Weak lensing analysis

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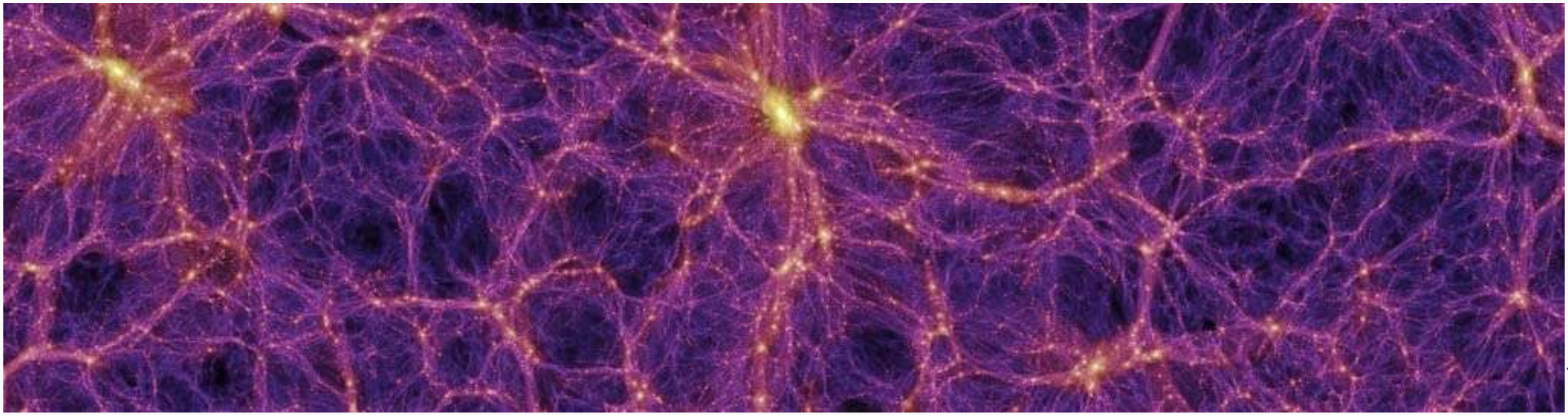
Plan of this talk

- basic of weak lensing
- measurement of shear and its error
- power spectrum analysis
- mass map analysis

Why gravitational lensing?



Millennium Simulation Project

- density fluctuations contain rich information

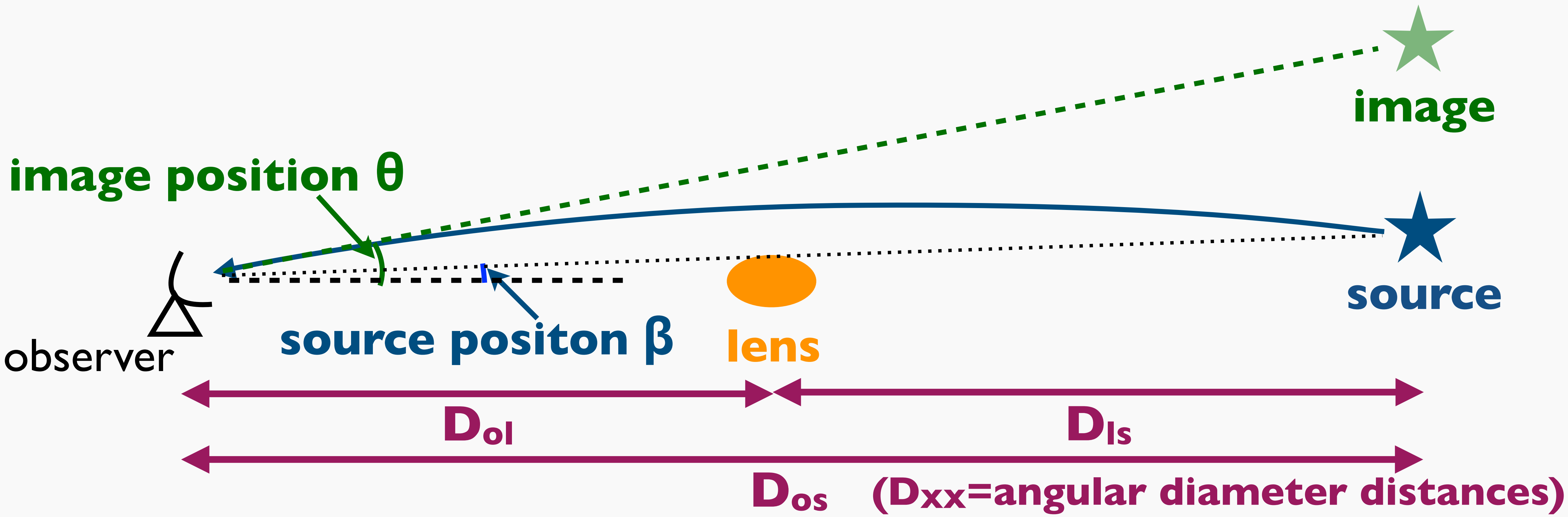
≈ **dark matter** density ⬅ **directly** probed by **gravitational lensing**!

Lens equation

- mapping from image position $\boldsymbol{\theta}$ to source position $\boldsymbol{\beta}$

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) = \boldsymbol{\theta} - \nabla_{\boldsymbol{\theta}} \boxed{\psi}$$ **lens potential**

$$\boxed{\psi(\boldsymbol{\theta})} = \frac{2}{c} \int_0^{z_{\rm s}} dz \frac{D_{\rm ls}}{H(z)(1+z)D_{\rm ol}D_{\rm os}} \boxed{\Phi(z, \boldsymbol{\theta})}$$ **gravitational potential**



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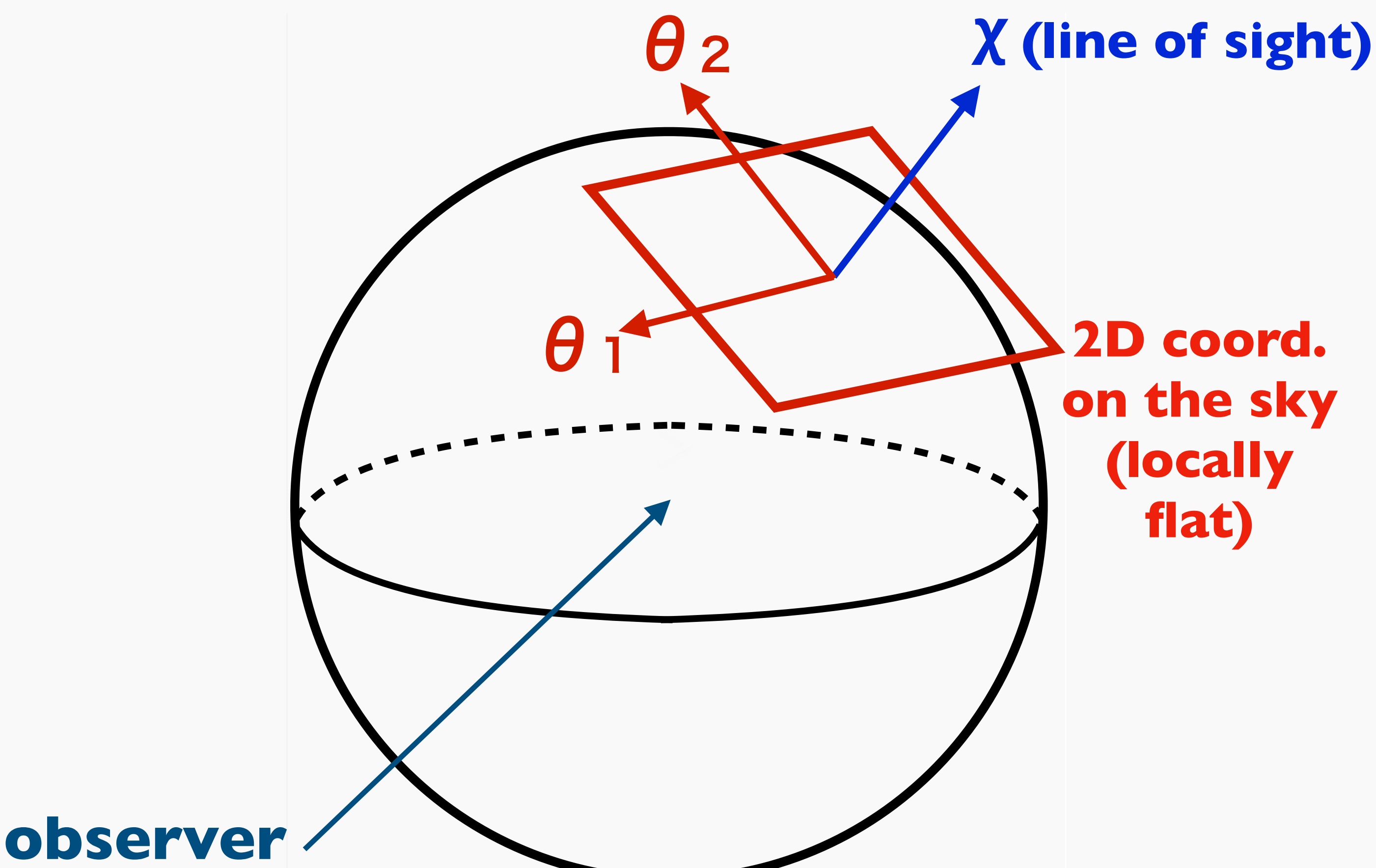
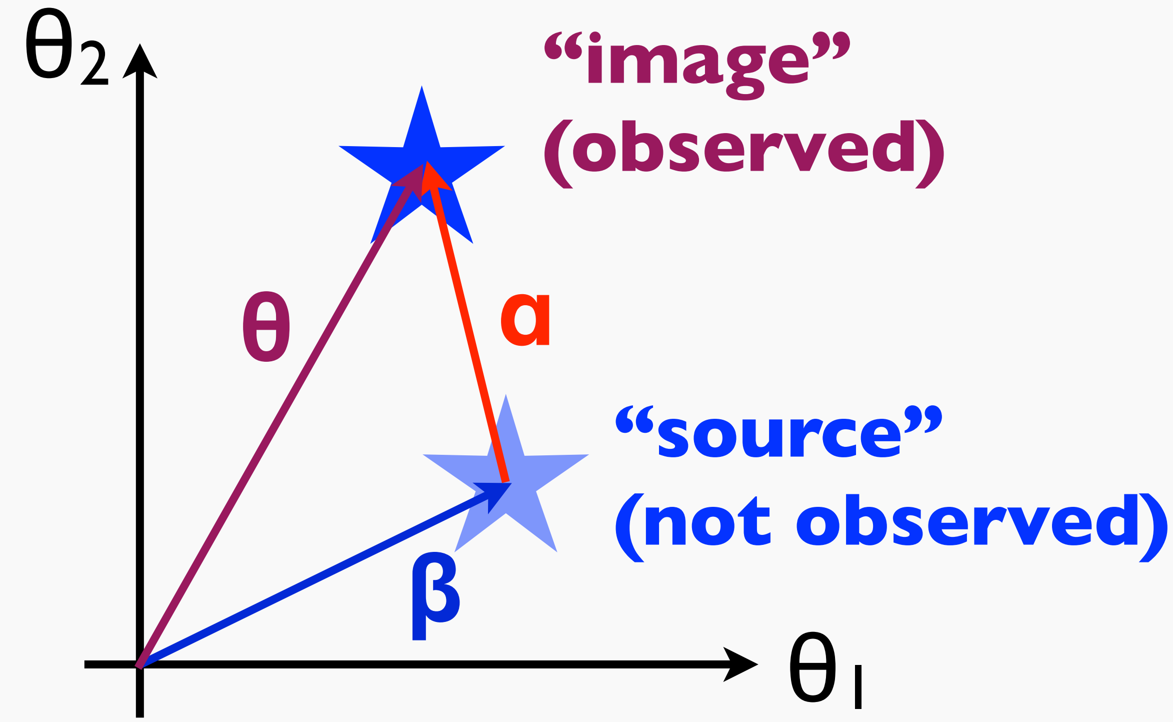
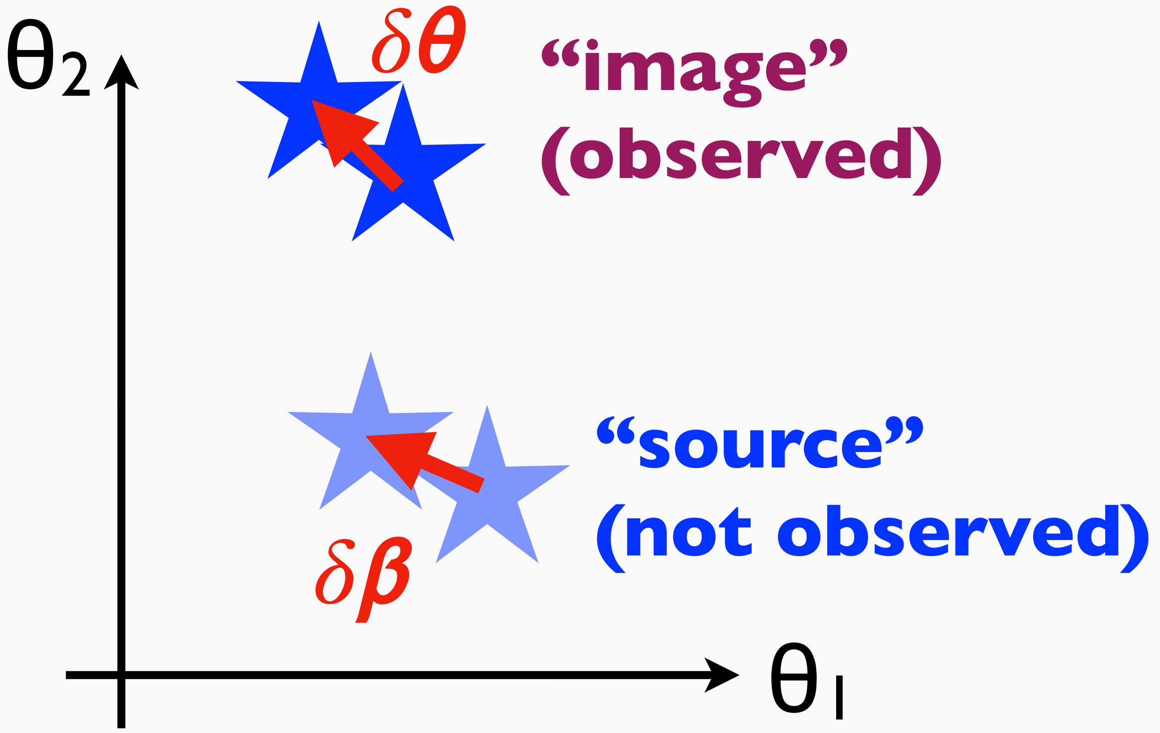




