# Indirect probes of muon EDM and spin force


Yohei Ema

University of Minnesota


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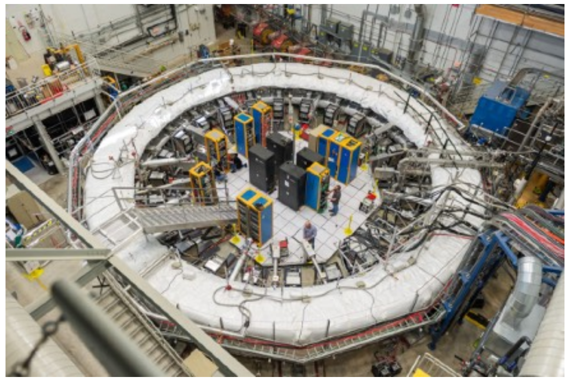
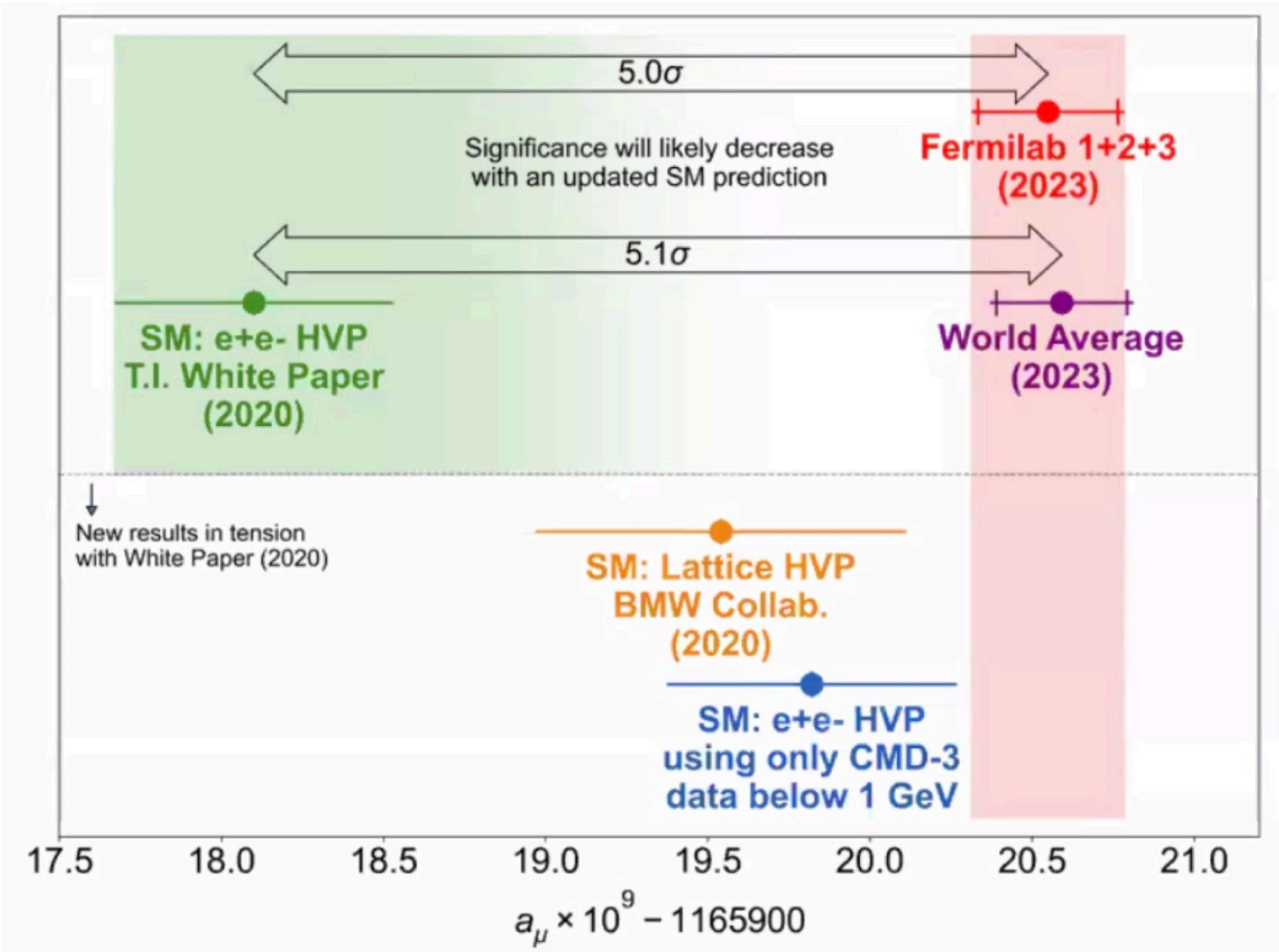
Based on 2108.05398, 2207.01679 and 2308.01356

with Ting Gao and Maxim Pospelov

# Muon magnetic moment

- August 2023, FNAL released their run 2 and 3 result:





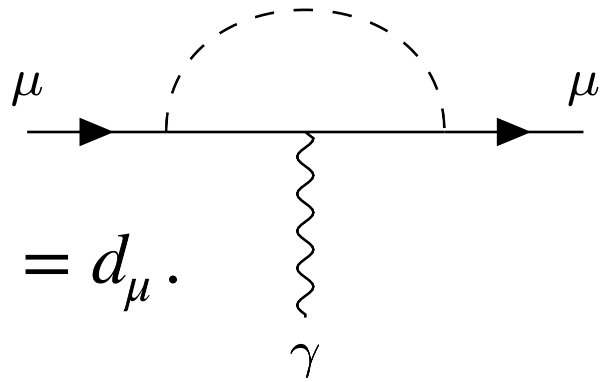

[FNAL 23]

- Caveat on hadronic vacuum polarization from lattice QCD, CMD-3, …

- But given this and other progress, exploring new physics in muon sector is interesting.

# Muon EDM and spin force

- Muon g-2 and EDM can be closely related:




$$\mathscr{L} = -\frac{c}{2}\bar{\psi}_R \sigma \cdot F \psi_L + \mathrm{h.c.} \Rightarrow \mathrm{Re}[c] = \frac{e\Delta a_\mu}{2m_\mu}, \quad \mathrm{Im}[c] = d_\mu .$$

$$d_\mu \simeq 2 \times 10^{-22}\, e\,\mathrm{cm} \times \tan\phi \times \left(\frac{\Delta a_\mu}{2.5 \times 10^{-9}}\right)$$ with $c = |c| e^{i\phi}$.

- Muon spin force: additional spin rotation to storage ring experiment.

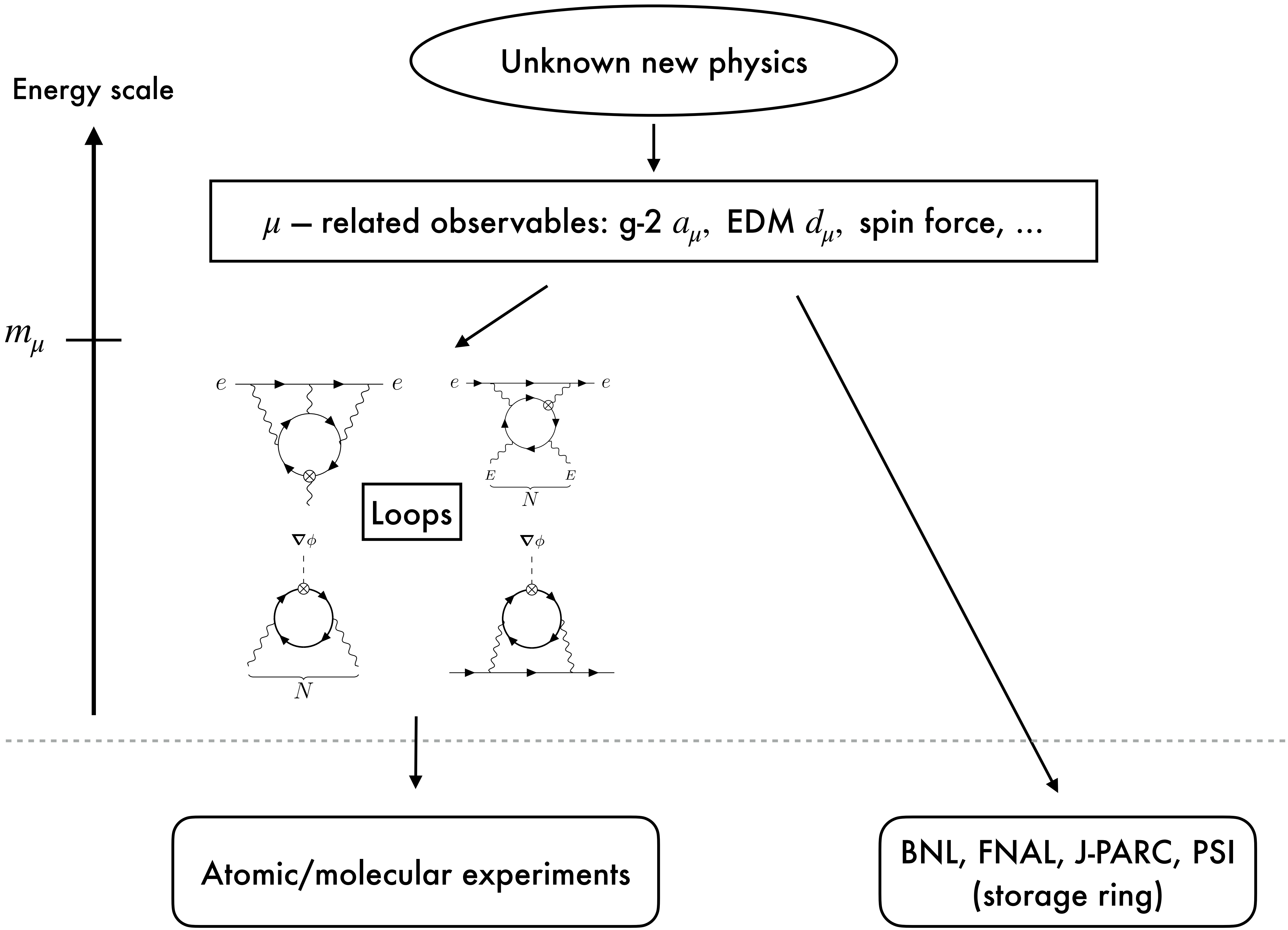
$$H = \Delta E_\mu (\vec{s} \cdot \vec{n}), \quad \Delta E_\mu = \frac{g_A^{(\mu)} + g_P^{(\mu)}}{m_\mu} |\vec{\nabla}\phi| .$$

$$\vec{\Omega} = \frac{e}{m}\left[a\vec{B} - \left(a - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right] + \frac{\Delta E_\mu}{\gamma} .$$

