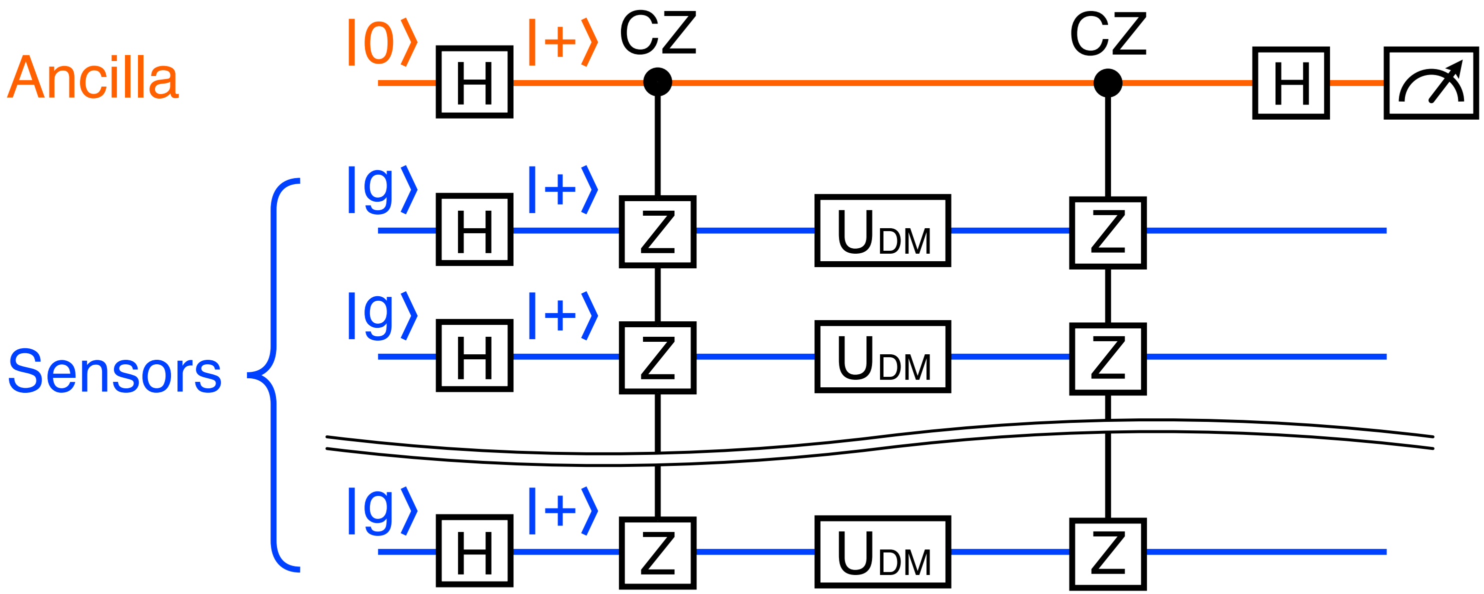
One measurement cycle for the signal enhancement



$$|\Psi(t_\mathrm{f})\rangle = \big(\cos\, N_\mathrm{q}\delta\,|0\rangle + i\sin\, N_\mathrm{q}\delta\,|1\rangle\big) \otimes |+\rangle^{\otimes N_\mathrm{q}}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0\to 1} \simeq \sin^2 N_\mathrm{q}\delta \simeq N_\mathrm{q}^2\delta^2$

The phase α is unknown in the actual search, but...