Weak gravitational lensing

- lens equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \nabla_{\boldsymbol{\theta}} \psi$$



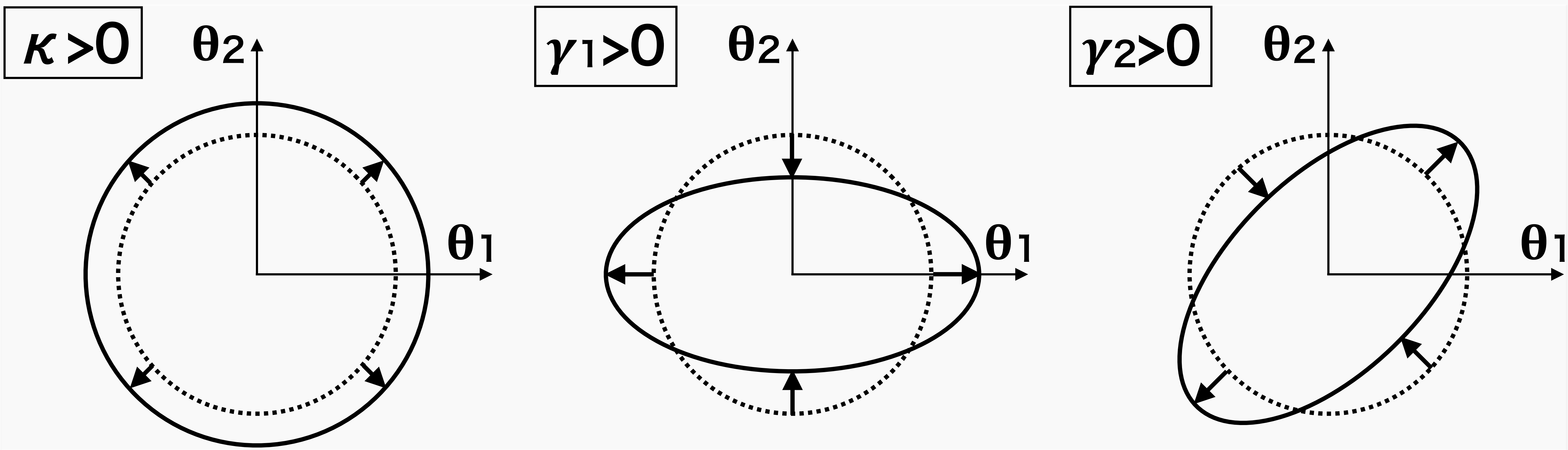
- image deformation

$$\delta\boldsymbol{\beta} = A(\boldsymbol{\theta})\delta\boldsymbol{\theta}$$

$$A(\boldsymbol{\theta}) := \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} 1 - \psi_{,\theta_1\theta_1} & -\psi_{,\theta_1\theta_2} \\ -\psi_{,\theta_1\theta_2} & 1 - \psi_{,\theta_2\theta_2} \end{pmatrix}$$

measure lensing signals from image deformations

Weak lensing distortions



convergence κ

not easy to measure

(**trace part of A**)

shear γ

measured from galaxy shapes

(**traceless part of A**)

Convergence and shear

lens potential (Born approximation)

$$\psi(\boldsymbol{\theta}) = \frac{2}{c}\int_0^{z_{\mathrm{s}}} dz \frac{D_{\mathrm{ls}}}{H(z)(1+z)D_{\mathrm{ol}}D_{\mathrm{os}}}\Phi(z,\boldsymbol{\theta})$$

2nd derivative

2nd derivative

$$\kappa := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} + \psi_{,\theta_2\theta_2}\right)$$

$$\gamma_1 := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} - \psi_{,\theta_2\theta_2}\right)$$

$$\gamma_2 := \psi_{,\theta_1\theta_2}$$

related

convergence κ

shear γ

Connection w/ density fluctuation

- from **lens potential** + **Poisson equation**

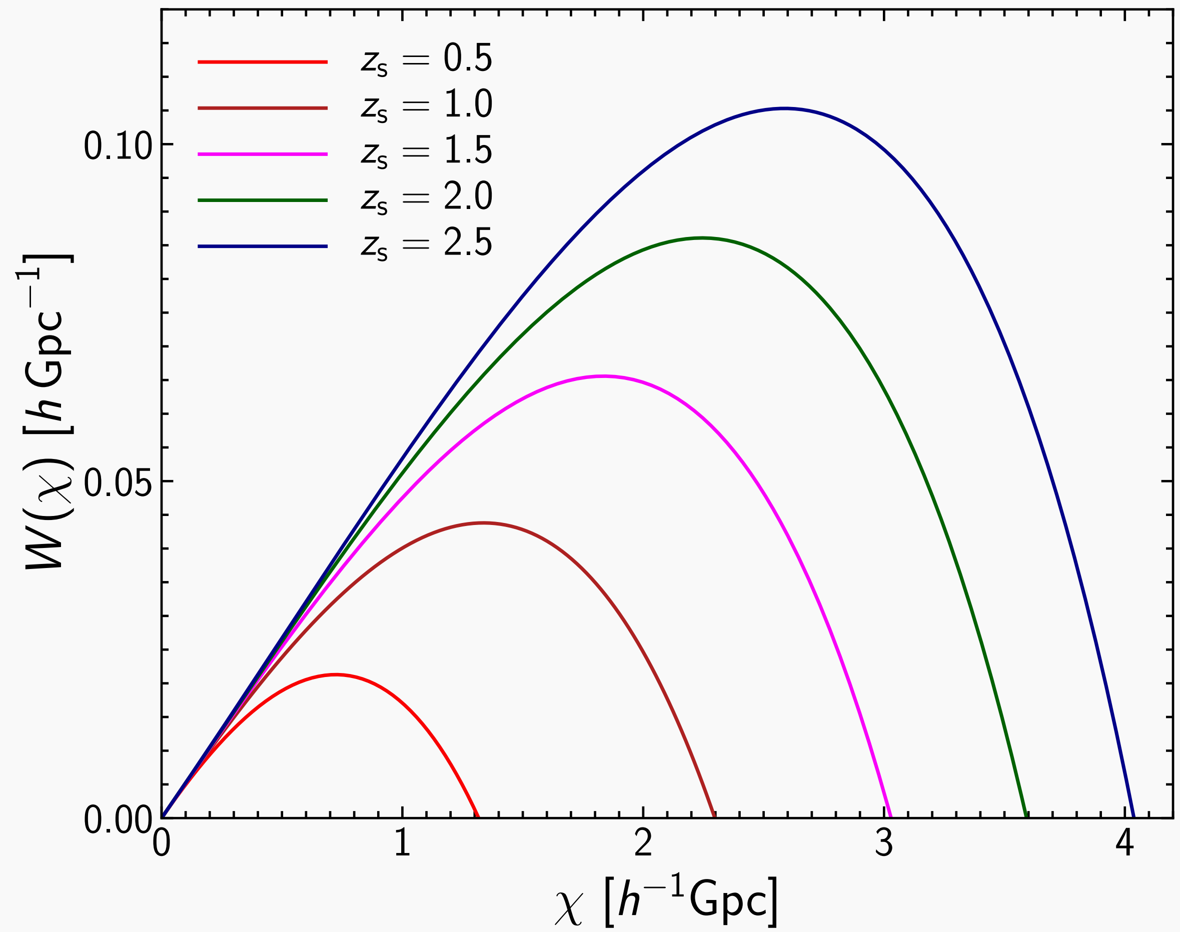
$$\kappa(\boldsymbol{\theta}) = \int_0^{\chi_\mathrm{s}} d\chi\, W(\chi)\delta_\mathrm{m}(\chi, \boldsymbol{\theta})$$

convergence
= projected surface density

$$W(\chi) := \frac{3\Omega_{\mathrm{m}0} H_0^2}{2c^2} \frac{f_K(\chi_\mathrm{s} - \chi) f_K(\chi)}{a\, f_K(\chi_\mathrm{s})}$$

$$= \frac{a\bar{\rho}_\mathrm{m}(\chi)}{\Sigma_\mathrm{cr}(\chi,\, \chi_\mathrm{s})}$$

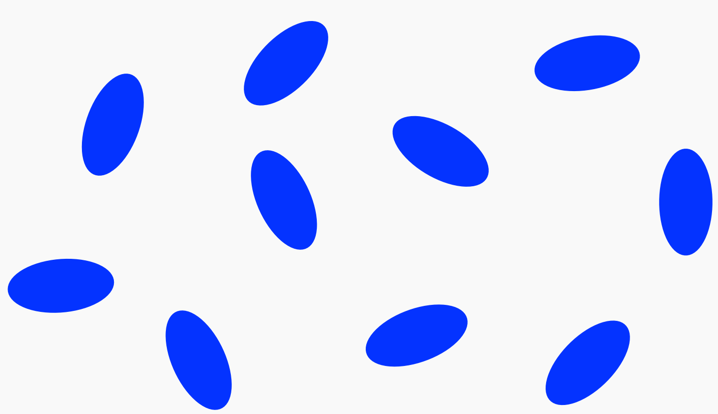
weight along line-of-sight



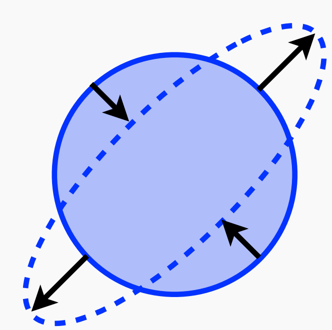
Measuring shear

- assuming **orientations of galaxies are random**

average of shapes of galaxies in some region → average shear γ in the region



average shapes



measured shear γ

Short summary of weak lensing

lens potential (Born approximation)

$$\psi(\boldsymbol{\theta}) = \frac{2}{c}\int_0^{z_{\rm s}} dz \frac{D_{\rm ls}}{H(z)(1+z)D_{\rm ol}D_{\rm os}}\Phi(z,\boldsymbol{\theta})$$

2nd derivative

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$$\kappa := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} + \psi_{,\theta_2\theta_2}\right)$$

related

$$\gamma_1 := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} - \psi_{,\theta_2\theta_2}\right)$$

$$\gamma_2 := \psi_{,\theta_1\theta_2}$$

shear γ

= measured from galaxy shapes

infer

convergence κ
= projected mass distribution

Error of measurement (1)

- it is useful to define complex shear/ellipticity

$$\gamma := \gamma_1 + i\gamma_2$$

$$\epsilon := \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}} \qquad Q_{ab} := \frac{\int d\boldsymbol{\theta} I(\boldsymbol{\theta})\theta_a\theta_b}{\int d\boldsymbol{\theta} I(\boldsymbol{\theta})}$$

2nd moment of surface brightness of galaxy

Error of measurement (2)

- relation btw source and image ellipticities

$$Q^{(\mathrm{s})}_{ab} := \frac{\int d\boldsymbol{\beta} I(\boldsymbol{\beta})\beta_a\beta_b}{\int d\boldsymbol{\beta} I(\boldsymbol{\beta})} \simeq A_{ac}A_{bd}Q_{cd}$$

$$\epsilon = \frac{\epsilon^{(\mathrm{s})} + 2g + g^2\epsilon^{(\mathrm{s})*}}{1 + |g|^2 + 2\mathrm{Re}(g\epsilon^{(\mathrm{s})*})}$$

$$\boxed{\langle\epsilon\rangle \simeq 2\langle g\rangle \simeq 2\langle\gamma\rangle}$$

average shapes of galaxies

average shear in the region

$$g := \frac{\gamma}{1-\kappa}$$

reduced shear

Error of measurement (3)

- order of magnitude

average shapes of galaxies

$$\boxed{\langle\epsilon\rangle \simeq \langle\epsilon^{(\mathrm{s})} + 2g\rangle \simeq 2\langle g\rangle \simeq 2\langle\gamma\rangle}$$

average shear in the region

$$\langle\epsilon^{(s)}\rangle = 0$$

$$\sigma_{\epsilon/2} \simeq 0.3$$

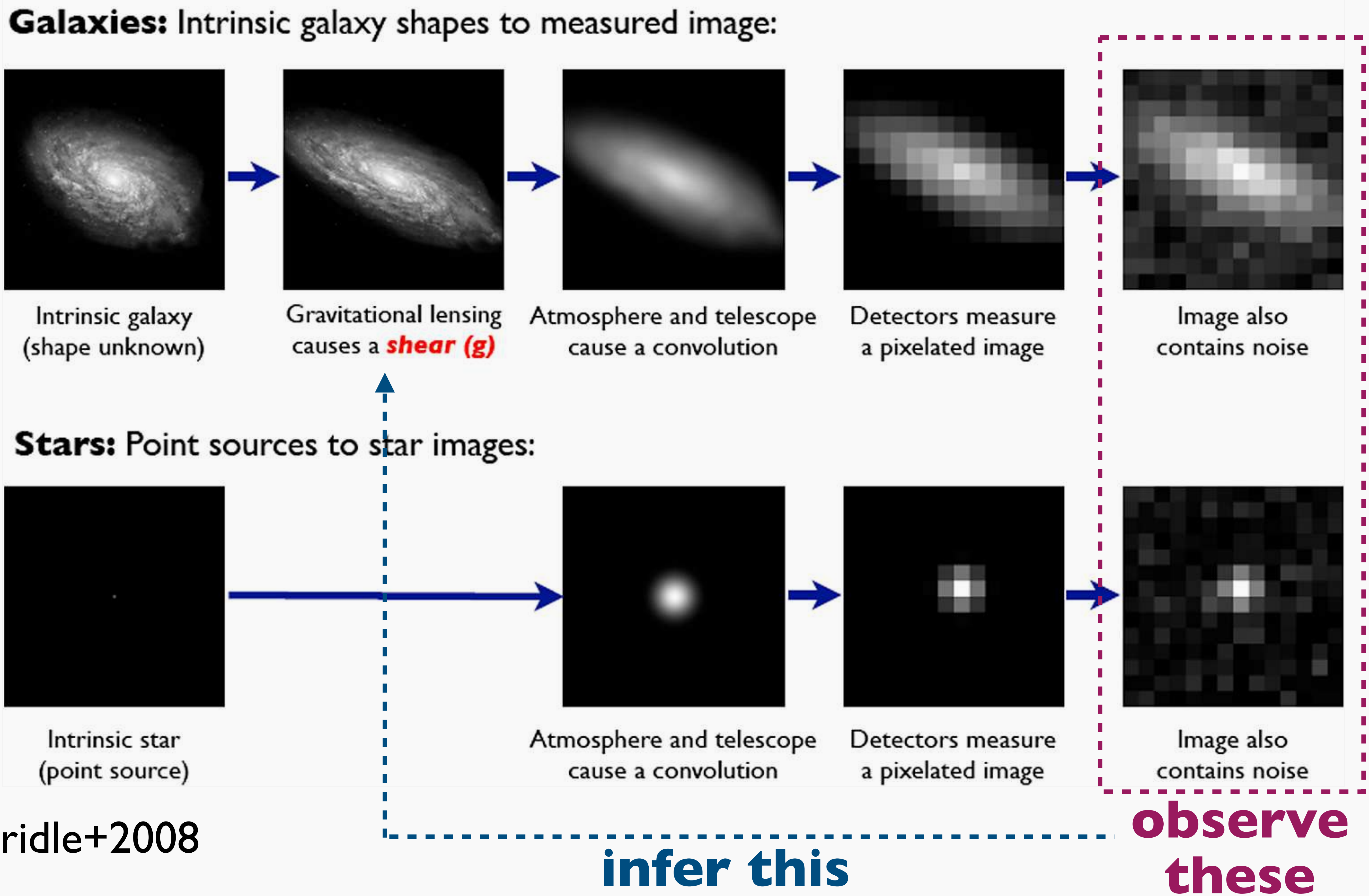
$$|\gamma| \sim 0.03 - 0.003$$

$$\gamma^{\mathrm{obs}} = \frac{1}{2N}\sum\epsilon \quad \Rightarrow \quad \frac{S}{N} \simeq \frac{|\gamma|}{\sigma_{\epsilon/2}/\sqrt{N}}$$

use *N* galaxies

need *N*~10$^{3\text{-}5}$ galaxies for detection!