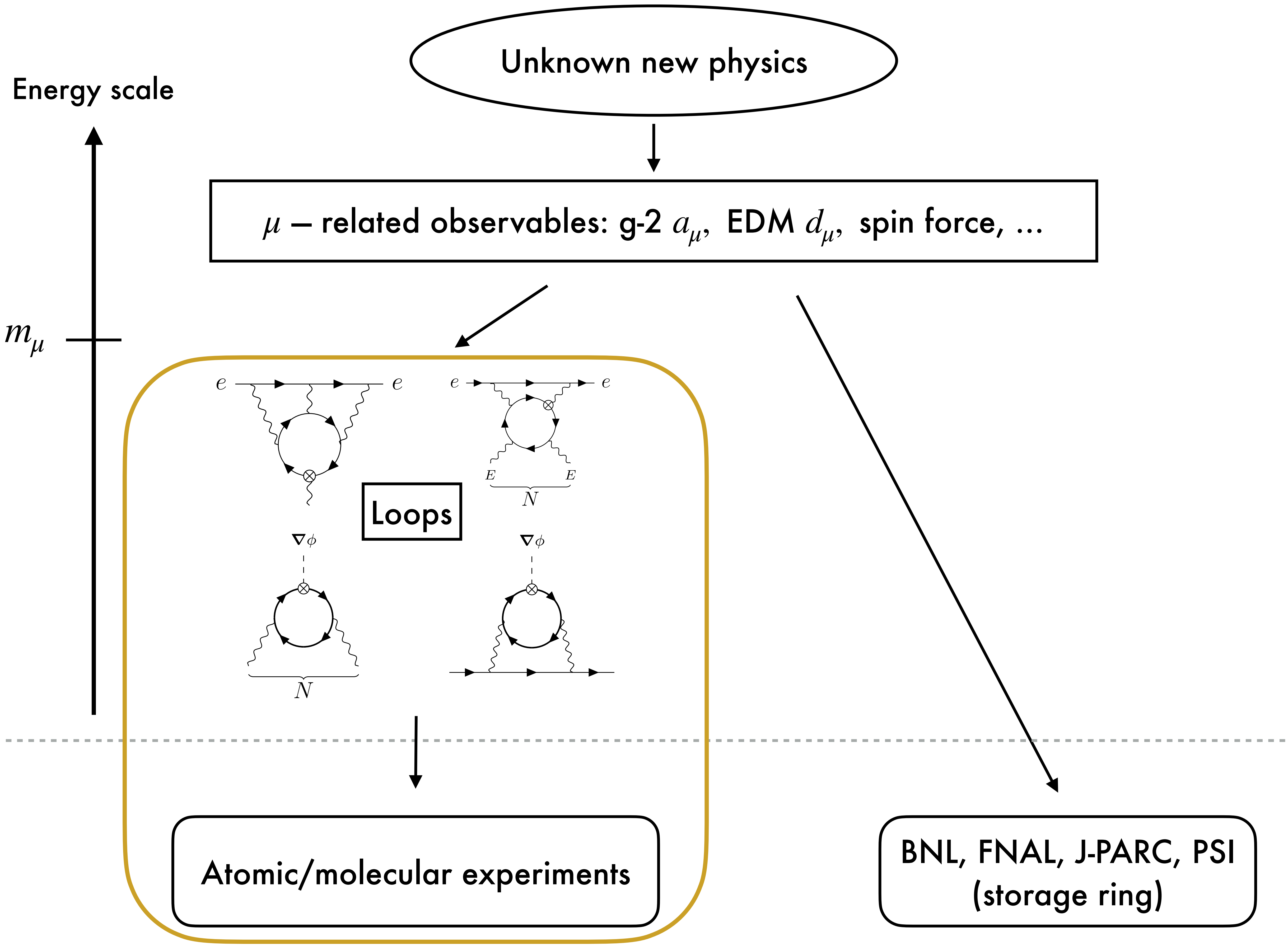
Muon g-2 explained if $\Delta E_\mu = 6 \times 10^{-14}\,\mathrm{eV}$ .

[Davoudiasl+ 22; Agrawal+ 22]

# Towards observables



# Towards observables



# Outline

1. Introduction

2. Indirect constraints on muon EDM

3. Muon spin force

4. Summary

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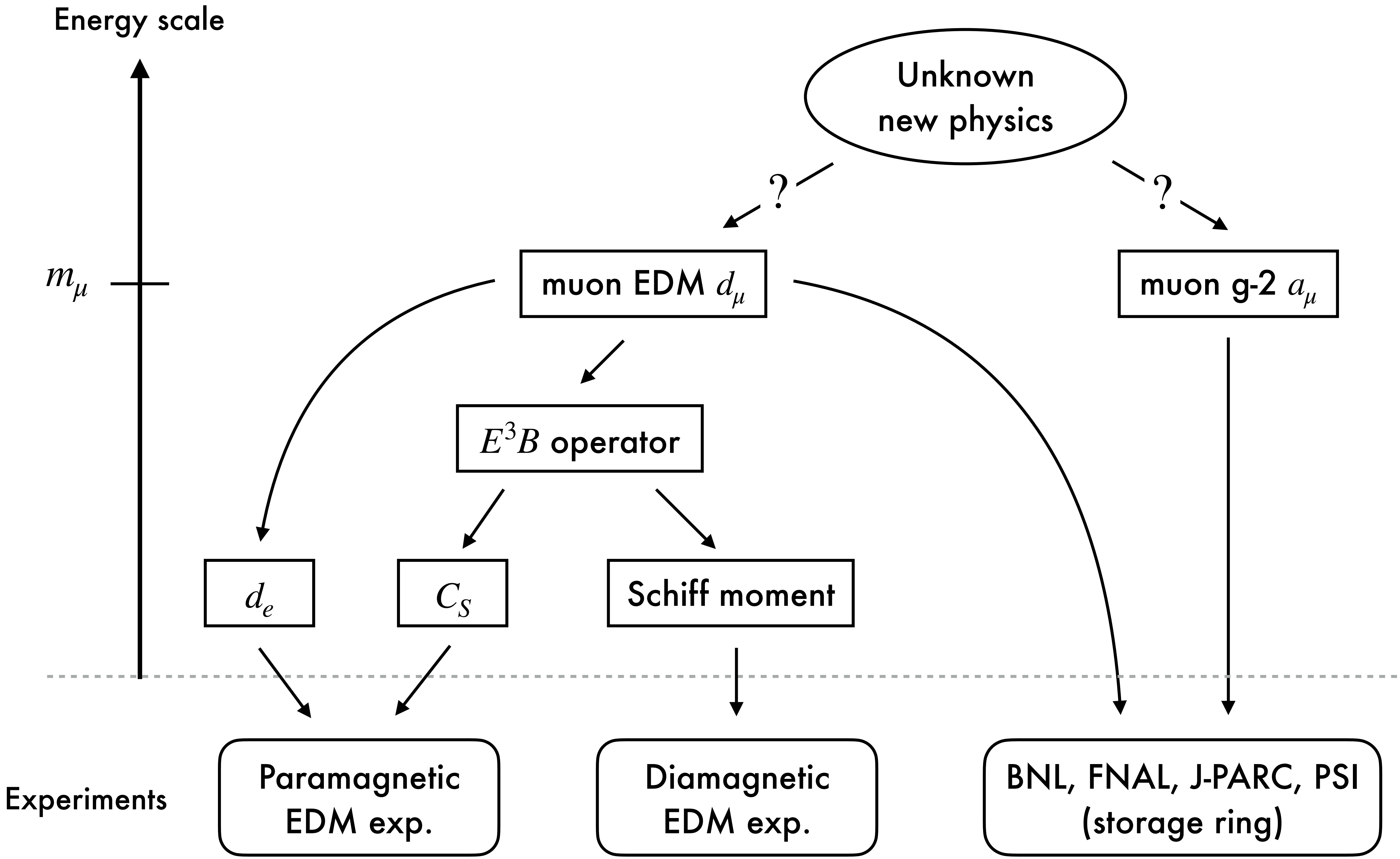
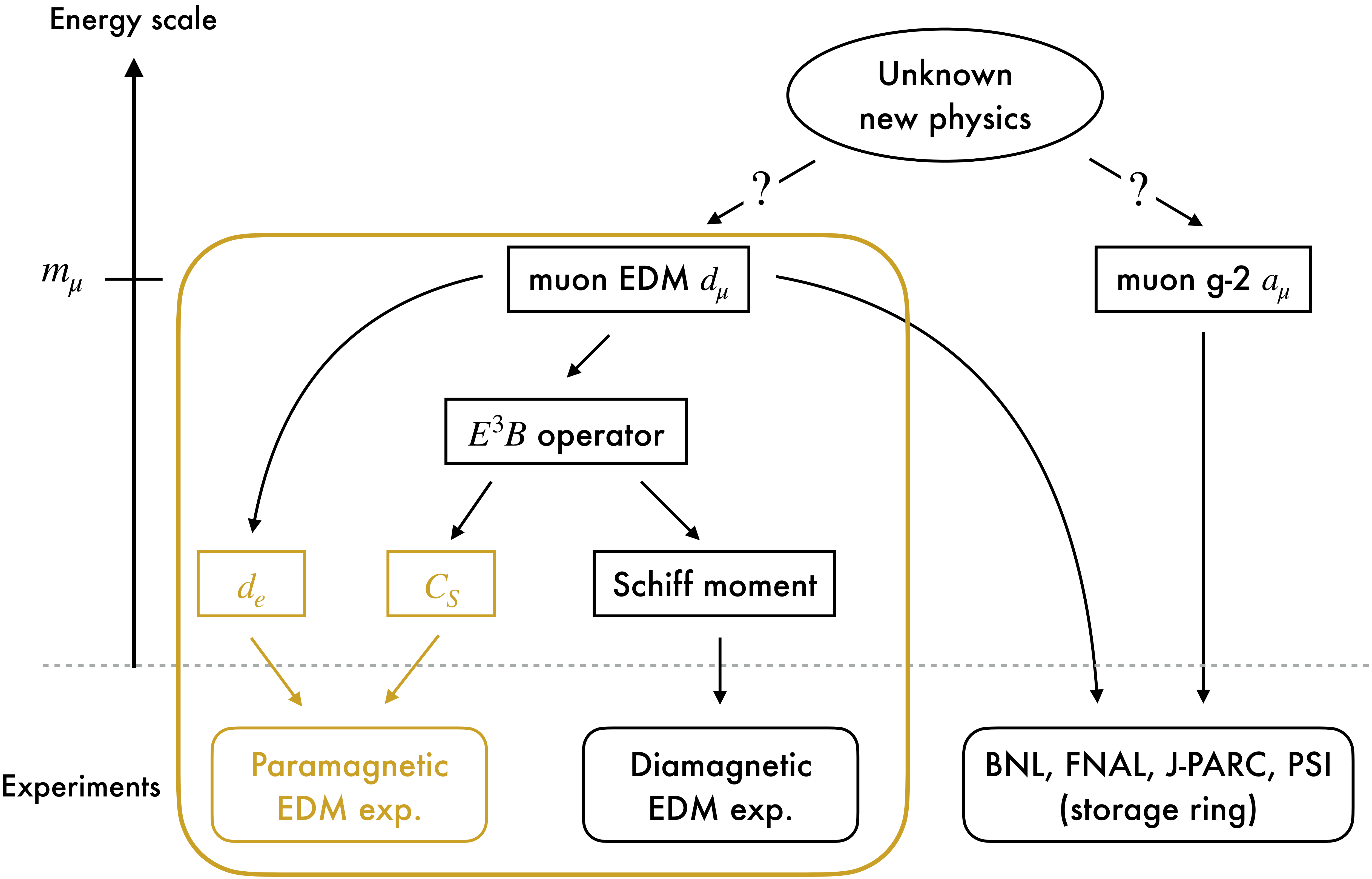
# Towards observables

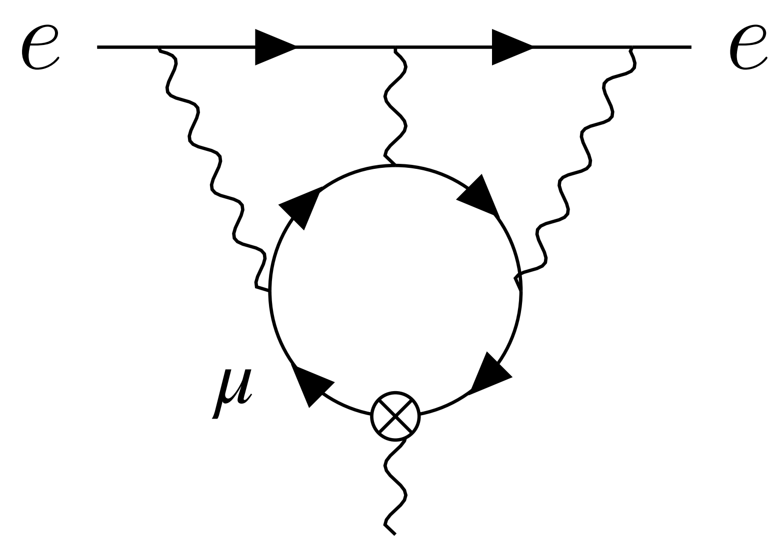


# Towards observables



# Electron EDM

- Muon EDM induces electron EDM at three-loop:




+ permutations (⊗: EDM operator)

- Two types of contributions:

$$i\mathscr{M} = i\tilde{F}^{\mu\nu}\,\bar{e}(p)\left[S^{(1)}m_e\sigma_{\mu\nu} + S^{(2)}\left\{\sigma_{\mu\nu}, \not{p}\right\}\right]e(p)\,.$$

[Grozin, Khriplovich, Rudenko 08] overlooked $S^{(2)}$.