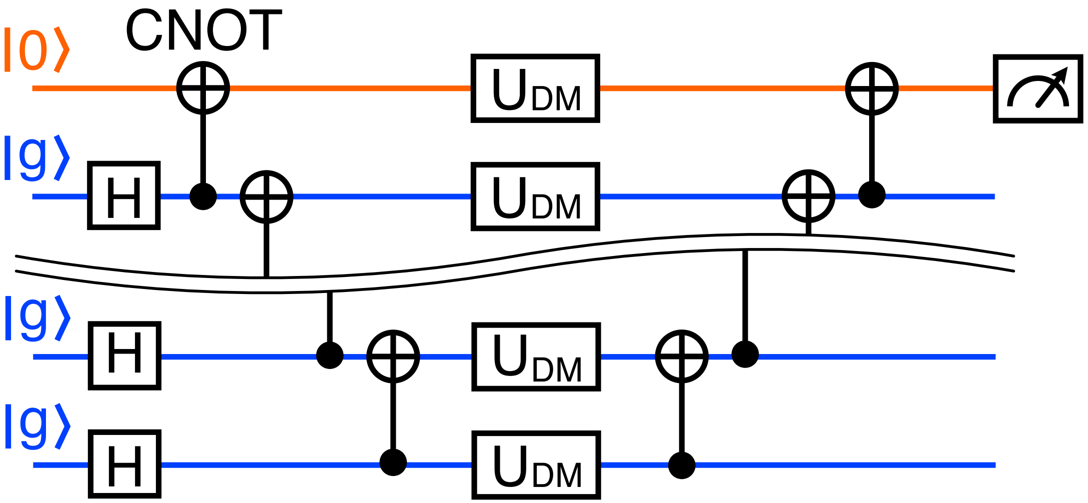


$$P_{0\to 1} \simeq N_{\mathrm{q}}^2 \delta^2 \cos^2 \alpha \to \frac{1}{2} N_{\mathrm{q}}^2 \delta^2$$

$\Rightarrow$ Signal rate can be of $O(N_{\mathrm{q}}^2)$

Circuit only with nearest neighbor interactions

$\Rightarrow$ (# of gates) $\sim O(N_\mathrm{q})$



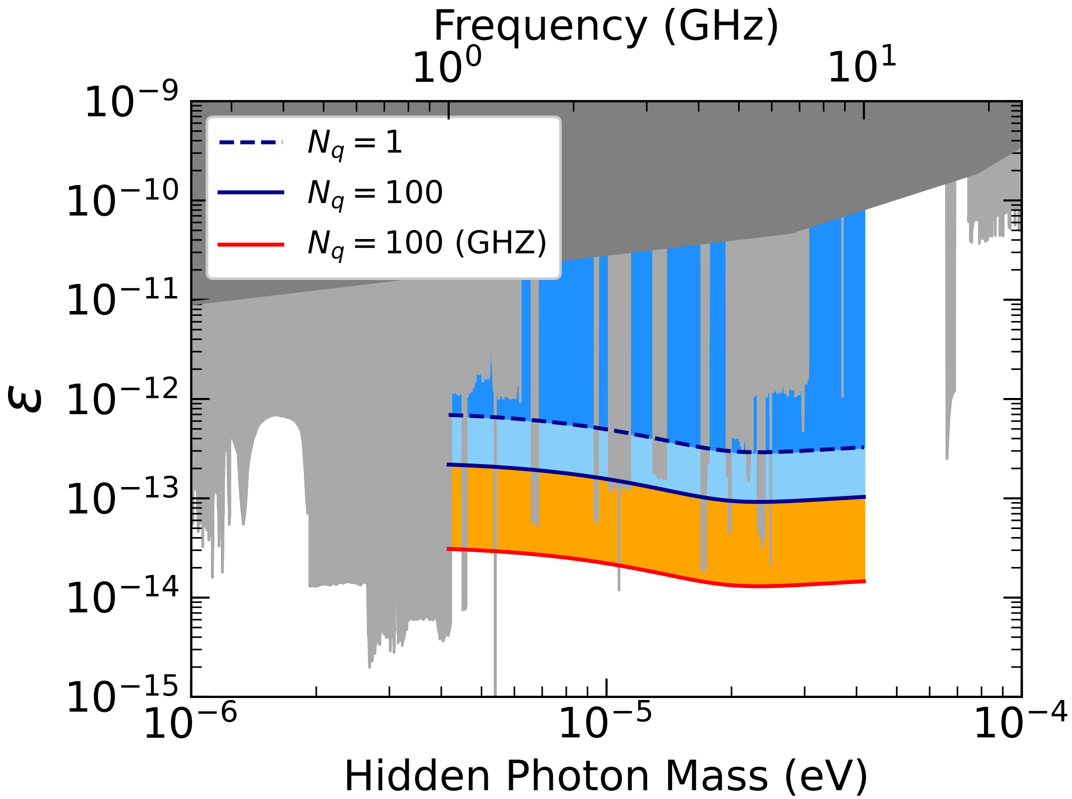
$\Rightarrow P_{0\to 1} \simeq \frac{1}{2} N_\mathrm{q}^2 \delta^2$

