Consistency relations of the kurtosis in large-scale structure and modified gravity

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Today's gravity

General Relativity (GR)

$$\mathcal{L}_{\mathrm{GR}} = (R - 2\Lambda)/(16\pi G)$$

GR predicted

- *Black Hole*
- *Big Bang Universe*
- *gravitational waves*

Problems and Possible solutions

Problems

- ⋆ *dark energy*
- ⋆ *dark matter*
- ⋆ *Hubble tension and S_8 tension*

  etc...

Solutions?

Modified gravity?

How to extend gravity? (1) [Shankaranarayanan and Johnson, 2204.06533]

Lovelock theorem(dimension$= 4 \Rightarrow$Einstein gravity)

- *Covariant equation of motion (EoM)*
- *second order $g_{\mu\nu}$ derivative EoM*
- *no extra field (only $g_{\mu\nu}$)*
- *Locality*

Which condition should be removed?

How to extend gravity? (2) [Shankaranarayanan and Johnson, 2204.06533]

Since rich scalar $R \sim [L^{-2}]$...

If higher order terms exist...

$$\mathcal{L}_{\text{Stelle}} = \kappa^{-2} R + \beta \underline{R^2} - \alpha \underline{R^{\mu\nu} R_{\mu\nu}} \qquad \text{(Stelle's quadratic gravity)}$$

small scale correction?

How to extend gravity? (3)

[Shankaranarayanan and Johnson, 2204.06533] [Bergè, 1809.00698]

Small scale study $V(r) = -G\frac{mM}{r}\left[1 + \alpha \exp\left(-r/\lambda\right)\right]$

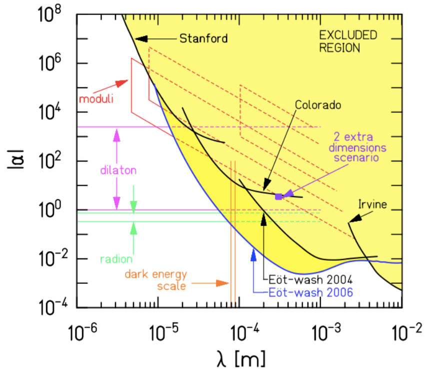


Figure: $\lambda < 1\text{cm}$

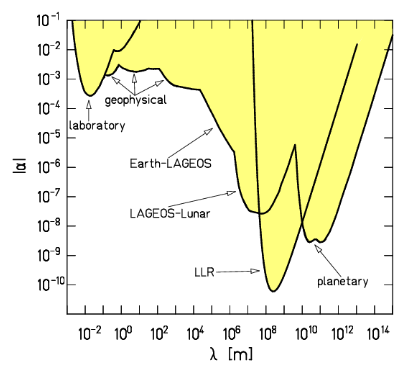


Figure: $\lambda > 1\text{cm}$

How to extend gravity? (4) [Shankaranarayanan and Johnson, 2204.06533]

Other models

$f(R)$ gravity

$$\mathcal{L} = f(R) + \mathcal{L}_{\rm matter}$$

Scalar tensor gravity

$$\mathcal{L} = \frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + \mathcal{L}_{\rm matter}$$

NOTE* : Almost theory can be transformed into scalar-tensor theory.

*[Fujii and Maeda, “The Scalar-Tensor Theory of Gravitation”, CAMBRIDGE UNIVERSITY PRESS]

General scalar-tensor theories (1)

[Horndeski. Int.J.Theor.Phys.10 (1974) 363-384] [Kobayashi et al., 1105.5723]

General EoM (2nd order time derivative)

Horndeski theory

$X = \phi^\mu \phi_\mu,$

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] + G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^\mu_\lambda\right] + \mathcal{L}_{\mathrm{matter}}$$

General scalar-tensor theories (2) [Langlois and Noui., 1510.06930]

General EoM (higher (≥ 2) order time derivative)

Degenerate higher-order scalar-tensor (DHOST) theory

$X = \phi^\mu \phi_\mu,$

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + F(\phi, X)R + a_1(\phi, X)\phi^{\mu\nu}\phi_{\mu\nu} + a_2(\phi, X)(\Box\phi)^2 \\ + a_3(\phi, X)(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu + a_4(\phi, X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu + a_5(\phi, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2 + \mathcal{L}_{\mathrm{matter}}$$

We try to constrain DHOST theory using a large-scale structure.

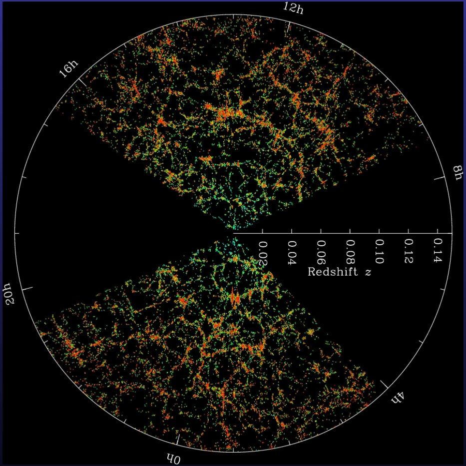
Large Scale Structure (LSS) of the Universe

LSS: Galaxies' distribution and structure

- The distribution is homogeneous and isotropic on the large scales.
- Matter density fluctuations grow in the matter-dominated era.

→ Gravity is the most important to determine the distribution.

We can investigate (modified) gravity theories using LSS !



Credit: M. Blanton and the Sloan Digital Sky Survey.

Our Purpose

- Investigate (modified) gravity theories using skewness and kurtosis parameters.
- [Yamauchi, Ishimaru, Matsubara, Takahashi. 2211.13453] derived skewness consistency relation

  $\longrightarrow$ can derive kurtosis consistency relation in the same manner?

Why skewness and kurtosis consistency relations?

- Initial density fluctuation $\delta_{\rm ini}(\boldsymbol{x}) \stackrel{\rm def}{:=} \dfrac{\rho_{\rm ini}(\boldsymbol{x}) - \bar{\rho}_{\rm ini}}{\bar{\rho}_{\rm ini}}$ can be described as Gaussian distribution approximately (and evolve as non-Gaussianity field).
- We want a simple method to investigate gravity theories.
- Skewness and kurtosis can be calculated with only $\delta(\boldsymbol{x})$ using perturbation.
- We can define multiple skewness and kurtosis.

Definitions of skewness and kurtosis parameters (1)

Multiple skewness and kurtosis parameters including some spatial derivatives.