Shape measurement



Bridle+2008

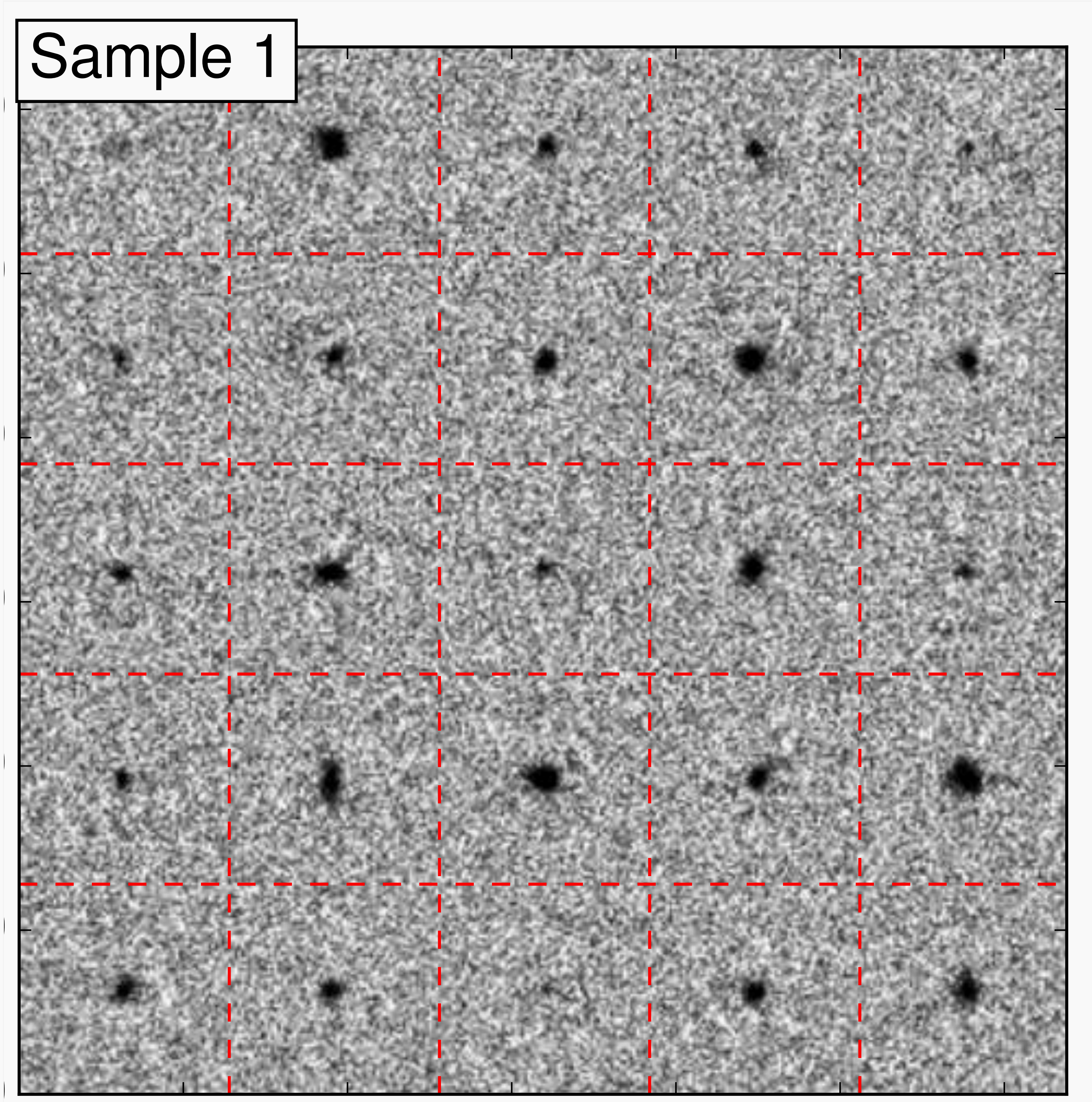
Calibration by image simulations

mock images of galaxies and stars

apply your shape measurements

compare input γ and inferred γ

calibrate residuals



Mandelbaum+2018

Power spectrum analysis

- how to quantify density fluctuations?

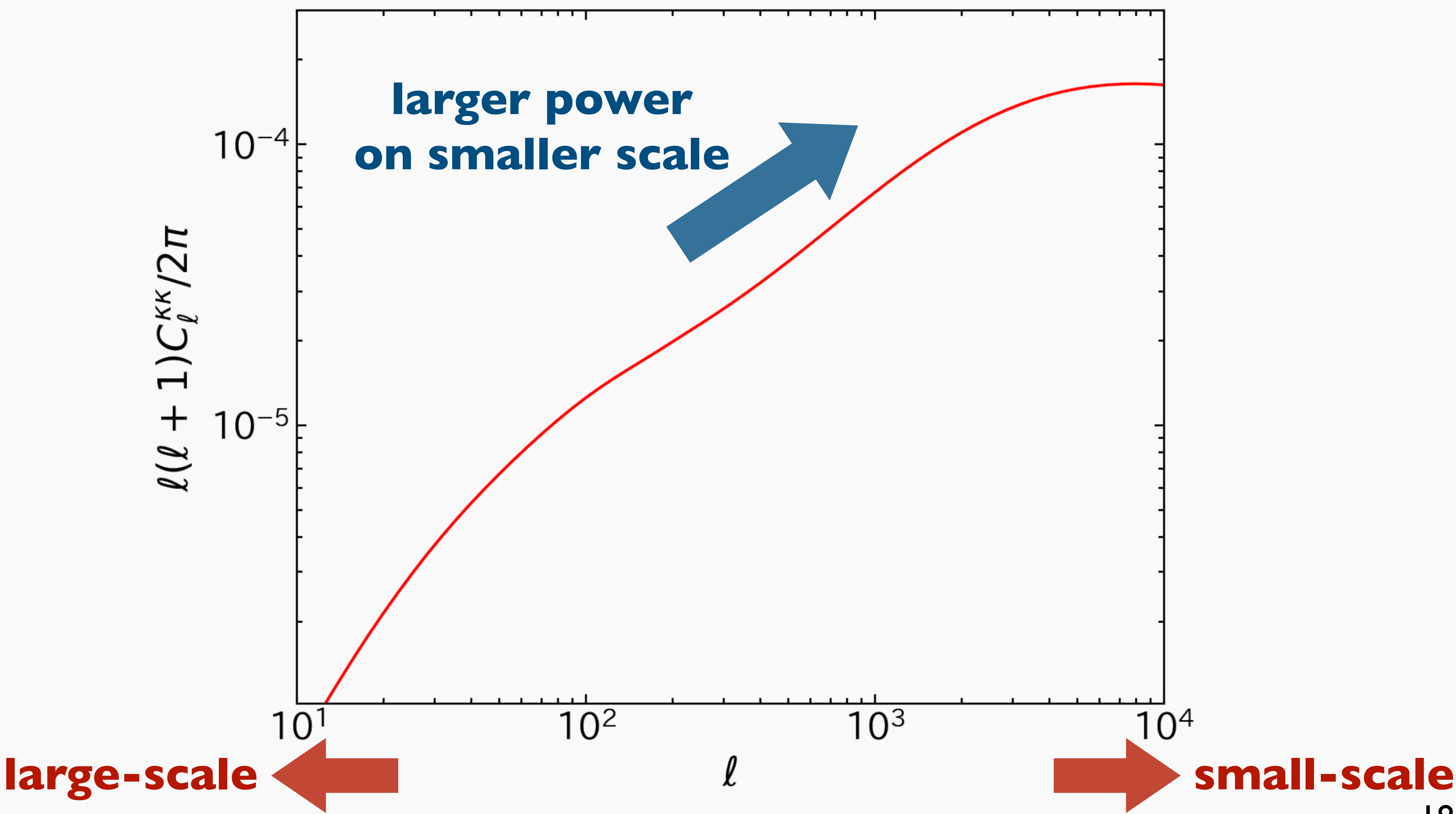
angular power spectrum

$$\tilde{\kappa}(\boldsymbol{\ell}) = \int d\boldsymbol{\theta}\, \kappa(\boldsymbol{\theta})\, e^{-i\boldsymbol{\ell}\cdot\boldsymbol{\theta}}$$

$$\langle \tilde{\delta}(\boldsymbol{\ell})\tilde{\delta}(\boldsymbol{\ell}')\rangle = (2\pi)^2 \delta^{\mathrm{D}}(\boldsymbol{\ell} + \boldsymbol{\ell}')\boxed{C_\ell}$$

(flat sky approximation)

Convergence power spectrum



From shear to convergence

$$\kappa := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} + \psi_{,\theta_2\theta_2}\right)$$

related

$$\gamma_1 := \frac{1}{2}\left(\psi_{,\theta_1\theta_1} - \psi_{,\theta_2\theta_2}\right)$$

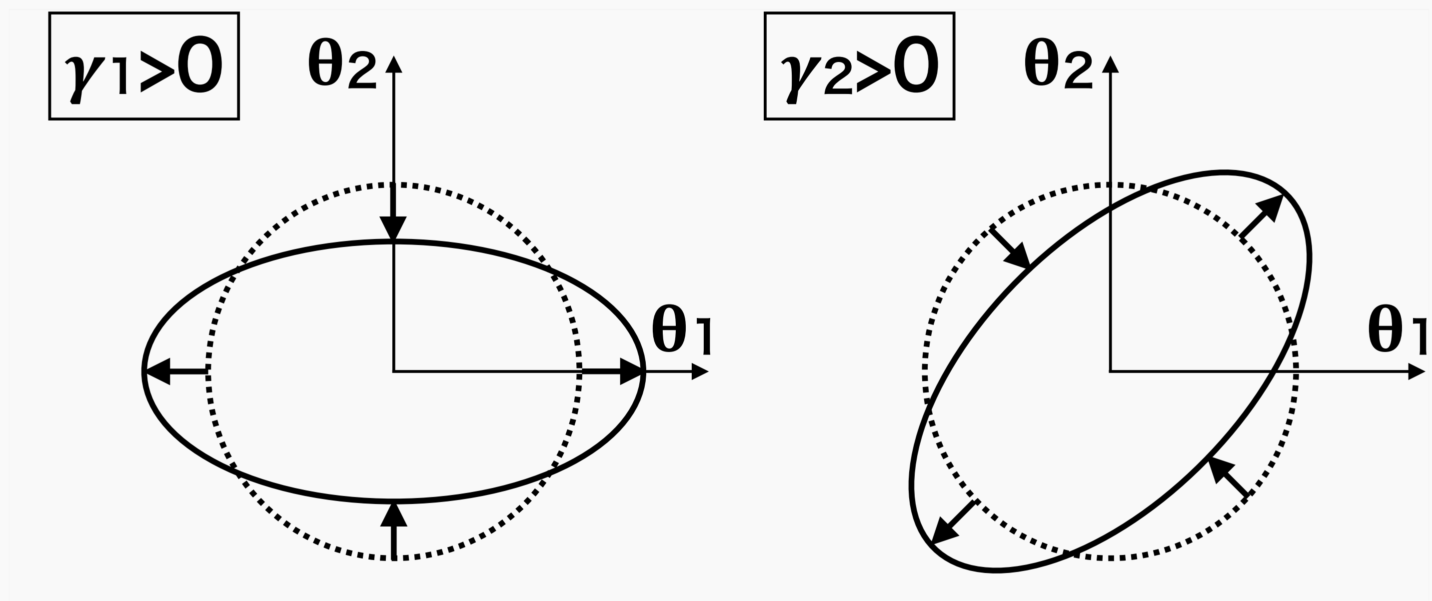
$$\gamma_2 := \psi_{,\theta_1\theta_2}$$