CNOT (Controlled-NOT) $= |g\rangle\langle g| \otimes \mathbf{1} + |e\rangle\langle e| \otimes X$

$\Rightarrow$ (# of signals) $\sim O(N_\mathrm{q}^2)$

$\Rightarrow$ (# of errors & noises) $\sim O(N_\mathrm{q}) \ll$ (# of signals), for $N_\mathrm{q} \gg 1$

The # of signal can be of $O(N_\mathrm{q}^2)$, not $O(N_\mathrm{q})$



Technical challenges:

- We need reliable quantum gates
- Frequencies of all the qubits should be equal to m_X

4. Summary

DM searches with qubits are interesting

- Transmon qubit can be an excellent DM detector
- Quantum enhancement of the signal is possible
- R&D is in progress for the DM detection with qubits

  $\Rightarrow$ Chen-san's talk next

Backup: Hidden Photon DM

Case of hidden photon X_μ

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2}\,\epsilon\, F'_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu$$

$F'_{\mu\nu}$: EM field (in gauge eigenstate)

Vector bosons in the mass eigenstates

$A_\mu \simeq A'_\mu - \epsilon X_\mu$ and X_μ

Interaction with electron

$$\mathcal{L}_{\rm int} = e\bar{\psi}_e \gamma^\mu A'_\mu \psi_e = e\bar{\psi}\gamma^\mu \psi (A_\mu + \epsilon X_\mu)$$

Hidden photon as dark matter

$$\vec{X} \simeq \bar{X}\vec{n}_X \cos m_X t$$

Energy density of hidden photon DM

$$\rho_{\rm DM} = \frac{1}{2}\dot{\vec{X}}^2 + \frac{1}{2}m_X^2\vec{X}^2 \simeq \frac{1}{2}m_X^2\bar{X}^2$$

$$\Leftrightarrow\ \rho_{\rm DM} \sim 0.45\ {\rm GeV/cm^3}$$

Effective electric field induced by the hidden photon

$$\vec{E}^{(X)} = -\epsilon\dot{\vec{X}} = \bar{E}^{(X)}\vec{n}_X \sin m_X t$$

$$\bar{E}^{(X)} = \epsilon m_X \bar{X} = \epsilon\sqrt{\rho_{\rm DM}}$$

Backup: Transmon Qubit

Hamiltonian

$$H_0 = \frac{1}{2C}Q^2 - J\cos\theta = \frac{1}{2Z}n^2 - J\cos\theta$$

$$Z \equiv (2e)^{-2}C$$

Transmon limit: $CJ \gg (2e)^2 \quad \Rightarrow \quad \langle\theta^2\rangle \ll 1$

[Koch et al. ('07); see also Roth, Ma & Chew (2106.11352)]

$$\Rightarrow\; H_0 = \frac{1}{2Z}n^2 + \frac{1}{2}J\theta^2 + O(\theta^4)$$

$$\hat{a} \equiv \frac{1}{\sqrt{2\omega Z}}(n - i\omega Z\theta), \quad \hat{a}^\dagger \equiv \frac{1}{\sqrt{2\omega Z}}(n + i\omega Z\theta)$$

$$\Rightarrow\; [\hat{a}, \hat{a}^\dagger] = 1$$

In the transmon limit, anharmonicity is small:

$$\Rightarrow \ |e\rangle \simeq \hat{a}^\dagger |g\rangle$$

$$\Rightarrow \ \omega_{21} \simeq \left(1 - \frac{1}{8}\frac{2e}{\sqrt{CJ}}\right)\omega$$