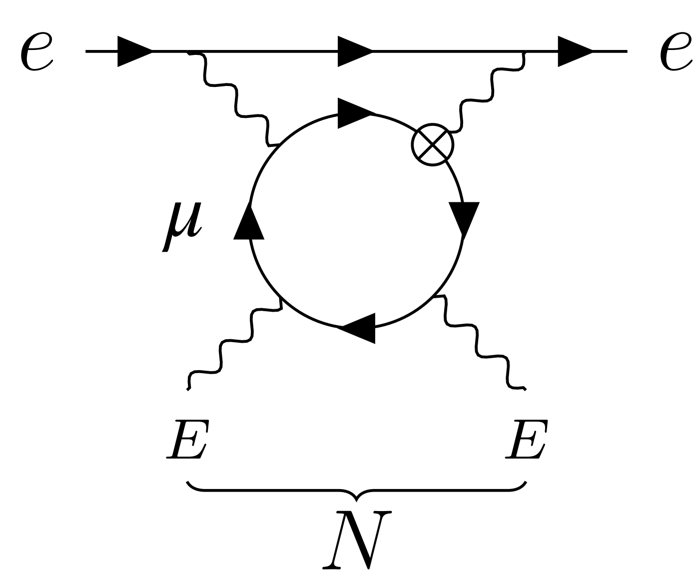
- Combining $S^{(1)}$ and $S^{(2)}$, the result is $40\,\%$ larger:

$$d_e = 2.75 \times d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \simeq 1.7 \times 10^{-10} \times d_\mu$$ (this is UV finite).

[YE, Gao, Pospelov 22]

# Semi-leptonic CP-odd operator

- Muon EDM induces




$$\sim d_\mu \times \bar{e} i \gamma_5 e \times E_N^2 .$$

- Nuclear electric field $E_N^2$ localized around nucleus.

$\bar{e} i \gamma_5 e \times E_N^2 \ \sim \ \bar{e} i \gamma_5 e \times n_N \sim \ \bar{e} i \gamma_5 e \times \bar{N} N$ : equivalent to $C_S$.

- Combining $C_S$ and $d_e$, ACME translated as

$$| d_\mu(\text{ThO}) | < 1.7 \times 10^{-20} \, e \, \text{cm}$$

[YE, Gao, Pospelov 21, 22]

Better than BNL bound: $| d_\mu(\text{BNL}) | < 1.8 \times 10^{-19} \, e \, \text{cm}$ .

- Recent Colorado result [Roussay+ 22] even stronger: $| d_\mu(\text{HfF}) | < 8.9 \times 10^{-21} \, e \, \text{cm}$ .

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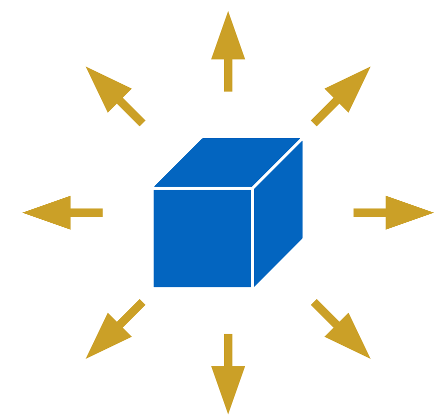
# Spin-mass coupling

- Spin-mass coupling mediated by light CP-violating scalar:

$$\mathcal{L} = -\, g_S \phi\, \bar{N} N + \frac{g_A}{2m} \partial^\alpha \phi\, \bar{\psi} \gamma_\alpha \gamma_5 \psi - g_P\, \phi\, \bar{\psi} i \gamma_5 \psi\,.$$

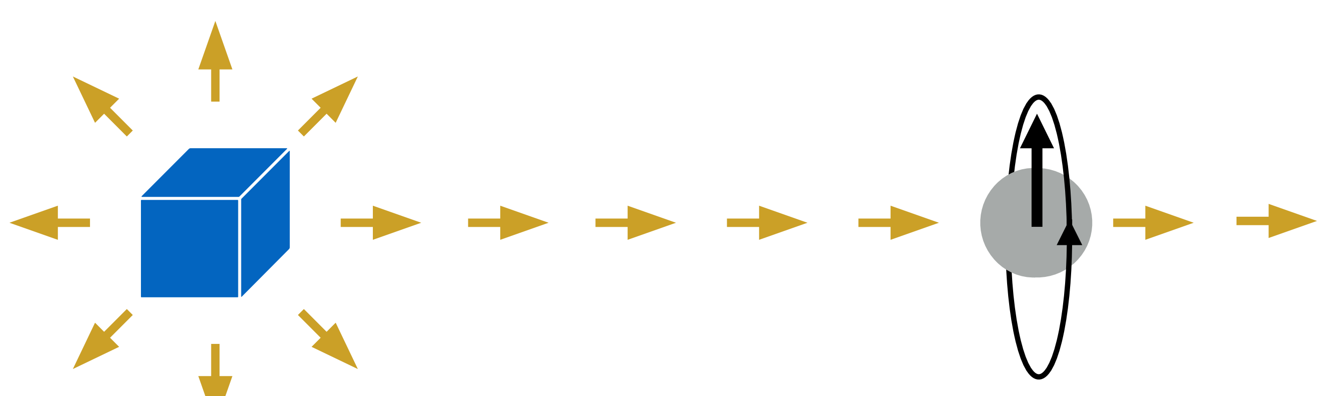
- Mass coupling $g_s$ creates classical $\phi$:

$$\left( \nabla^2 - m_\phi^2 \right) \phi = g_S\, n_N \quad \Rightarrow \quad \phi(r) = -\frac{g_S N_N}{4\pi r} e^{-m_\phi r}$$



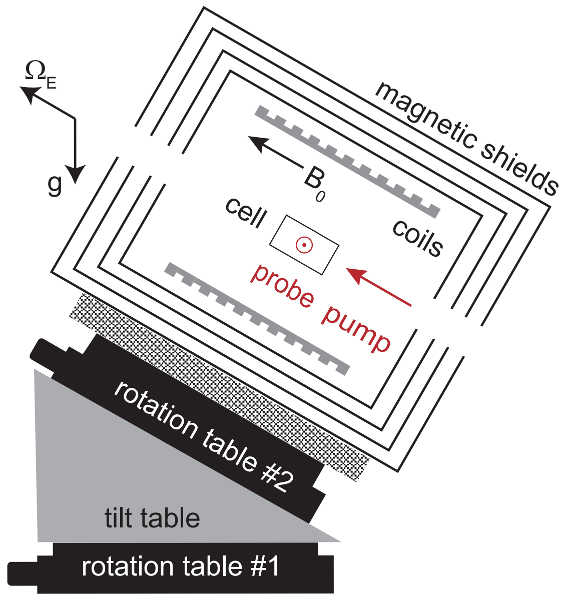
- Couples to the spin via $g_P$, $g_A$, act as "pseudo" magnetic field:

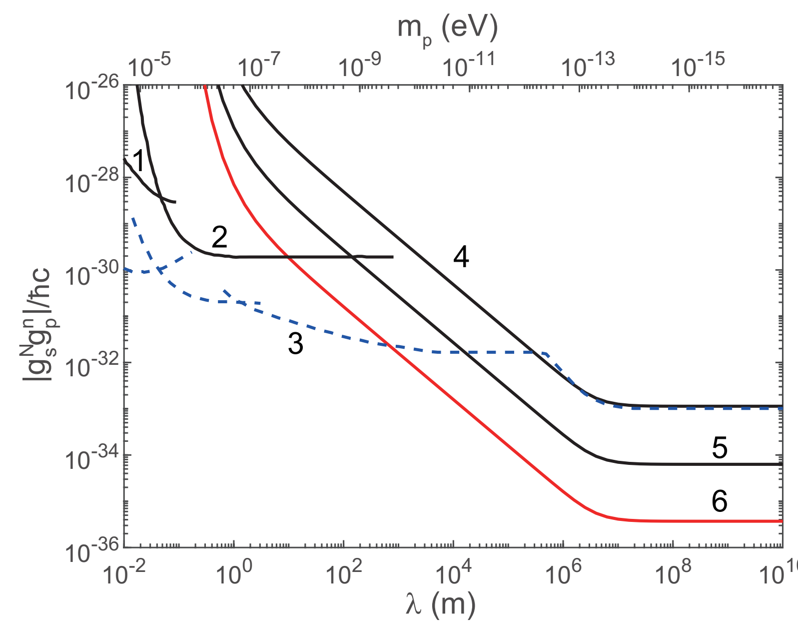
$$H = -\frac{g_A + g_P}{2m} \vec{\sigma} \cdot \vec{B}_\phi \ \text{ with } \ \text{“}\vec{B}_\phi\text{”} = -\vec{\nabla} \phi.$$



# Spin force experiments

- Spin force experiments probing "gravity" and nuclear spin couplings.




[Zhang+ 23]

- Co-magnetometer technique to suppress magnetic field noise.

$$H = \vec{I} \cdot \left( -g\mu_N \vec{B} + \underline{\Delta E\, \vec{n}} \right) \quad \Rightarrow \quad \vec{\Omega}_1 - \frac{g_1}{g_2}\vec{\Omega}_2 = \vec{n}\left( \Delta E_1 - \frac{g_1}{g_2}\Delta E_2 \right).$$

$\propto \vec{\nabla}\phi$

(Evaluation of $\Delta E$ requires the nuclear physics input.)

- Measured by $^{129/131}\mathrm{Xe}$ and $^{199/201}\mathrm{Hg}$ systems:

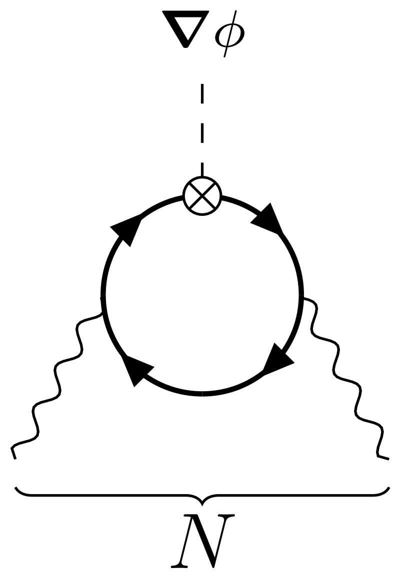
$$\begin{cases} \text{Hg: } |\Delta E_{201} + 0.369139\Delta E_{199}| < 3.0 \times 10^{-21}\,\mathrm{eV}, & \text{[Venema+ 92]} \\ \text{Xe: } |\Delta E_{129} + 3.37337\Delta E_{131}| < 1.7 \times 10^{-22}\,\mathrm{eV}. & \text{[Zhang+ 23]} \end{cases}$$

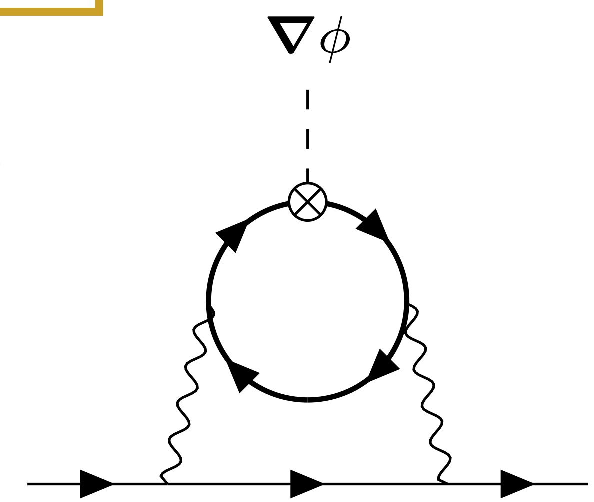
c.f. $\Delta E_\mu = 6 \times 10^{-14}\,\mathrm{eV}$ for muon g-2.