Fourier transform

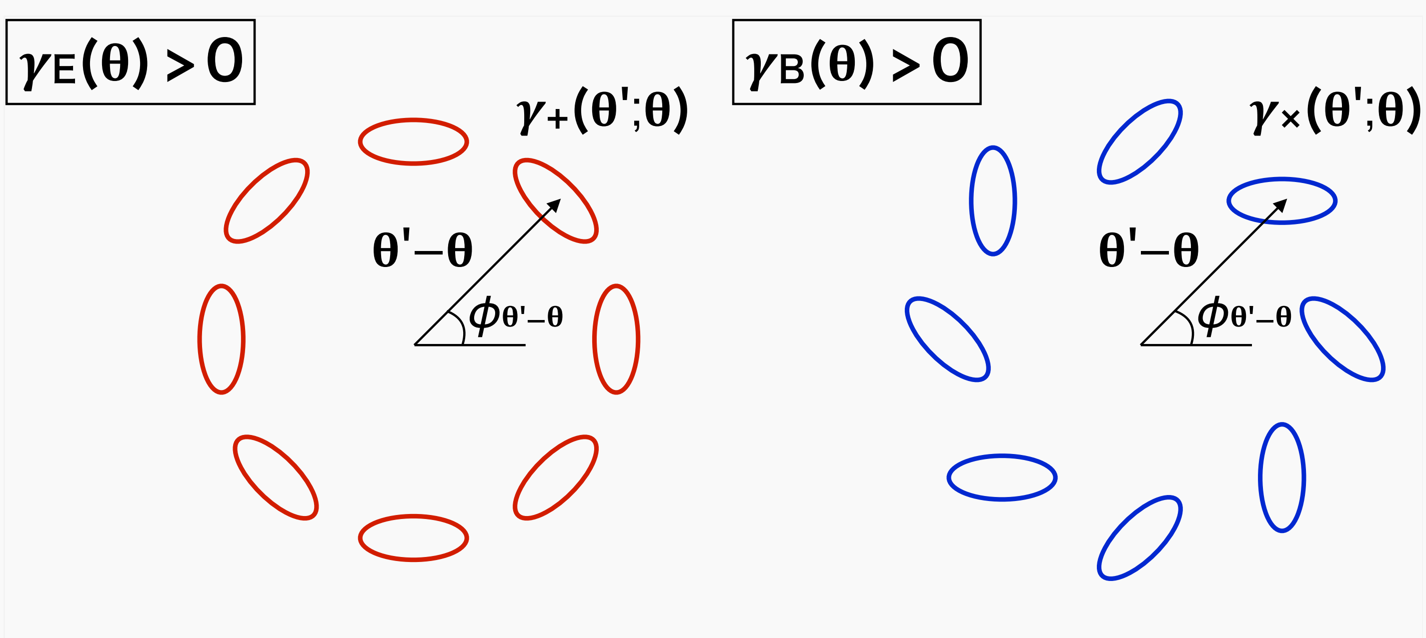
$\nabla_\theta \rightarrow -i\,\boldsymbol{\ell}$

$$\tilde{\kappa} = \frac{\ell_1^2 - \ell_2^2 - 2i\ell_1\ell_2}{\ell^2}\tilde{\gamma} = \mathrm{Re}(e^{-2i\varphi_\ell})\tilde{\gamma}$$

E/B decomposition



$$\tilde{\gamma}_{\rm E} + i\tilde{\gamma}_{\rm B} = e^{-2i\phi_{\ell}}\tilde{\gamma}$$



γ_1, γ_2

local

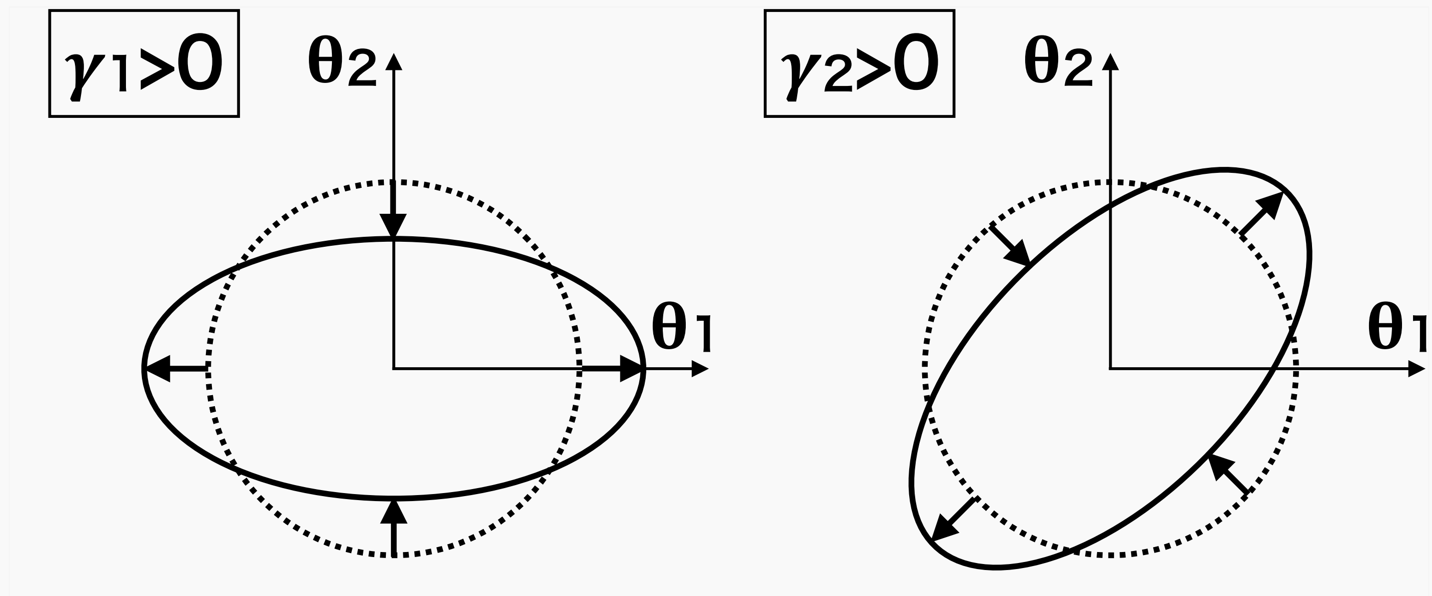
coordinate-dependent

γ_E, γ_B

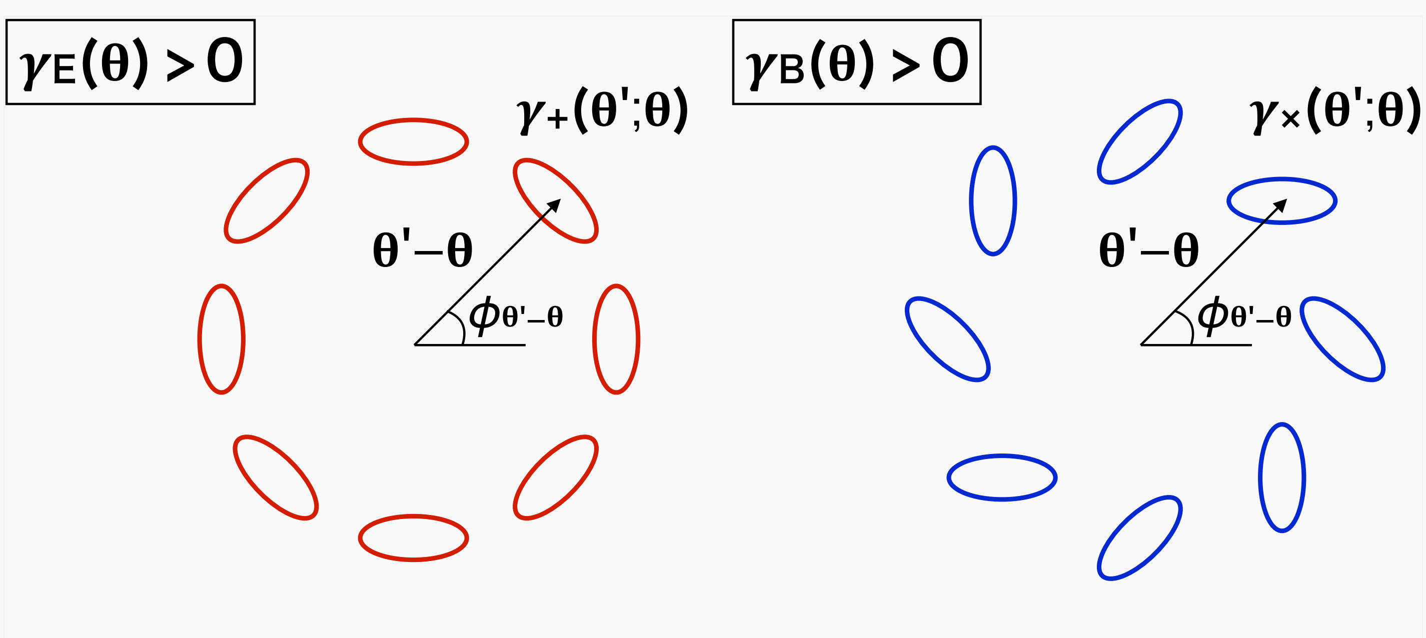
non-local

coordinate-independent

E/B decomposition



$$\tilde{\gamma}_{\rm E} + i\tilde{\gamma}_{\rm B} = e^{-2i\phi_\ell}\tilde{\gamma}$$



γ_1, γ_2

local

coordinate-dependent

(Born approximation)

$\gamma_E = \kappa$ $\gamma_B = 0$

γ_E, γ_B

non-local

coordinate-independent

Basic procedure of analysis

(over-simplified…)

- measure galaxy shapes
- construct shear field
- E/B decomposition → convergence filed
- measure power spectrum
- compare it with theoretical model to extract e.g., amplitude of density fluctuations

Estimating power spectrum (1)

- divide the sky into small cells with each area $\Delta\Omega$
- shear filed from discrete galaxy sample (N_i=0 or 1)

$$\gamma^{\rm obs}(\boldsymbol{\theta}) := \frac{1}{\bar{n}} \sum_i N_i \gamma_i^{\rm obs} \delta^{\rm D}(\boldsymbol{\theta}-\boldsymbol{\theta}_i)$$

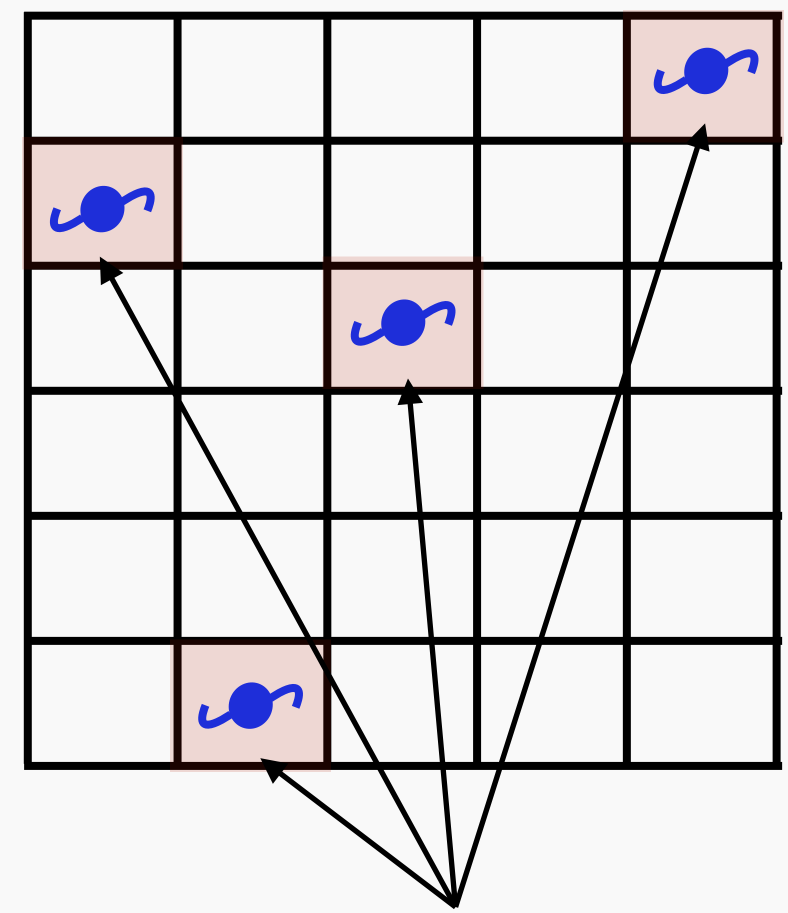
galaxy number density

run over all cells

$$\langle \gamma^{\rm obs}(\boldsymbol{\theta}) \rangle = \sum_i \Delta\Omega\, \gamma(\boldsymbol{\theta}_i) \delta^{\rm D}(\boldsymbol{\theta}-\boldsymbol{\theta}_i)$$

$$\simeq \int d\Omega\, \gamma(\boldsymbol{\theta}_i) \delta^{\rm D}(\boldsymbol{\theta}-\boldsymbol{\theta}_i)$$

$$= \gamma(\boldsymbol{\theta})$$



N_i=1

(N_i=0 for empty cell)

Estimating power spectrum (2)

- Fourier transform of the shear field

$$\tilde{\gamma}^{\rm obs}(\boldsymbol{\ell}) = \frac{1}{\bar{n}} \sum_i N_i \gamma_i^{\rm obs} e^{-i\boldsymbol{\ell}\cdot\boldsymbol{\theta}_i}$$

- compute E-mode power spectrum

$$\begin{aligned}
\langle \tilde{\gamma}_{\rm E}^{\rm obs}(\boldsymbol{\ell}) \tilde{\gamma}_{\rm E}^{\rm obs}(\boldsymbol{\ell}') \rangle &= \frac{1}{\bar{n}^2} \sum_{i,j} \langle N_i N_j \gamma_{{\rm E},i}^{\rm obs} \gamma_{{\rm E},j}^{\rm obs} \rangle e^{-i(\boldsymbol{\ell}\cdot\boldsymbol{\theta}_i + \boldsymbol{\ell}'\cdot\boldsymbol{\theta}_j)} \\
&= \frac{1}{\bar{n}} \sum_i \Delta\Omega \frac{\sigma_{\epsilon/2}^2}{2} e^{-i(\boldsymbol{\ell}+\boldsymbol{\ell}')\cdot\boldsymbol{\theta}_i} \\
&\quad + \sum_{i,j} (\Delta\Omega)^2 \langle \gamma_{\rm E}(\boldsymbol{\theta}_i) \gamma_{\rm E}(\boldsymbol{\theta}_j) \rangle e^{-i(\boldsymbol{\ell}\cdot\boldsymbol{\theta}_i + \boldsymbol{\ell}'\cdot\boldsymbol{\theta}_j)} \\
&= (2\pi)^2 \delta^{\rm D}(\boldsymbol{\ell} + \boldsymbol{\ell}') \left(\underbrace{\frac{\sigma_{\epsilon/2}^2}{2\bar{n}}}_{\text{shot noise}} + C_\ell^{\gamma_{\rm E}\gamma_{\rm E}} \right)
\end{aligned}$$

Shot noise

- power spectrum estimation from discrete galaxy sample is always affected by shot noise

$$C_\ell^{\gamma_E \gamma_E,\mathrm{obs}} = C_\ell^{\gamma_E \gamma_E} + \underbrace{\frac{\sigma_{\epsilon/2}^2}{2\bar{n}}}_{\textbf{shot noise}}$$

- higher galaxy number density leads to smaller shot noise

Covariance

- measurement error under Gaussian approx.