Charge operator in the transmon limit

$$Q = 2en = \sqrt{\frac{C\omega}{2}}\left(\hat{a} + \hat{a}^\dagger\right) \simeq \sqrt{\frac{C\omega}{2}}\Big(\,|g\rangle\langle e| + |e\rangle\langle g|\,\Big)$$

Interaction Hamiltonian

$$H_{\rm int} = QdE^{(\rm ext)} \simeq \sqrt{\frac{C\omega}{2}}dE^{(\rm ext)}\Big(\,|g\rangle\langle e| + |e\rangle\langle g|\,\Big)$$

Backup: Schrd̈inger Equation

Effective Hamiltonian

$$H = \omega|e\rangle\langle e| + 2\eta \sin m_X t \big(|e\rangle\langle g| + |g\rangle\langle e| \big)$$

η: Small parameter

Schrödinger equation:

$$i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$|\Psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t}\,\psi_e(t)|e\rangle$$

$$\Rightarrow\; i\frac{d}{dt}\begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix} = 2\eta \sin m_X t \begin{pmatrix} 0 & e^{-i\omega t} \\ e^{i\omega t} & 0 \end{pmatrix}\begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix}$$

Solution with $|\Psi(0)\rangle = |g\rangle$ (for $|\omega \pm m_X|^{-1} \ll t \ll \eta^{-1}$)

$$\psi_g(t) \simeq 1 + O(\eta^2)$$

$$\psi_e(t) \simeq \eta \left(\frac{e^{i(\omega - m_X)t} - 1}{i(\omega - m_X)} - \frac{e^{i(\omega + m_X)t} - 1}{i(\omega + m_X)} \right)$$

Resonance limit: $\omega \to m_X$

$\Rightarrow \psi_e(t) \to \eta t +$ (non-growing)

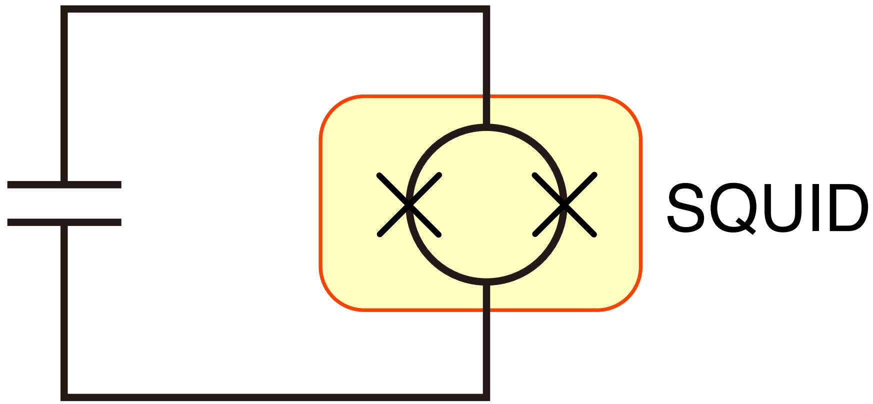
$|g\rangle \to |e\rangle$ transition rate (for $t \ll \eta^{-1}$)

$$P_{ge} = |\psi_e(t)|^2 \simeq \begin{cases} \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \\ \eta^2 t^2 & : \omega = m_X \end{cases}$$