Skewness parameters [Matsubara., astro-ph/0006269] [Matsubara et al., 2012.00203] :

$$S_g^{(0)} \overset{\text{def}}{:=} \frac{\langle \delta_s^3 \rangle_c}{\sigma_0^4}, \quad S_g^{(1)} \overset{\text{def}}{:=} \frac{3}{2} \cdot \frac{\langle \delta_s \,|\, \nabla \delta_s \,|^2 \rangle_c}{\sigma_0^2 \sigma_1^2}, \quad S_g^{(2)} \overset{\text{def}}{:=} -\frac{9}{4} \cdot \frac{\langle \,|\, \nabla \delta_s \,|^2 \Delta \delta_s \rangle_c}{\sigma_1^4}.$$

δ_s : Smoothed matter density fluctuation.

($\star$) variances : $\sigma_0 \overset{\text{def}}{:=} \langle \delta_s^2 \rangle^{1/2}, \quad \sigma_1 \overset{\text{def}}{:=} \langle \nabla \delta_s \cdot \nabla \delta_s \rangle^{1/2}$

Definitions of skewness and kurtosis parameters (2)

Kurtosis parameters [Matsubara et al., 2012.00203] :

$$K_g^{(0)} \overset{\text{def}}{:=} \frac{\langle \delta_s^4 \rangle_c}{\sigma_0^6}, \qquad K_g^{(1)} \overset{\text{def}}{:=} 2 \cdot \frac{\langle \delta_s^2 \, |\nabla \delta_s|^2 \rangle_c}{\sigma_0^4 \sigma_1^2},$$

$$K_{g1}^{(2)} \overset{\text{def}}{:=} -\frac{3}{5} \cdot \frac{5 \langle \delta_s \, |\nabla \delta_s|^2 \Delta \delta_s \rangle_c + \langle |\nabla \delta_s|^4 \rangle_c}{\sigma_0^2 \sigma_1^4},$$

$$K_{g2}^{(2)} \overset{\text{def}}{:=} -\frac{3}{5} \cdot \frac{5 \langle \delta_s \, |\nabla \delta_s|^2 \Delta \delta_s \rangle_c + 3 \langle |\nabla \delta_s|^4 \rangle_c}{\sigma_0^2 \sigma_1^4},$$

$$K_g^{(3)} \overset{\text{def}}{:=} 9 \cdot \frac{\langle |\nabla \delta_s|^2 (\Delta \delta_s)^2 \rangle_c - \langle |\nabla \delta_s|^2 \delta_{s,ij} \delta_{s,ij} \rangle_c}{\sigma_1^6}.$$

δ_s : Smoothed matter density fluctuation.

($\star$) variances : $\sigma_0 \overset{\text{def}}{:=} \langle \delta_s^2 \rangle^{1/2}$, $\sigma_1 \overset{\text{def}}{:=} \langle \nabla \delta_s \cdot \nabla \delta_s \rangle^{1/2}$

Matter density fluctuation δ (1)

[Matsubara, "Large scale structure of the Universe", KYORITSU SHUPPAN CO., LTD., 2014(in japanese)]

Skewness and kurtosis parameters include $\boxed{\delta(\boldsymbol{x}) \overset{\text{def}}{:=} \dfrac{\rho(\boldsymbol{x}) - \bar{\rho}}{\bar{\rho}}}$. $\Longrightarrow$ Need $\delta(\boldsymbol{x})$ evolution!

Fluid equation in expanding Universe (GR case)

$$\frac{\partial}{\partial \tau}\delta + \nabla \cdot \left[(\delta + 1)\boldsymbol{V}\right] = 0, \quad \frac{\partial}{\partial \tau}\boldsymbol{V} + \frac{1}{2} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} + \frac{3}{2}\nabla\varphi = 0, \quad \Delta\varphi = \delta$$

$\downarrow$ Get a perturbative solution in Fourier space ...

Matter density fluctuation δ (2)

[Matsubara (2014), "Large scale structure of the Universe", KYORITSU SHUPPAN CO., LTD.(in japanese)]

$\delta(\boldsymbol{x})$ and $\theta(\boldsymbol{x}) \overset{\text{def}}{:=} -\nabla\cdot\boldsymbol{V}$ evolution (GR case)

$$\frac{\partial}{\partial\tau}\tilde{\delta}(\boldsymbol{k}) - \tilde{\theta}(\boldsymbol{k}) = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\left(1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_1^2}+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_2^2}\right)\tilde{\theta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2)$$

$$\frac{\partial}{\partial\tau}\tilde{\theta}(\boldsymbol{k}) - \frac{3}{2}\tilde{\delta}(\boldsymbol{k}) + \frac{1}{2}\tilde{\theta}(\boldsymbol{k}) = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{|\boldsymbol{k}_1+\boldsymbol{k}_2|^2\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2\boldsymbol{k}_1^2\boldsymbol{k}_2^2}\tilde{\theta}(\boldsymbol{k}_1)\tilde{\theta}(\boldsymbol{k}_2)$$

$\downarrow$ Solve by the perturbative method

Matter density fluctuation δ (3) [Hirano et al., 2008.02798]

GR ($\Omega_m = 1$) case up to the 3rd order ($\tilde{\delta}_{\rm L}$ is linear order solution $\leftarrow$ Gaussian) :

$$\tilde{\delta}(\boldsymbol{k}) = \tilde{\delta}_{\rm L}(\boldsymbol{k}) + \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!}\tilde{\delta}_{\rm L}(\boldsymbol{k}_1)\tilde{\delta}_{\rm L}(\boldsymbol{k}_2)$$
$$+ \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{F_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!}\tilde{\delta}_{\rm L}(\boldsymbol{k}_1)\tilde{\delta}_{\rm L}(\boldsymbol{k}_2)\tilde{\delta}_{\rm L}(\boldsymbol{k}_3)$$

- $\star \quad \dfrac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} = \dfrac{5}{7} + \dfrac{1}{2}\left(\dfrac{k_1}{k_2}+\dfrac{k_2}{k_1}\right)\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2 + \dfrac{2}{7}(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2$
- $\star \quad \dfrac{F_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!} = \alpha\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - \dfrac{4}{7}\alpha\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - \dfrac{2}{21}\gamma\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \dfrac{1}{9}\xi_c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$

$\alpha\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$, $\alpha\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3), \ldots$ are already known, $\boldsymbol{k}$'s functions.

Matter density fluctuation (4) [Hirano et al., 2008.02798]

DHOST kernel

$$\frac{F_2(t,\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} = \kappa\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2) - \frac{2}{7}\lambda\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)$$

$$\frac{F_3(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!} = d_{\alpha\alpha}\alpha\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - \frac{4}{7}d_{\alpha\gamma}\alpha\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - \frac{2}{21}d_{\gamma\gamma}\gamma\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$
$$+ \frac{1}{9}d_\xi\xi_c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + d_{\alpha\alpha-}\alpha\alpha_-(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + d_{\alpha\gamma-}\alpha\gamma_-(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + d_\zeta\zeta_c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

$\kappa(t),\ \lambda(t),\ d_{\alpha\alpha}(t),\ d_{\alpha\gamma}(t),\ d_{\gamma\gamma}(t),\ d_\xi(t),\ d_{\alpha\alpha-}(t),\ d_{\alpha\gamma-}(t),\ d_\zeta(t)$ are model parameters.