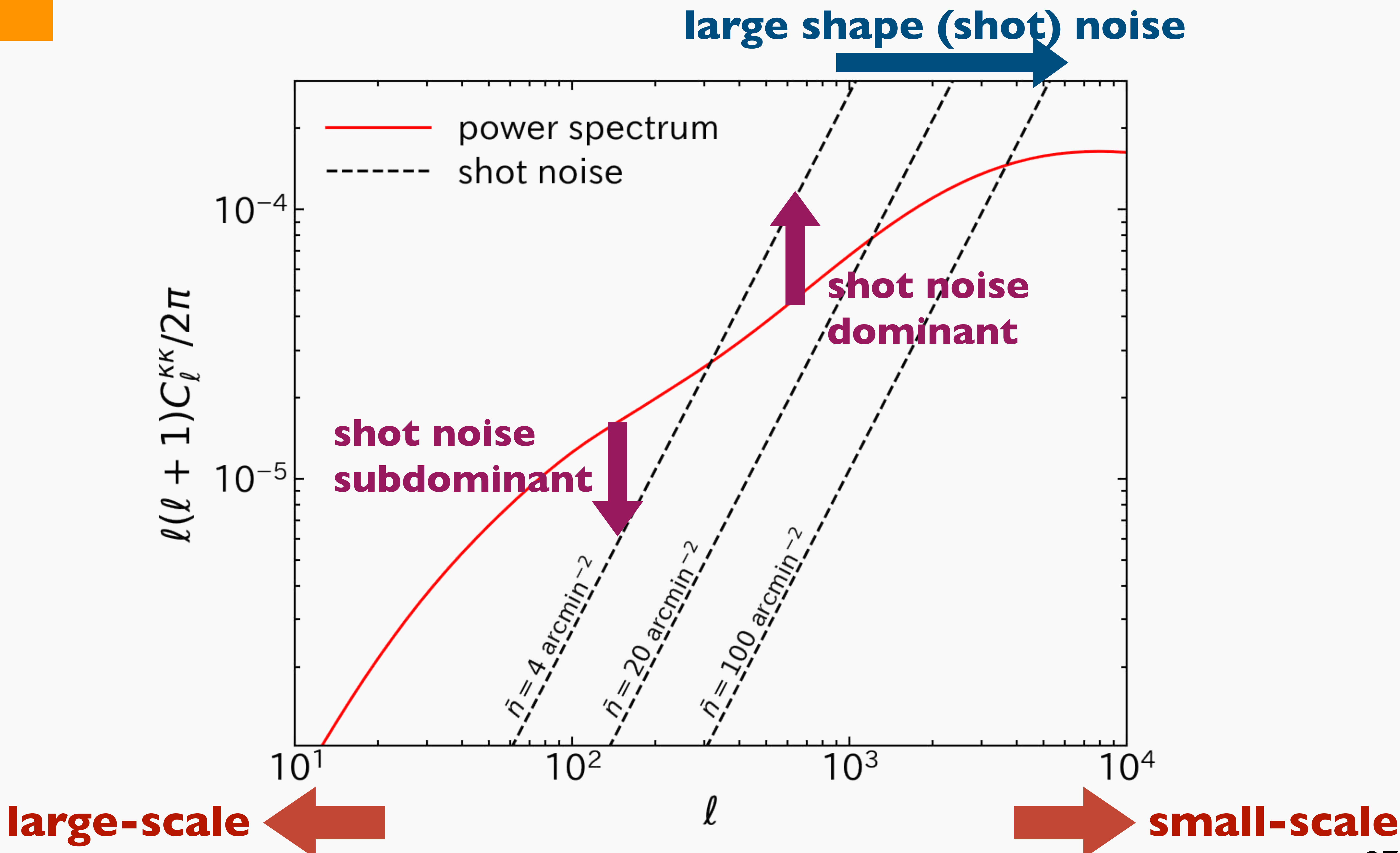
$$\left[\mathrm{Cov}(\hat{C}_\ell^{\gamma_E\gamma_E})\right]_{ij} = \frac{2\delta_{ij}}{N_{\mathrm{mode},i}}\Bigg(\underbrace{C_{\ell,i}^{\gamma_E\gamma_E}}_{\text{cosmic variance}} + \underbrace{\frac{\sigma_{\epsilon/2}^2}{2\bar{n}}}_{\text{shot noise}}\Bigg)^2$$

$$N_{\mathrm{mode},i} := \frac{\pi\left(\ell_{i,\max}^2 - \ell_{i,\min}^2\right)}{\Delta\ell^2} = \underbrace{f_{\mathrm{sky}}}_{=\Omega_s/4\pi \text{ survey area}}\left(\ell_{i,\max}^2 - \ell_{i,\min}^2\right)$$

- non-Gaussian error also important

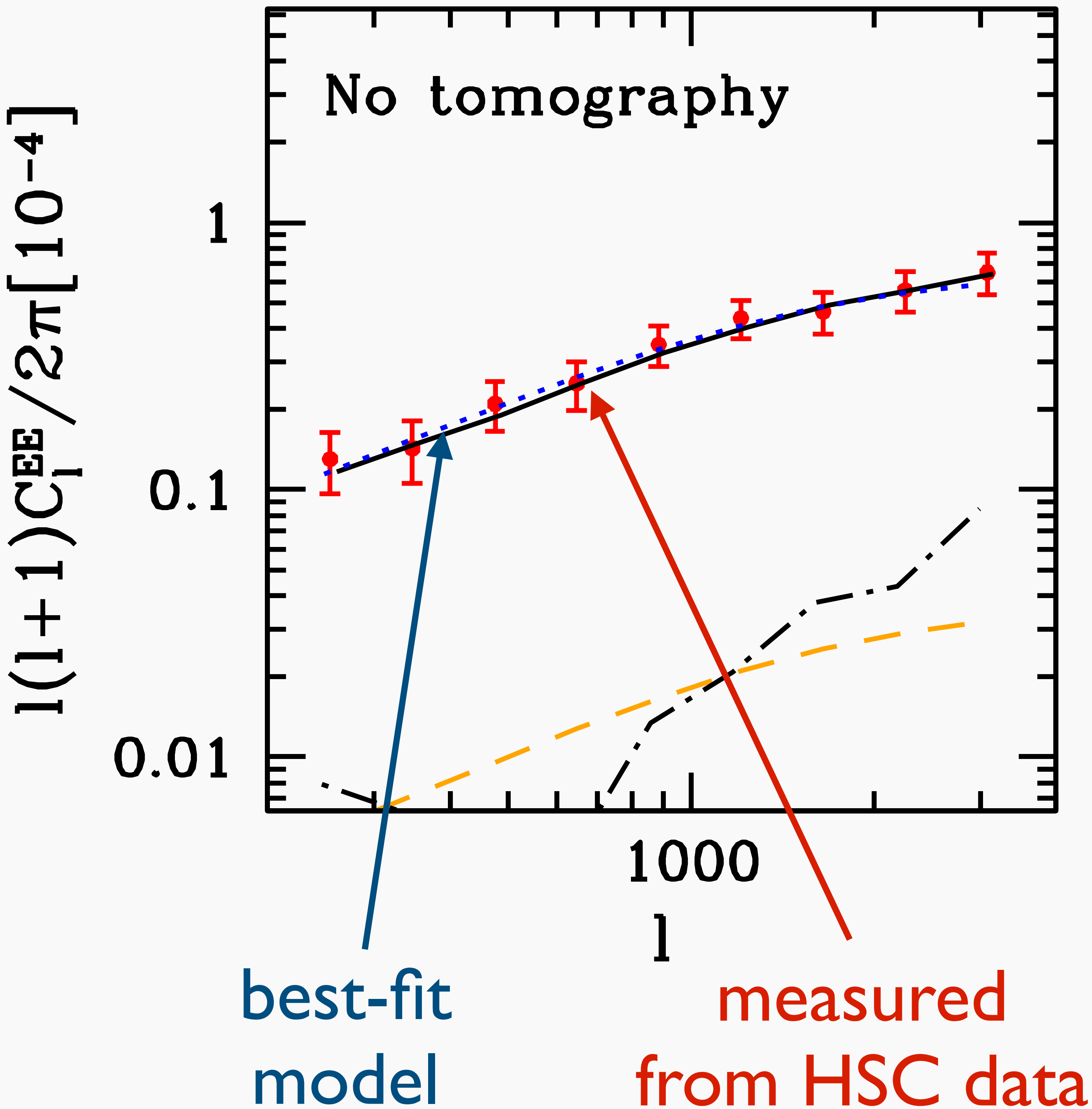
(e.g., Takada & Jain 2009; Takada & Hu 2013)

Effect of shot noise

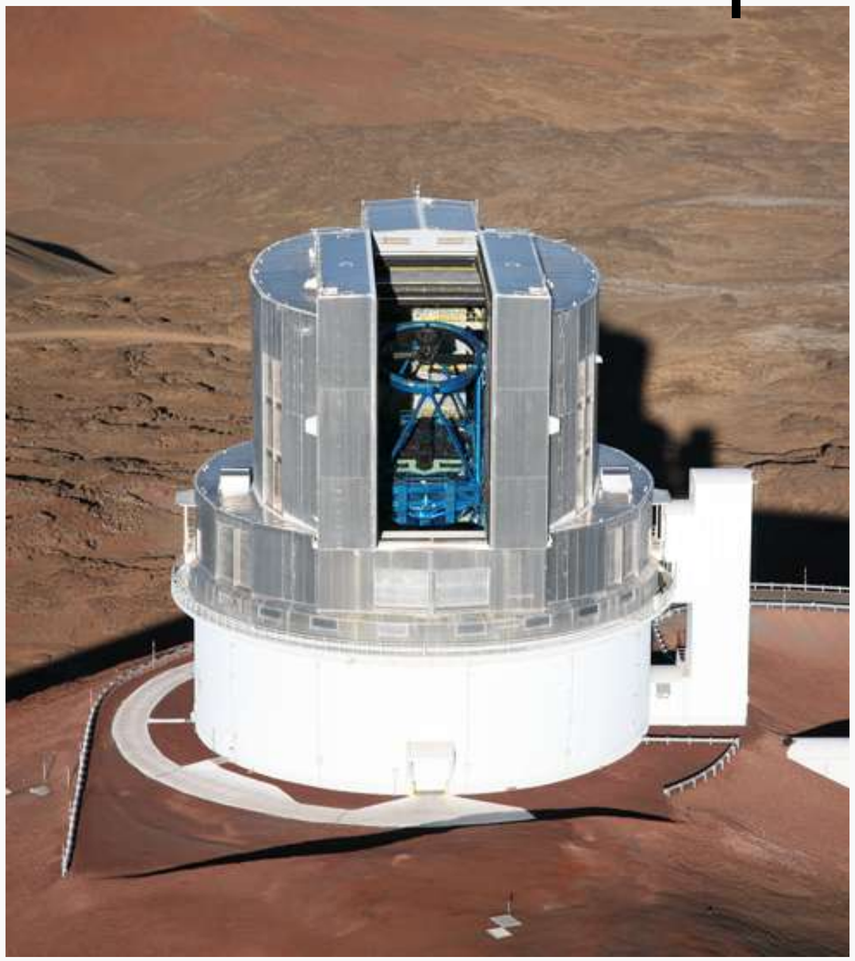


Hikage, MO+ PASJ **71** (2019)43

Example of measurement



HSC/Subaru telescope



NAOJ, https://www.naoj.org/

measurement with shapes of 9 million galaxies

Mass map analysis

- how to quantify density fluctuations?

mass (convergence) map

$$\tilde{\kappa} = \frac{\ell_1^2 - \ell_2^2 - 2i\ell_1\ell_2}{\ell^2}\tilde{\gamma} = \mathrm{Re}(e^{-2i\varphi_\ell})\tilde{\gamma}$$

inverse Fourier transform
(multiplication → convolution)

$$\kappa(\boldsymbol{\theta}) = \frac{1}{\pi}\int d\boldsymbol{\theta}'\gamma(\boldsymbol{\theta}')D^*(\boldsymbol{\theta} - \boldsymbol{\theta}')$$

$$D(\boldsymbol{\theta}) := \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1\theta_2}{|\boldsymbol{\theta}|^4}$$

Effect of shot noise

- error of mass map due to shot noise

$$\langle\{\kappa^{\mathrm{obs}}(\boldsymbol{\theta})\}^2\rangle_{\mathrm{shot}} \propto \int \frac{d\boldsymbol{\ell}}{(2\pi)^2}\frac{\sigma_{\epsilon/2}}{2\bar{n}} \to \infty$$

- need **smoothing** to suppress small-scale shot noise contribution

$$\gamma^{\mathrm{s}}(\boldsymbol{\theta}) = \int d\boldsymbol{\theta}'\gamma(\boldsymbol{\theta}')W_{\mathrm{s}}(\boldsymbol{\theta}-\boldsymbol{\theta}')$$ **smoothed shear filed**

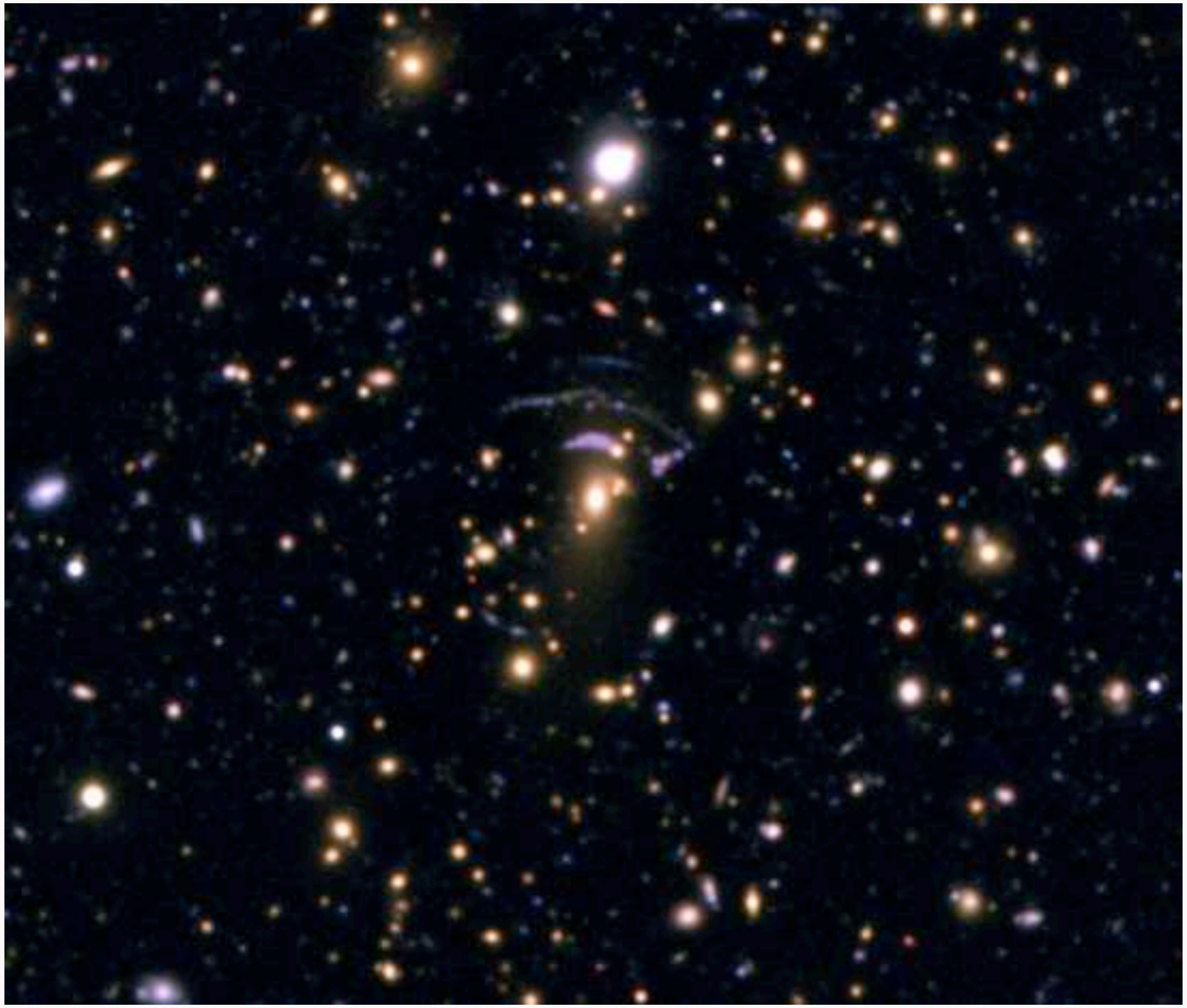
$$W_{\mathrm{s}}(\boldsymbol{\theta}) = \frac{1}{\pi\sigma_{\mathrm{s}}}\exp\left(-\frac{|\boldsymbol{\theta}|^2}{\sigma_{\mathrm{s}}^2}\right)$$