# Muons at loop level

- By closing muon loops, several different spin-mass couplings are generated.

$$\frac{g_A}{2m}\partial^\alpha\phi\,\bar{\psi}\gamma_\alpha\gamma_5\psi - g_P\,\phi\,\bar{\psi}i\gamma_5\psi$$




"photon spin coupling"

$$\frac{e^2\vec{\nabla}\phi}{4\pi^2 m_\mu}\cdot\left(g_P^{(\mu)}\vec{B}\,A_0 - \frac{g_A^{(\mu)}}{12m_\mu^2}\vec{B}\,\rho_N\right)$$

(Dominated by IR → $g_A^{(\mu)}$ suppressed due to decoupling.)

nucleon spin coupling

$$-g_A^{(\mu)}\frac{3}{4}\left(\frac{\alpha}{\pi}\right)^2 Q^2\log\left(\frac{\Lambda_{\rm UV}^2}{\Lambda_{\rm IR}^2}\right)\partial^\alpha\phi\times\bar{\psi}\gamma_\alpha\gamma_5\psi$$

(photon "spin" is more important for $g_P$.)

These couplings probed by the co-magnetometer experiments.

# Existing constraints

- Indirect constraints on muon spin force derived from loop effects. [YE, Gao, Pospelov 23]

- For $g_P^{(\mu)}$, photon "spin" coupling is more important:

$$|\Delta E_\mu| < 6 \times 10^{-17}\,\mathrm{eV}$$

- For $g_A^{(\mu)}$, both are important:

$$\begin{cases} |\Delta E_\mu| < 4 \times 10^{-14}\,\mathrm{eV} & \text{from photon spin,} \\ |\Delta E_\mu| < 6 \times 10^{-15}\,\mathrm{eV} & \text{from nucleon spin.} \end{cases}$$

Note that $\Delta E_\mu = 6 \times 10^{-14}\,\mathrm{eV}$ to explain muon g-2.

But subject to nuclear physics uncertainty (co-magnetometer → cancellation possible).

# A new $\mu$SR experiment?

Muon spin: a useful tool to measure local magnetic fields, "$\mu$SR".

- Widely used in the context of condensed matter physics, chemistry, etc.

  - PSI, TRIUMF, MuSIC: continuous muon beam,
  - J-PARC, ISIS: pulsed muon beam.

- Muon phase rotation from $\vec{\nabla}\phi$ within one muon lifetime:

$$\Delta\psi = \Delta E_\mu \times \tau_\mu = 2 \times 10^{-4} \quad \Rightarrow \quad N_\mu > (\Delta\psi)^{-2} \sim 10^8 \text{ needed.}$$

- Changing $B$ can extract $\Delta E_\mu$:

$$-B\frac{d\omega_\mu/\omega_p}{dB} = 3.4 \times 10^{-4} \times \frac{\Delta E_\mu}{6 \times 10^{-14}\,\mathrm{eV}} \times \frac{10\,\mathrm{G}}{B}.$$

(Taking proton as a co-magnetometer, for example.)

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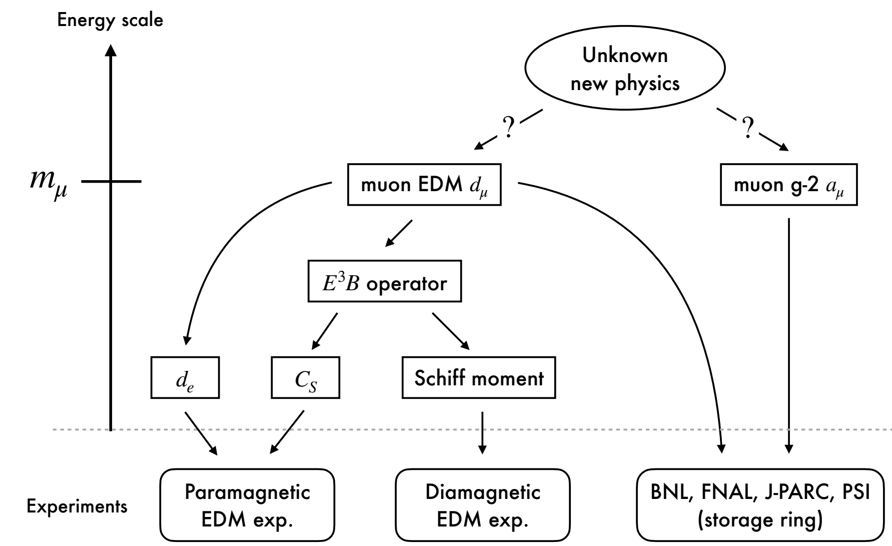
# Summary

## Indirect constraints on muon EDM:

- Muon EDM is interesting, given the muon $g-2$ anomaly.
- From ThO $|d_\mu(\mathrm{ThO})| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}$.
- From $^{199}$Hg $|d_\mu(\mathrm{Hg})| < 6.4 \times 10^{-20}\, e\,\mathrm{cm}$.
- Similar constraints on tau/charm/bottom EDMs.




## Indirect constraints on muon spin force:

- Muon spin force motivated by muon g-2 anomaly.
- Muon loops induce operators probed by nuclear spin experiments.
- Muon spin force explaining g-2 disfavored, but with large nuclear uncertainty.
- A new $\mu$SR experiment seems feasible, probing interesting parameter region.

# Back up on EDM

# Storage ring experiments

$$\vec{\Omega} = -\frac{e}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} + \frac{\eta_\mu}{2}\left(\vec{\beta}\times\vec{B} + \vec{E}\right)\right]$$

where $a_\mu = (g_\mu - 2)/2, \ \ d_\mu = e\eta_\mu/2m_\mu \, .$

- BNL and FNAL: "magic momentum"

$$p_\mu = 3.094\,\mathrm{GeV} \to a_\mu - \frac{1}{\gamma^2 - 1} \simeq 0.$$

- J-PARC: "ultra-cold muon beam"

Super-low emittance $\to \vec{E} = 0.$

- PSI: "frozen spin"

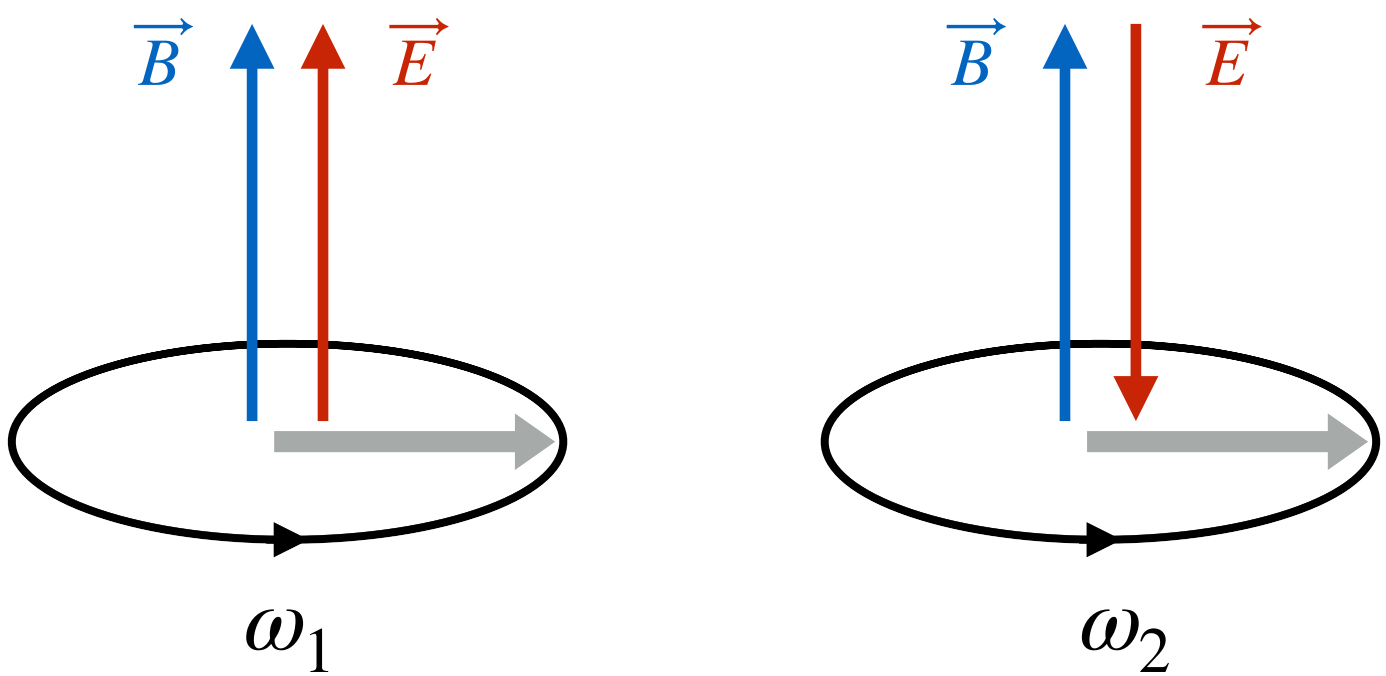
$a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} = 0$, dedicated to muon EDM.

# Spin precession

- EDM observable: spin precession

$$\frac{d\vec{s}}{dt} = \vec{\omega} \times \vec{s}, \quad \vec{\omega} = 2\mu \vec{B} + 2d\vec{E}\,.$$

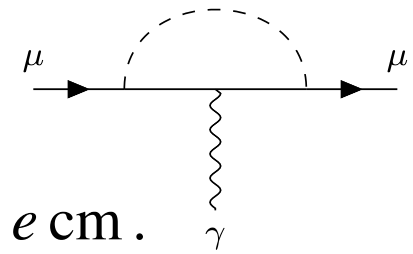
- Extract EDM by flipping $\vec{E}$:




$$\Delta\omega = \omega_1 - \omega_2 = 4dE\,.$$

# Muon EDM

- Muon g-2 and EDM can be closely related.

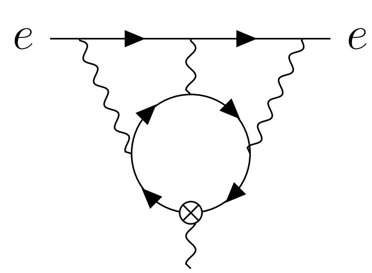



➡ Current/future muon EDM measurements:

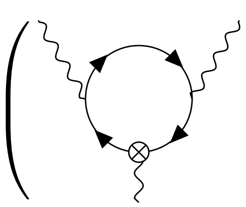
$$\begin{cases} \text{BNL (existing limit): } |d_\mu| < 1.8 \times 10^{-19}\, e\,\text{cm}\,. \\ \text{FNAL, J-PARC: } |d_\mu| \lesssim 10^{-21}\, e\,\text{cm}\,. \\ \text{PSI ("frozen spin"): } |d_\mu| < 6 \times 10^{-23}\, e\,\text{cm}\,. \end{cases}$$

- Muon EDM probed also by atomic/molecular EDM experiments.

chirality flipping

E.g. $d_e \sim$ 

$\sim d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 10^{-10} \times d_\mu .$

(  $\sim F_\mu{}^\nu F_\nu{}^\rho \tilde{F}_\rho{}^\mu = 0$ )

➡ From $|d_e^{(\text{equiv})}| < 4.1 \times 10^{-30}\, e\,\text{cm}$, we expect $|d_\mu| < \mathcal{O}(10^{-19} - 10^{-20})\, e\,\text{cm}\,.$

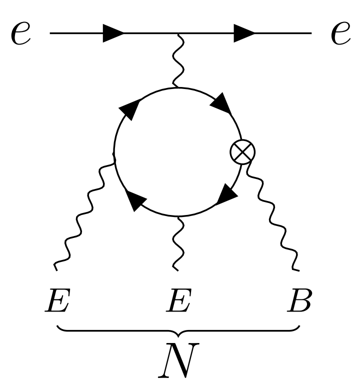
Limits from atomic/molecular EDM important → compute more precisely.