Horndeski limit

$$\kappa = d_{\alpha\alpha} = 1$$
$$d_{\alpha\alpha-} = d_{\alpha\gamma-} = d_\zeta = 0$$

GR($\Omega_m = 1$) limit

$$\kappa = \lambda = d_{\alpha\alpha} = d_{\alpha\gamma} = d_{\gamma\gamma} = d_\xi = 1,$$
$$d_{\alpha\alpha-} = d_{\alpha\gamma-} = d_\zeta = 0.$$

Bias effect (1) [McDonald and Roy., 0902.0991] [Desjacques et al., 1611.09787]

Bias: Differences between galaxy distribution and matter distribution.

Complicated phenomena, star and galaxy formation, can be simplified using bias models.

The most easy model

$$\delta_g(\boldsymbol{x}) = \int d^3x \; b_1(\boldsymbol{x} - \boldsymbol{x}_1)\delta(\boldsymbol{x})$$

In large scale, b_1 converges into a constant.

Bias effect (2) [McDonald and Roy., 0902.0991] [Desjacques et al., 1611.09787]

Smoothing is also a bias model.

Gaussian smoothing (in Fourier space)

$$\tilde{\delta}(\boldsymbol{x}) \longrightarrow \delta_s(\boldsymbol{x}, R) = \int d^3x' \delta_{\mathrm{g}}(\boldsymbol{x}') W_R(|\boldsymbol{x} - \boldsymbol{x}'|) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \tilde{\delta}_{\mathrm{g}}(\boldsymbol{k}) e^{-k^2R^2/2}$$

$$W_R(x) \stackrel{\mathrm{def}}{:=} \frac{1}{(2\pi R^2)^{3/2}} \exp[-x^2/(2R)^2]$$

Bias effect (3) [McDonald and Roy., 0902.0991] [Desjacques et al., 1611.09787]

Local bias models (The tidal term is neglected)

$$\delta_{\rm g}(\boldsymbol{x}) = b_1 \Bigg\{ \delta(\boldsymbol{x}) + \frac{1}{2!}\beta_2 \delta^2(\boldsymbol{x}) + \frac{1}{3!}\beta_3 \delta^3(\boldsymbol{x})$$

$$+ \beta_{K^2} \sum_{i,j} \left[\left(\frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3}\delta_{ij} \right) \delta(\boldsymbol{x}) \right]^2$$

$$+ \beta_{K^3} \sum_{i,j,k} \left[\left(\frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3}\delta_{ij} \right) \delta(\boldsymbol{x}) \times \left(\frac{\partial_j \partial_k}{\partial^2} - \frac{1}{3}\delta_{jk} \right) \delta(\boldsymbol{x}) \times \left(\frac{\partial_k \partial_i}{\partial^2} - \frac{1}{3}\delta_{ki} \right) \delta(\boldsymbol{x}) \right]$$

$$+ \beta_{\delta K^2} \delta(\boldsymbol{x}) \sum_{i,j} \left[\left(\frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3}\delta_{ij} \right) \delta(\boldsymbol{x}) \right]^2 + \cdots \Bigg\}.$$

Bias effect (4) [McDonald and Roy., 0902.0991] [Desjacques et al., 1611.09787]

Local bias models in Fourier space

$$
\begin{aligned}
\tilde{\delta}_{\rm g}(\boldsymbol{k}) &= b_1 \tilde{\delta}(\boldsymbol{k}) \\
&+ b_1 \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{12}-\boldsymbol{k}) \\
&\quad \times \left[\frac{\beta_2}{2!} + \beta_{K^2}\left(\frac{k_{1i}k_{1j}}{k_1^2}-\frac{1}{3}\delta_{ij}\right)\left(\frac{k_{2j}k_{2i}}{k_2^2}-\frac{1}{3}\delta_{ji}\right)\right]\tilde{\delta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2) \\
&+ b_1 \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{123}-\boldsymbol{k}) \\
&\quad \times \Bigg\{\left[\frac{\beta_3}{3!} + \beta_{K^3}\left(\frac{k_{1i}k_{1j}}{k_1^2}-\frac{1}{3}\delta_{ij}\right)\left(\frac{k_{2j}k_{2k}}{k_2^2}-\frac{1}{3}\delta_{jk}\right)\left(\frac{k_{1k}k_{1i}}{k_1^2}-\frac{1}{3}\delta_{ki}\right)\right] \\
&\qquad + \beta_{\delta K^2}\left(\frac{k_{1i}k_{1j}}{k_1^2}-\frac{1}{3}\delta_{ij}\right)\left(\frac{k_{2j}k_{2i}}{k_2^2}-\frac{1}{3}\delta_{ji}\right)\Bigg\}\tilde{\delta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2)\tilde{\delta}(\boldsymbol{k}_3) + \mathcal{O}(\delta^4)
\end{aligned}
$$

$\tilde{\delta}(\boldsymbol{k}) \longleftarrow$ DHOST solution

Bias in Fourier space

Transformed δ_g in Fourier space, $\tilde{\delta}_g$

$$\tilde{\delta}_{\rm g}(\boldsymbol{k}) = b_1 \Bigg[\tilde{\delta}_{\rm L}(\boldsymbol{k}) + \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{12}-\boldsymbol{k})\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!b_1}\tilde{\delta}_{\rm L}(\boldsymbol{k}_1)\tilde{\delta}_{\rm L}(\boldsymbol{k}_2)$$
$$+ \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{123}-\boldsymbol{k})\frac{Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!b_1}\tilde{\delta}_{\rm L}(\boldsymbol{k}_1)\tilde{\delta}_{\rm L}(\boldsymbol{k}_2)\tilde{\delta}_{\rm L}(\boldsymbol{k}_3) + \cdots \Bigg]$$

$Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)$ & $Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ include (DHOST + bias) informations.

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where
$$\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!b_1} = \frac{\beta_2}{2!} + \frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} + \beta_{K^2}\left[\left(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2\right)^2 - \frac{1}{3}\right],$$
$$\frac{Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!b_1} = \frac{\beta_3}{3!} + \frac{F_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!} + \beta_2\frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} + \cdots.$$

Skewness and kurtosis (DHOST + bias) (1)

($\star$) Showing definitions again.