typically 1 (or several) arcmin

Example of mass map

- cluster SDSSJ1138 at z=0.45

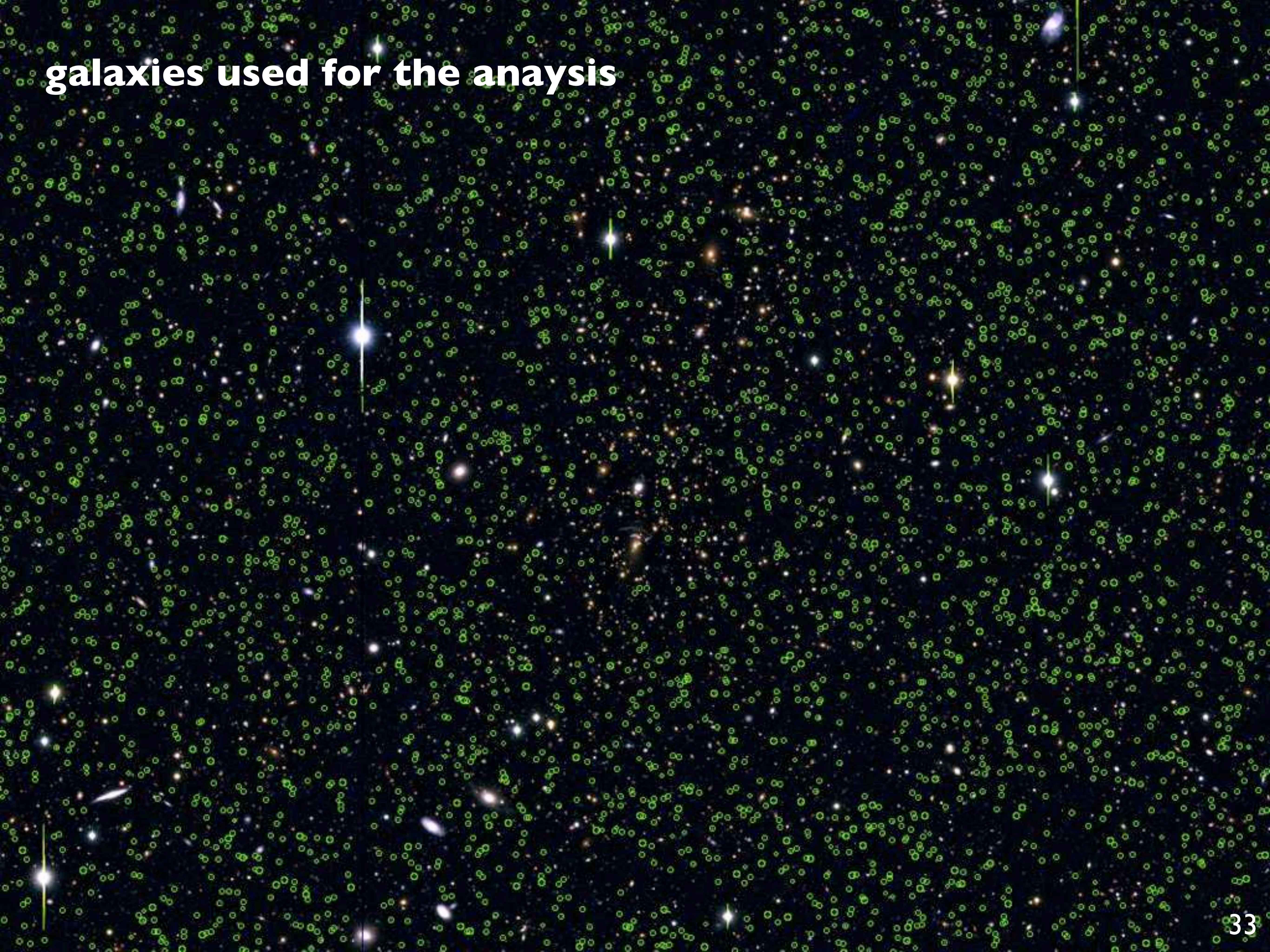
- analysis of Subaru Suprime-cam image



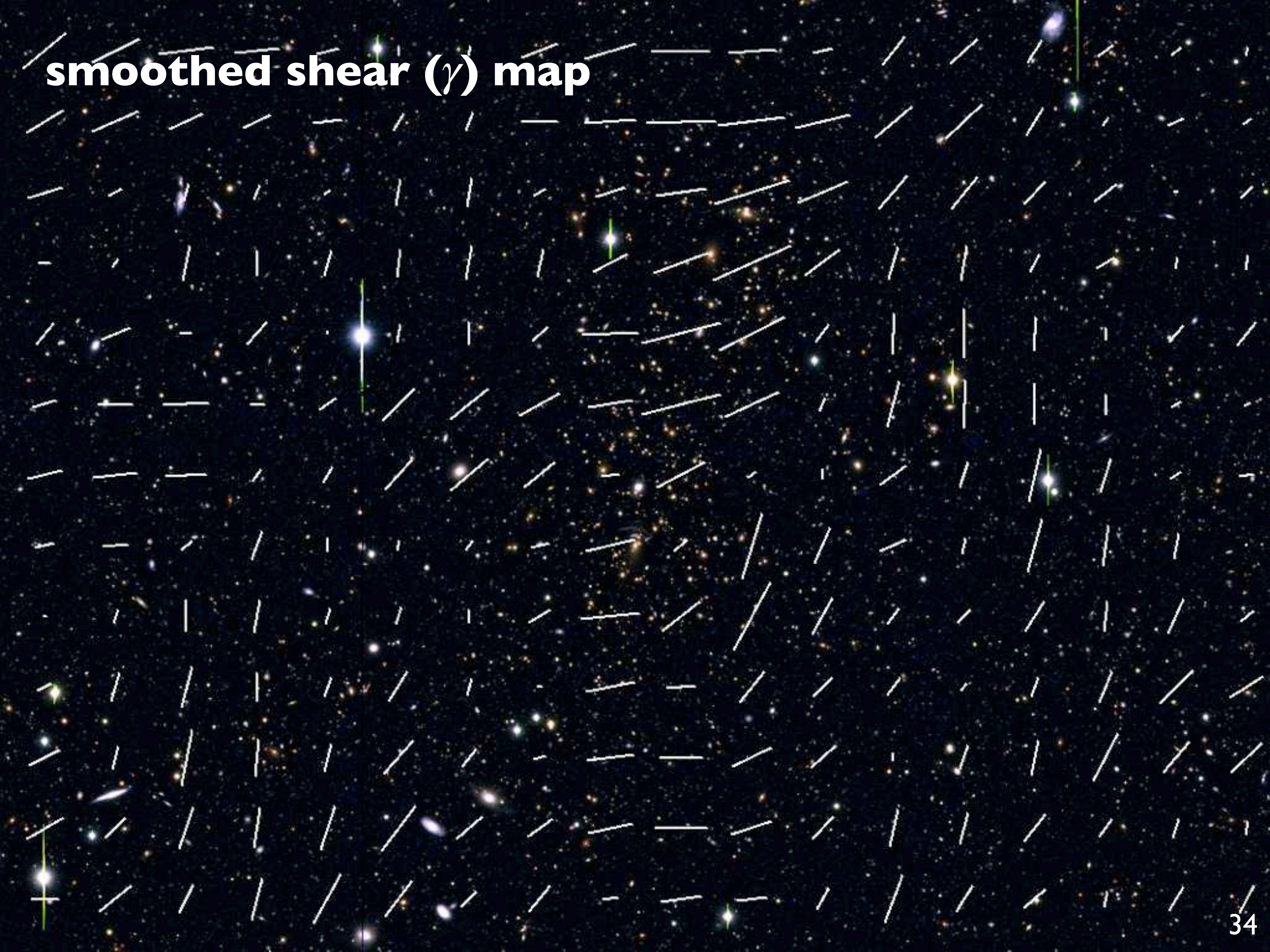
Subaru/Suprime-cam gri-band (MO+2012)