Backup: Frequency Scan

Frequency scan

Frequency scan is possible with qubit consisting of SQUID and capacitor

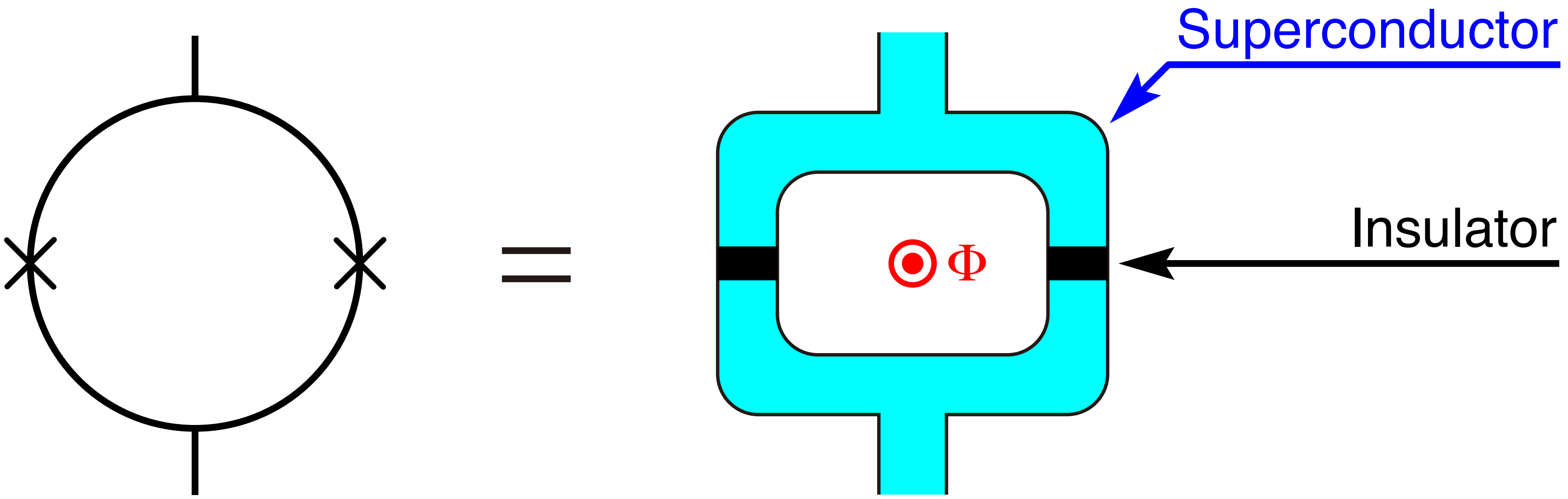


SQUID: superconducting quantum interference device

- Quantum device sensitive to magnetic flux

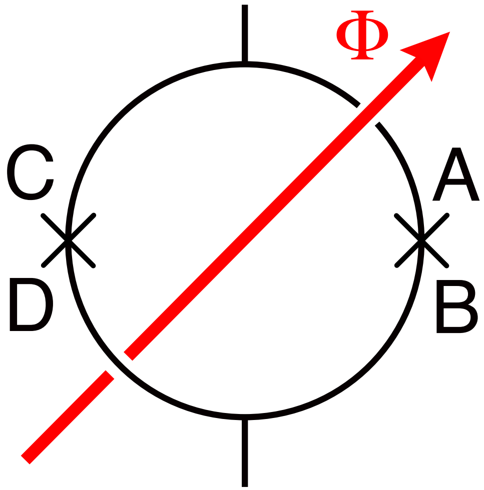
SQUID

- Loop-shaped superconductors separated by insulating layers



- We consider the case with external magnetic flux Φ going through the loop

Phases in the presence of magnetic flux



$$\theta_C - \theta_A = (2e) \int_{A\to C} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_B - \theta_D = (2e) \int_{D\to B} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_{BA} - \theta_{DC} = (2e) \oint \vec{A}(\vec{x})\, d\vec{x} = (2e)\, \Phi = \frac{2\pi}{\Phi_0}\Phi$$

$\theta_{YX} = \theta_Y - \theta_X$

$\Phi_0 = \dfrac{h}{2e}$: magnetic flux quantum

Define: $\theta \equiv (\theta_{BA} + \theta_{DC})/2$

$$H_{\rm SQUID} \simeq -J\left(\cos\theta_{BA} + \cos\theta_{DC}\right) = -2J\cos(e\Phi)\cos\theta$$

Based on the previous analysis with $J \to 2J\cos(e\Phi)$

$$\omega \simeq \sqrt{\frac{2J}{Z}\cos(e\Phi)}$$

$$Z = (2e)^{-2}C$$

The excitation energy depends on Φ

$\Rightarrow$ Frequency scan is possible with varying the external magnetic field

Backup: Comments on Backgrounds

Backgrounds (dark counts)