Skewness parameters :

$$\underline{S^{(0)} \stackrel{\text{def}}{:=} \left\langle \delta_s^3 \right\rangle_c / \sigma_0^4}, \quad \underline{S^{(1)} \stackrel{\text{def}}{:=} 3/2 \cdot \left\langle \delta_s \, |\nabla \delta_s|^2 \right\rangle_c / \sigma_0^2 \sigma_1^2}, \quad \underline{S^{(2)} \stackrel{\text{def}}{:=} -9/4 \cdot \left\langle |\nabla \delta_s|^2 \Delta \delta_s \right\rangle_c / \sigma_1^4}$$

Kurtosis parameters :

$$\underline{K^{(0)} \stackrel{\text{def}}{:=} \left\langle \delta_s^4 \right\rangle_c / \sigma_0^6}, \quad \underline{K^{(1)} \stackrel{\text{def}}{:=} 2 \cdot \left\langle \delta_s^2 \, |\nabla \delta_s|^2 \right\rangle_c / \sigma_0^4 \sigma_1^2},$$

$$\underline{K_1^{(2)} \stackrel{\text{def}}{:=} -3/5 \cdot \left[5 \left\langle \delta_s \, |\nabla \delta_s|^2 \Delta \delta_s \right\rangle_c + \left\langle |\nabla \delta_s|^4 \right\rangle_c \right] \Big/ \sigma_0^2 \sigma_1^4},$$

$$\underline{K_2^{(2)} \stackrel{\text{def}}{:=} -3/5 \cdot \left[5 \left\langle \delta_s \, |\nabla \delta_s|^2 \Delta \delta_s \right\rangle_c + 3 \left\langle |\nabla \delta_s|^4 \right\rangle_c \right] \Big/ \sigma_0^2 \sigma_1^4},$$

$$\underline{K^{(3)} \stackrel{\text{def}}{:=} 9 \left[\left\langle |\nabla \delta_s|^2 (\Delta \delta_s)^2 \right\rangle_c - \left\langle |\nabla \delta_s|^2 \delta_{s,ij} \delta_{s,ij} \right\rangle_c \right] \Big/ \sigma_1^6}.$$

Skewness and kurtosis (DHOST + bias) (2)

Calculate the tree level.

Skewness parameter $S_g^{(0)}$

$$\left\langle \delta_{gs}^3 \right\rangle_c = \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9} e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{x}} e^{-(k_1^2+k_2^2+k_3^2)R^2/2} \left\langle \tilde{\delta}_g(\boldsymbol{k}_1)\tilde{\delta}_g(\boldsymbol{k}_2)\tilde{\delta}_g(\boldsymbol{k}_3) \right\rangle_c$$

$$= \cdots \left\langle (\tilde{\delta}_1^{(1)} + \tilde{\delta}_1^{(2)}) \cdot (\tilde{\delta}_2^{(1)} + \tilde{\delta}_2^{(2)}) \cdot (\tilde{\delta}_3^{(1)} + \tilde{\delta}_3^{(2)}) \right\rangle_c$$

$$= \cdots \left[\left\langle \tilde{\delta}_1^{(2)} \cdot \tilde{\delta}_2^{(1)} \cdot \tilde{\delta}_3^{(1)} \right\rangle_c + \left\langle \tilde{\delta}_1^{(1)} \cdot \tilde{\delta}_2^{(2)} \cdot \tilde{\delta}_3^{(1)} \right\rangle_c + \left\langle \tilde{\delta}_1^{(1)} \cdot \tilde{\delta}_2^{(1)} \cdot \tilde{\delta}_3^{(2)} \right\rangle_c \right]$$

$$\left\langle \tilde{\delta}_1^{(2)} \cdot \tilde{\delta}_2^{(1)} \cdot \tilde{\delta}_3^{(1)} \right\rangle_c = \int \frac{d^3p_1 d^3p_2}{(2\pi)^3} \delta_{\mathrm{D}}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{k}_1) \frac{Z_2(\boldsymbol{p}_1, \boldsymbol{p}_2)}{2!} \left\langle (\tilde{\delta}_{\mathrm{L}p1}\tilde{\delta}_{\mathrm{L}p2}) \cdot b_1\tilde{\delta}_{\mathrm{L}2} \cdot b_1\tilde{\delta}_{\mathrm{L}3} \right\rangle_c$$

$$\left\langle (\tilde{\delta}_{\mathrm{L}p1}\tilde{\delta}_{\mathrm{L}p2}) \cdot \tilde{\delta}_{\mathrm{L}2} \cdot \tilde{\delta}_{\mathrm{L}3} \right\rangle_c = \left\langle \tilde{\delta}_{\mathrm{L}p1} \cdot \tilde{\delta}_{\mathrm{L}2} \right\rangle_c \left\langle \tilde{\delta}_{\mathrm{L}p2} \cdot \tilde{\delta}_{\mathrm{L}3} \right\rangle_c + \left\langle \tilde{\delta}_{\mathrm{L}p1} \cdot \tilde{\delta}_{\mathrm{L}3} \right\rangle_c \left\langle \tilde{\delta}_{\mathrm{L}p1} \cdot \tilde{\delta}_{\mathrm{L}2} \right\rangle_c$$

$$= (2\pi)^6 \left[\delta_{\mathrm{D}}(\boldsymbol{p}_1 + \boldsymbol{k}_2)\delta_{\mathrm{D}}(\boldsymbol{p}_2 + \boldsymbol{k}_3) + \delta_{\mathrm{D}}(\boldsymbol{p}_1 + \boldsymbol{k}_3)\delta_{\mathrm{D}}(\boldsymbol{p}_2 + \boldsymbol{k}_2) \right] P_{\mathrm{L}}(\boldsymbol{k}_2) P_{\mathrm{L}}(\boldsymbol{k}_3)$$

Skewness and kurtosis (DHOST + bias) (3)

Skewness parameter

$$S_g^{(a)} = \frac{6b_1^3}{\sigma_0^{4-2a}\sigma_1^{2a}} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{123})e^{-(k_1^2+k_2^2+k_3^2)R^2/2} \times s^{(a)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\left[\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!b_1}P_{\rm L}(k_1)P_{\rm L}(k_2)\right] \qquad (a=0,1,2)$$

($\star$) exponential factor is a smoothing factor, , $P_{\rm L}(k)$ is power spectrum.