Subaru wide-field image

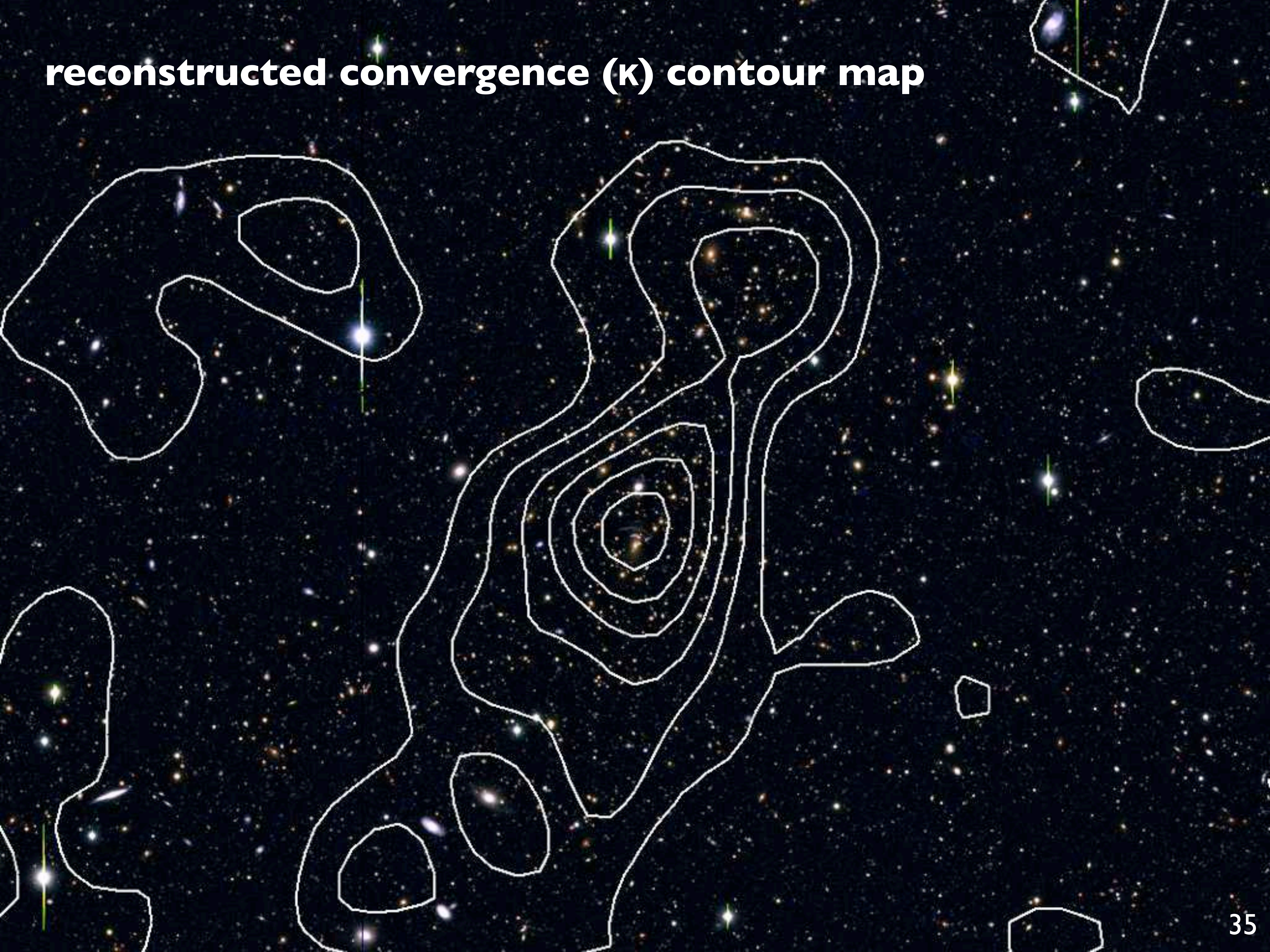
galaxies used for the anaysis



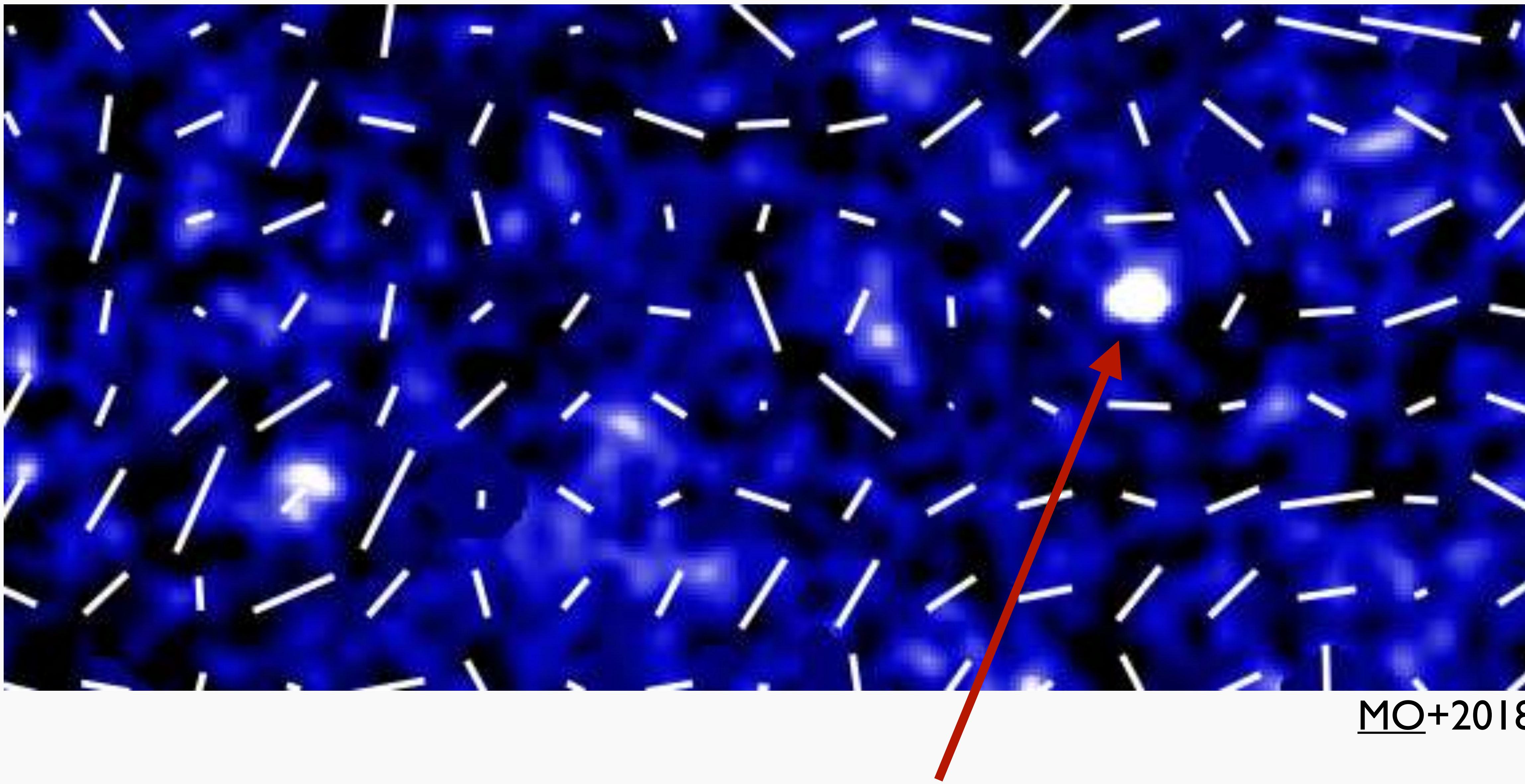
smoothed shear (γ) map



reconstructed convergence (κ) contour map



Weak lensing (WL) selected clusters

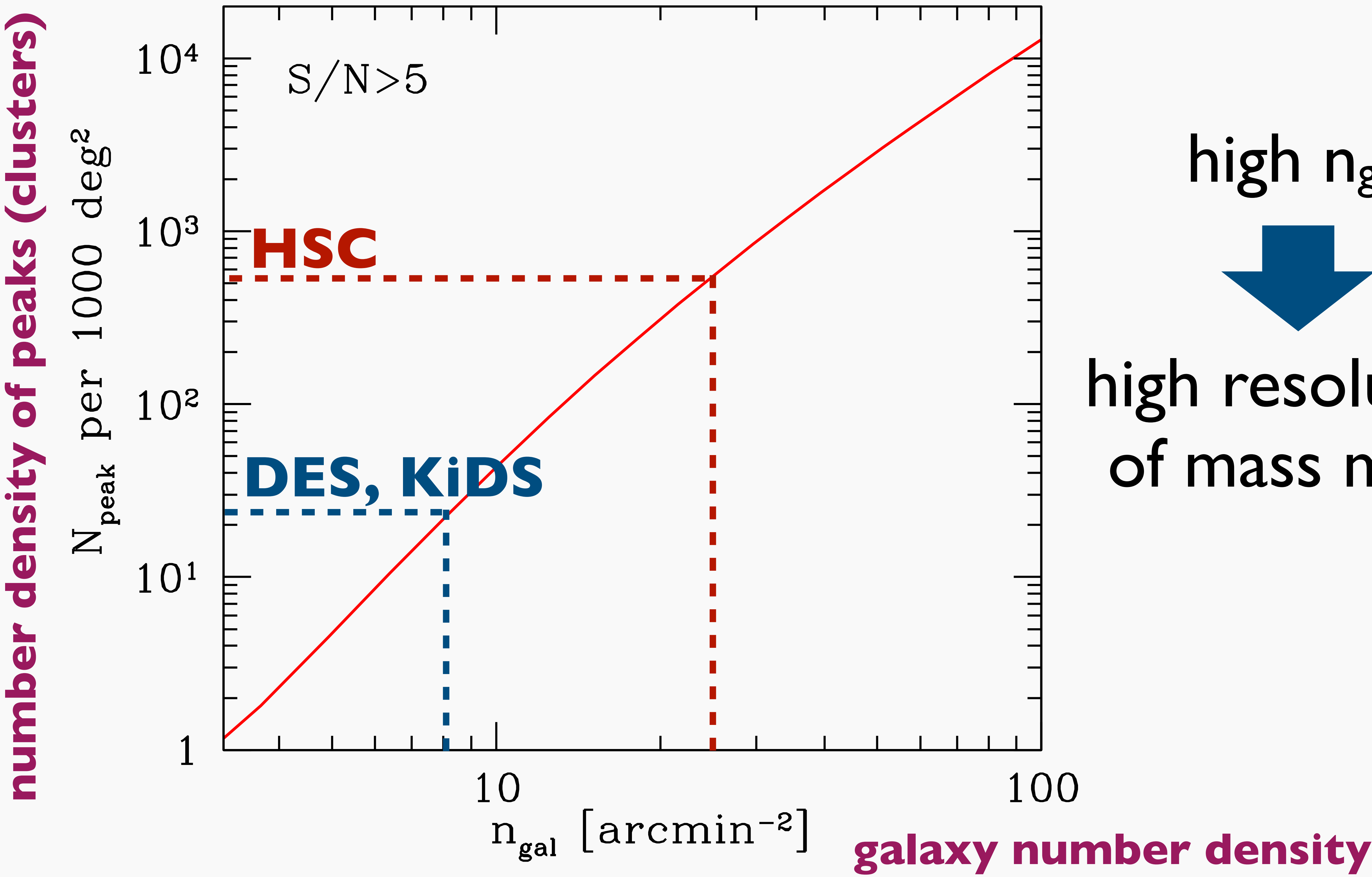


MO+2018

clusters from **peaks** in mass map
[**purely gravitational selection!**]