# Schiff moment

- Schiff moment: $\mathscr{H}_{\rm int} = -4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)\,.$

$$\vec{d}_A = \sum_{i=1}^{Z}\langle\Psi|e\vec{r}_i|\Psi\rangle = -\sum_{n\neq 0}\frac{2}{E_0-E_n}\sum_{i=1}^{Z}\langle 0_e|e\vec{r}_i|n_e\rangle\langle n_e|4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_i\delta^{(3)}(\vec{r}_i)|0_e\rangle\,.$$

- $E^3B$ with two $E_N$ and one $B_N$ induces effective EDM distribution:




$$= \int d^3r\left(\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|}\right)\cdot\frac{\vec{d}_N(\vec{r})}{e},\quad \vec{d}_N \propto d_\mu\left(2\vec{E}_N(\vec{E}_N\cdot\vec{B}_N)+\vec{B}_N E_N^2\right).$$

- Difference between EDM and charge distribution gives Schiff moment:

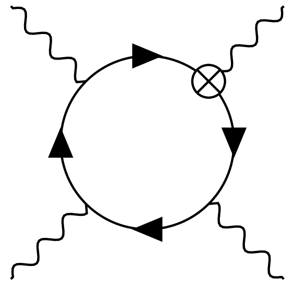
$$\mathscr{H}_{\rm eff} = \int d^3r\left(\frac{\vec{d}_N}{e}-\rho_q\frac{\langle\vec{d}_N\rangle}{e}\right)\cdot\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|} = -4\pi\alpha\frac{\vec{S}}{e}\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)+\cdots.$$

- $^{199}$Hg constraint: $|S_{^{199}\mathrm{Hg}}| < 3.1\times 10^{-13}\,e\mathrm{fm}^3\,.$ [Graner et.a. 16]

$$|d_\mu(\mathrm{Hg})| < 6.4\times 10^{-20}\,e\mathrm{cm}$$ [**YE**, Gao, Pospelov 21]

# CP-odd photon operator

- Muon EDM induces CP-odd photon operator at one-loop:



$$= -\frac{e^3 d_\mu}{96\pi^2 m_\mu^3}\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

where cross-dot: muon EDM $d_\mu \bar{\mu}\sigma \cdot \tilde{F}\mu$ insertion.

- Atomic EDM exp. has large $Z \rightarrow$ strong nuclear electric field.

➡ $\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} \ni E^3 B$ can induce sizable CP-odd effects.

- In particular this operator induces

$$\begin{cases} \text{Schiff moment} \rightarrow \text{diamagnetic EDM (Hg)} \\ \text{semi-leptonic CP-odd operator} \rightarrow \text{paramagnetic EDM (ThO)} \end{cases}$$

# Nuclear electric field

- Nuclear electric field given by the charge distribution inside nuclei:

$$e\vec{E}_N(\vec{r}) = \frac{Ze^2}{4\pi}\int d^3r_N \rho_q(\vec{r}_N)\,\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}_N|}\,.$$

- We simply take the charge distribution as

$$\rho_q(r_N) = \frac{3}{4\pi R_N^3}\Theta\left(R_N - r_N\right), \quad R_N = \sqrt{\frac{5}{3}}r_c, \quad r_c = 5.45\,\mathrm{fm}\ \ \text{for}\ {}^{199}\mathrm{Hg}.$$

- The Woods-Saxon shape different only within $10\,\%$ in the final result.

# Nuclear magnetic field

- $^{199}$Hg has an unpaired outermost neutron with $2p_{1/2}$ $(n=2, l=1, j=1/2)$ .

  ➡ $\vec{B}_N$ dominantly provided by this neutron.

- As a result $\vec{B}_N$ is given by

$$e\vec{B}_N(\vec{r}) = \frac{2e\mu_n}{3}\psi_n^\dagger(\vec{r})\vec{\sigma}\psi_n(\vec{r}) + \frac{e\mu_n}{4\pi}\left[\vec{\nabla}\left(\vec{\nabla}\cdot\right) - \frac{\vec{\nabla}^2}{3}\right]\int d^3r_n \frac{\psi_n^\dagger(\vec{r}_n)\vec{\sigma}\psi_n(\vec{r}_n)}{|\vec{r}_n - \vec{r}|}$$

$$= e\mu_n \frac{\left|R(r)\right|^2}{4\pi}\chi^\dagger\left[\left(\vec{n}\cdot\vec{\sigma}\right)\vec{n} - \vec{\sigma}\right]\chi + \frac{e\mu_n}{4\pi}\int_0^\infty dr_n\, r_n^2 \left|R(r_n)\right|^2 \chi^\dagger \vec{g}(\vec{r}, r_n)\chi,$$

where $\mu_n \simeq -1.91\frac{e}{2m_p}$ : neutron magnetic moment, $\psi_n$ : neutron wave function,

$R_n$: neutron radial wave function, $\chi$ neutron spinor, $\vec{n} = \vec{r}/r$ .

- We use the nuclear shell model to obtain $\psi_n$.

# Fudge factor from $E_N^2$ to $\bar{N}N$

- $E_N^2$ and $\bar{N}N$ are both localized around nuclei, but not exactly the same.

- Relevant electron transition between $s_{1/2}$ and $p_{1/2}$ states.

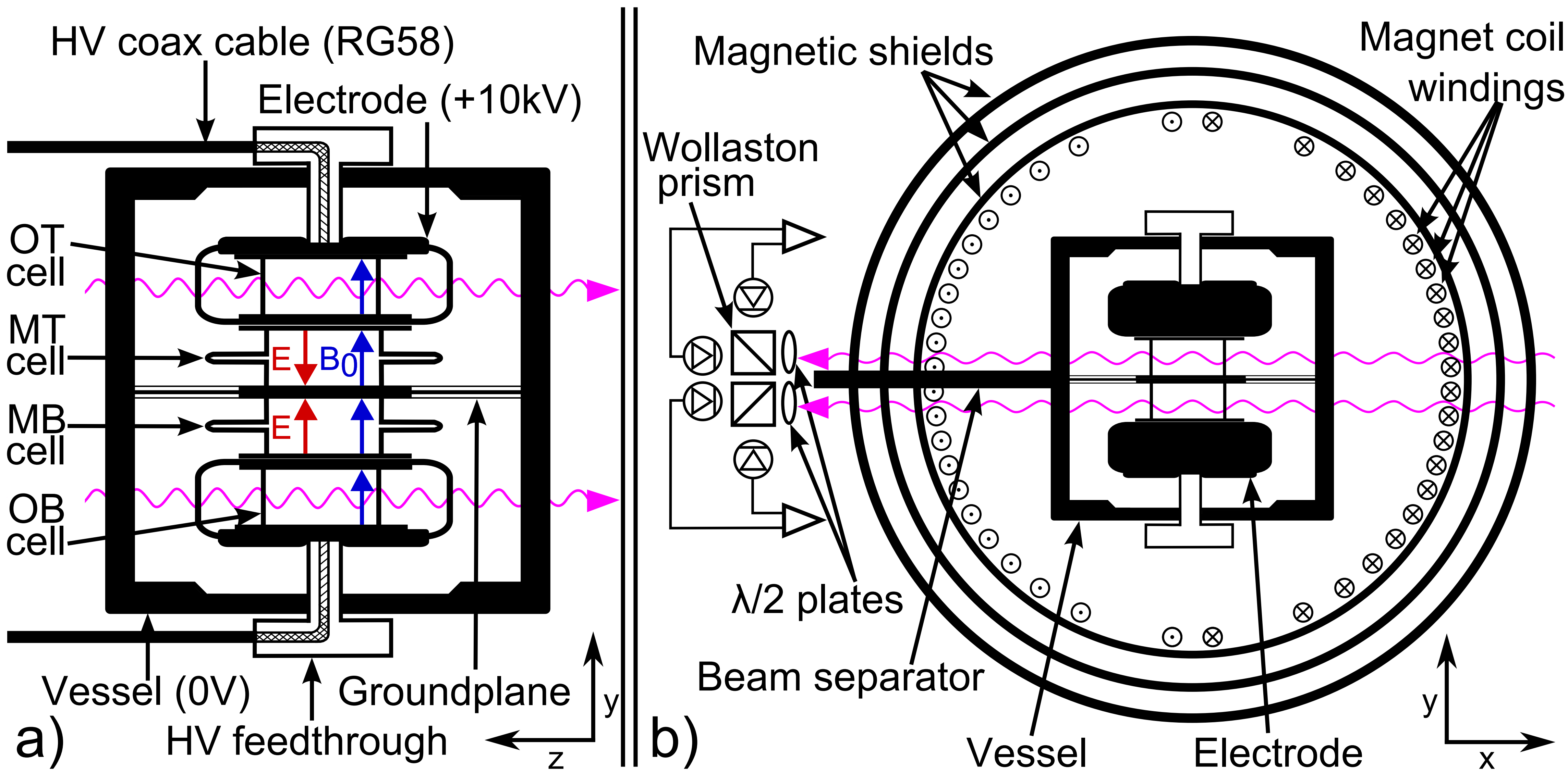
  ➡ matrix element:

$$\begin{cases} \int d^3r \rho_N \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_N \left( f_p g_s + f_s g_p \right) & \text{for} \quad \bar{N}N\bar{e}i\gamma_5 e, \\ \int d^3r \, |\overrightarrow{E}_N|^2 \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_{E^2} \left( f_p g_s + f_s g_p \right) & \text{for} \quad E_N^2 \bar{e} i \gamma_5 e \,. \end{cases}$$