- Thermal excitation
- Readout error

Our (simple) criterion for DM detection

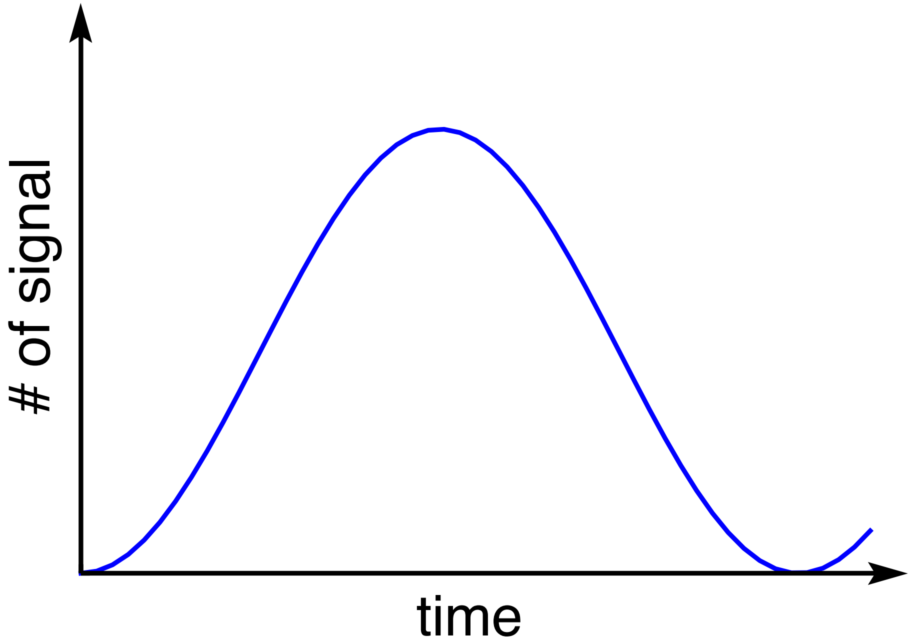
$$N_{\rm sig} > \max(3, 5\sqrt{N_{\rm bkg}})$$

Example: 1 year scan of $1 \leq f \leq 10$ GHz

- Scan time for each frequency: ~ 14 sec (for $Q = 10^6$)
- $N_{\rm rep} \sim O(10^4 - 10^5)$

Comment on the background

- $p_{g \to e}(t) \simeq \sin^2 \eta t$
- Signal and Bkg may be distinguished by observing the time dependence predicted by Rabi oscillation



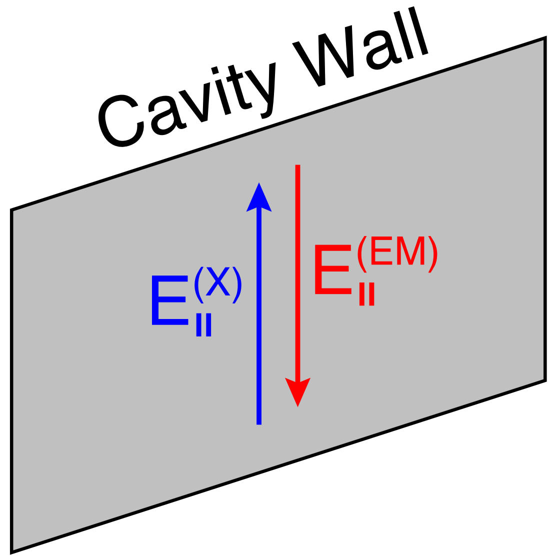
Backup: Cavity Effect

Qubits are usually set in a "microwave cavity"

$\Rightarrow$ Qubits are surrounded by metals

$\Rightarrow$ $\vec{E}_{\parallel}^{(\mathrm{eff})}$ should vanish at the cavity wall

"Effective" electric field: $\vec{E}^{(\mathrm{eff})} = \vec{E}^{(\mathrm{EM})} + \vec{E}^{(X)}$



$\Leftrightarrow$ $\vec{E}^{(\mathrm{eff})}$ induces the qubit excitation

Equations to be solved to obtain $\vec{E}^{(\mathrm{EM})}$ for given $\vec{E}^{(X)}$