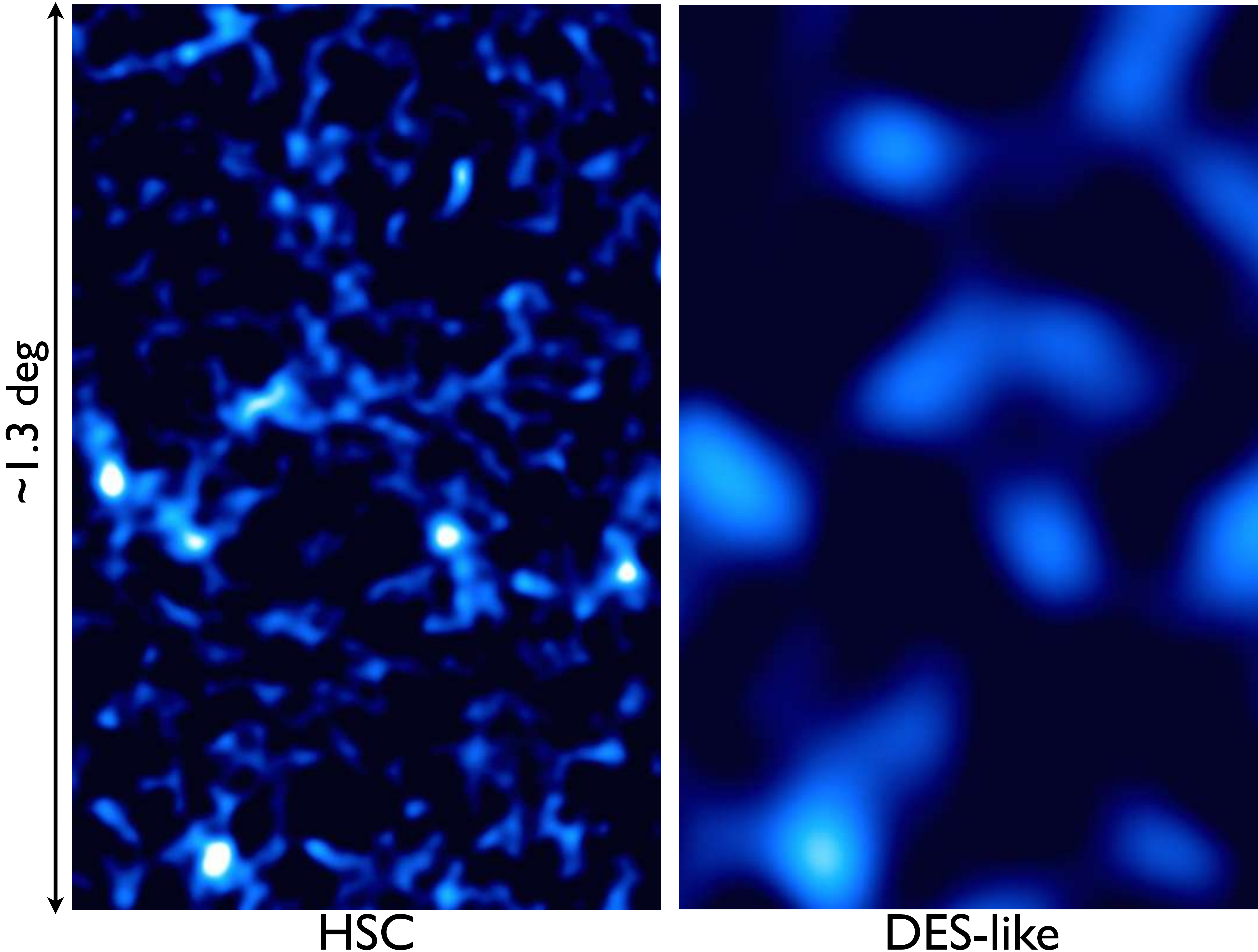
Miyazaki, MO+ PASJ **70**(2018)S27

Depth is important

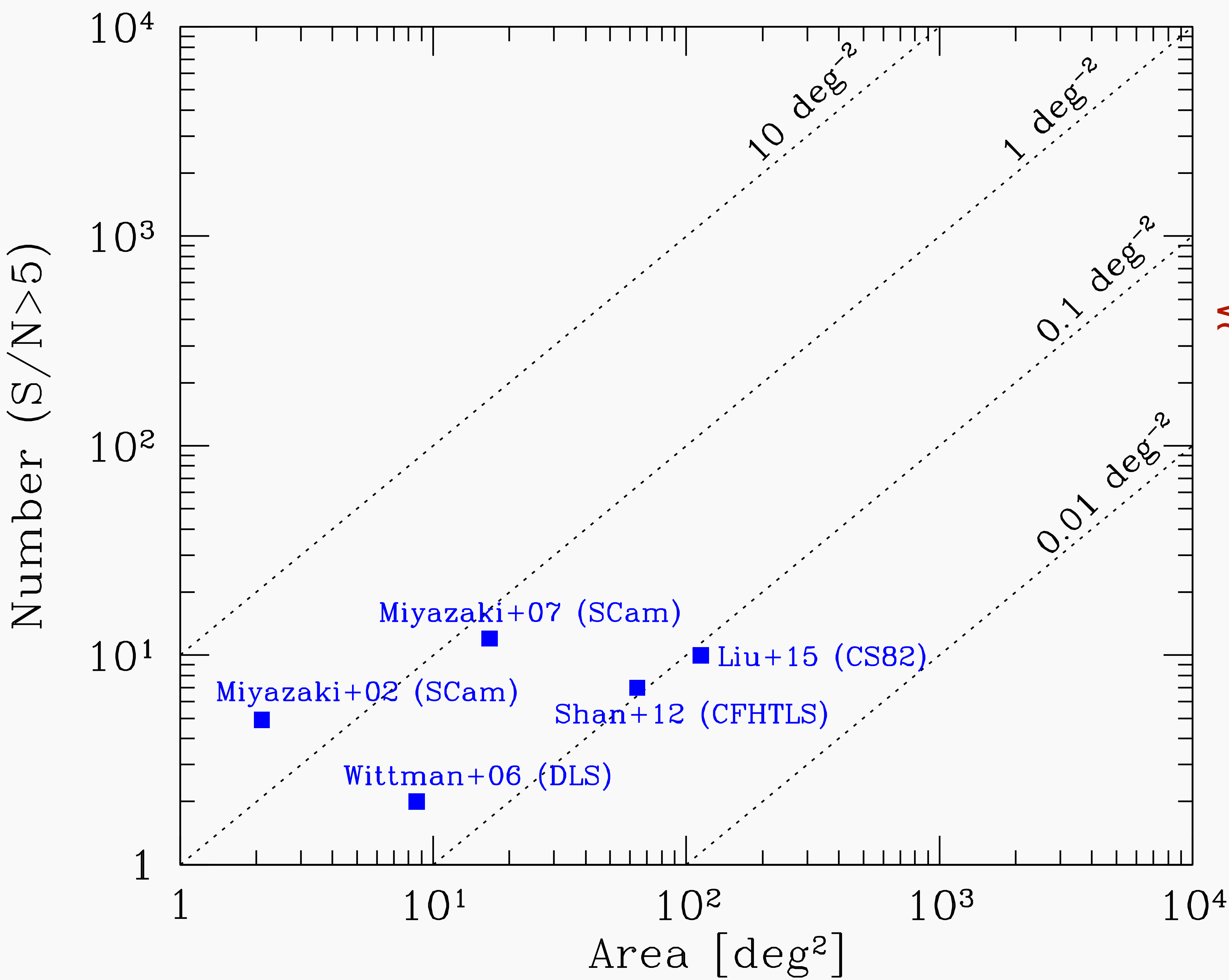


high n_{gal}

high resolution of mass maps


~1.3 deg
HSC
DES-like

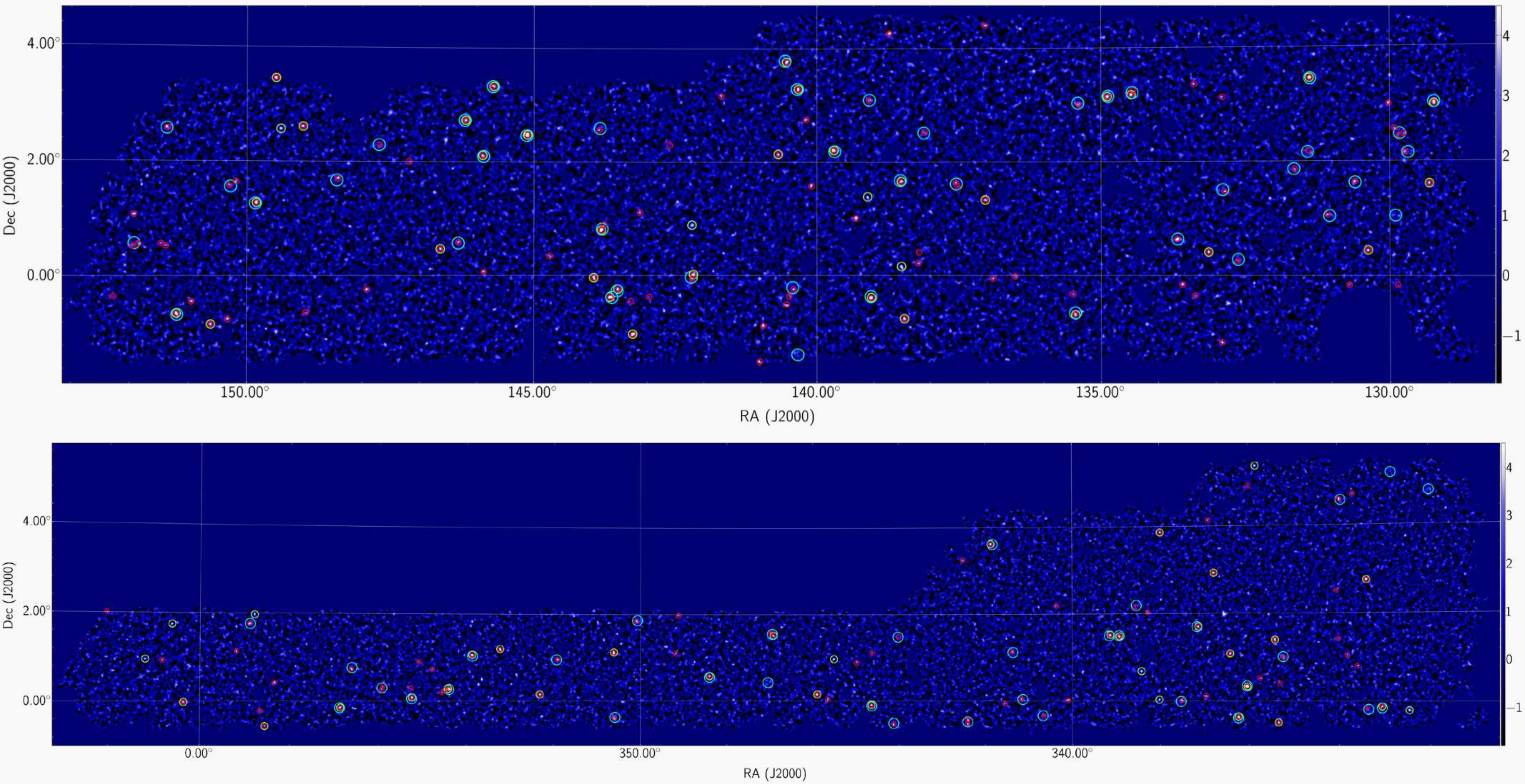
Challenge: deep *and* wide imaging



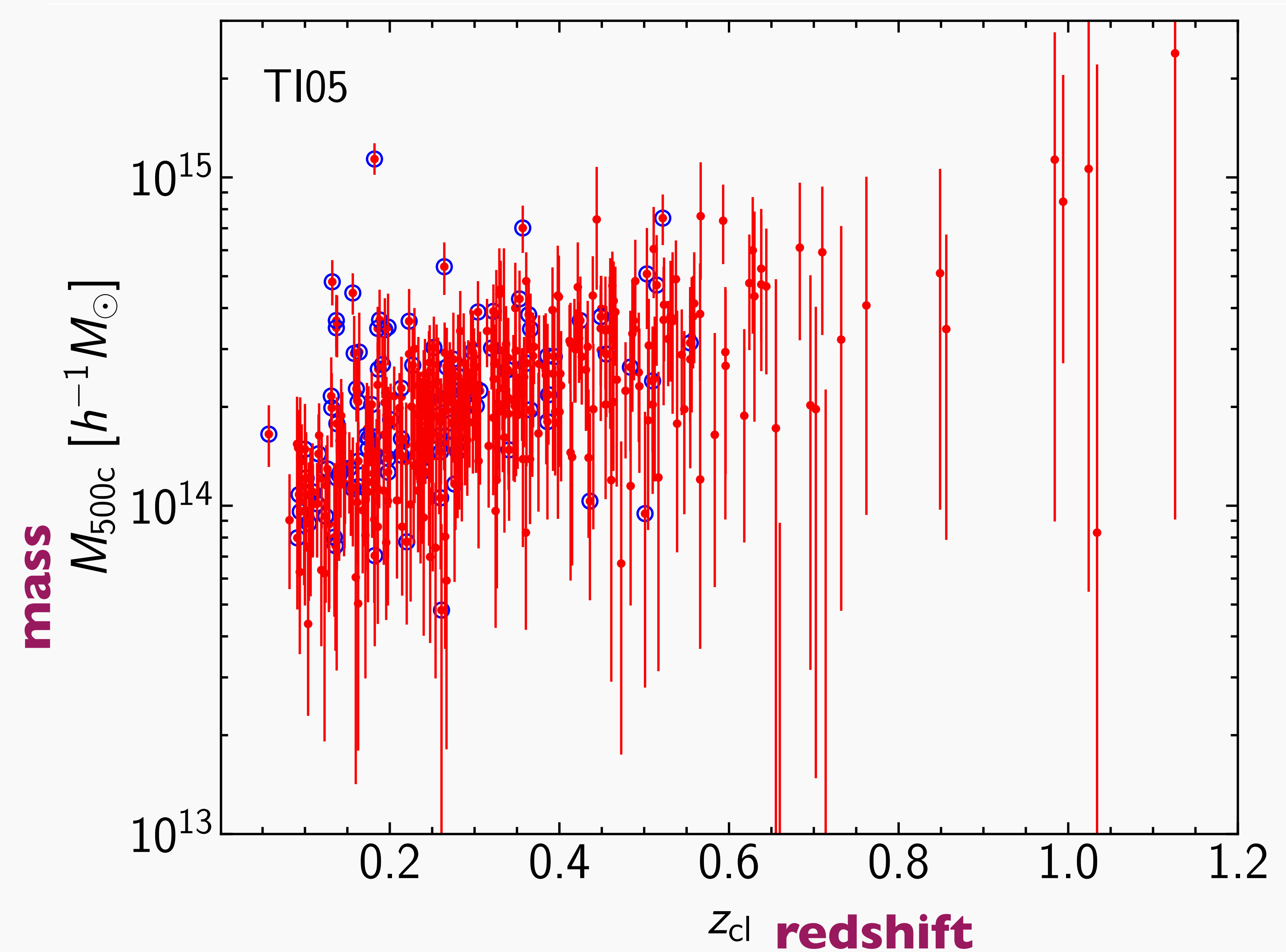
$\lesssim$ **10 clusters** before HSC-SSP

MO, Miyazaki+ PASJ **73**(2021)817

WL selected clusters from HSC



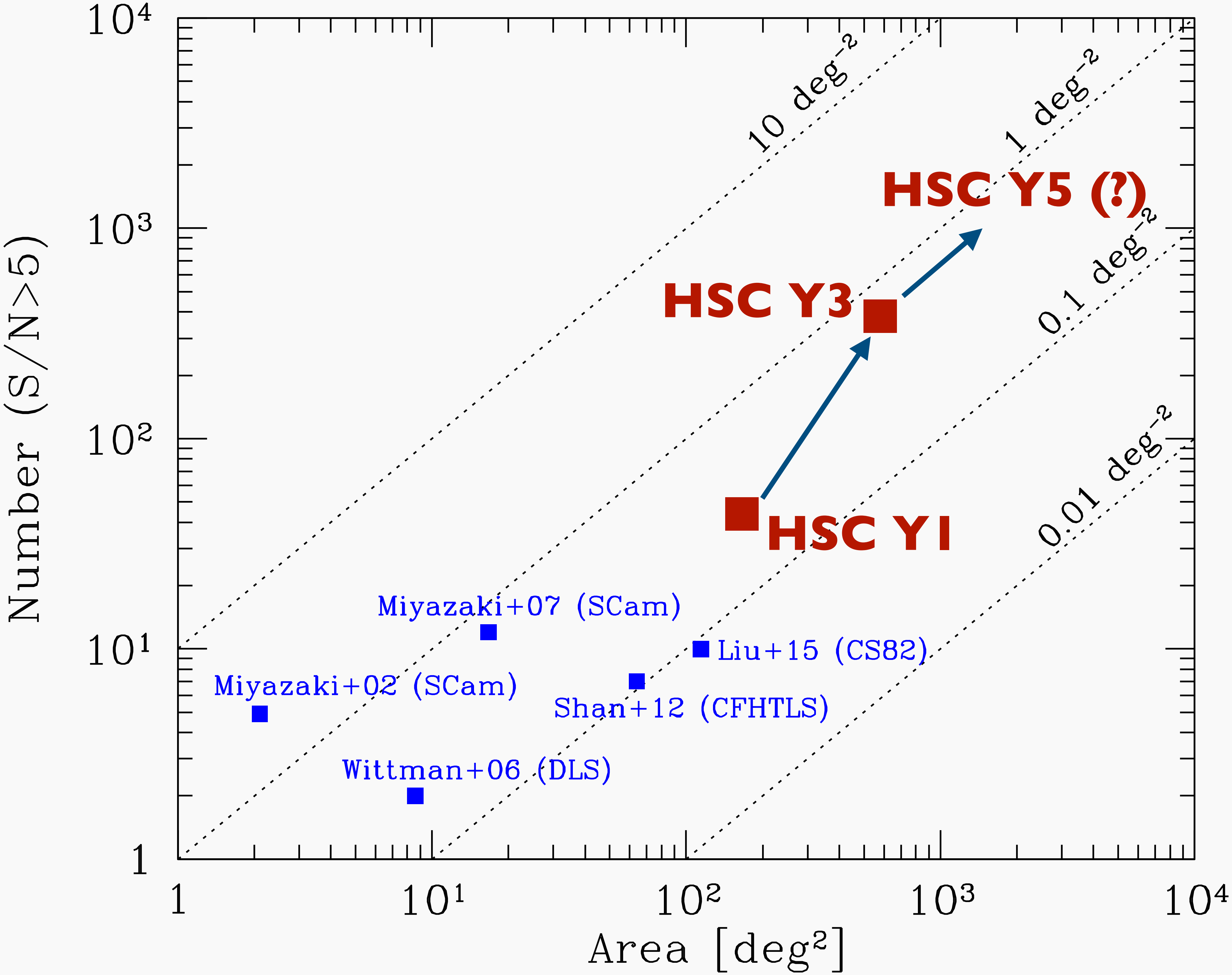
HSC WL selected cluster sample



418 clusters
with
S/N > 4.7

significantly large sample for statistical studies!

WL selected clusters



Chiu, Chen, MO+, in prep.

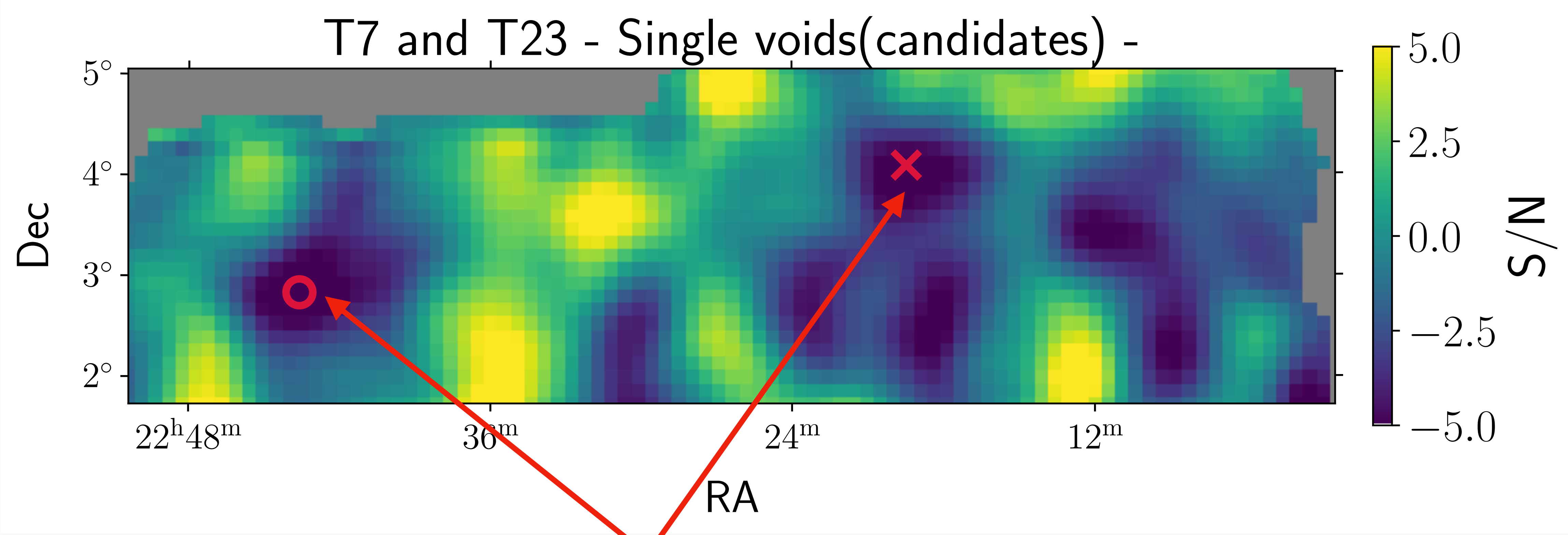
Cosmological analysis

- injection sim. to quantify selection function
- blind analysis
- 4% constraint on S_8 from 130 clusters

stay tuned!

Void search with mass map

- void = empty region in large-scale structure
- useful probe of modified gravity, neutrino, …
- can search for voids with weak lensing!



negative peaks in HSC Y3 mass map → voids

Summary

- weak lensing measures mass distribution from galaxy shapes
- power spectrum measured from discrete galaxy sample is affected by shot noise
- massive clusters (and voids) can be selected from peaks in mass maps
- note: there are many details I didn't explain
  - masking, inhomogeneous galaxy sample, residual shape error, intrinsic alignment, …