- We compute the fudge factor $\kappa$ by solving the Dirac equation and get

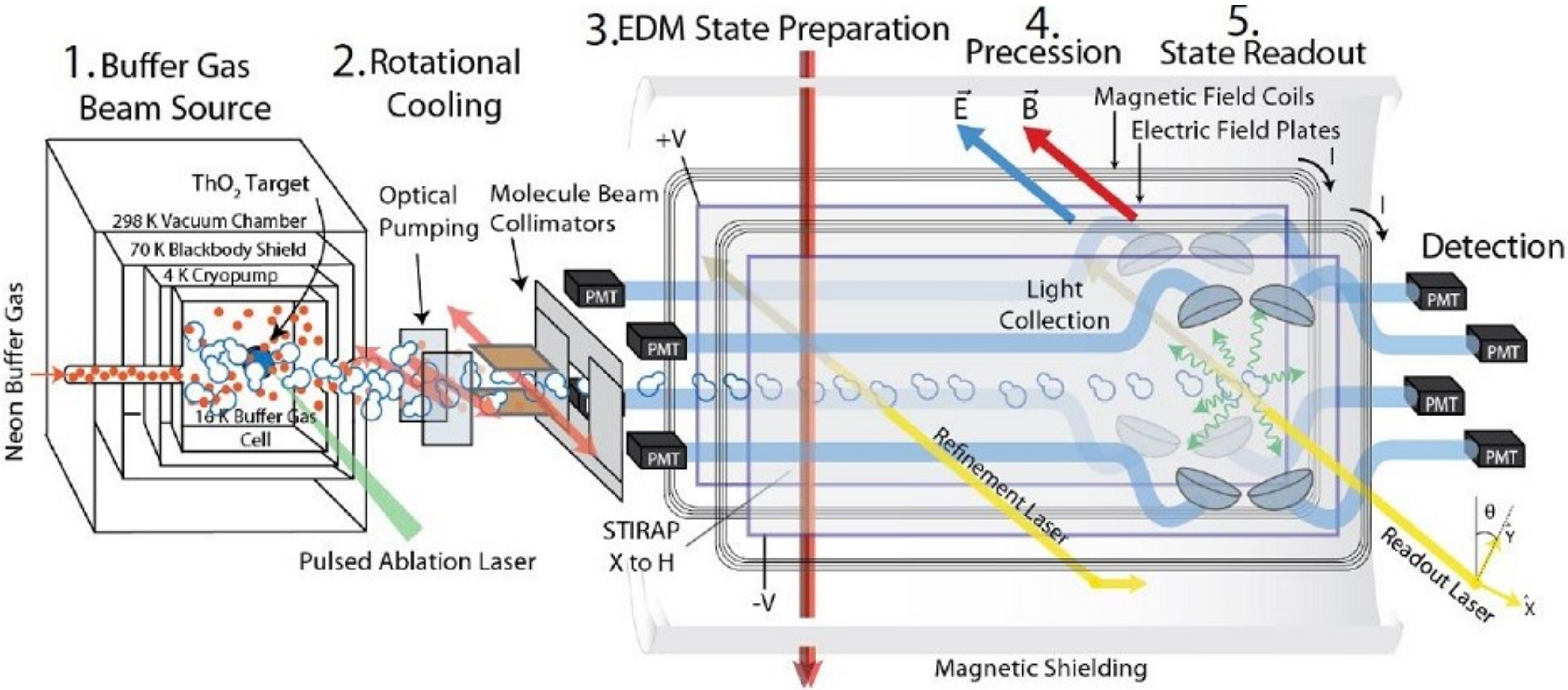
$$\kappa = \frac{\int dr\, r^2 \bar{\rho}_{E^2} \left( f_p g_s + f_s g_p \right)}{\int dr\, r^2 \bar{\rho}_N \left( f_p g_s + f_s g_p \right)} \simeq 0.66.$$

# $^{199}$Hg experiment




[Graner et.a. 16]

# ACME ThO experiment




[ACME 18]

# Tau/charm/bottom EDMs

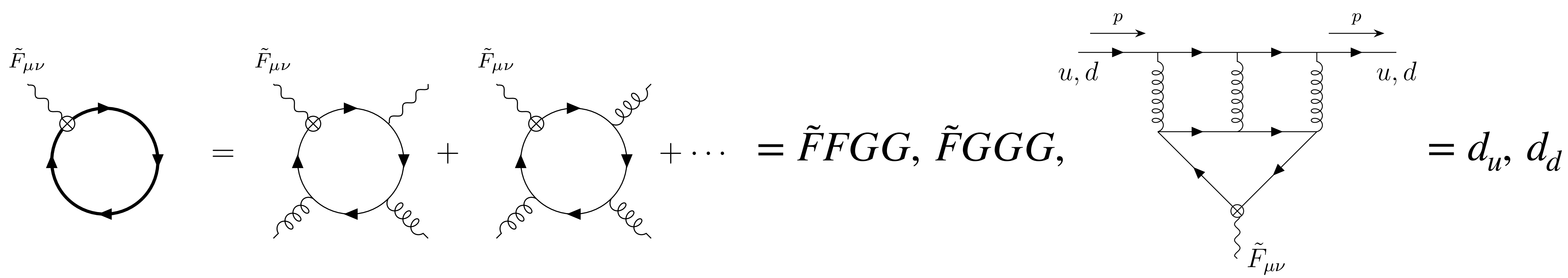
- Tau EDM constraint obtained by $m_\mu \to m_\tau$ :

$$|d_\tau(\text{ThO})| < 1.1 \times 10^{-18}\, e\,\text{cm}.$$

[**YE**, Gao, Pospelov 22b]

$$\begin{cases} \text{Belle}: -2.2\times 10^{-17} < \text{Re}(d_\tau)/e\,\text{cm} < 4.5\times 10^{-17}, \ -2.5\times 10^{-17} < \text{Im}(d_\tau)/e\,\text{cm} < 8.0\times 10^{-19}, \\ \text{Belle-II (future)}: |\text{Re}(d_\tau)|, |\text{Im}(d_\tau)| \lesssim \mathcal{O}(10^{-18} - 10^{-19})\, e\,\text{cm}. \end{cases}$$

- Similar constraints on charm/bottom quark EDMs.




$$\begin{cases} |d_c| < 1.3\times 10^{-20}\, e\,\text{cm}, \quad |d_b| < 7.6\times 10^{-19}\, e\,\text{cm} & \text{from paramagnetic EDM,} \\ |d_c| < 6\times 10^{-22}\, e\,\text{cm}, \quad |d_b| < 2\times 10^{-20}\, e\,\text{cm} & \text{from neutron EDM.} \end{cases}$$

[**YE**, Gao, Pospelov 22c]

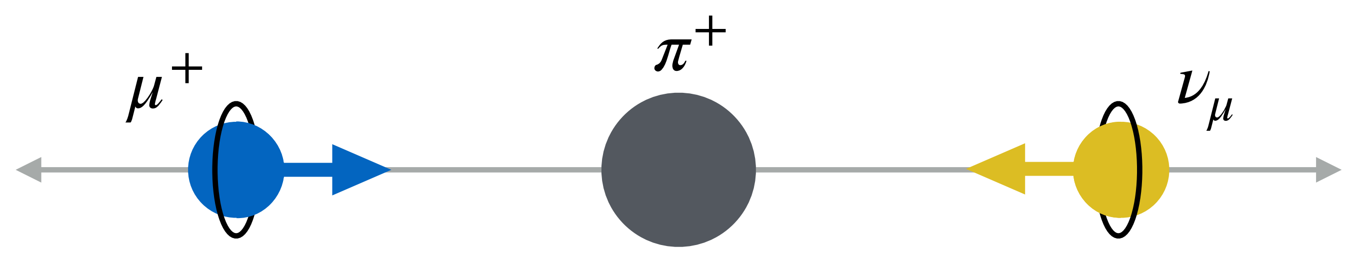
Constraints from $d_n$ stronger but with more hadronic uncertainties.

# Back up on spin force

# Muon spin rotation

- Muon has two advantages for spin spectroscopy.

  1. Surface muon from pion decay 100% polarized:




  2. Positron from decay telling muon spin direction:

$$\frac{d^2\Gamma}{dx\,d\cos\theta} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[3 - 2x + (2x-1)\underline{P_\mu \cos\theta}\right] x^2, \quad x = 2E_e/m_\mu\,.$$

➡ A useful tool to measure local magnetic fields etc. = $\mu$SR.

- Widely used in the context of condensed matter physics, chemistry, etc.

{ PSI, TRIUMF, MuSIC: continuous muon beam,
J-PARC, ISIS: pulsed muon beam.

# Magnetic quadrupole moment

- Magnetic quadrupole moment (MQM) also violates P and CP:

$$\mathscr{H}_{\rm eff} = -\frac{M}{6}\nabla_j B_i I_{ij}, \quad I_{ij} \equiv \frac{3}{2I(I-1)}\left(I_i I_j + I_j I_i - \frac{2}{3}\delta_{ij} I(I+1)\right).$$

- The $E^3B$ operator converts EQM $Q$ to MQM as

$$\frac{M}{e} \simeq -\frac{Z^2\alpha^3 d_\mu/e}{5\pi m_\mu^2 R_N^3}\frac{Q}{e} \simeq 1.1\times 10^{-4}\,{\rm fm}\left(\frac{Q/e}{300\,{\rm fm}^2}\right)\times d_\mu/e\,.$$

- $Q$ can be large in nuclei with $I \geq 1$ and large deformation.

➡ can be an interesting observable in future.

# Caveat on nuclear physics

- Excluding $g_P^{(\mu)}$ is probably robust, given $\mathcal{O}(10^3)$ margin, but $g_A^{(\mu)}$ is not.

- Co-magnetometer technique combines two isotopes.

$$H = \vec{I} \cdot \left( -g\mu_N \vec{B} + \underbrace{\Delta E \vec{n}}_{\propto \vec{\nabla}\phi} \right) \quad \Rightarrow \quad \vec{\Omega}_1 - \frac{g_1}{g_2}\vec{\Omega}_2 = \vec{n}\left( \Delta E_1 - \frac{g_1}{g_2}\Delta E_2 \right).$$

⇒ Constraints completely gone if $\Delta E_{129} = \dfrac{g_{129}}{g_{131}} \Delta E_{131}$.

- Our modeling of $\vec{B}_N$ in photon "spin" coupling turns out to be very close to this:

$$\Delta E_{129} = -8.9 \times 10^{-8} \times \Delta E_\mu, \quad \frac{g_{129}}{g_{131}} \Delta E_{131} = -9.2 \times 10^{-8} \times \Delta E_\mu .$$

- Shell model computation also uncertain, not quite reproducing magnetic moment.

⇓

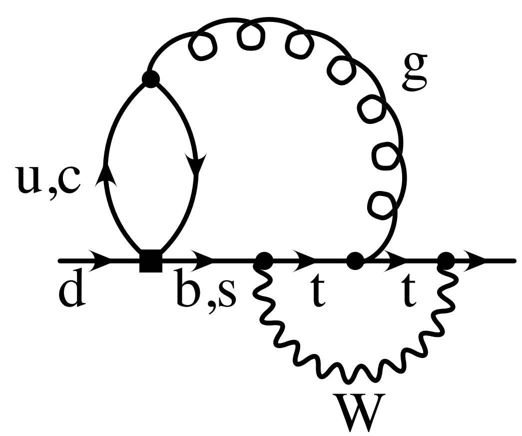
Observable free from nuclear uncertainty desired = $\mu$SR.

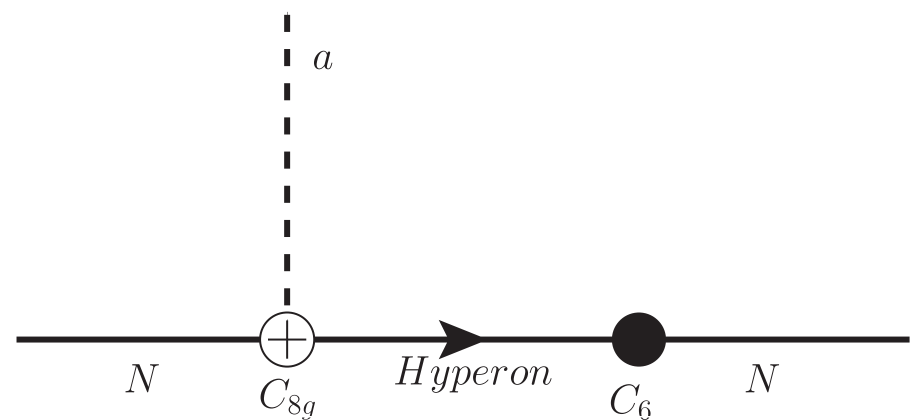
(Electron spin experiments not sensitive enough to exclude $\Delta E_\mu = 6 \times 10^{-14}\,\mathrm{eV}$.)

# CP violation and QCD axion?

- CKM phase violates CP, inducing QCD axion-nucleon scalar coupling $g_S^{(N)}$.

- Both short and long distance contributions exit:




...