- $\Box \vec{E}^{(\mathrm{EM})} = 0$ and $\vec{\nabla}\vec{E}^{(\mathrm{EM})} = 0$
- $[\vec{E}_{\|}^{(\mathrm{EM})} + \vec{E}_{\|}^{(X)}]_{\mathrm{wall}} = 0$

  $\vec{E}^{(X)}$ is unaffected by the cavity and is homogeneous

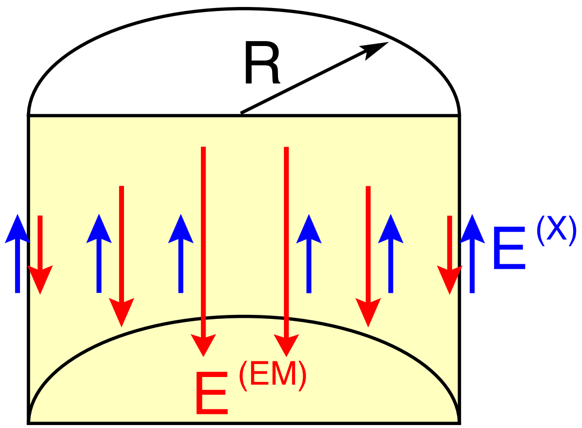
$\vec{E}^{(\mathrm{EM})}$ at the position of the qubit depends on:

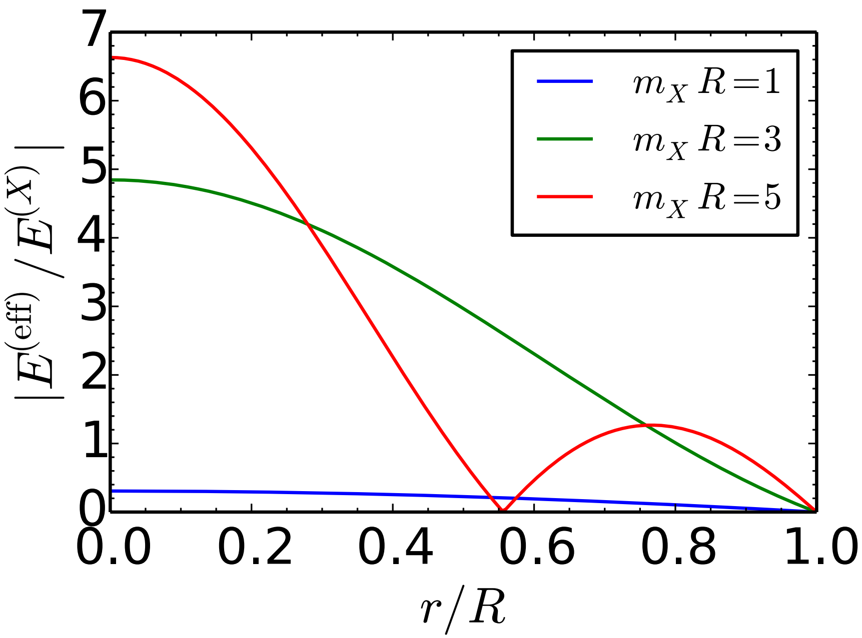
- Geometry of the cavity
- Location of the qubit

  $\Rightarrow$ No excitation, if the qubit is located on the wall

Cylinder-shaped cavity (with $\vec{E}^{(X)} //$ cylinder axis)

$$\vec{E}^{(\mathrm{eff})} \equiv \vec{E}^{(\mathrm{EM})} + \vec{E}^{(X)} = \left[1 - \frac{J_0(m_X r)}{J_0(m_X R)}\right] \vec{E}^{(X)}$$