$S^{(0)},\ S^{(1)},\ S^{(2)}$: 3 Conditions

$\beta_2,\ \beta_{K^2}$:2 Biases

→ $3-2=1$ consistency relations

Skewness and kurtosis (DHOST + bias) (4)

Skewness parameters are Z_2's integral:

Z_2 term

$$\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!b_1} = \frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} + \frac{1}{2}\beta_2 + \beta_{K^2}\left[\left(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2\right)^2 - \frac{1}{3}\right]$$
$$= \frac{1}{2}\beta_2 + \left[(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 - \frac{1}{3}\right]\beta_{K^2} + \left[1 + \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)\right]\kappa + \frac{2}{7}\left[(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 - 1\right]\lambda$$

Fixing gravity theory, unknown parameters are only biases' (β_2, β_{K^2}).

[Yamauchi et al., 2211.13453]

$$\{\beta_2, \beta_{K^2}\} \quad \leftrightarrow \quad \{S^{(0)}, S^{(1)}, S^{(2)}\} \quad \Longrightarrow \quad \left[S_g^{(0)},\ S_g^{(1)},\ S_g^{(2)}\right] \text{'s linear function} = 0$$

derived a skewness consistency relation !

Skewness and kurtosis (DHOST + bias) (5)

Calculate the tree level.

Kurtosis parameter $K_g^{(0)}$

$$\left\langle \delta_{gs}^4 \right\rangle_c = \int \frac{d^3k_1 \cdots d^3k_4}{(2\pi)^{12}} e^{i(\boldsymbol{k}_1+\cdots+\boldsymbol{k}_4)\cdot\boldsymbol{x}} e^{-(k_1^2+\cdots+k_4^2)R^2/2} \left\langle \tilde{\delta}_g(\boldsymbol{k}_1)\cdot\tilde{\delta}_g(\boldsymbol{k}_2)\cdot\tilde{\delta}_g(\boldsymbol{k}_3)\cdot\tilde{\delta}_g(\boldsymbol{k}_4) \right\rangle_c$$

$$= \cdots \left\langle (\tilde{\delta}_1^{(1)}+\tilde{\delta}_1^{(2)}+\tilde{\delta}_1^{(3)})\cdot(\tilde{\delta}_2^{(1)}+\tilde{\delta}_2^{(2)}+\tilde{\delta}_2^{(3)})\cdot(\tilde{\delta}_3^{(1)}+\tilde{\delta}_3^{(2)}+\tilde{\delta}_3^{(3)})\cdot(\tilde{\delta}_4^{(1)}+\tilde{\delta}_4^{(2)}+\tilde{\delta}_4^{(3)}) \right\rangle_c$$

$$= \cdots \left\{ \left[\left\langle \tilde{\delta}_1^{(2)}\cdot\tilde{\delta}_2^{(2)}\cdot\tilde{\delta}_3^{(1)}\cdot\tilde{\delta}_4^{(1)} \right\rangle_c + \cdots \right] + \left[\left\langle \tilde{\delta}_1^{(3)}\cdot\tilde{\delta}_2^{(1)}\cdot\tilde{\delta}_3^{(1)}\cdot\tilde{\delta}_4^{(1)} \right\rangle_c + \cdots \right] \right\}$$

$$\left\langle \tilde{\delta}_1^{(2)}\cdot\tilde{\delta}_2^{(2)}\cdot\tilde{\delta}_3^{(1)}\cdot\tilde{\delta}_4^{(1)} \right\rangle_c = \int \frac{d^3p_1 d^3p_2 d^3q_1 d^3q_2}{(2\pi)^6} \delta_{\rm D}(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{k}_1)\delta_{\rm D}(\boldsymbol{q}_1+\boldsymbol{q}_2-\boldsymbol{k}_2)$$
$$\times \frac{Z_2(\boldsymbol{p}_1,\boldsymbol{p}_2)}{2!}\frac{Z_2(\boldsymbol{q}_1,\boldsymbol{q}_2)}{2!} \left\langle (\tilde{\delta}_{{\rm L}p1}\tilde{\delta}_{{\rm L}p2})\cdot(\tilde{\delta}_{{\rm L}q1}\tilde{\delta}_{{\rm L}q2})\cdot b_1\tilde{\delta}_{\rm L3}\cdot b_1\tilde{\delta}_{\rm L4} \right\rangle_c$$

$$\left\langle \tilde{\delta}_1^{(3)}\cdot\tilde{\delta}_2^{(1)}\cdot\tilde{\delta}_3^{(1)}\cdot\tilde{\delta}_4^{(1)} \right\rangle_c = \int \frac{d^3p_1 d^3p_2 d^3p_3}{(2\pi)^3} \delta_{\rm D}(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3-\boldsymbol{k}_1)$$
$$\times \frac{Z_3(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3)}{3!} \left\langle (\tilde{\delta}_{{\rm L}p1}\tilde{\delta}_{{\rm L}p2}\tilde{\delta}_{{\rm L}p3})\cdot b_1\tilde{\delta}_{\rm L2}\cdot b_1\tilde{\delta}_{\rm L3}\cdot b_1\tilde{\delta}_{\rm L4} \right\rangle_c$$

Skewness and kurtosis (DHOST + bias) (6)

Kurtosis parameters

$$K_g^{(a)} = \frac{24b_1^4}{\sigma_0^{6-2a}\sigma_1^{2a}} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_{1234})e^{-(k_1^2+k_2^2+k_3^2+k_4^2)R^2/2}$$
$$\times\, \kappa^{(a)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)\left[2\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3)}{2!b_1}\frac{Z_2(\boldsymbol{k}_2,-\boldsymbol{k}_2-\boldsymbol{k}_3)}{2!b_1}P_{\rm L}(|\boldsymbol{k}_2+\boldsymbol{k}_3|)P_{\rm L}(k_1)P_{\rm L}(k_2)\right.$$
$$\left. + \frac{Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!b_1}P_{\rm L}(k_1)P_{\rm L}(k_2)P_{\rm L}(k_3)\right] \qquad (a = 0, 1, 2_1, 2_2, 3)$$

(⋆) exponential factor is a smoothing factor , $P_{\rm L}(k)$ is power spectrum.