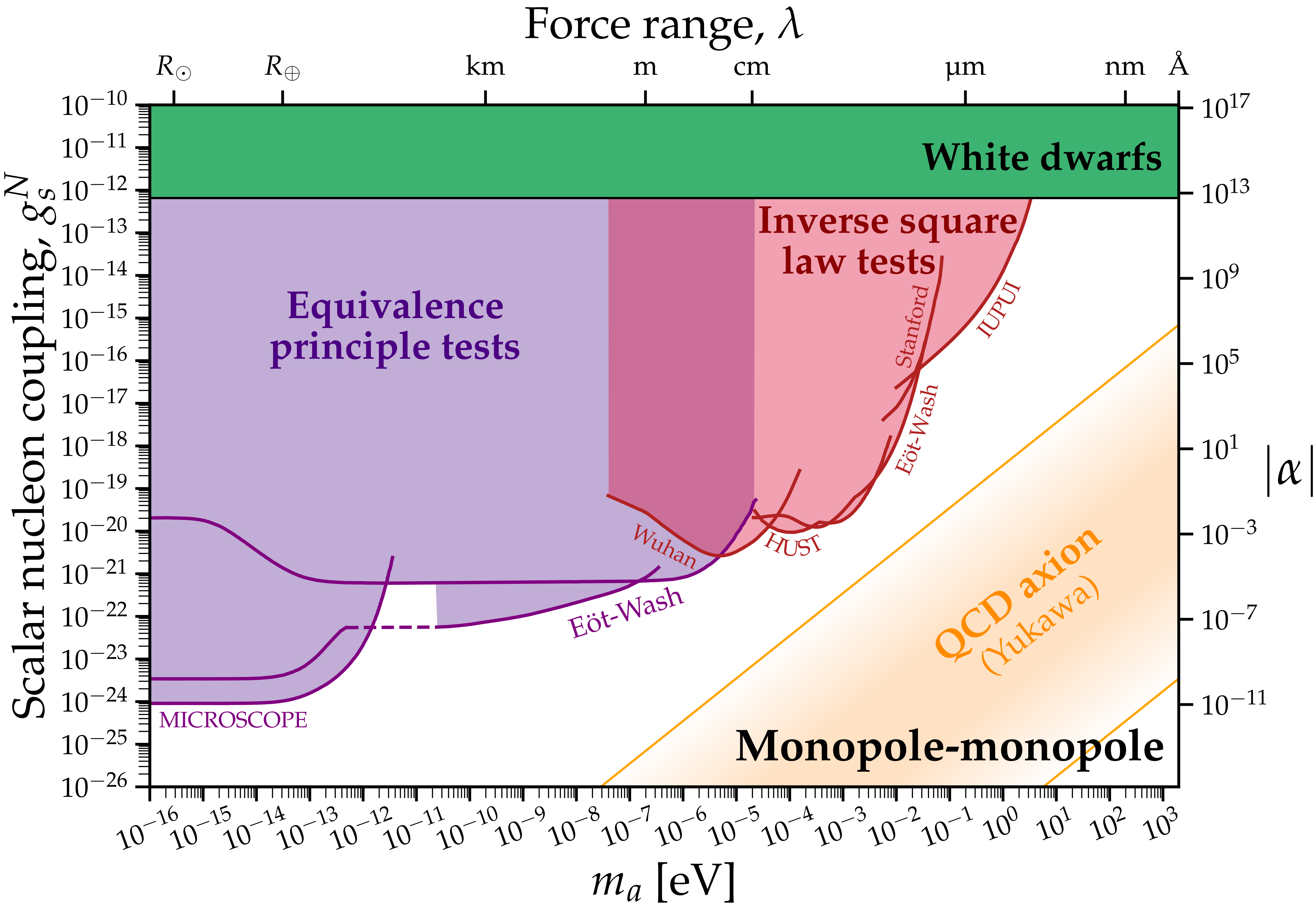
$$g_S^{(N)} \sim 10^{-31} \left( \frac{10^{10}\,\mathrm{GeV}}{f_a} \right).$$

[Czarnecki+ 97; Okawa+ 21]

Too small for muon g-2 since we need $g_S^{(N)} g_P^{(\mu)} \gtrsim 10^{-31}$ .

Cannot be a QCD axion, has to be an "ALP".

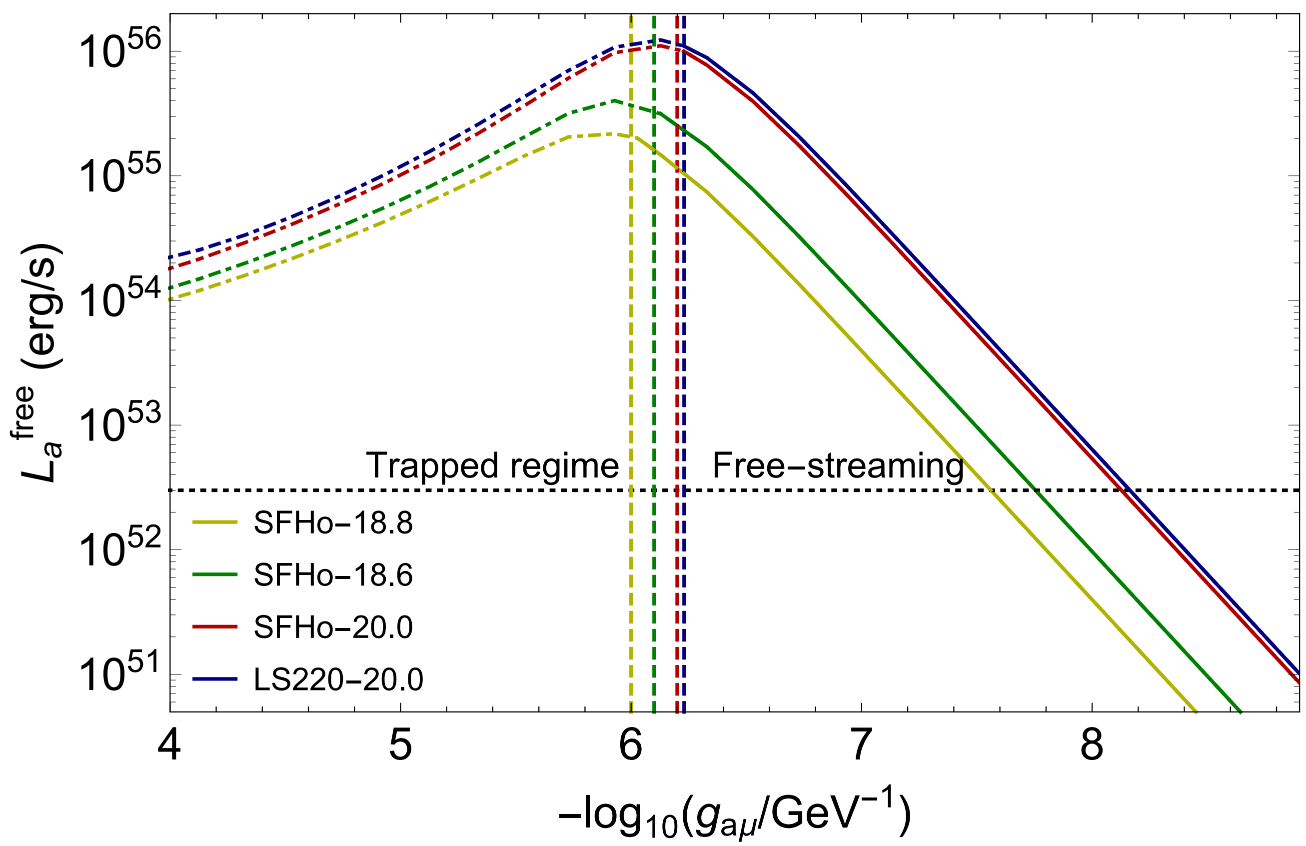
# Fifth-force constraints



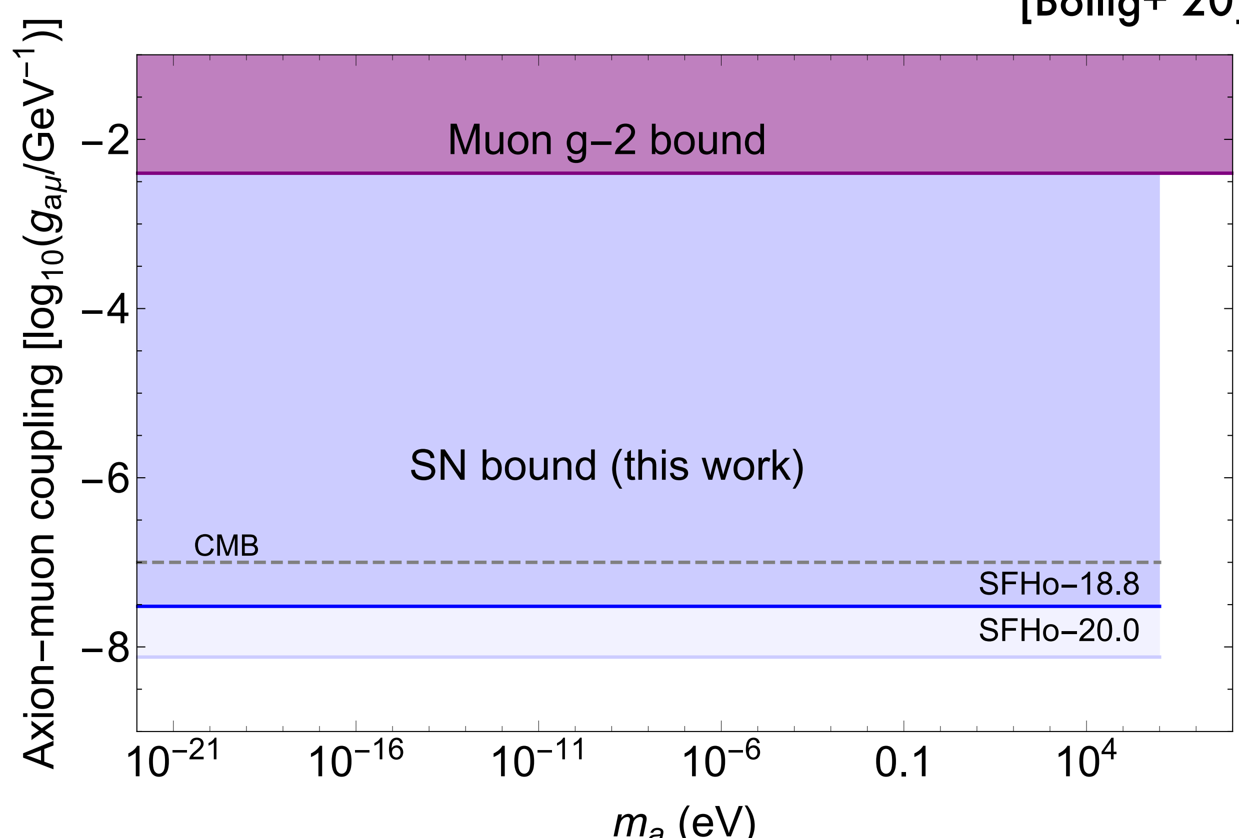

[From AxionLimits]

# Astrophysical constraints

Super-nova cooling constraint:




[Bollig+ 20]