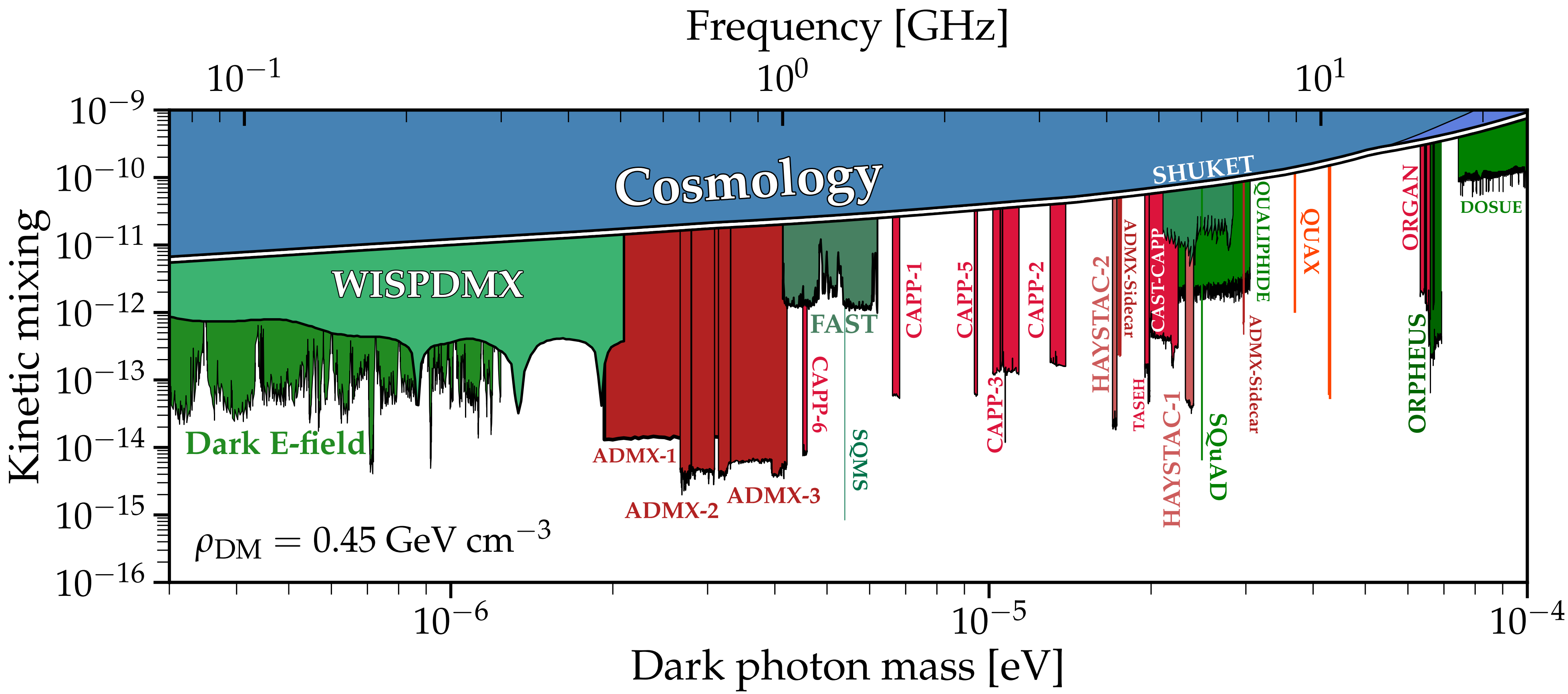


$\Rightarrow$ $|\vec{E}^{(\mathrm{eff})}| \gtrsim |\vec{E}^{(X)}|$ is possible if $R \gtrsim m_X^{-1}$

$\Leftrightarrow$ Sensitivity we have seen before: $|\vec{E}^{(\mathrm{eff})}| = |\vec{E}^{(X)}|$

Backup: Misc.

Constraints on hidden photon DM



[Caputo, Millar, O'Hare & Vitagliano (2105.04565)]

For $t \gtrsim \tau$, coherence is lost

Coherence time: $\tau = \dfrac{2\pi \mathcal{Q}}{\omega}$ (with $\mathcal{Q}$ = quality factor)

Decoherence of DM due to its velocity dispersion

$$\mathcal{Q}_{\rm DM} = \frac{\omega}{\delta\omega} \sim v_{\rm DM}^{-2} \sim 10^6$$

Decoherence of qubit

$$\mathcal{Q}_{\rm qubit} \sim 10^{(5-6)}$$

For our numerical analysis, we take

$$\mathcal{Q} = 10^6$$