$K^{(0)},\ K^{(1)},\ K_1^{(2)},\ K_2^{(2)},\ K^{(3)}$: 5 Conditions

$\beta_3,\ \beta_{K^3},\ \beta_{\delta K^2}$: 3 Biases

→ $5-3=2$ consistency relations

Skewness and kurtosis (DHOST + bias) (7)

Integrand Z_2 product

$$
\begin{aligned}
&\frac{Z_2(\boldsymbol{k}_1, \boldsymbol{k}_2+\boldsymbol{k}_3)}{2!b_1}\frac{Z_2(\boldsymbol{k}_2, -\boldsymbol{k}_2-\boldsymbol{k}_3)}{2!b_1} \\
&\qquad = \frac{1}{(k_1 k_2)^2|\boldsymbol{k}_2+\boldsymbol{k}_3|^4} \\
&\qquad\quad \times \left\{(\boldsymbol{k}_2+\boldsymbol{k}_3)^2\left[-h_1(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)+(h_0-h_1){k_2}^2\right]-\boldsymbol{k}_2.(\boldsymbol{k}_2+\boldsymbol{k}_3)\left[-h_2(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)+(h_1-h_2)k_2^2\right]\right\} \\
&\qquad\quad \times \left\{(\boldsymbol{k}_2+\boldsymbol{k}_3)^2\left[h_1\boldsymbol{k}_1\cdot(\boldsymbol{k}_2+\boldsymbol{k}_3)+h_0k_1^2\right]+\boldsymbol{k}_1\cdot(\boldsymbol{k}_2+\boldsymbol{k}_3)\left[h_2\boldsymbol{k}_1\cdot(\boldsymbol{k}_2+\boldsymbol{k}_3)+h_1k_1^2\right]\right\}.
\end{aligned}
$$

where

$$
h_0 = \beta_2/2-\beta_{K^2}/3+\kappa-2\lambda/7, \qquad h_1 = \kappa/2, \qquad h_2 = \beta_{K^2}+2\lambda/7
$$

A biases' quadratic function

Skewness and kurtosis (DHOST + bias) (8)

Integrand Z_3 term

$$
\begin{aligned}
&\frac{Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!b_1} \\
&\quad = 1/126\,[126c_1 - 72c_2 - 12c_3 + 14c_4 + \{6(7\kappa - 2\lambda)(3\beta_2 - 2\beta_{K^2}) + 7(3\beta_3 + 4\beta_{K^3} - 6\beta_{\delta K^2})\}] \\
&\quad\; + \mu_{12}(k_1^2 + k_2^2)(42k_1k_2)^{-1}\left[42c_1 - 12c_2 + 21c_4 + 7\kappa(3\beta_2 - 2\beta_{K^2} + 6\beta_{K^2}\mu_{12}^2)\right] \\
&\quad\; + \mu_{12}^2/21\,[12c_2 + 2c_3 - 7c_4 + 21c_5 + 6\lambda\beta_2 + \left(42\kappa - 16\lambda + 12\lambda\mu_{12}^2\right)\beta_{K^2} - 21\beta_{K^3} + 21\beta_{\delta K^2}] \\
&\quad\; + \mu_{12}\mu_{23}\left[\frac{1}{2k_1k_3}\{c_1(k_1^2 + k_2^2 + k_3^2) + 2c_5(k_2^2 + k_3^2)\} + \mu_{23}\frac{k_2}{7k_1}(4c_2 + 14c_5) + \mu_{31}\left(\beta_{K^3} + \frac{2}{9}c_4\right)\right] \\
&\quad\; + (1 - \mu_{23}^2)(21|\boldsymbol{k}_2 + \boldsymbol{k}_3|^2)^{-1}\,[3k_1k_2\mu_{12}(7c_1 - 4c_2) + 4k_2^2\mu_{12}^2c_3 + 4k_2k_3\mu_{12}\mu_{31}c_3]
\end{aligned}
$$

where $\boxed{\mu_{ij} \stackrel{\text{def}}{:=} \hat{\boldsymbol{k}}_i \cdot \hat{\boldsymbol{k}}_j}$ and parameters $c_1, \ldots, c_5$ are

$$
c_1 \stackrel{\text{def}}{:=} d_{\alpha\alpha} + d_{\alpha\alpha-}, \quad c_2 \stackrel{\text{def}}{:=} d_{\alpha\gamma} - \frac{4}{7}d_{\alpha\gamma-}, \quad c_3 \stackrel{\text{def}}{:=} d_{\gamma\gamma}, \quad c_4 \stackrel{\text{def}}{:=} d_{\xi}, \quad c_5 \stackrel{\text{def}}{:=} d_{\zeta}
$$

Just a biases' linear function !

Consistency relations (smoothing $R = 10\ \mathrm{Mpc}/h$) (1)

Biased skewness and kurtosis parameters

$$S_g^{(0)} = b_1^{-1}\left(3.488\beta_2 + 0.28\beta_{K^2} + 4.814\kappa - 1.249\lambda\right)$$

$$S_g^{(1)} = b_1^{-1}\left(3.405\beta_2 + 0.2308\beta_{K^2} + 4.862\kappa - 1.231\lambda\right)$$

$$S_g^{(2)} = b_1^{-1}\left(4.255\beta_2 - 0.4642\beta_{K^2} + 5.411\kappa - 1.754\lambda\right)$$

$$K_g^{(0)} = b_1^{-2}\left(5.951\beta_3 - 0.112\beta_{K^3} + 0.8749\beta_{\delta K^2}\right) + \dots$$

$$K_g^{(1)} = b_1^{-2}\left(5.720\beta_3 - 0.09287\beta_{K^3} + 0.7784\beta_{\delta K^2}\right) + \dots$$

$$K_{g1}^{(2)} = b_1^{-2}\left(7.246\beta_3 - 0.1364\beta_{K^3} + 1.065\beta_{\delta K^2}\right) + \dots$$

$$K_{g2}^{(2)} = b_1^{-2}\left(5.729\beta_3 - 0.2991\beta_{K^3} - 1.58\beta_{\delta K^2}\right) + \dots$$

$$K_g^{(3)} = b_1^{-2}\left(10.06\beta_3 + 2.066\beta_{K^3} - 5.336\beta_{\delta K^2}\right) + \dots$$

Consistency relations (smoothing $R = 10$ Mpc$/h$) (2)

$\beta_2, \beta_{K^2} \longleftrightarrow S_g^{(0)}, S_g^{(1)}$

$$\beta_2 = -1.687\kappa + 0.381\lambda - 1.555 b_1 S_g^{(0)} + 1.886 b_1 S_g^{(1)}$$

$$\beta_{K^2} = 3.822\kappa - 0.2857\lambda + 22.93 b_1 S_g^{(0)} - 23.49 b_1 S_g^{(1)}$$

Consistency relations (smoothing $R = 10$ Mpc$/h$) (3)