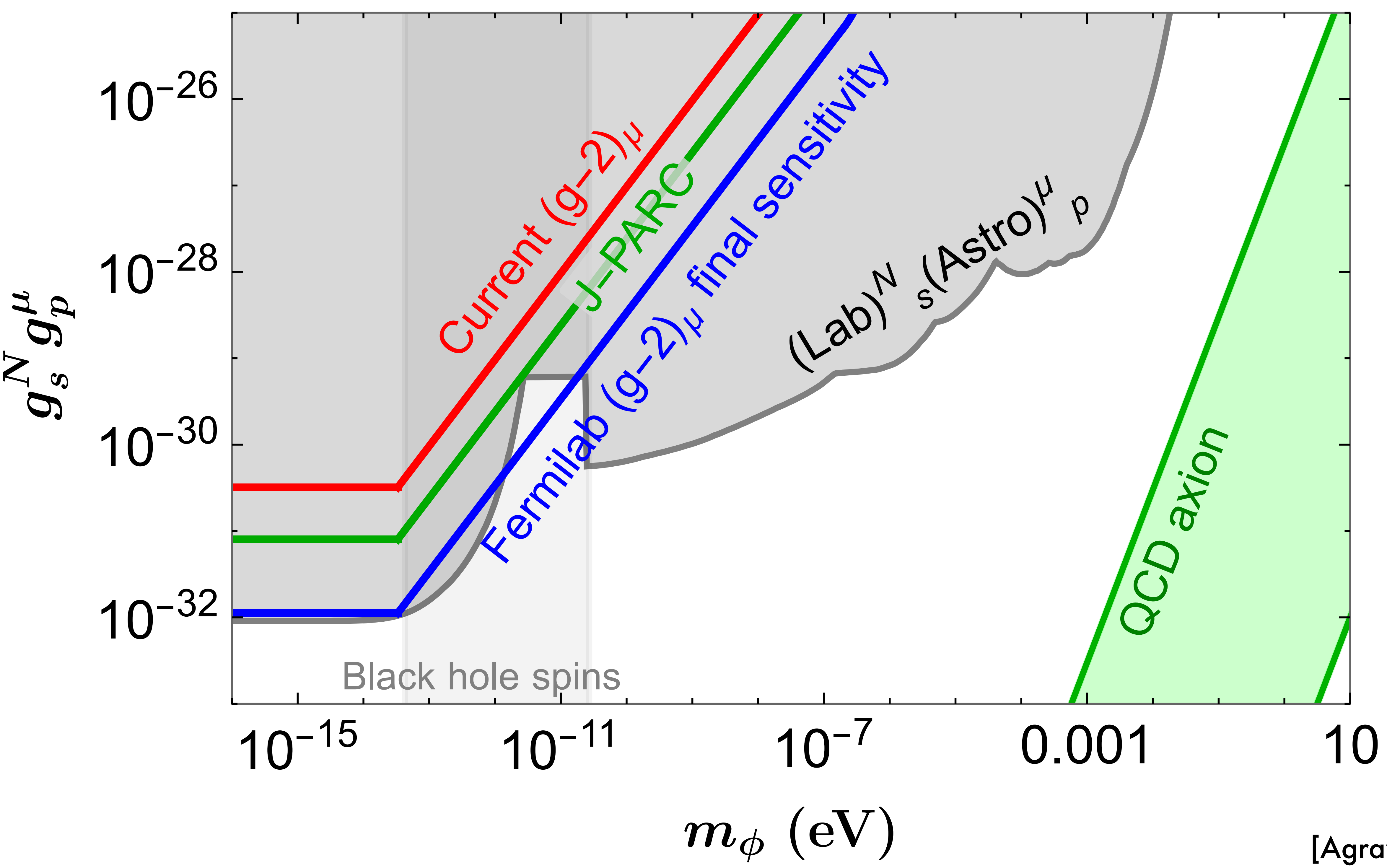



$(g_{a\mu} = g_P^{(\mu)}/2m_\mu)$

- SN modeling (muon population inside SN) uncertainty exits.
- Can be avoided by model building, e.g. by increasing mass with finite density.

[DeRocco+ 20; Davoudiasl+ 22]

# Couplings for different masses




[Agrawal+ 22]

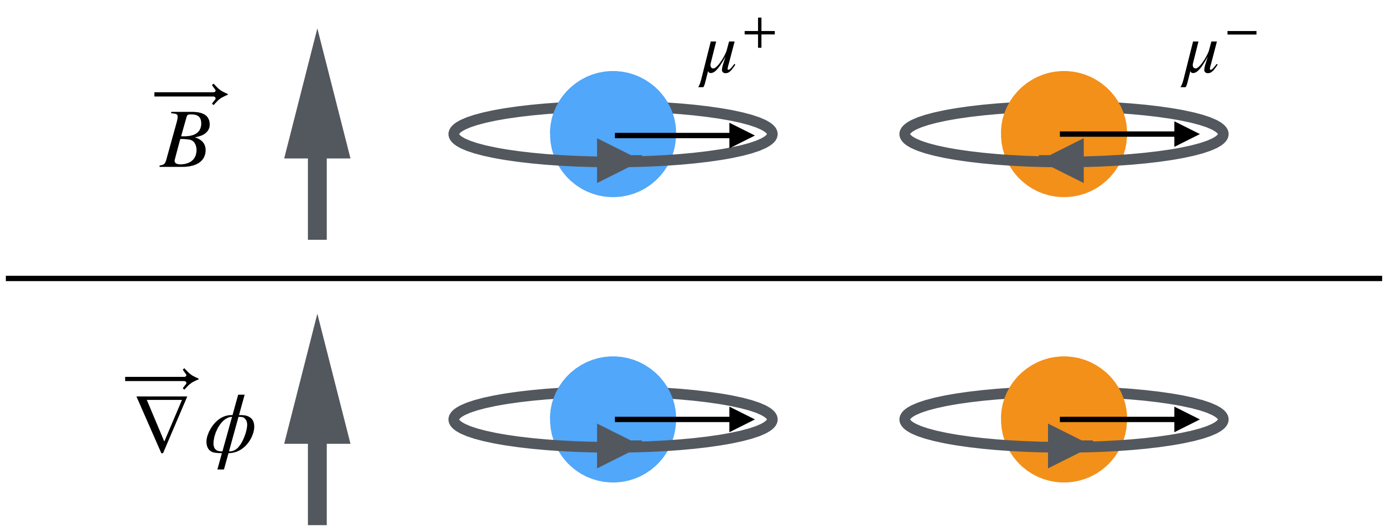
# Spin force vs. magnetic field

- Different transformation under boost:

$$\begin{cases} \vec{B}_{\parallel} \to \vec{B}_{\parallel}, \quad \vec{B}_{\perp} \to \gamma\left(\vec{B}_{\perp} - \vec{\beta} \times \vec{E}\right), \\ \vec{\nabla}_{\parallel}\phi \to \gamma \vec{\nabla}_{\parallel}\phi, \quad \vec{\nabla}_{\perp}\phi \to \vec{\nabla}_{\perp}\phi. \end{cases}$$

Effect gets bigger for smaller $\gamma$ ("maximal" for muon at rest).

- Act differently on $\mu^+$ and $\mu^-$: $\bar{\psi}\gamma_\mu\psi \xrightarrow{C} -\bar{\psi}\gamma_\mu\psi, \quad \bar{\psi}\gamma_\mu\gamma_5\psi \xrightarrow{C} +\bar{\psi}\gamma_\mu\gamma_5\psi\,.$



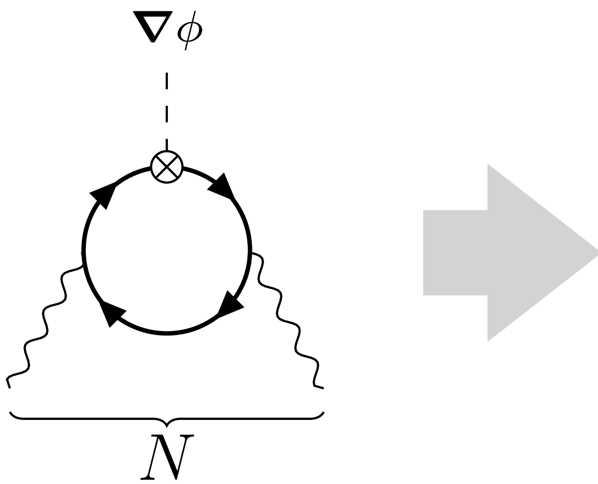

BNL used both $\mu^\pm$ but flipping $\vec{B}$, FNAL rotates in the same direction as BNL
→ consistent with the above.

[Agrawal+ 22]

($\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi \xrightarrow{C} -\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi$ so vector field with EDM-like coupling to muon does not work.)

# Photon "spin" coupling

- Photon "spin" coupling induced at one-loop, sourced by nuclear EM fields:




$$\mathscr{L}_{\text{eff}} = \frac{e^2 \vec{\nabla}\phi}{4\pi^2 m_\mu} \cdot \left( g_P^{(\mu)} \vec{B} A_0 - \frac{g_A^{(\mu)}}{12 m_\mu^2} \vec{B} \rho_N \right).$$

(Diagram dominated by IR → $g_A^{(\mu)}$ suppressed due to decoupling.)

- Nuclear magnetic field not very well understood:

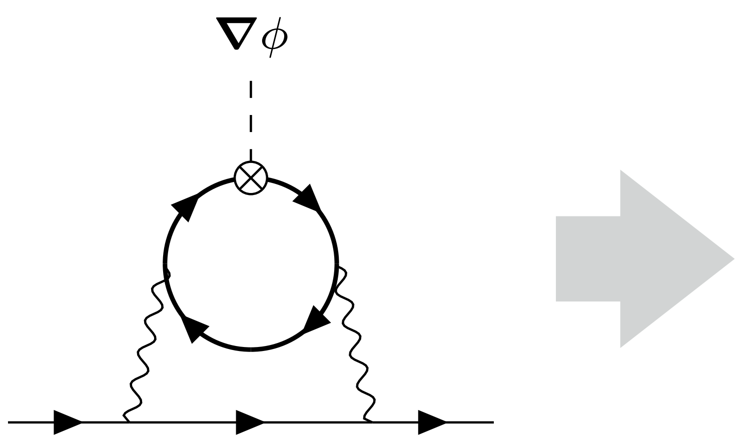
| | $^{199}$Hg | $^{201}$Hg | $^{129}$Xe | $^{131}$Xe |
|---|---|---|---|---|
| Valence $n$ | $-0.33\mu_n$ | $\mu_n$ | $\mu_n$ | $-0.6\mu_n$ |
| Experiment | $-0.26\mu_n$ | $0.29\mu_n$ | $0.41\mu_n$ | $-0.36\mu_n$ |

- We model $\vec{B}$ as valence neutron + "core" contributions,

$$\vec{B} = \vec{B}_n + "\vec{B}_{\text{core}}" \quad \text{with} \quad \vec{B}_{\text{core}} = \frac{\mu_c}{4\pi} \frac{3\hat{r}(\hat{r}\cdot\vec{I}) - \vec{I}}{r^3} \Theta(r - R_N) + \frac{\mu_c \vec{I}}{2\pi R_N^3} \Theta(R_N - r).$$

# Nucleon spin coupling

- Quark spin coupling induced at two-loop:




$$\mathscr{L}_{\rm eff} = -\, g_A^{(\mu)} \times \frac{3}{4}\left(\frac{\alpha}{\pi}\right)^2 Q^2 \log\left(\frac{\Lambda_{\rm UV}^2}{\Lambda_{\rm IR}^2}\right) \partial^\alpha\phi \times \bar{\psi}\gamma_\alpha\gamma_5\psi.$$

(photon "spin" is more important for $g_P$.)

- Quark spin to nucleon spin by lattice: [FLAG 2021]

$$g_A^{(p)} = 0.777 g_A^{(u)} - 0.438 g_A^{(d)} - 0.053 g_A^{(s)}, \quad g_A^{(n)} = -\,0.438 g_A^{(u)} + 0.777 g_A^{(d)} - 0.053 g_A^{(s)}.$$

- Nucleon spin to nuclear spin by shell model (available only for Xe): [Klos+ 2013]

$$\Delta E_{129} + 0.37337 \Delta E_{131} = (3-7) \times 10^{-8} \times \Delta E_\mu \times \frac{\log(\Lambda_{\rm UV}^2/\Lambda_{\rm IR}^2)}{\log(m_\tau^2/m_\mu^2)}.$$

Different values corresponding to different models, again nuclear physics uncertainty.