$\beta_3, \beta_{K^3}, \beta_{\delta K^2} \longleftrightarrow K_g^{(0)}, K_g^{(1)}, K_g^{(3)}$

$$\beta_3 = -6.024c_1 + 2.571c_2 + 0.2919c_3 - 0.1495c_4 - 1.271c_5 + 10.98\kappa^2 - 5.487\kappa\lambda + 0.5267\lambda^2$$
$$+ b_1 \Big[21.62S_g^{(0)}\kappa - 24.11S_g^{(1)}\kappa - 8.911S_g^{(0)}\lambda + 9.466S_g^{(1)}\lambda$$
$$+b_1 \left(-4.655K_g^{(0)} + 5.062K_g^{(1)} - 0.0249K_g^{(3)} + 76.71S_g^{(0)2} - 147.9S_g^{(0)}S_g^{(1)} + 70.71S_g^{(1)2}\right)\Big]$$

$$\beta_{K^3} = 124.7c_1 - 43.08c_2 - 1.98c_3 - 0.1438c_4 + 50.14c_5 - 34.34\kappa^2 + 16.9\kappa\lambda - 0.241\lambda^2$$
$$+ b_1 \Big[-61.6S_g^{(0)}\kappa + 93.22S_g^{(1)}\kappa + 8.935S_g^{(0)}\lambda - 9.258S_g^{(1)}\lambda$$
$$+b_1 \left(160.5K_g^{(0)} - 169.7K_g^{(1)} + 1.558K_g^{(3)} - 3274.S_g^{(0)2} + 6403.S_g^{(0)}S_g^{(1)} - 3123.S_g^{(1)2}\right)\Big]$$

$$\beta_{\delta K^2} = 40.85c_1 - 14.34c_2 - 0.7511c_3 + 0.1305c_4 + 16.33c_5 - 13.47\kappa^2 + 7.152\kappa\lambda - 0.2446\lambda^2$$
$$+ b_1 \Big[-27.72S_g^{(0)}\kappa + 36.5S_g^{(1)}\kappa + 8.649S_g^{(0)}\lambda - 8.971S_g^{(1)}\lambda$$
$$+b_1 \left(53.36K_g^{(0)} - 56.16K_g^{(1)} + 0.3689K_g^{(3)} - 1029.S_g^{(0)2} + 1999.S_g^{(0)}S_g^{(1)} - 967.9S_g^{(1)2}\right)\Big]$$

Consistency relations (smoothing $R = 10$ Mpc$/h$) (4)

Skewness and kurtosis consistency relations

$$0 = -17.26 S_g^{(0)} + 18.93 S_g^{(1)} - \frac{3.542\kappa}{b_1} - S_g^{(2)} \qquad \text{(eq1)}$$

$$\begin{aligned} 0 = {} & b_1^2 \left(129.1 {S_g^{(0)}}^2 - 244.4 S_g^{(1)} S_g^{(0)} + 114.7 {S_g^{(1)}}^2 - 1.218 K_g^{(0)} + K_{g1}^{(2)}\right) \\ & + b_1 \left(108.2 S_g^{(0)} \kappa - 108.3 S_g^{(1)} \kappa - 10.56 S_g^{(0)} \lambda + 11.1 S_g^{(1)} \lambda\right) \\ & + 15.33\kappa^2 - 4.435\lambda\kappa + 0.4796\lambda^2 + 1.476 c_1 + 0.4876 c_2 + 0.2862 c_3 - 0.1966 c_4 + 3.041 c_5 \end{aligned} \qquad \text{(eq2)}$$

$$\begin{aligned} 0 = {} & b_1^2 \left(-2697 {S_g^{(0)}}^2 + 5214 S_g^{(1)} S_g^{(0)} - 2510. {S_g^{(1)}}^2 + 159. K_g^{(0)} - 168.5 K_g^{(1)} + K_{g2}^{(2)} + 1.192 K_g^{(3)}\right) \\ & + b_1 \left(119.7 S_g^{(0)} \kappa - 97.62 S_g^{(1)} \kappa + 0.2565 S_g^{(0)} \lambda + 0.2213 S_g^{(1)} \lambda\right) \\ & - 2.239\kappa^2 + 8.909\lambda\kappa + 0.5928\lambda^2 + 118.5 c_1 - 41.76 c_2 - 1.962 c_3 + 0.06199 c_4 + 44.03 c_5 \end{aligned} \qquad \text{(eq3)}$$

How to constraint theories?

functions

$$\mathcal{S} \overset{\text{def}}{:=} \text{RHS of eq1},$$

$$\mathcal{K}_1 \overset{\text{def}}{:=} \text{RHS of eq2},$$

$$\mathcal{K}_2 \overset{\text{def}}{:=} \text{RHS of eq3}$$

Assuming $c_1 = c_2 = c_3 = c_4 = 1, c_5 = 0.$

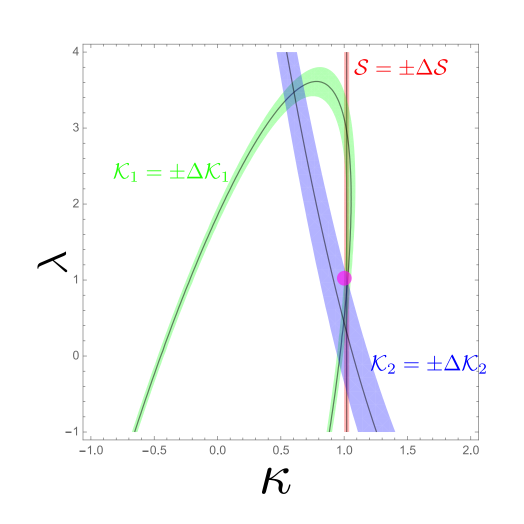
If we derive observational value (EdS) :

$S_g^{(0)} = 3.56, S_g^{(1)} = 3.63, S_g^{(2)} = 3.66,$

$K_g^{(0)} = 23.2, K_g^{(1)} = 23.8,$

$K_{g1}^{(2)} = 30.6, K_{g2}^{(2)} = 18.8, K_g^{(3)} = 25.1.$

How κ and λ constrained? (Preliminary)



Summary

- ⋆ We derived skewness and kurtosis (DHOST + bias) and discussed the relation of them.

- ⋆ Eliminating bias parameters, we derived independent three consistency relations.

- ⋆ The consistency relations would be useful for probing gravity theories.

$S^{(0)},\ S^{(1)},\ S^{(2)},\ K^{(0)},\ K^{(1)},\ K_1^{(2)},\ K_2^{(2)},\ K^{(3)}$

$3+5$ Conditions

5 Biases

$\beta_2,\ \beta_{K^2},\ \beta_3,\ \beta_{K^3},\ \beta_{\delta K^2}$

→ $8-5=3$ consistency relations

$s^{(a)}, \kappa^{(a)}$ の具体的表式

$$\begin{pmatrix} s^{(0)} \\ s^{(1)} \\ s^{(2)} \end{pmatrix} \stackrel{\text{def}}{:=} \begin{pmatrix} 1 \\ (k_1^2 + k_2^2 + \boldsymbol{k}_1 \cdot \boldsymbol{k}_2)/2 \\ 3[k_1^2 k_2^2 - (\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2]/2 \end{pmatrix}$$

$$\begin{pmatrix} \kappa^{(0)} \\ \kappa^{(1)} \\ \kappa_1^{(2)} \\ \kappa_2^{(2)} \\ \kappa^{(3)} \end{pmatrix} \stackrel{\text{def}}{:=} \begin{pmatrix} 1 \\ 1/3 \cdot (k_1^2 + k_2^2 + k_3^2 + \boldsymbol{k}_1 \cdot \boldsymbol{k}_2 + \boldsymbol{k}_2 \cdot \boldsymbol{k}_3 + \boldsymbol{k}_3 \cdot \boldsymbol{k}_1) \\ 1/20 \cdot \{5[k_1^2 k_2^2 - (\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2] - 6(\boldsymbol{k}_1 \cdot \boldsymbol{k}_3)(\boldsymbol{k}_2 \cdot \boldsymbol{k}_3) + 2(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2) k_3^2\} + 5\text{perms} \\ 1/20 \cdot \{5[k_1^2 k_2^2 - (\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2] + 2(\boldsymbol{k}_1 \cdot \boldsymbol{k}_3)(\boldsymbol{k}_2 \cdot \boldsymbol{k}_3) + 6(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2) k_3^2\} + 5\text{perms} \\ 3/4 \cdot [k_1^2 k_2^2 k_3^2 + 2(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)(\boldsymbol{k}_2 \cdot \boldsymbol{k}_3)(\boldsymbol{k}_3 \cdot \boldsymbol{k}_1) - 3(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2 k_3^2] + 5\text{perms} \end{pmatrix}$$

各関数の定義

$$\alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) \overset{\text{def}}{:=} 1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_1^2}$$

$$\gamma(\boldsymbol{k}_1, \boldsymbol{k}_2) \overset{\text{def}}{:=} 1 - (\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2)^2$$

$$\alpha_s(\boldsymbol{k}_1, \boldsymbol{k}_2) \overset{\text{def}}{:=} \frac{1}{2}[\alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) + \alpha(\boldsymbol{k}_2, \boldsymbol{k}_1)]$$

$$\alpha_{as}(\boldsymbol{k}_1, \boldsymbol{k}_2) \overset{\text{def}}{:=} \frac{1}{2}[\alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) - \alpha(\boldsymbol{k}_2, \boldsymbol{k}_1)]$$

$$\alpha\alpha(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \alpha_s(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\alpha_s(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\alpha\gamma(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \alpha_s(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\gamma(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\gamma\alpha(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \gamma(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\alpha_s(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\gamma\gamma(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \gamma(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\gamma(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\alpha\alpha_-(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \alpha_a s(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\alpha_s(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\alpha\gamma_-(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} \alpha_a s(\boldsymbol{k}_1, \boldsymbol{k}_2 + \boldsymbol{k}_3)\gamma(\boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$\begin{aligned}\xi_c(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} &\ 1 - 3(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2)^2 \\ &+ 2(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2)(\hat{\boldsymbol{k}}_2 \cdot \hat{\boldsymbol{k}}_3)(\hat{\boldsymbol{k}}_3 \cdot \hat{\boldsymbol{k}}_1)\end{aligned}$$

$$\begin{aligned}\zeta_c(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \overset{\text{def}}{:=} &\ (\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2)\left[(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2) + \frac{k_2}{k_1}\right] \\ &+ \left[2\frac{k_2}{k_3} + \frac{k_2^2 + k_3^2}{k_3 k_1}\right](\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2)(\hat{\boldsymbol{k}}_2 \cdot \hat{\boldsymbol{k}}_3)\end{aligned}$$

(DHOST + bias) におけるゆらぎの積分核 (Z_2 & Z_3)

$$\frac{Z_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!b_1} = \frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} + \frac{1}{2!}\frac{b_2}{b_1} + \frac{b_{K^2}}{b_1}\left[\left(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2\right)^2 - \frac{1}{3}\right]$$

$$\frac{Z_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!b_1} = \frac{F_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{3!} + \frac{b_2}{b_1}\frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!} + \frac{2b_{K^2}}{b_1}\frac{F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)}{2!}\left[(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 - \frac{1}{3}\right] + \frac{1}{3!}\frac{b_3}{b_1}$$
$$+ \frac{b_{K^3}}{b_1}\left[(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{k}}_3)(\hat{\boldsymbol{k}}_3\cdot\hat{\boldsymbol{k}}_1) - (\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 + \frac{2}{9}\right] + \frac{b_{\delta K^2}}{b_1}\left[(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 - \frac{1}{3}\right]$$

$1\mathrm{Gpc}^3$におけるシミュレーション [Matsubara et al.(2022), 2012.00203]

上段は摂動の予言．下段はシミュレーションの結果.

R [Mpc/h]	10	20	30	40
σ_0	0.385 0.3804 ± 0.0001	0.193 0.1899 ± 0.0001	0.121 0.1194 ± 0.0001	0.0845 0.08374 ± 0.0001
σ_1	0.0367 0.03652 ± 0.00001	0.0101 0.009918 ± 0.000004	0.00441 0.004352 ± 0.000003	0.00240 0.002371 ± 0.000002
$S^{(0)}$	3.56 3.762 ± 0.004	3.40 3.46 ± 0.01	3.33 3.36 ± 0.02	3.28 3.28 ± 0.05
$S^{(1)}$	3.63 3.932 ± 0.003	3.45 3.531 ± 0.006	3.36 3.41 ± 0.01	3.31 3.34 ± 0.03
$S^{(2)}$	3.66 4.499 ± 0.004	3.68 3.887 ± 0.006	3.71 3.81 ± 0.01	3.72 3.78 ± 0.03
$K^{(0)}$	23.2 26.66 ± 0.09	20.9 21.6 ± 0.2	19.9 20.2 ± 0.5	19.2 19 ± 1
$K^{(1)}$	23.8 28.77 ± 0.09	21.3 22.2 ± 0.1	20.2 20.6 ± 0.3	19.5 19.8 ± 0.8
$K_1^{(2)}$	30.6 41.7 ± 0.1	28.3 30.4 ± 0.2	27.2 28.1 ± 0.4	26.7 27.5 ± 0.8
$K_2^{(2)}$	18.8 26.7 ± 0.1	17.7 19.2 ± 0.1	17.3 18.0 ± 0.2	17.0 17.8 ± 0.6
$K^{(3)}$	25.1 45.5 ± 0.2	25.6 29.6 ± 0.2	26.1 27.6 ± 0.4	26.4 27 ± 1

誤差の見積もり

- 摂動の精度（シミュレーションを見るに，20 Mpc/h で 2%くらい？）